

Finite volume matrix elements of two-body states

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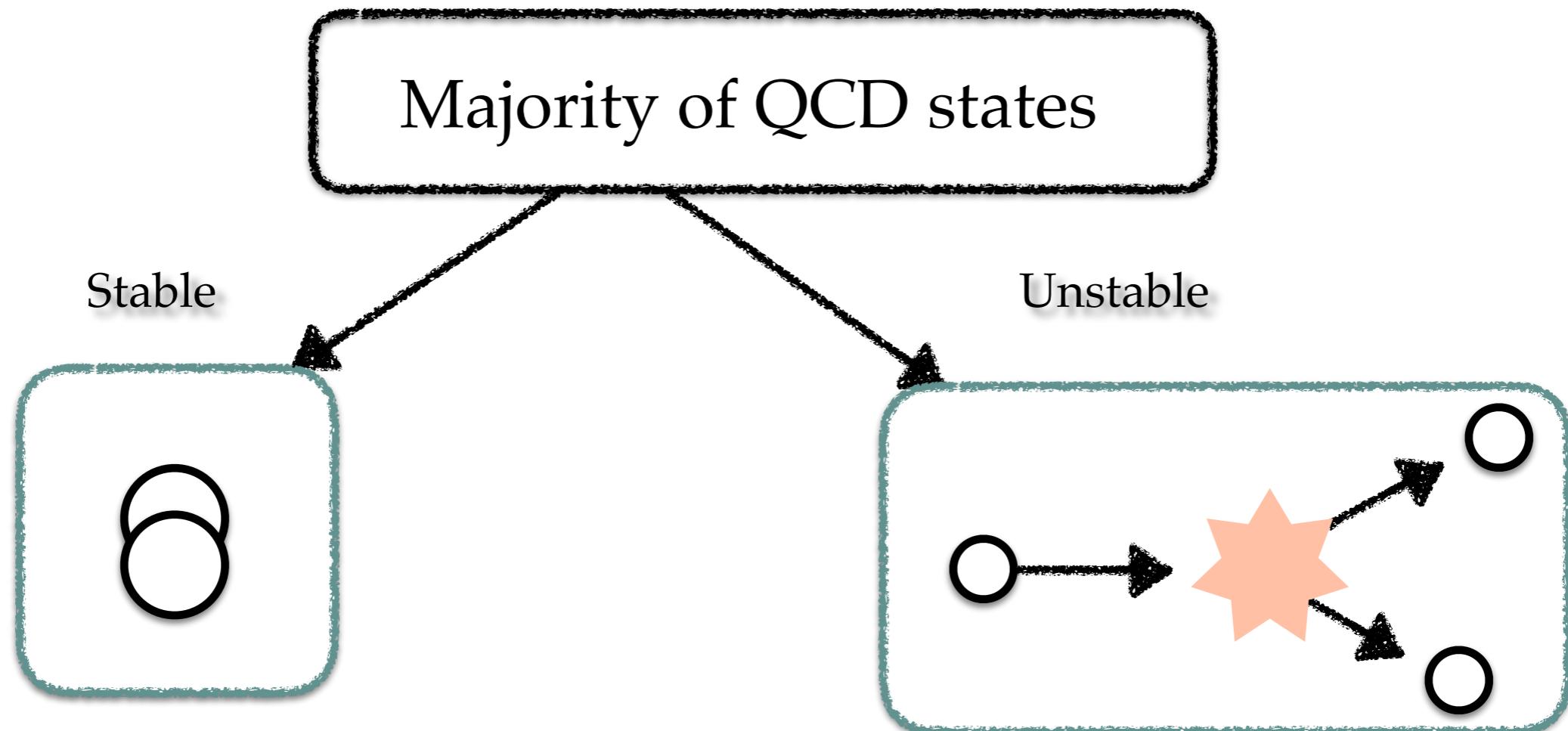
F. G. Ortega
William and Mary



D. J. Wilson
Trinity

Lattice Conference 2018
Michigan State University

Composite states



- Bound states
 - nuclei (deuteron ...)
 - hypernuclei
 - ...

- Resonances
 - $\rho \rightarrow \pi\pi$
 - the Roper ($N\pi, N\pi\pi, \dots$)
 - ...

Goals

- Developing a framework for studying :
 - structure of composite states
 - structure of resonances
 - structure of the deuteron
 - Weak processes involving few-hadron systems
 - parity violation
 - p-p fusion, neutrino-nucleus

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See Dimitra Pefkou's talk

Goals

- Developing a framework for studying :

- structure of composite states

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- structure of the deuteron

- Weak processes involving few-hadron systems

- parity violation, p-p fusion

- neutrino-nucleus

See Dimitra Pefkou's talk

LQCD inputs for
low energy nuclear physics

Goals

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 - structure of composite states
 - structure of resonances
 - structure of the deuteron
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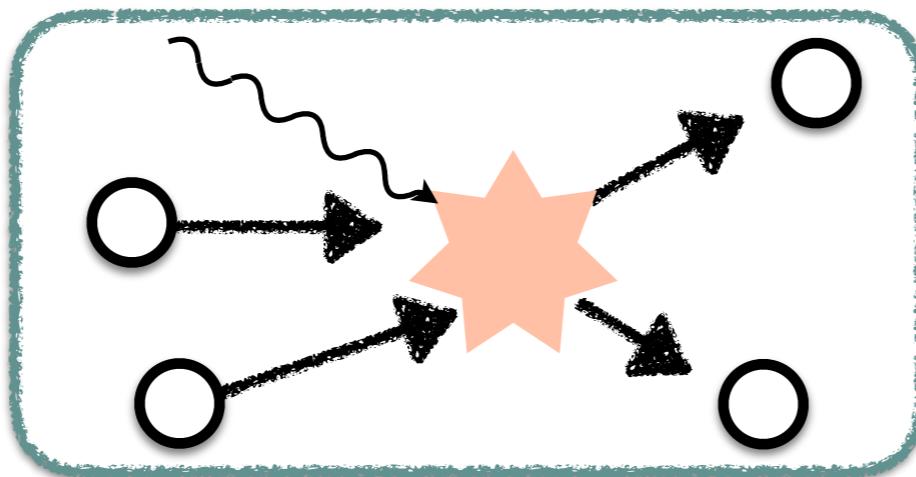
See Dimitra Pefkou's talk

LQCD inputs for
low energy nuclear physics

LQCD inputs for neutrino-nucleus scattering

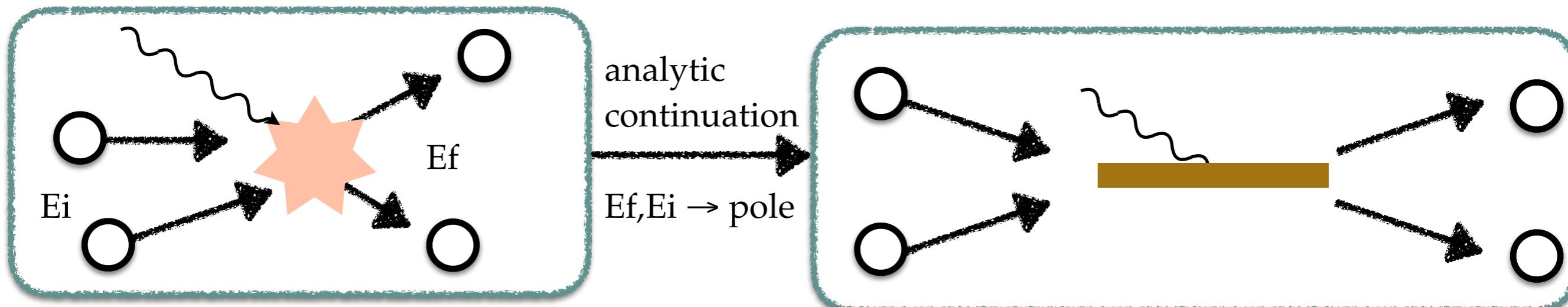
Form factor of a resonance

- Resonance are not asymptotic states → Form factor?

