The strange quark contribution to the spin of the nucleon

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Edinburgh – RIKEN (Kobe) – Leipzig – FZ (Jülich) – Liverpool – DESY – Hamburg – Adelaide

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[Wednesday 25/7/18 17:10 (Kellogg Hotel and Conference Center, room 106)]

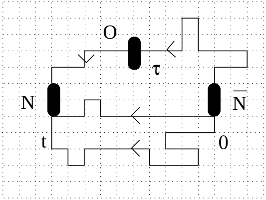
Papers:

- ‘A Lattice Study of the Glue in the Nucleon’
  arXiv:1205.6410 (PLB)
- ‘A Feynman-Hellmann approach to the spin structure of hadrons’
  arXiv:1405.3019 (PRD)
- ‘A novel approach to nonperturbative renormalization of singlet and nonsinglet lattice operators’
  arXiv:1410.3078 (PLB)
- ‘Disconnected contributions to the spin of the nucleon’
  arXiv:1508.06856 (PRD)

Lattice conferences:

- ‘Connected and disconnected quark contributions to hadron spin’
  arXiv:1412.6569, (Lattice 2014, June 2014, Columbia University, New York, USA)
- ‘Applications of the Feynman-Hellmann theorem in hadron structure’
  arXiv:1511.07090, (Lattice 2015, July 2015, Kobe, Japan)

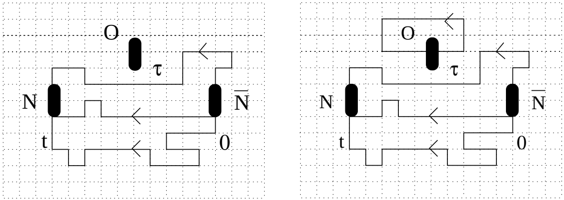
(Quark line) connected matrix elements $\mathcal{O} \sim \overline{\psi}\psi$



$$R(t,\tau;\vec{p}) = \frac{\langle N(t;\vec{p})O(\tau;\vec{0})\overline{N}(0;\vec{p})\rangle}{\langle N(t;\vec{p})\overline{N}(0;\vec{p})\rangle}$$ 3-point correlation function

$$\propto \langle N(\vec{p})|\widehat{O}|N(\vec{p})\rangle$$ $\frac{1}{2}T \gg t \gg \tau \gg 0$

(Quark line) disconnected matrix elements $\mathcal{O} \sim FF$ or $\mathcal{O} \sim \overline{\psi}\psi$



All t and τ allowed but:

- Gluon $\mathcal{O}$
  - short distance quantity
  - large fluctuations

  – huge number of configurations required $\sim O(10^6)$
- Fermion $\mathcal{O}$
  - All-to-all propagators unfeasible – $O(V)$ inversions needed
  - Stochastic estimators – still many inversions

Alternative approach (to both con and dis): Feynman–Hellmann

Feynman–Hellmann

If $S(\lambda) = S + \lambda O$

then

$$\frac{\partial E_N(\lambda)}{\partial \lambda} = \frac{1}{2E_N(\lambda)} \left\langle N \middle| : \widehat{O} : \middle| N \right\rangle_\lambda$$

- E_N from 2-point correlation functions
- Thus by suitably choosing O and by identifying numerically the gradient of $E_N(\lambda)$ at $\lambda = 0$ we can determine the desired matrix element
- [: . . . : means that the vacuum term has been subtracted]

FH application:

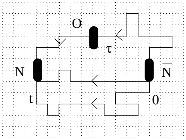
Modification location determines the contributions we access

Modify Dirac matrix before quark propagator inversion

$$D'^{-1} = [D + \lambda O]^{-1}$$

Inserts connected contributions on every line:

$$\frac{\partial}{\partial \lambda} D'^{-1} \bigg|_{\lambda=0} = D^{-1} O D^{-1}$$



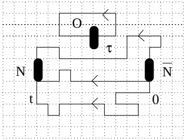
Easy to implement

Modify field weighting during HMC

$$\det D' e^{-S_g} = \det[D + \lambda O]\, e^{-S_g}$$

Access disconnected contributions:

$$\frac{\partial}{\partial \lambda} \det D' \bigg|_{\lambda=0} = \mathrm{tr}(D^{-1} O) \det D$$



