Progress in the lattice simulations of Sp(2N) gauge theories

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✤ Motivation: BSM phenomenology

Novel strong dynamics has been receiving quite large attention in the context of BSM phenomenology.

- (Walking) Technicolor
- Composite Higgs
- Top partner via partial compositeness
- Composite Dark Matter

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→ ***Sp(2N) gauge theory***

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2) Mesons in Sp(4) with Nf=2 fund. reps. (dynamical)

3) Mesons in Sp(4) Anti-sym. reps (quenched)

4) Glueballs in Sp(6)

5) Conclusions & Outlook

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✤ Sp(2N) composite Higgs

Sp(2N) group is pseudoreal: $U\Omega U^T = \Omega, \quad \Omega = \begin{bmatrix} 0 & \mathbb{I}_N \\ -\mathbb{I}_N & 0 \end{bmatrix}$

Most relevant model: N_f=2 fund. & N_f=3 anti-sym., Sp(4)

Barnard, Gherghetta & Ray (2014)

SM Electroweak $SU(4)/Sp(4)$ ~ SO(6) / SO(5) $\times$ $SU(6)/SO(6)$ ***SM Strong***

$SU(2)_L \times U(1)_Y \subset$ Sp(4) $\quad$ $SU(3)_c$ x $U(1)_Y$ $\subset$ SO(6)

Higgs = pseudo NG boson $\quad$ *Top partner = Chimera baryon*

(Extra) pseudo NG bosons could be used for Dark Matter pheno.

$\hat{\Psi}^{ab\alpha} \equiv \left(q^a \chi^\alpha q^b\right)$ ***carry color charge***

✤ Simulation details

- Standard Wilson gauge and fermion actions

- Gauge configurations: pure Sp(4) using HB & dyn. Sp(4) using HMC

~200 configurations for each ensemble

- Modified HiRep code *Del Debbio, Patella, Pica (2010)*

  - Resymplecitization: Gram-Schmidt procedure
  - Reduced size of configurations by half
  - Implemented anti-symmetric reps.

- Thermalization and autocorrelations are monitored by measuring average plaquette values.

- Scale setting: Luscher's gradient flow scales

$$\mathcal{W}(t) \equiv t\frac{\mathrm{d}\mathcal{E}(t)}{\mathrm{d}t} \qquad\qquad \mathcal{W}(t)|_{t=w_0^2} = \mathcal{W}_0$$

✤ Mass & Decay constant of mesons

Operators for mesons

Label	Operator	Meson	J^P
PS	$\overline{u}\gamma_5 d$	π	0^-
V	$\overline{u}\gamma_\mu d$	ρ	1^-
AV	$\overline{u}\gamma_5\gamma_\mu d$	a_1	1^+

Two-point correlation function

$$C_{\mathcal{O}_M}(t) \xrightarrow{t\to\infty} \langle 0|\mathcal{O}_M|M\rangle\langle 0|\mathcal{O}_M|M\rangle^* \frac{1}{m_M L^3}\left[e^{-m_M t} + e^{-m_M(T-t)}\right]$$

Parametrization of the mesonic matrix element

$$\begin{aligned}
\langle 0|\overline{Q_1}\gamma_5\gamma_\mu Q_2|PS\rangle &= i f_\pi p_\mu, \\
\langle 0|\overline{Q_1}\gamma_\mu Q_2|V\rangle &= i f_\rho m_\rho \epsilon_\mu, \\
\langle 0|\overline{Q_1}\gamma_5\gamma_\mu Q_2|AV\rangle &= i f_{a_1} m_{a_1} \epsilon_\mu,
\end{aligned}$$

$$C_{\mathcal{O}_V}(t) \xrightarrow{t\to\infty} \frac{m_\rho f_\rho^2}{L^3}\left[e^{-m_\rho t} + e^{-m_\rho(T-t)}\right],$$

$$C_{\mathcal{O}_{AV}}(t) \xrightarrow{t\to\infty} \frac{m_{a_1} f_{a_1}^2}{L^3}\left[e^{-m_{a_1} t} + e^{-m_{a_1}(T-t)}\right]$$

For the pseudoscalar decay constants

$$\begin{aligned}
C_\Pi(\vec{p},t) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0|[\overline{Q_1}\gamma_5\gamma_\mu Q_2(\vec{x},t)]\,[\overline{Q_1}\gamma_5 Q_2(\vec{0},0)]|0\rangle \\
&\xrightarrow{t\to\infty} \frac{i f_\pi \langle 0|\mathcal{O}_{PS}|PS\rangle^*}{L^3}\left[e^{-m_\pi t} - e^{-m_\pi(T-t)}\right].
\end{aligned}$$

Decay constants are converted to the continuum ones by perturbative one-loop matching.