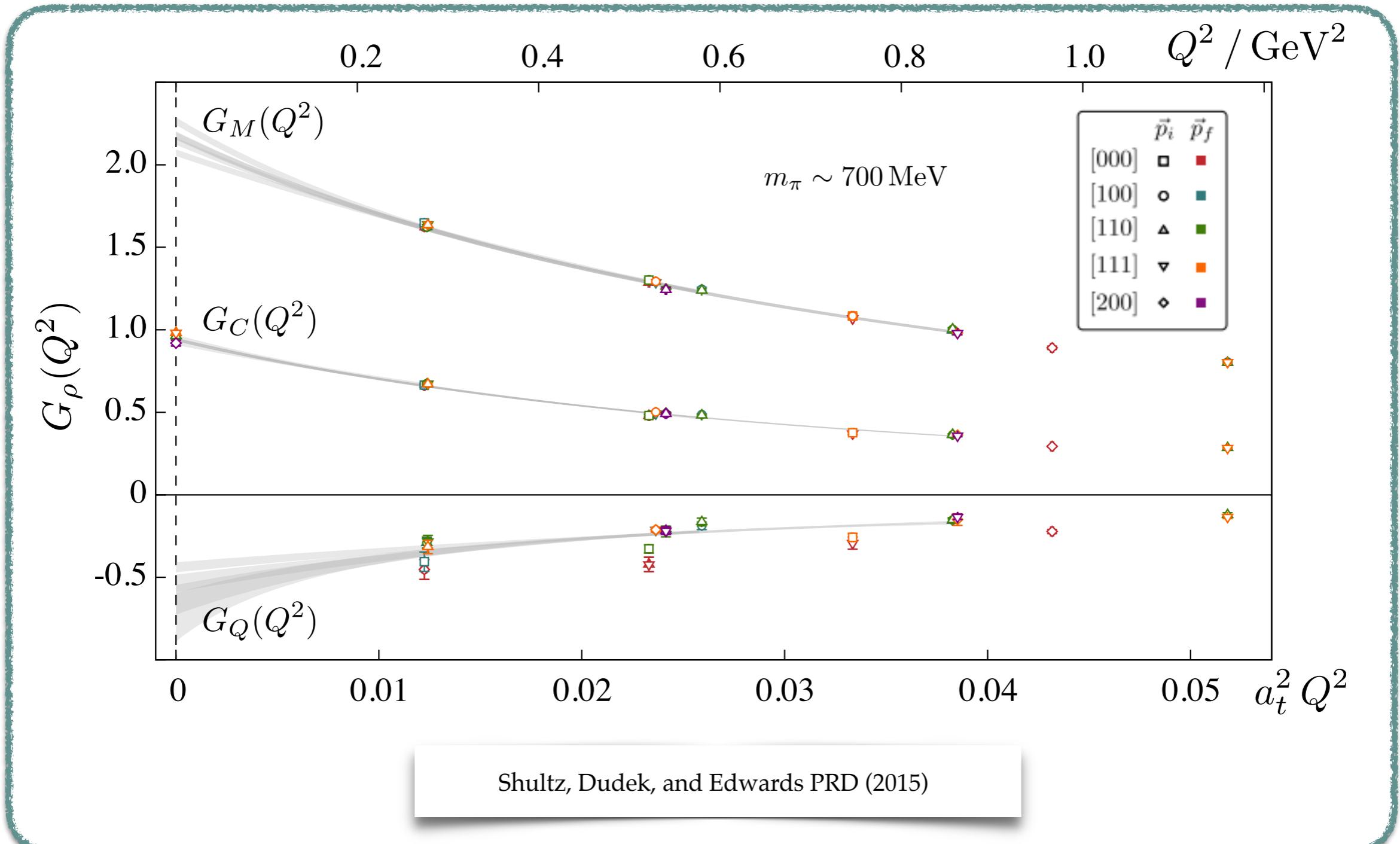


Standard LSZ theorem
cannot be used

- We can get the form factors from amplitudes

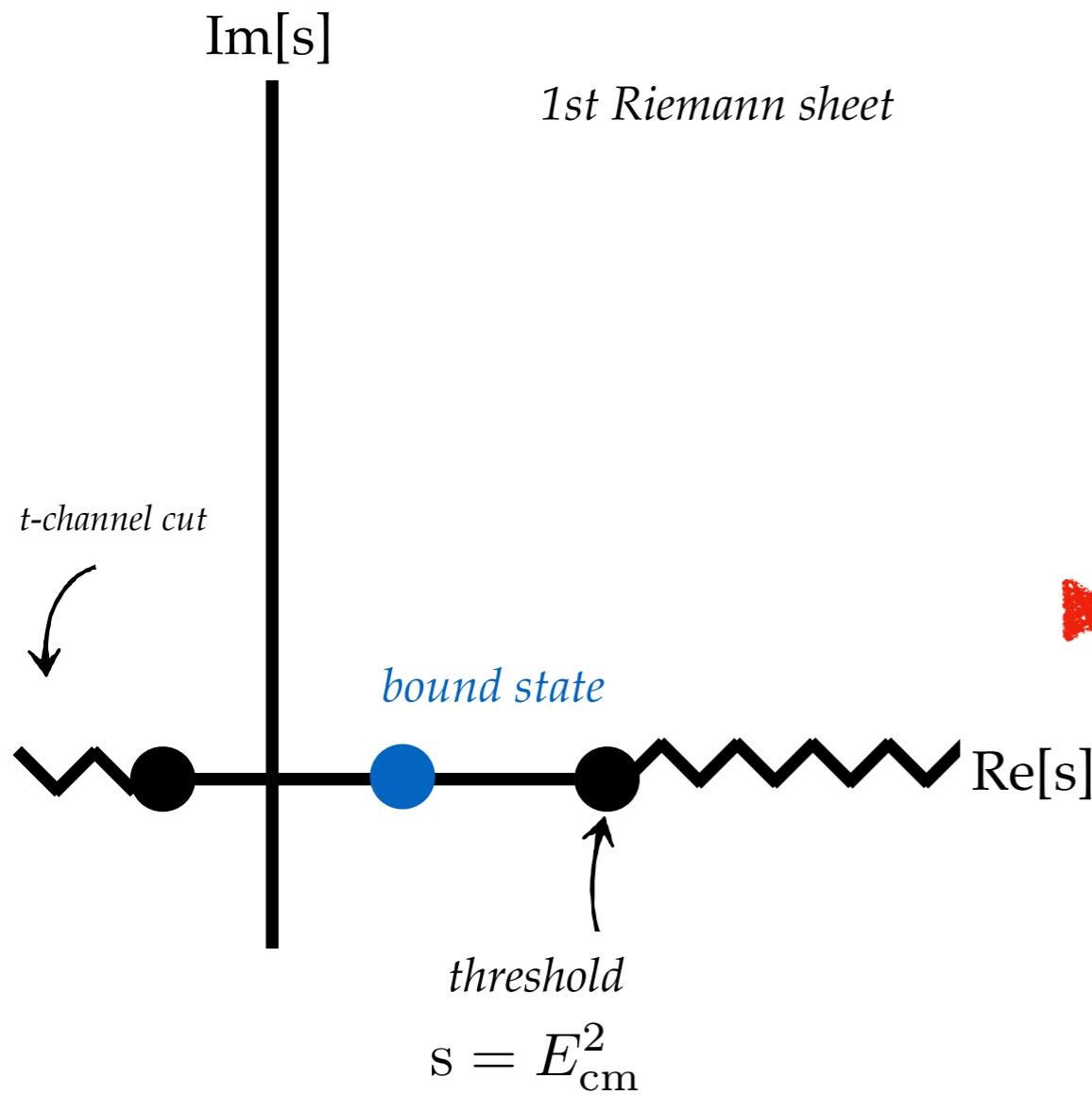


Form factor of a stable “resonance”