Need to generate new gauge ensembles

Or do both modifications: connected and disconnected terms

Nucleon spin

Ji gauge invariant decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- quark spin $\Delta\Sigma = \Delta\Sigma_{\rm con} + \Delta\Sigma_{\rm dis}$ with

$$\begin{aligned} \Delta\Sigma_{\rm con} &= \Delta u_{\rm con} + \Delta d_{\rm con} \\ \Delta\Sigma_{\rm dis} &= \Delta u_{\rm dis} + \Delta d_{\rm dis} + \Delta s \end{aligned}$$

- L_q – quark orbital angular momentum
- J_g – gluon angular momentum [do not split or discuss further here]
- [Axial charge: $g_A = \Delta u - \Delta d$]

'Spin crisis': $\Delta\Sigma$ small $\sim 35\%$ of total spin

July 2015

Nucleon polarised in the z-direction

$$\langle N,\sigma| i\,\overline{q}\gamma_3\gamma_5 q|N,\sigma\rangle = 2M_N\sigma\Delta q \qquad \sigma = \pm 1$$

Achieved by projection operators

$$\Gamma_\sigma = \frac{1}{2}\,(1+\gamma_4)\,(1+i\sigma\gamma_3\gamma_5)$$

and

$$C_\sigma(\lambda,t) \equiv \mathrm{tr}\,(\Gamma_\sigma C(\lambda,t)) \equiv (\Gamma_\sigma)_{\beta\alpha}\langle N_\alpha(t)\overline{N}_\beta(0)\rangle_\lambda$$

Δq connected contributions

Nucleon polarised in the z-direction ($\sigma = \pm 1$)

$$\langle N, \sigma | i\,\overline{q} \gamma_3 \gamma_5 q | N, \sigma \rangle = 2 M_N \sigma \Delta q$$

with [consider each quark separately]

$$D' = D \pm\, i\lambda \sum_x \overline{q}(x) \gamma_3 \gamma_5 q(x)$$

and FH

$$\left. \frac{\partial E_N(\lambda)}{\partial \lambda} \right|_{\lambda=0} = \pm \frac{1}{2M_N} \langle N, \sigma | i\,\overline{q} \gamma_3 \gamma_5 q | N, \sigma \rangle$$

gives

$$\Delta q_{\mathrm{conn}} = \pm \sigma \left. \frac{\partial E_N(\lambda)}{\partial \lambda} \right|_{\lambda=0}$$

ie change of sign of $\lambda \equiv$ changing spin polarisation

$\Delta\Sigma_{\mathrm{dis}} = \Delta u_{\mathrm{dis}} + \Delta d_{\mathrm{dis}} + \Delta s$ disconnected contributions

Recall

- need to modify action
- generate configurations – here $2+1$ flavours with $\overline{m} = \mathrm{const.}$

Problem:
fermion matrix in action must be γ_5 - hermitian for HMC, ie need

$$S = S_g + \lambda \sum_{q\,x} \overline{q}(x)\gamma_3\gamma_5 q(x)$$

(Here $\sum_q$ so determine $\Delta\Sigma = \Delta u + \Delta d + \Delta s$)

Imaginary energy shifts

Correlation function develops imaginary parts:

$$C_\sigma(\lambda, t) = A_N(\sigma\lambda) e^{i\delta(\sigma\lambda)} e^{-[E_N(\sigma\lambda)+i\phi(\sigma\lambda)]t}$$

Form ratio

$$R(\lambda, t) = \frac{\mathrm{Im}\, C_+(\lambda, t) - \mathrm{Im}\, C_-(-\lambda, t)}{\mathrm{Re}\, C_+(\lambda, t) - \mathrm{Re}\, C_-(-\lambda, t)} = -\tan(\phi(\lambda)t - \delta(\lambda))$$

Effective phase shift

$$\phi_{\rm eff}(\lambda) = \frac{1}{t} \tan^{-1}\left(-R(\lambda, t)\right) \to \phi(\lambda) = \phi_0\lambda + \phi_1\lambda^3 + \ldots$$

So have

$$\Delta\Sigma^{LAT} = \left.\frac{\partial\phi(\lambda)}{\partial\lambda}\right|_{\lambda=0}$$