Scattering amplitude

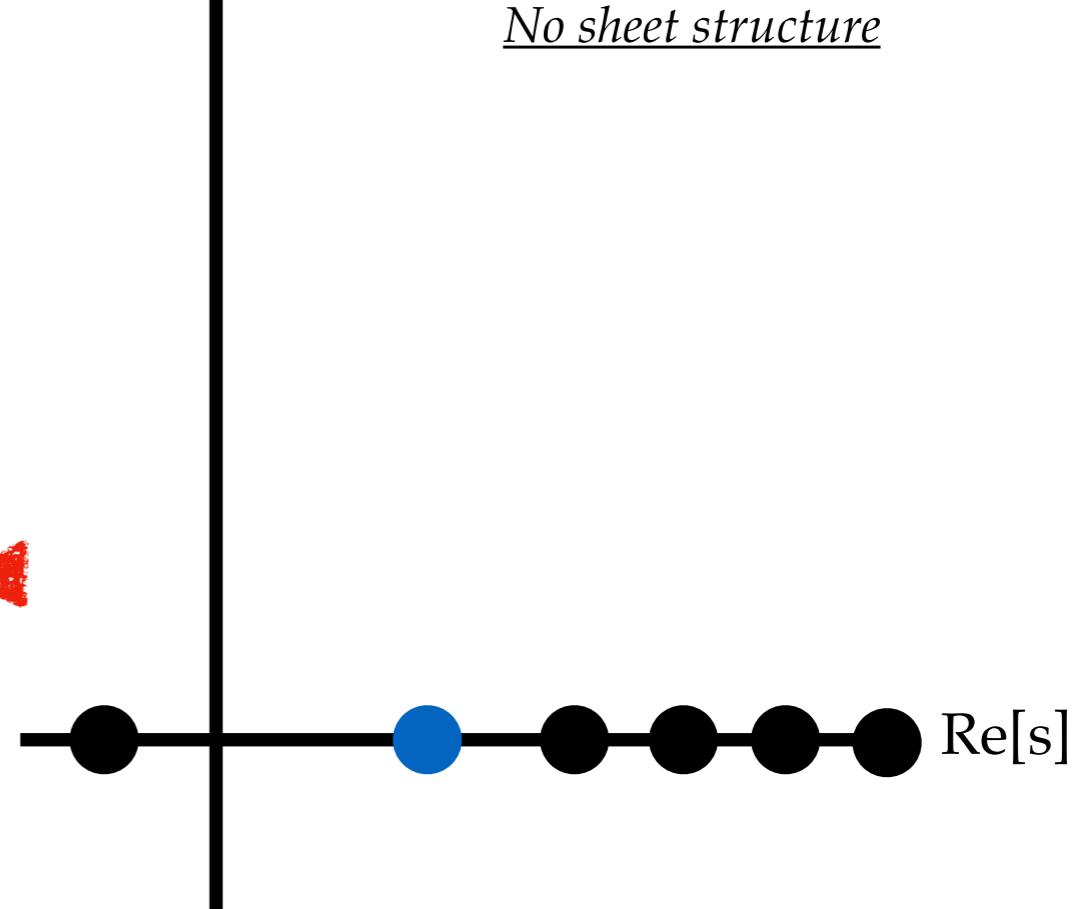
Infinite volume



Finite volume

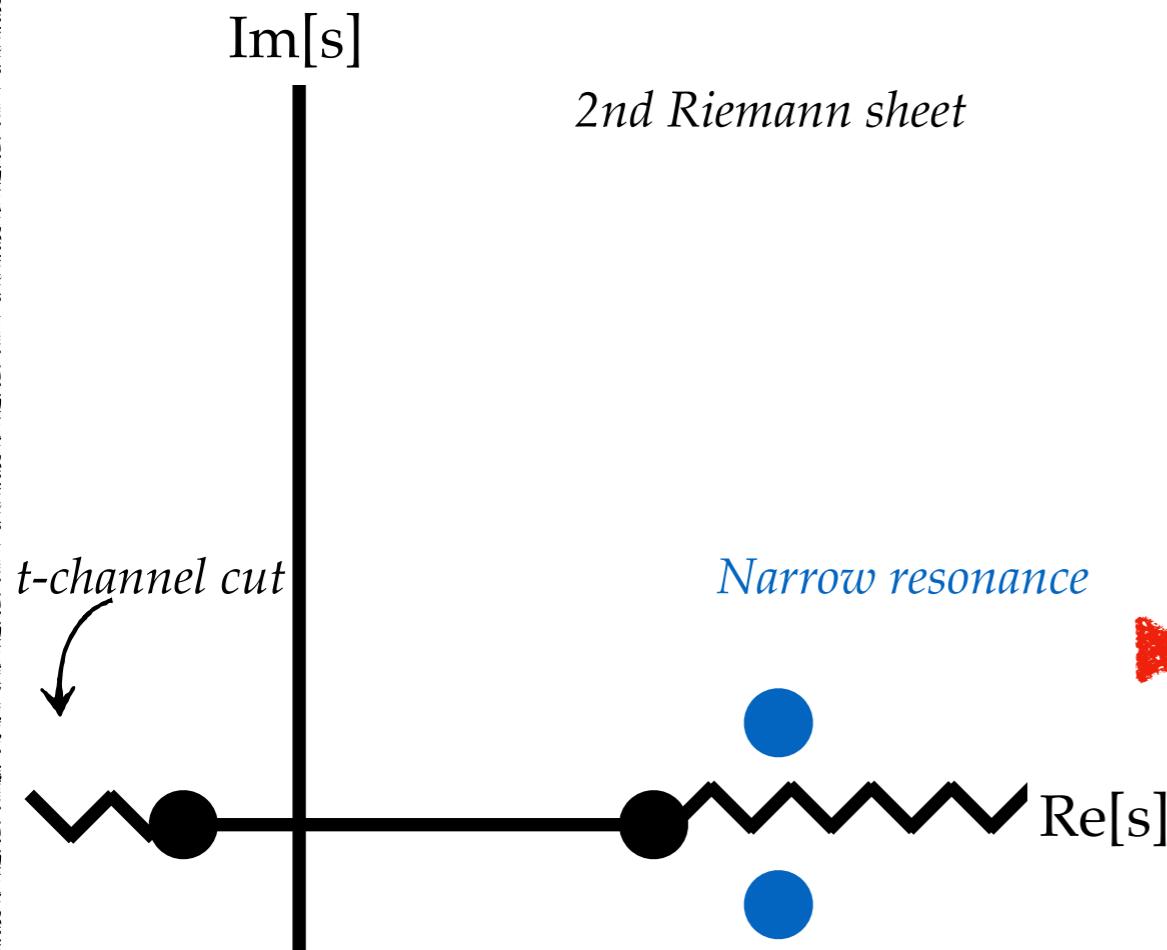
$\text{Im}[s]$

No sheet structure



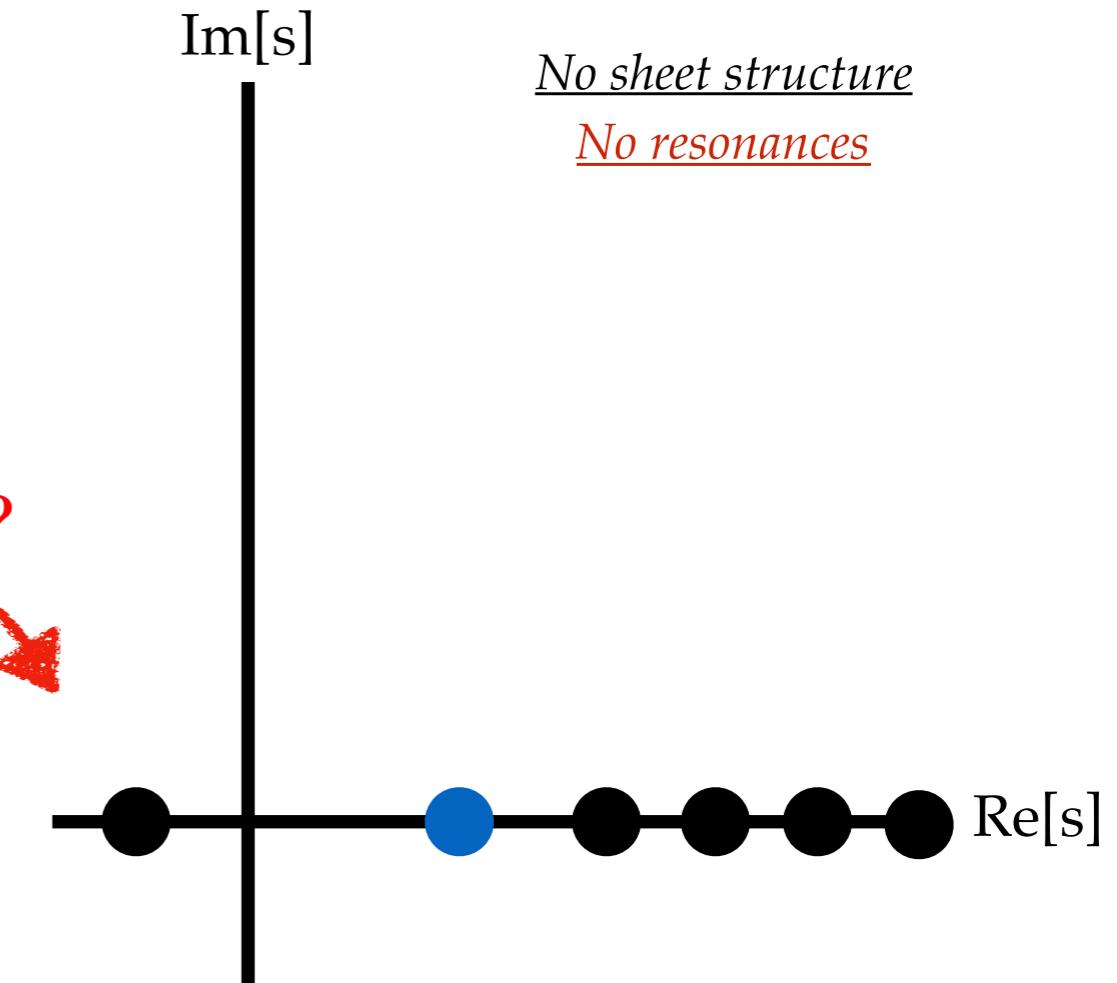
Scattering amplitude

Infinite volume



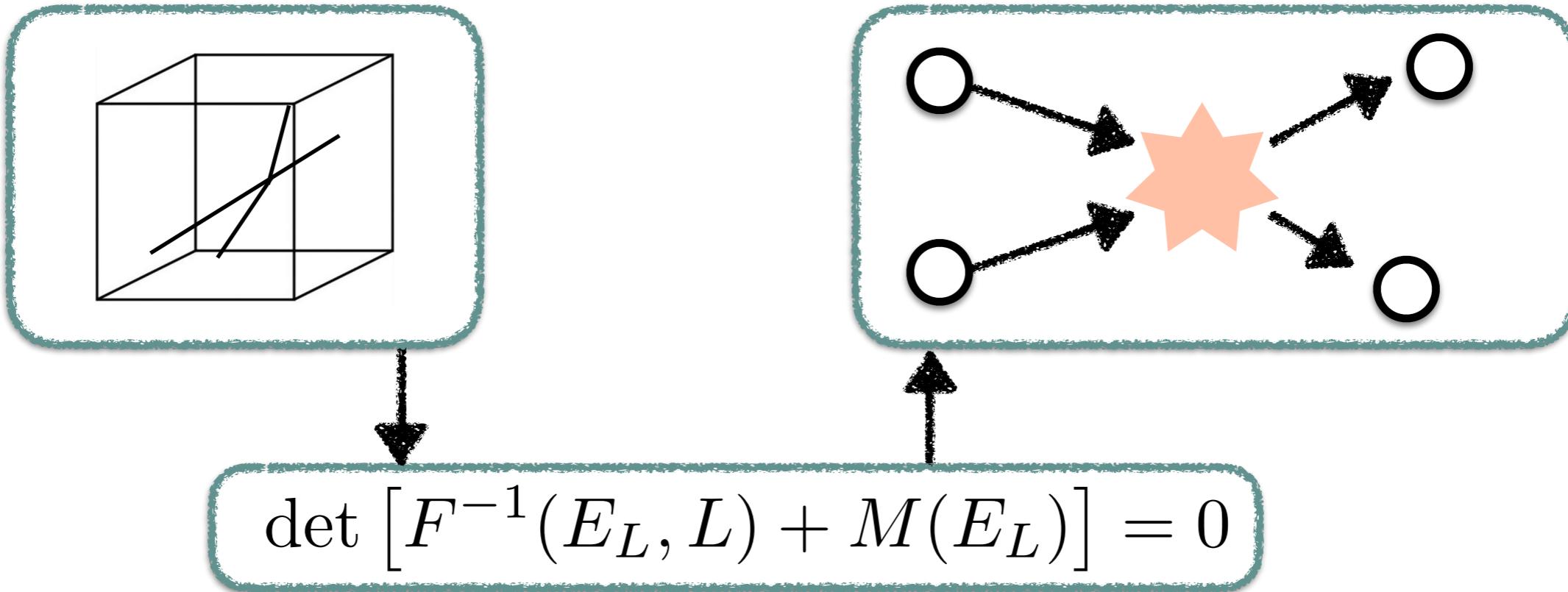
$$s_R = \left(E_R - \frac{i}{2}\Gamma_R\right)^2$$

Finite volume



$2 \rightarrow 2$

- FV spectra to infinite volume purely hadronic amplitudes
- Holds for a generic QFT with hadronic d.o.f, up to multi-particle thresholds

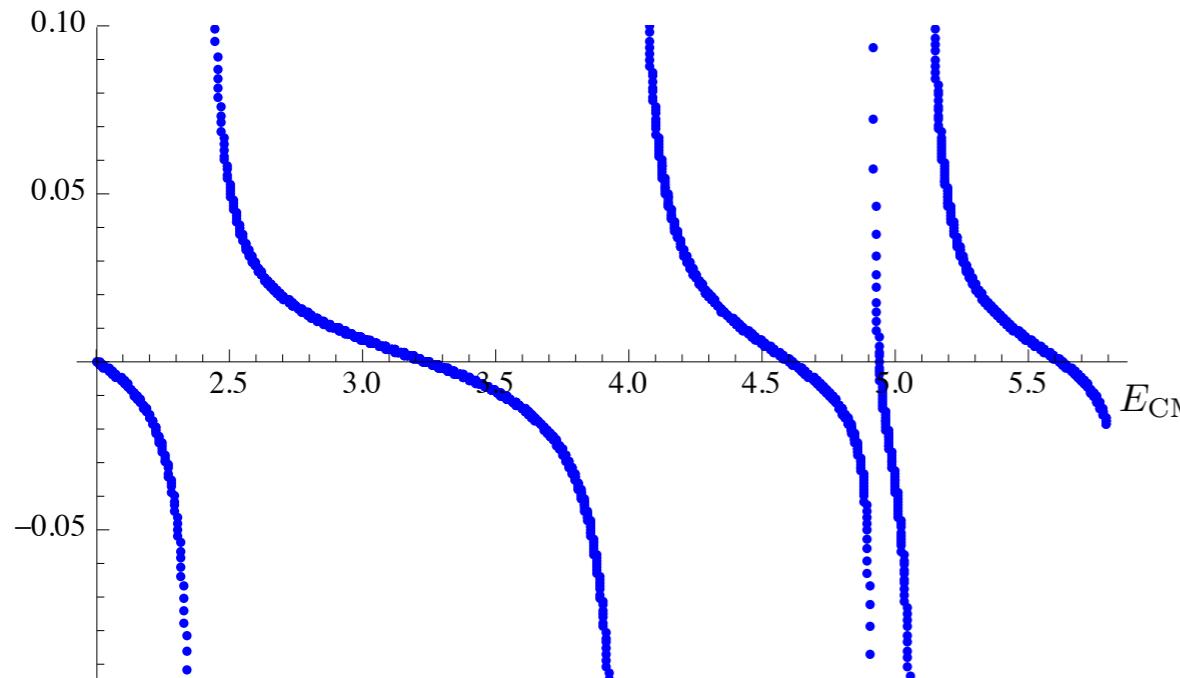


- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / Briceño & Davoudi (2012) [moving inelastic scalar bosons]

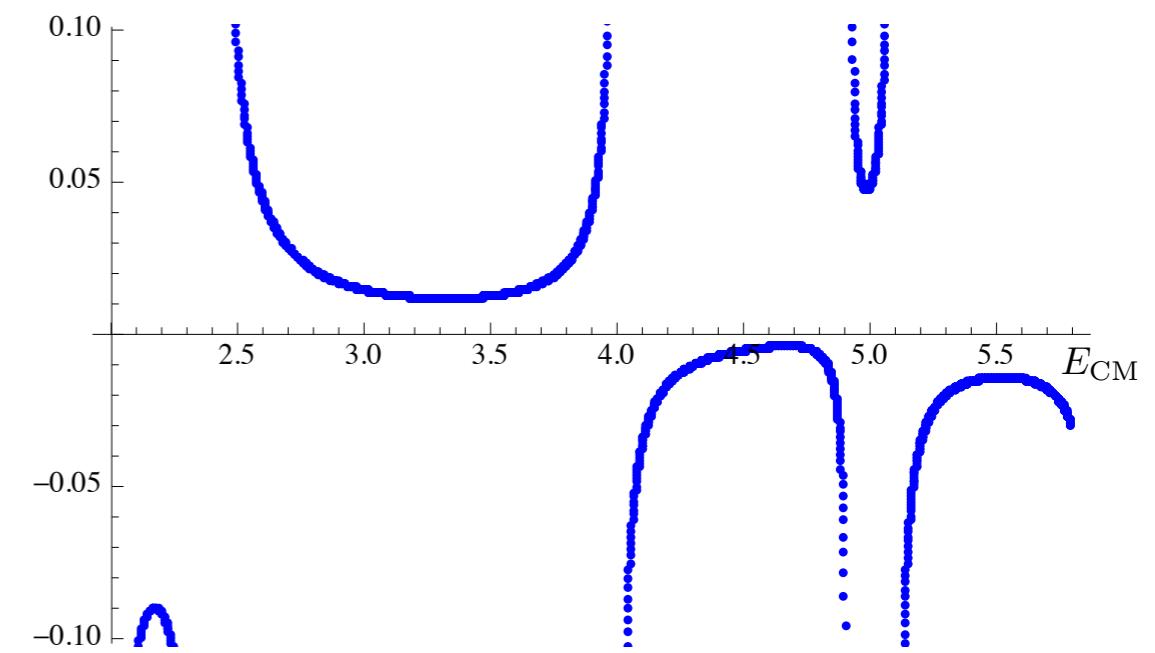
$$F_{l,m,l'm'}(E_L, L) = \text{Diagram} - \text{Diagram} = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\cdots)$$