Configurations

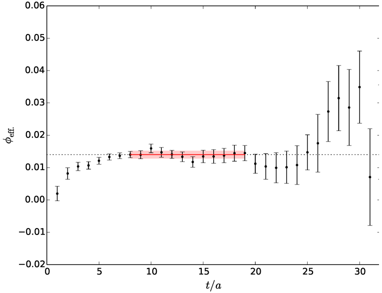
$2+1$ flavours with $\overline{m} = \text{const.}$

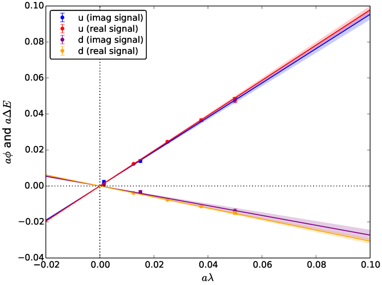
κ_l	κ_s	λ_l	λ_s
0.120900		-0.00625	
0.120900		-0.0125	
0.120900		0.0300	
0.121040	0.120620	-0.0750	
0.121095	0.120512	0.0000	0.0500
0.121095	0.120512	-0.0250	
0.121095	0.120512	0.0500	
0.121095	0.120512	-0.0750	

Flavour symmetric point, $M_\pi \sim 460\,\text{MeV}$ down to $\sim 300\,\text{MeV}$,
$a \sim 0.074\,\text{fm}$, $32^3 \times 64$

Following: Very preliminary analysis

Test: calculate connected parts using imaginary signal



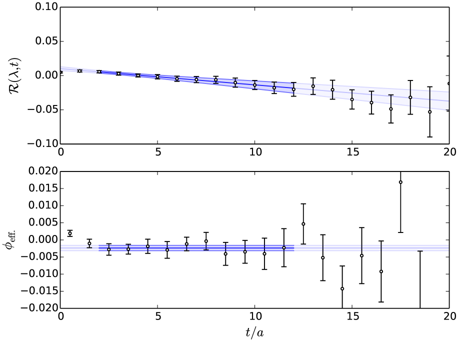


- LH plot: plateau seen
- RH plot: comparison between 'real/imaginary' λ

Conclusion: Using 'imaginary' λ works

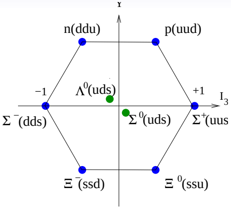
[But better to consider real λ, ie $D' = D \pm i\lambda \sum_x \overline{q}(x)\gamma_3\gamma_5 q(x)$ when no imaginary parts develop]

Imaginary signal



- Both Re/Im plots for $\lambda_{\Delta\Sigma} = 0.03$, $(\kappa_l, \kappa_s) = (0.120900, 0.120900)$ ($SU(3)$ flavour symmetric point) $M_\pi \sim 460\,\text{MeV}$
- Signal seen

$SU(3)$ flavour symmetry breaking quark mass expansion



For the singlet operators, need to consider $8 \times 1 \times 8$ tensors (similarly to masses) so to LO have expansion

$$\begin{aligned}
\Delta\Sigma^{LAT}_{N\,\mathrm{dis}} &= \Delta\Sigma_{0\,\mathrm{dis}} + 3A_{1\,\mathrm{dis}}\delta m_l + O(\delta m_l^2) \\
\Delta\Sigma^{LAT}_{\Sigma\,\mathrm{dis}} &= \Delta\Sigma_{0\,\mathrm{dis}} - 3A_{2\,\mathrm{dis}}\delta m_l + O(\delta m_l^2) \\
\Delta\Sigma^{LAT}_{\Xi\,\mathrm{dis}} &= \Delta\Sigma_{0\,\mathrm{dis}} - 3(A_{1\,\mathrm{dis}} - A_{2\,\mathrm{dis}})\delta m_l + O(\delta m_l^2) \\
\Delta\Sigma^{LAT}_{N_s\,\mathrm{dis}} &= \Delta\Sigma_{0\,\mathrm{dis}} - 6A_{1\,\mathrm{dis}}\delta m_l + O(\delta m_l^2)
\end{aligned}$$

- Here: $2+1$ quark flavours, $m_u = m_d \equiv m_l$, m_s
- $\delta m_l = m_l - \overline{m}$
  Distance from (point on) SU(3) flavour symmetric line ($m_l = m_s$).
  Also $\overline{m}$ average quark mass, held constant in simulations
- Similarly for con and renormalised quantities