$$\mathbf{P} = [0, 0, 2\pi/L], \quad m_\pi L = 4$$

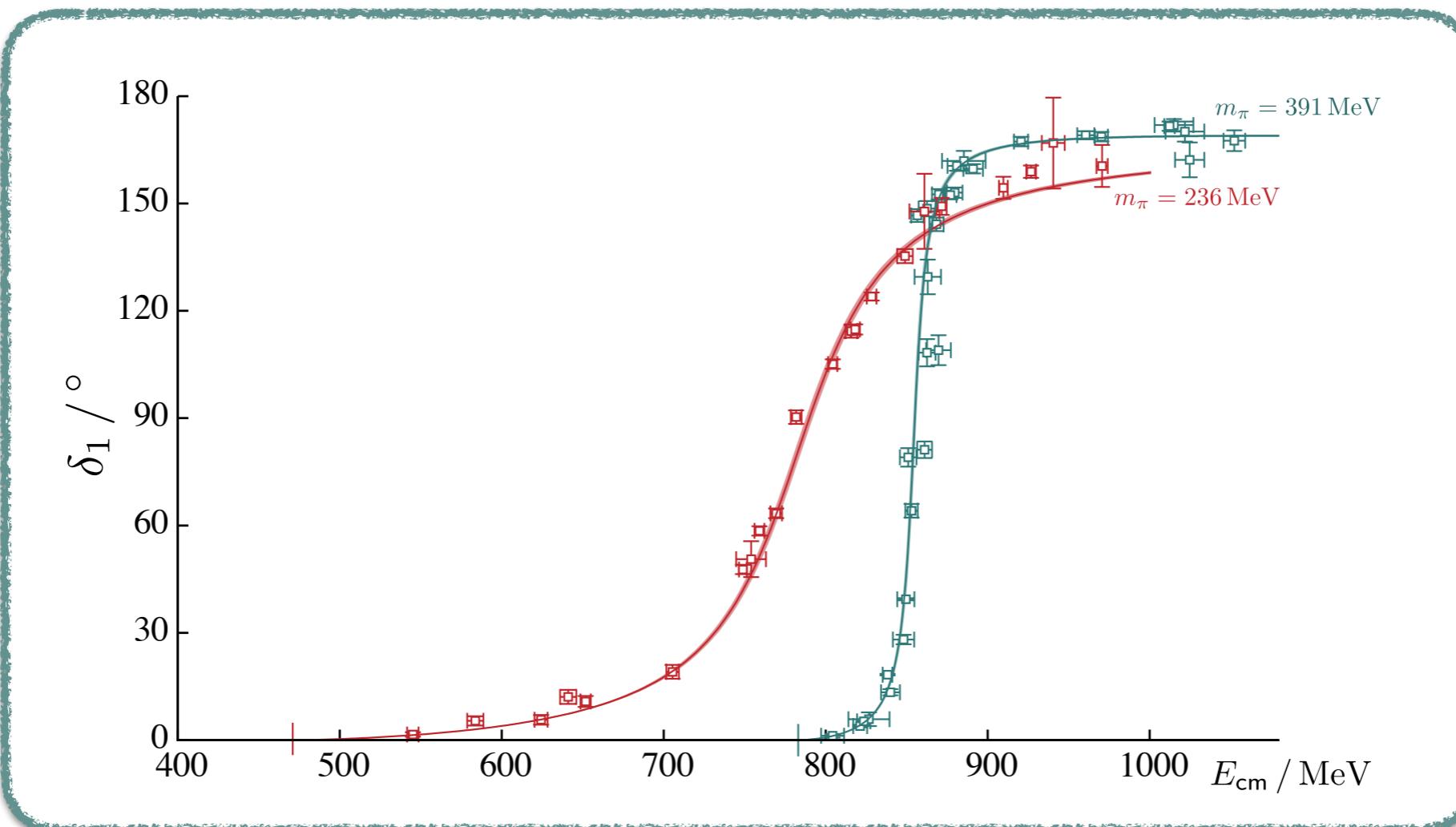
$\text{Re } F_{00,00}$



$\text{Re } F_{00,20}$



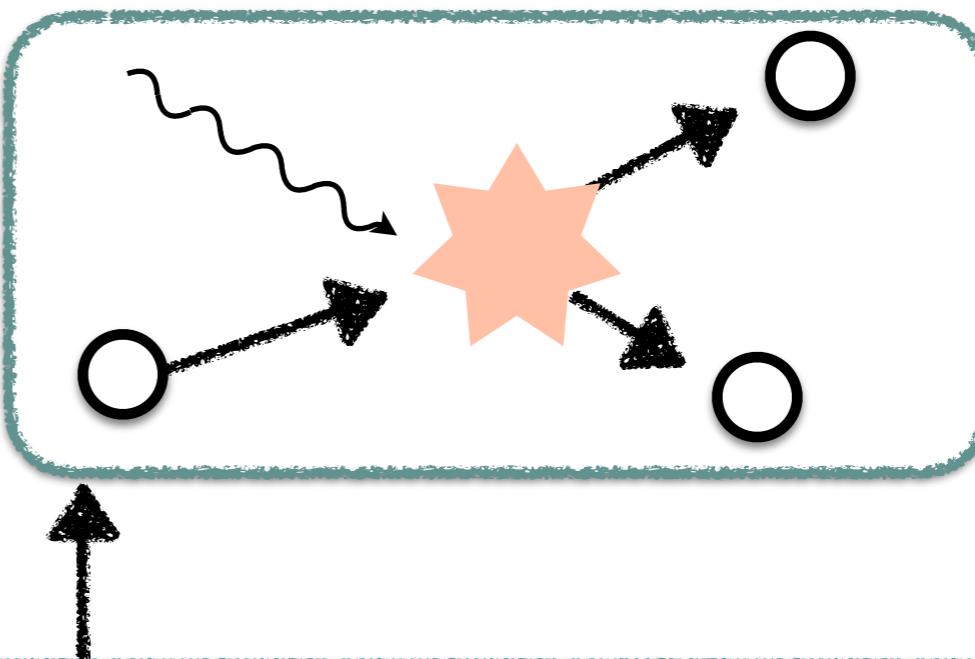
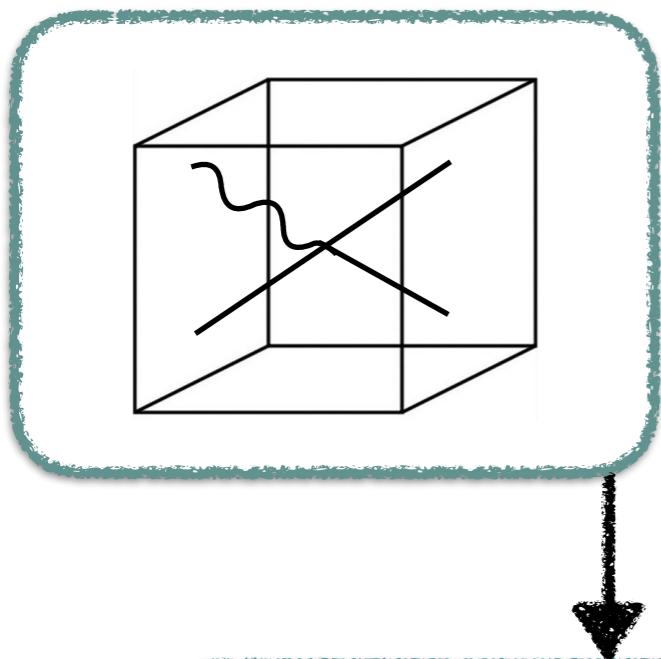
$2 \rightarrow 2$



● Wilson, Briceño, Dudek, Edwards, and Thomas PRD (2015)

$$1 + \mathcal{J} \rightarrow 2$$

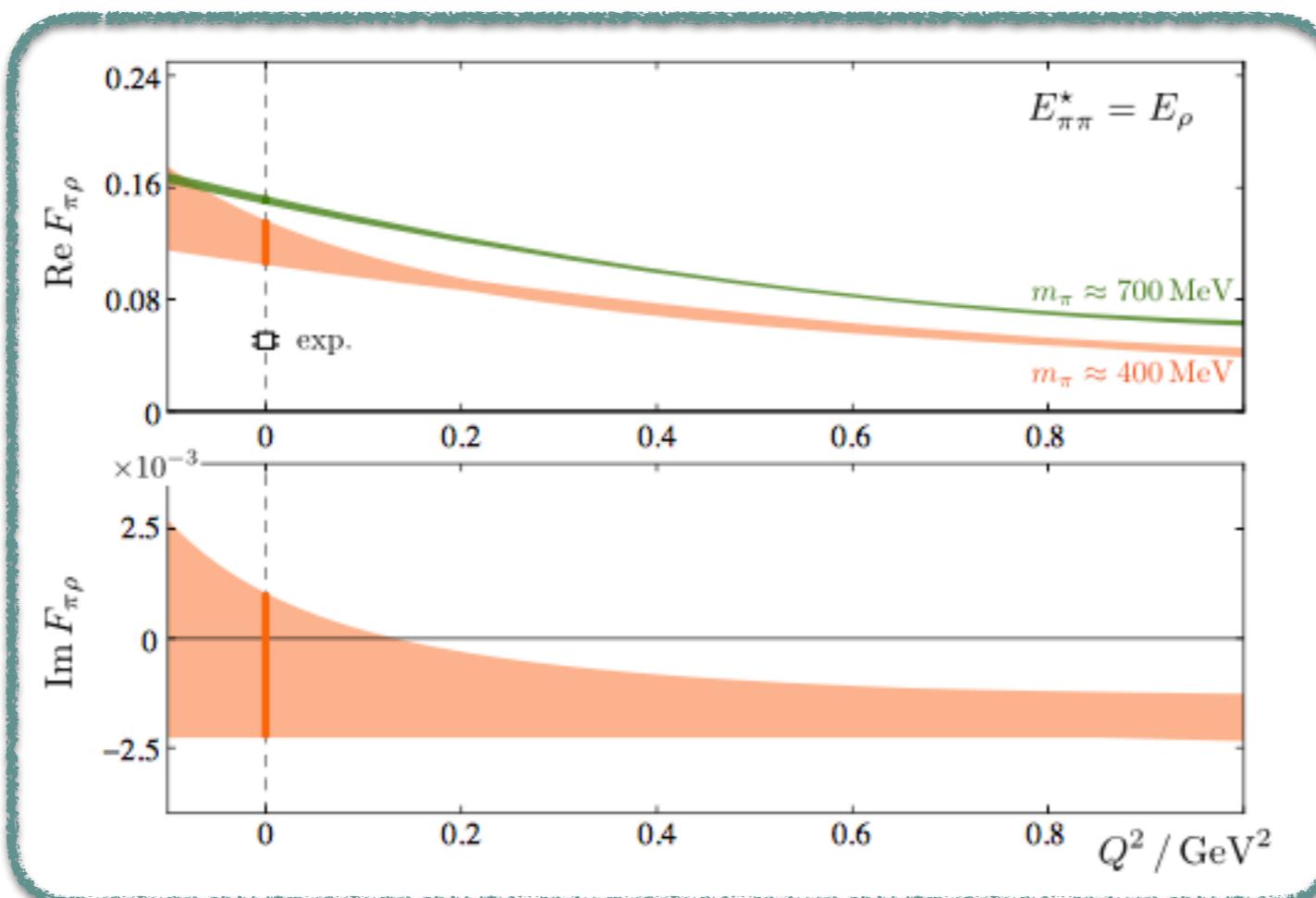
FV matrix elements to infinite volume electroweak amplitudes



$$|\langle 2|\mathcal{J}|1\rangle|_L = \frac{1}{L^3} \sqrt{\langle 1|\mathcal{J}|2\rangle_\infty R(E_L, L) \langle 2|\mathcal{J}|1\rangle_\infty}$$

- Lellouch & Lüscher (2000) [K-to- $\pi\pi$ at rest]
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005) [moving K-to- $\pi\pi$]
- Hansen & Sharpe (2012) [D-to- $\pi\pi$ /KK]
- Briceño, Hansen Walker-Loud / Briceño & Hansen(2014-2015)[general 1-to-2]

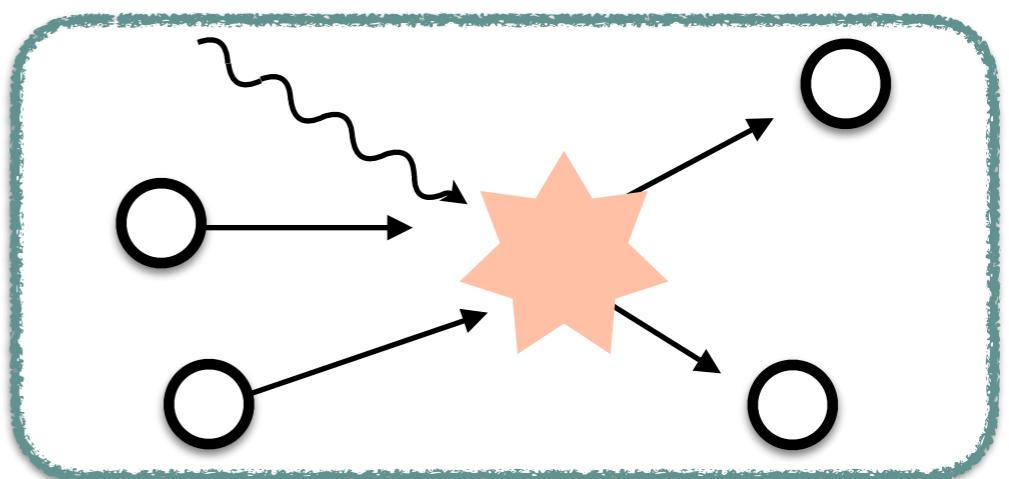
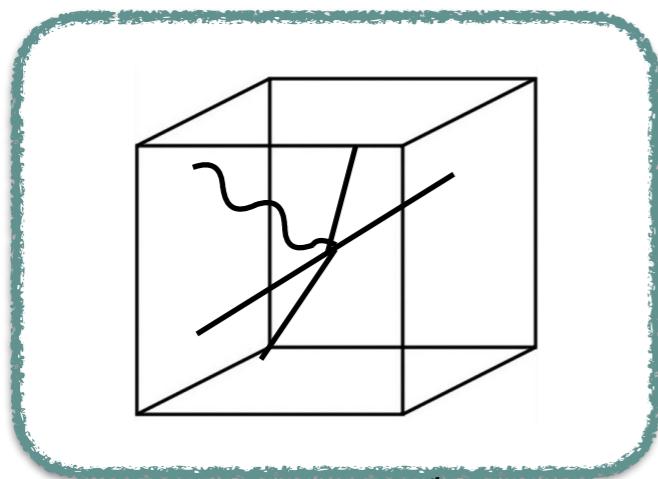
$$1 + \mathcal{J} \rightarrow 2$$



Briceño, Dudek, Edwards, Shultz, Thomas and Wilson PRL (2016)

$$2 + \mathcal{J} \rightarrow 2$$

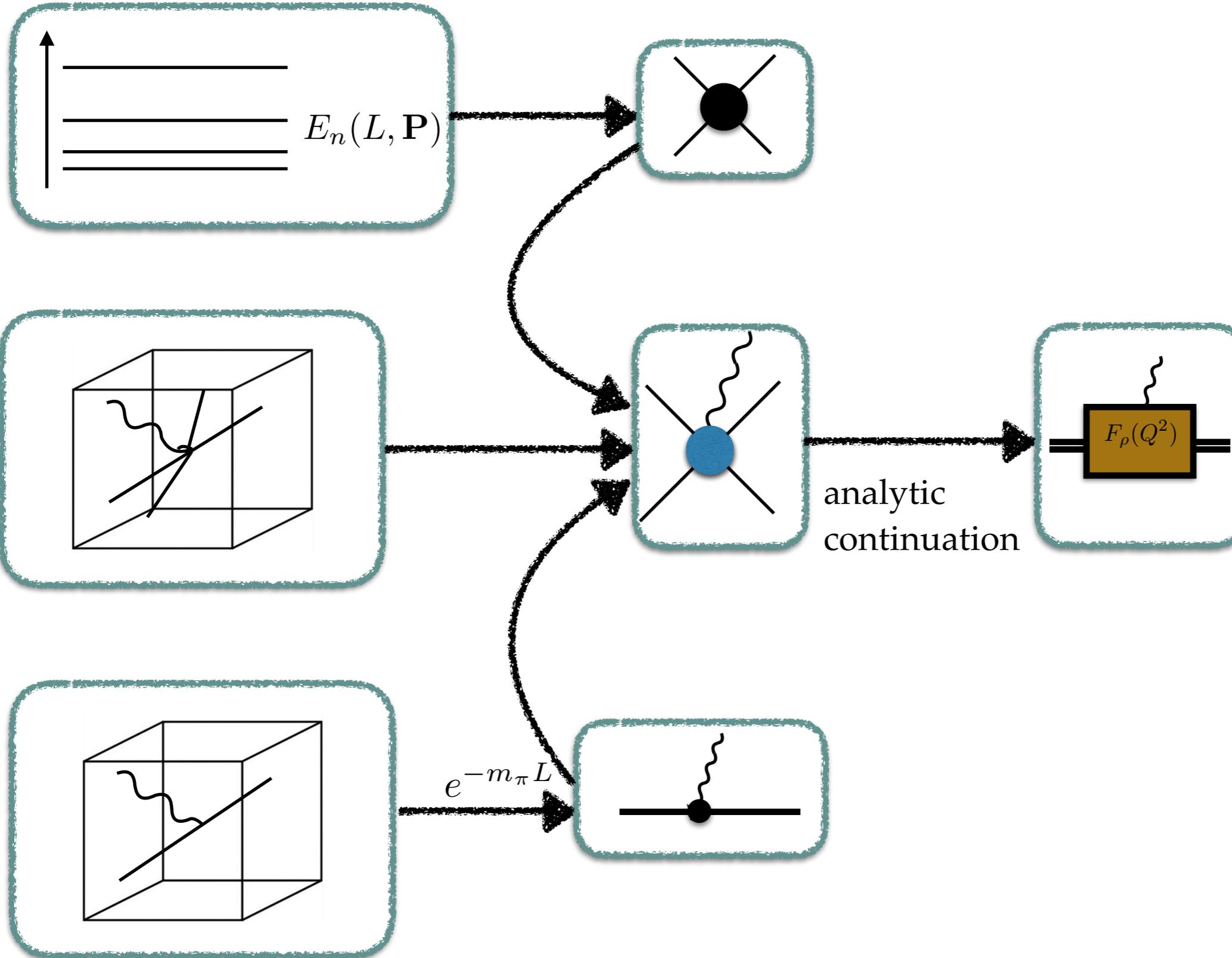
FV matrix elements to infinite volume electroweak amplitudes



$$\langle 2 | \mathcal{J} | 2 \rangle |_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

● Briceño & Hansen (2016)

Workflow



$2 + \mathcal{J} \rightarrow 2$

$$\langle 2 | \mathcal{J} | 2 \rangle |_L^2 = \frac{1}{L^6} \text{Tr} [R(E_L, L) W_{L,\text{df}} R(E_L, L) W_{L,\text{df}}]$$

$$W_{L,\text{df}} = W_{\text{df}} + MG(L, w)M$$

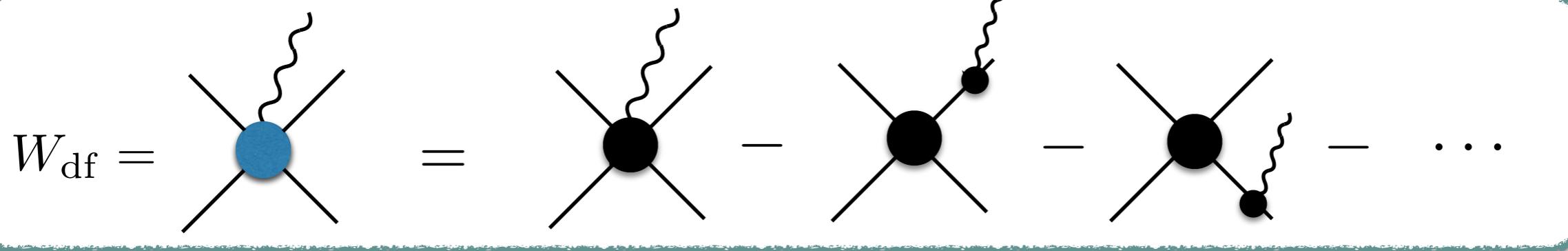
$$W_{\text{df}} = \text{Diagram with blue dot} = \text{Diagram with black dot} - \text{Diagram with black dot and wavy line} - \text{Diagram with black dot and wavy line} - \dots$$

$$w = \text{Diagram with wavy line}$$

$$M = \text{Diagram with black dot}$$

$$\begin{aligned} G(L, w) &= \text{Diagram with V} - \text{Diagram with infinity} \\ &= \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int d\mathbf{k} \right] (\dots) \end{aligned}$$

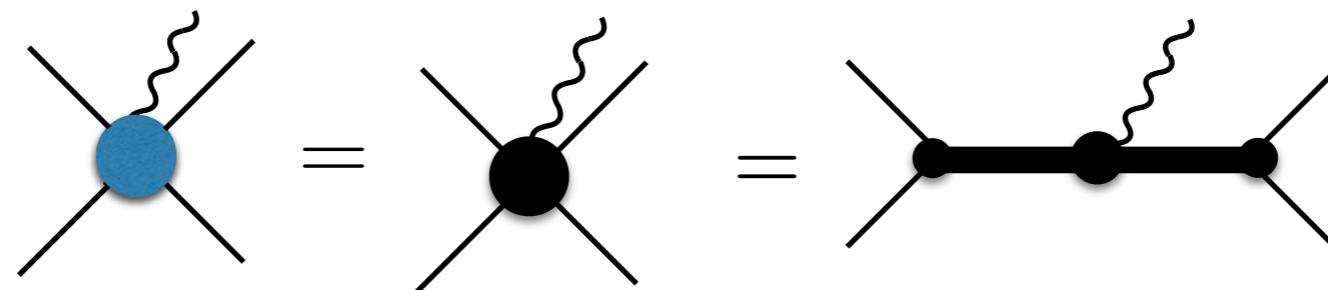
A detail



Kinematic poles cancel

But dynamical poles associated with resonances can still appear

$$\lim_{E_i^{\text{cm}}, E_f^{\text{cm}} \rightarrow E_R}$$



In the resonance region

Outline

$$G(P_i, P_f, L) = \left(\frac{1}{L^3} \sum_{\mathbf{k}} - \int_{\mathbf{k}} \right) f(P_i, P_f, \mathbf{k})$$

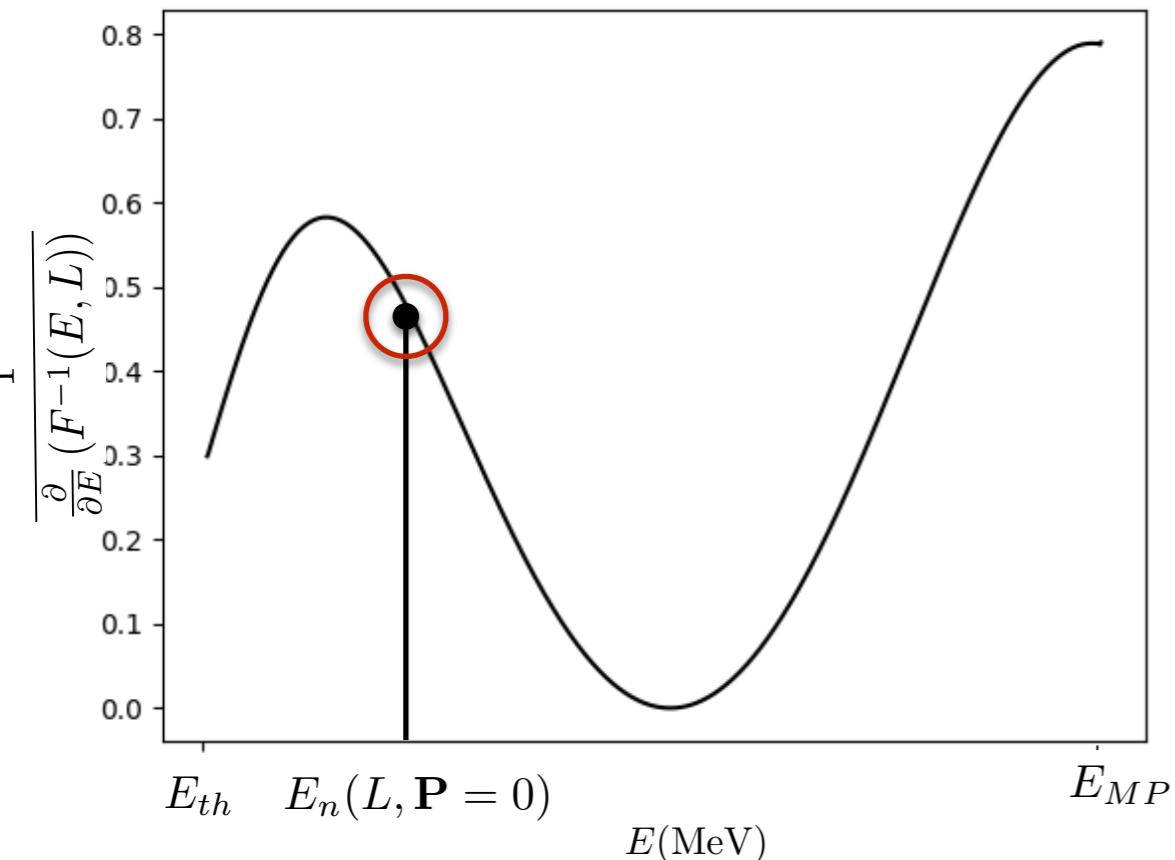
- The sum is “easy”
- The integral is highly not trivial (spectator particle goes on-shell)
 - integrand singularities are two surfaces in three-dimension
 - standard principal value techniques in one dimension fail
 - techniques from other fields are not suitable
 - using mathematical trickery we can isolate the singularities, treat them with standard field theory techniques, and be left with a 3D **smooth** integral