Singlet of singlets

Define a Singlet operator

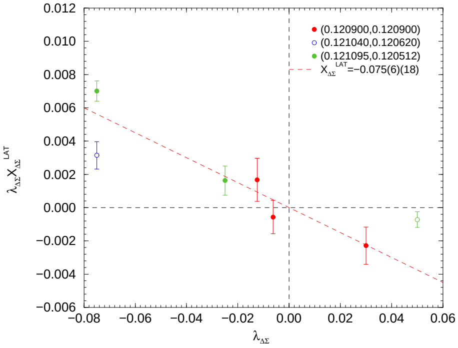
$$X^{LAT}_{\Delta\Sigma\,\mathrm{dis}} = \frac{1}{3}(\Delta\Sigma^{LAT}_{N\,\mathrm{dis}} + \Delta\Sigma^{LAT}_{\Sigma\,\mathrm{dis}} + \Delta\Sigma^{LAT}_{\Xi\,\mathrm{dis}})$$

With $SU(3)$ flavour breaking quark mass expansion

$$X^{LAT}_{\Delta\Sigma\,\mathrm{dis}} = \Delta\Sigma_{0\,\mathrm{dis}} + O(\delta m_l^2)$$

Can form quantities $\Delta\Sigma^{LAT}_{\mathrm{dis}}/X^{LAT}_{\Delta\Sigma\,\mathrm{dis}}$

$X^{LAT}_{\Delta\Sigma\,\mathrm{dis}}$



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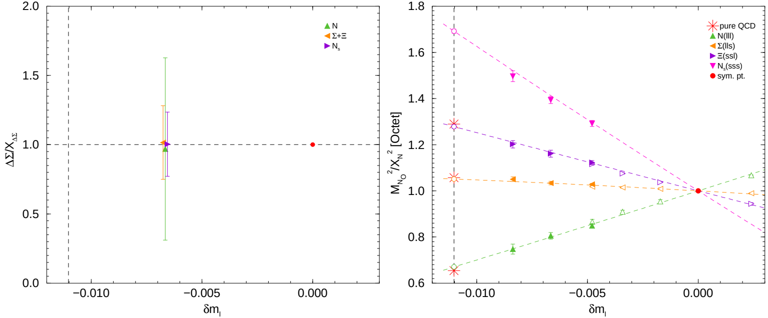
$$\phi_{\Delta\Sigma} = \lambda_{\Delta\Sigma} X_{\Delta\Sigma} \qquad \text{or} \qquad X^{LAT}_{\Delta\Sigma\,\mathrm{dis}} = \frac{\partial \phi_{\Delta\Sigma}}{\partial \lambda_{\Delta\Sigma}}$$

- Can use all quark masses
- Flavour symmetric point, $M_\pi \sim 460\,\mathrm{MeV}$ down to $\sim 300\,\mathrm{MeV}$
  $a \sim 0.074\,\mathrm{fm}$, $32^3 \times 64$

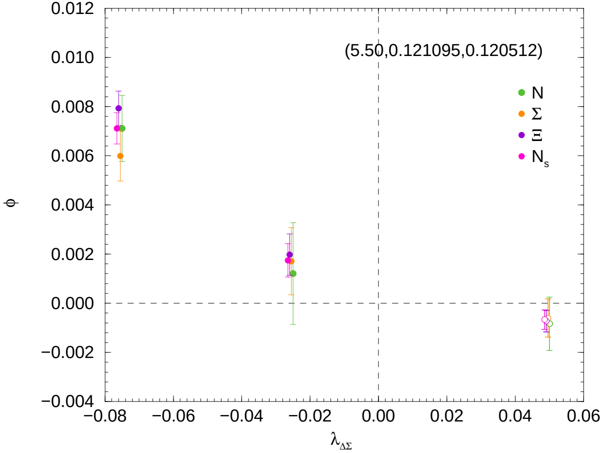
$\Delta\Sigma^{LAT}_{\text{dis}}/X^{LAT}_{\Delta\Sigma\,\text{dis}}$



- LH plot: 'fan' plot for $\Delta\Sigma^{LAT}_{\text{dis}}/X^{LAT}_{\Delta\Sigma\,\text{dis}}$
  Only evaluate where more than lambda available (pre-average)
- RH plot: comparison for baryon masses (what you would like to have/see)