Kinematic functions I

$$R(E_L, L) = \frac{1}{\frac{\partial}{\partial E} (F^{-1}(E, L) + M(E))} \Big|_{E=E_L}$$

$$m_\pi L = 4$$

$$G_{l_i m_i, l_f m_f}$$

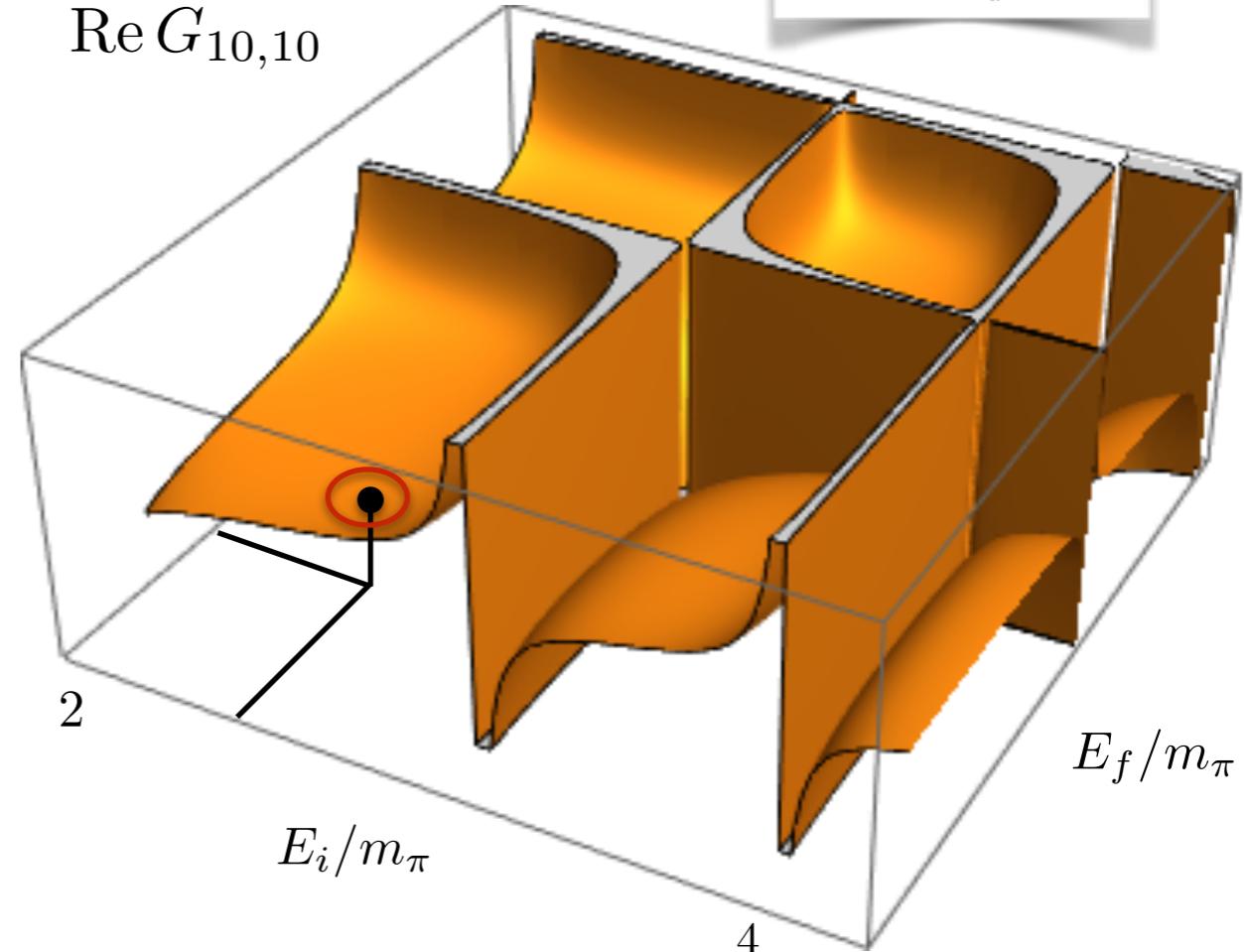


E_{th} = threshold energy

E_{MP} = multiparticle states energy

$$\text{Re } G_{10,10}$$

$$\mathbf{P}_i = \mathbf{P}_f = \mathbf{0}$$

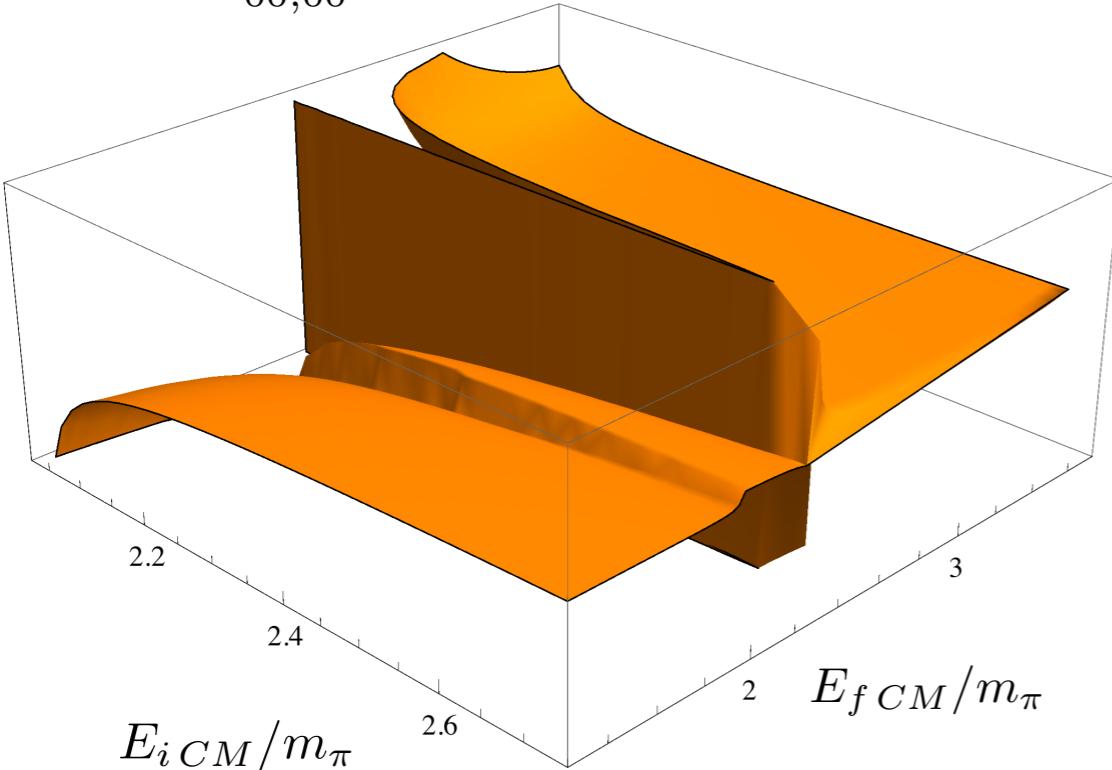


Singularities at free particle energies

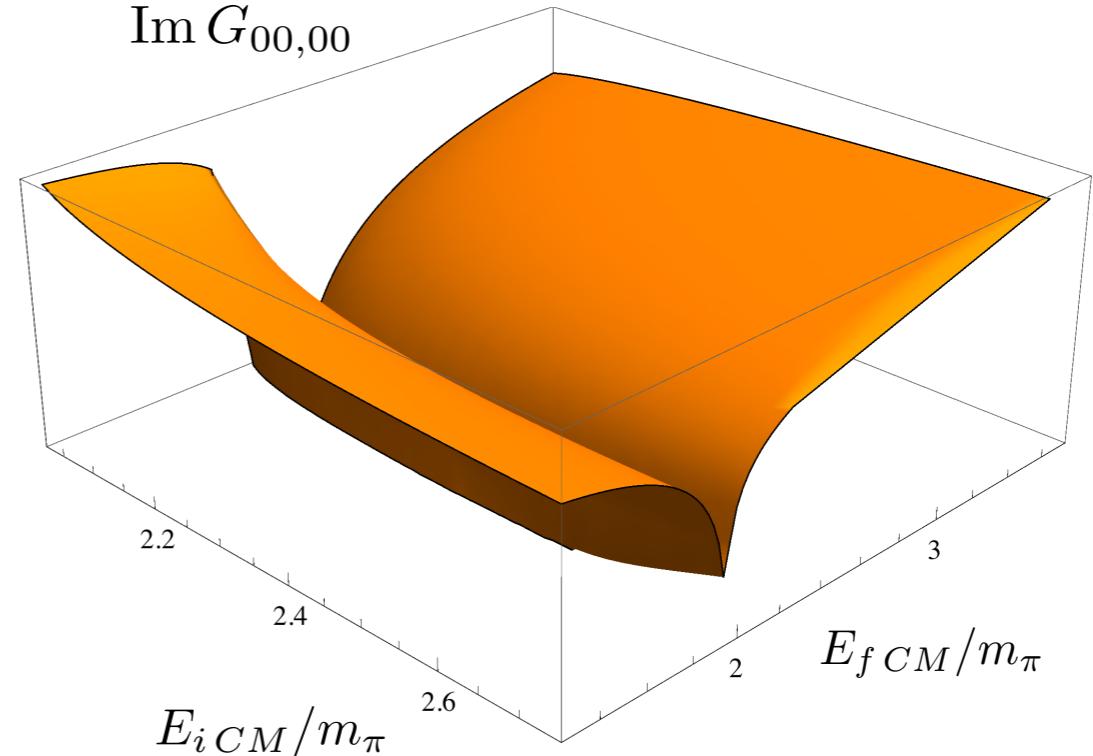
Kinematic functions II

$$\mathbf{P}_i = \mathbf{0}, \quad \mathbf{P}_f = [0, 0, 2\pi/L]$$
$$m_\pi L = 4$$

$\text{Re } G_{00,00}$



$\text{Im } G_{00,00}$



More plots in



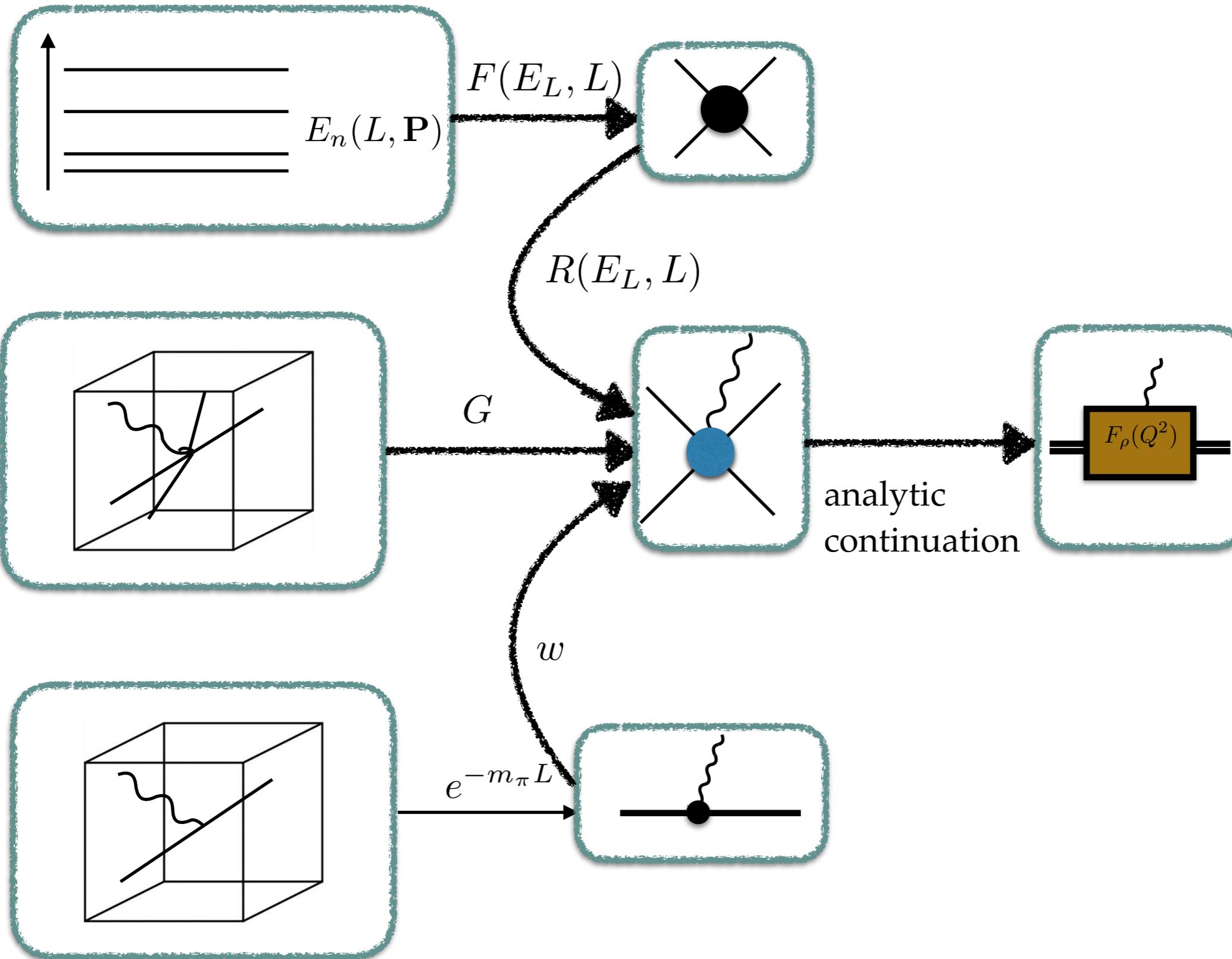
AB, R. A. Briceño, M. T. Hansen, F. Ortega, D.J. Wilson (2018)
In preparation



- ✓ General, relativistic, EFT independent, finite volume formalism to treat processes relevant for many electroweak processes (nuclear and not only) derived
- ✓ A crucial ingredient the new kinematic function is closed to the general complete numerical implementation
- Test/use this framework in an actual LQCD calculation

Thank you!

Workflow

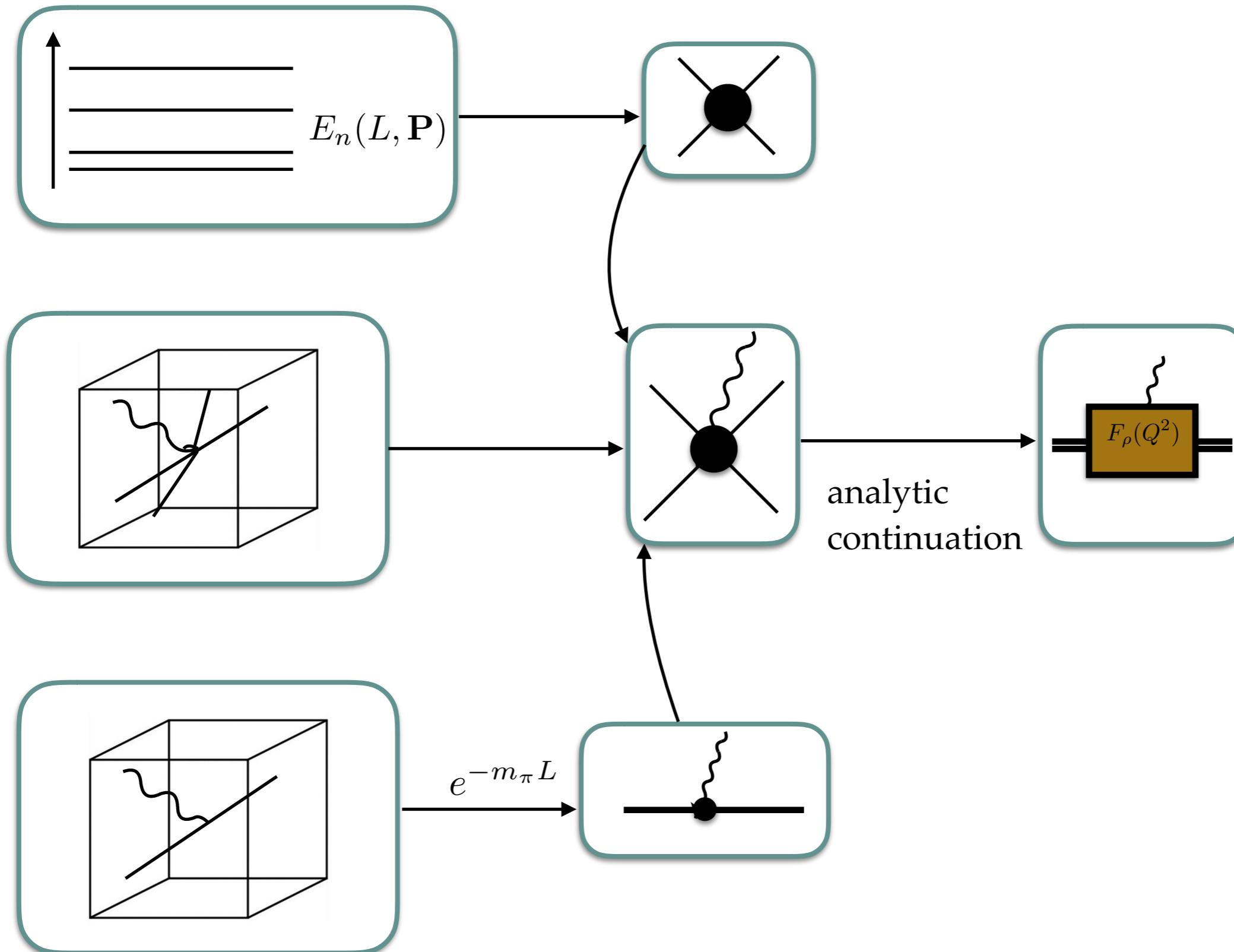


Some challenges

- A spin 1 particle between non-degenerate states has four form factors
- There is not a one-to-one mapping between matrix elements and amplitudes
 - Solved problem for spectrum analysis
- Analytical continuation of the amplitudes

Backup slides

Workflow 101



Steps left

$$\langle 2|\mathcal{J}|2\rangle_{\text{FV}} \rightarrow \langle 2|\mathcal{J}|2\rangle_\infty$$

- Evaluate kinematic functions for every value of energy and momenta
- Understand how to extract the form factors, mixing of waves.....

- From $\langle 2|\mathcal{J}|2\rangle_\infty$ how do we get the four form factors?
 - Analytic continuation