Tentative conclusion: little sign of $SU(3)$ flavour symmetry breaking

A single quark mass



- Confirmation: little evidence of different baryon effects

Renormalisation I

We have:

$$\begin{aligned} \Delta q_{\rm con} &= Z_A^{NS} \Delta q_{\rm con}^{LAT} \\ \Delta q_{\rm dis} &= Z_A^{NS} \Delta q_{\rm dis}^{LAT} + \frac{1}{3}(Z_A^{S\,\overline{MS}} - Z_A^{NS})(\Delta\Sigma_{\rm con}^{LAT} + \Delta\Sigma_{\rm dis}^{LAT}) \end{aligned}$$

giving

$$\begin{aligned} \Delta\Sigma_{\rm con} &= Z_A^{NS} \Delta\Sigma_{\rm con}^{LAT} \\ \Delta\Sigma_{\rm dis} &= Z_A^{S\,\overline{MS}} \Delta\Sigma_{\rm dis}^{LAT} + (Z_A^{S\,\overline{MS}} - Z_A^{NS})\Delta\Sigma_{\rm con}^{LAT} \end{aligned}$$

or

$$\Delta\Sigma_{\rm con} + \Delta\Sigma_{\rm dis} = Z_A^{S\,\overline{MS}}(\Delta\Sigma_{\rm con}^{LAT} + \Delta\Sigma_{\rm dis}^{LAT})$$

Additionally at $SU(3)$ symmetric point

$$\Delta s = \frac{1}{3}\,\Delta\Sigma_{\rm dis}$$

Renormalisation II

We have:

$$\begin{aligned}\Delta q_{\rm con} &= Z_A^{NS}\Delta q_{\rm con}^{LAT}\\ \Delta q_{\rm dis} &= Z_A^{NS}\Delta q_{\rm dis}^{LAT} + \frac{1}{3}(Z_A^{S\,\overline{MS}} - Z_A^{NS})(\Delta\Sigma_{\rm con}^{LAT} + \Delta\Sigma_{\rm dis}^{LAT})\end{aligned}$$

Have computed Z_A^{NS}, $Z_A^{S\,\overline{MS}}$ at 2 GeV in arXiv:1410.3078 [RI-MOM and FH]

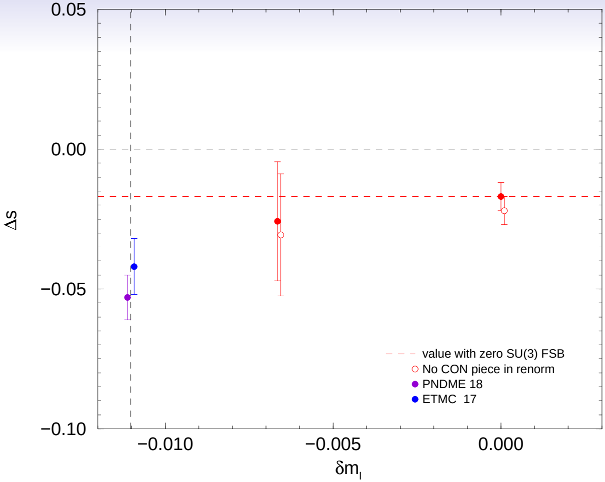
$$Z_A^{NS} = 0.8458(8) \qquad Z_A^{S\,\overline{MS}} = 0.8662(34)$$

Note, this gives

$$(Z_A^{S\,\overline{MS}} - Z_A^{NS})/Z_A^{S\,\overline{MS}} \sim 2\%$$

Also (partially) computed $\Delta\Sigma_{\rm con}^{LAT}$ in arXiv:1508.06856

Δs



- previously: little evidence of quark mass effects (little $SU(3)$ flavour symmetry breaking)
- $SU(3)$ flavour symmetric point + one other (dedicated run with λ_s)

$$\Delta s^{\overline{MS}}(2\,\text{GeV}) = -0.017(2)(5)$$

Conclusions

- 'Disconnected' quantities are notoriously difficult quantities to compute
  - short distance quantity
  - large fluctuations
- As alternative to more standard 'stochastic' approaches have introduced a method using the Feynman–Hellmann theorem
- $\Delta\Sigma$ – nucleon spin – $2+1$ flavours
  - Connected and disconnected 3-point functions
  - real and imaginary components to the energy
  - Application to axial current
  - Renormalisation
  - Δs