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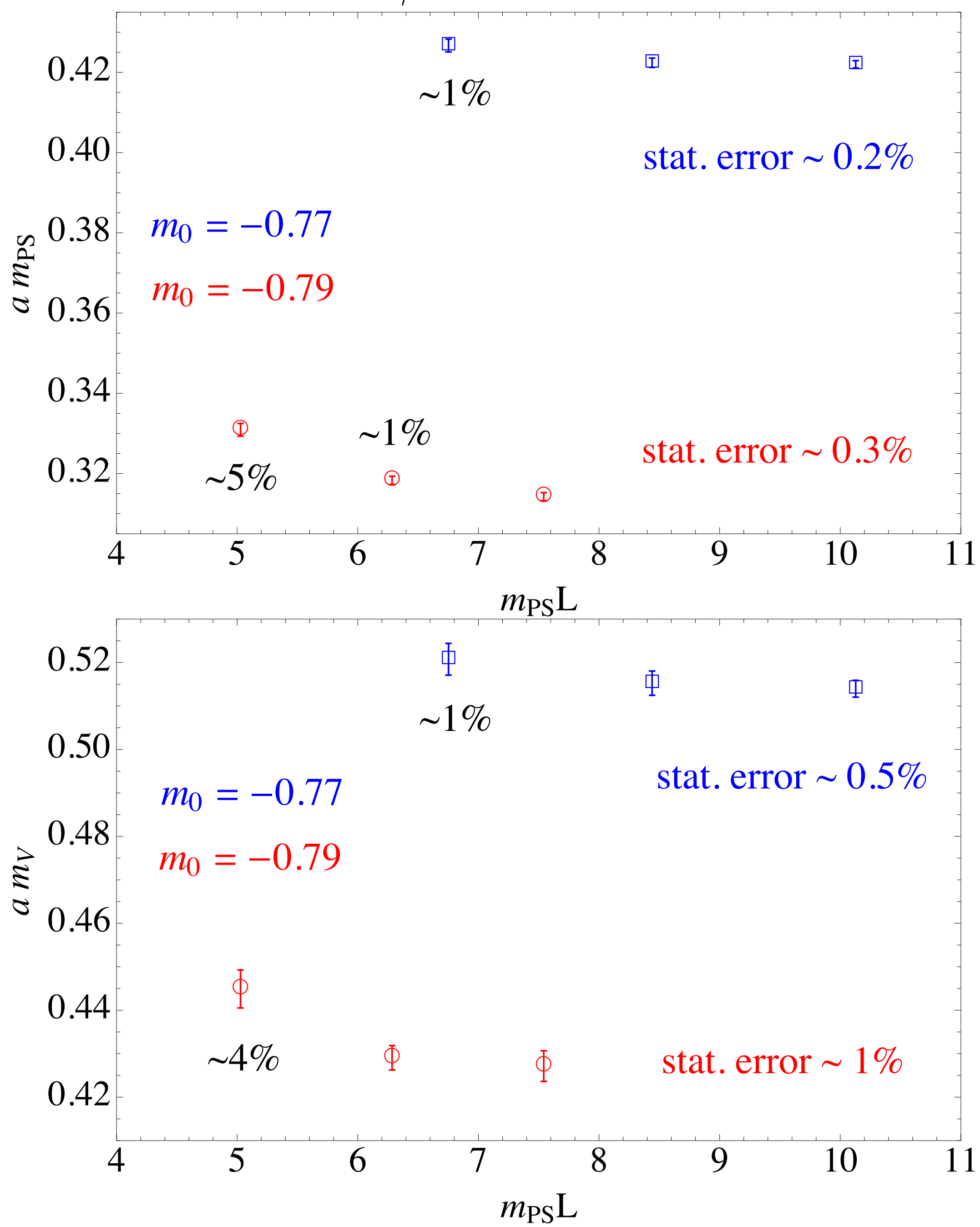
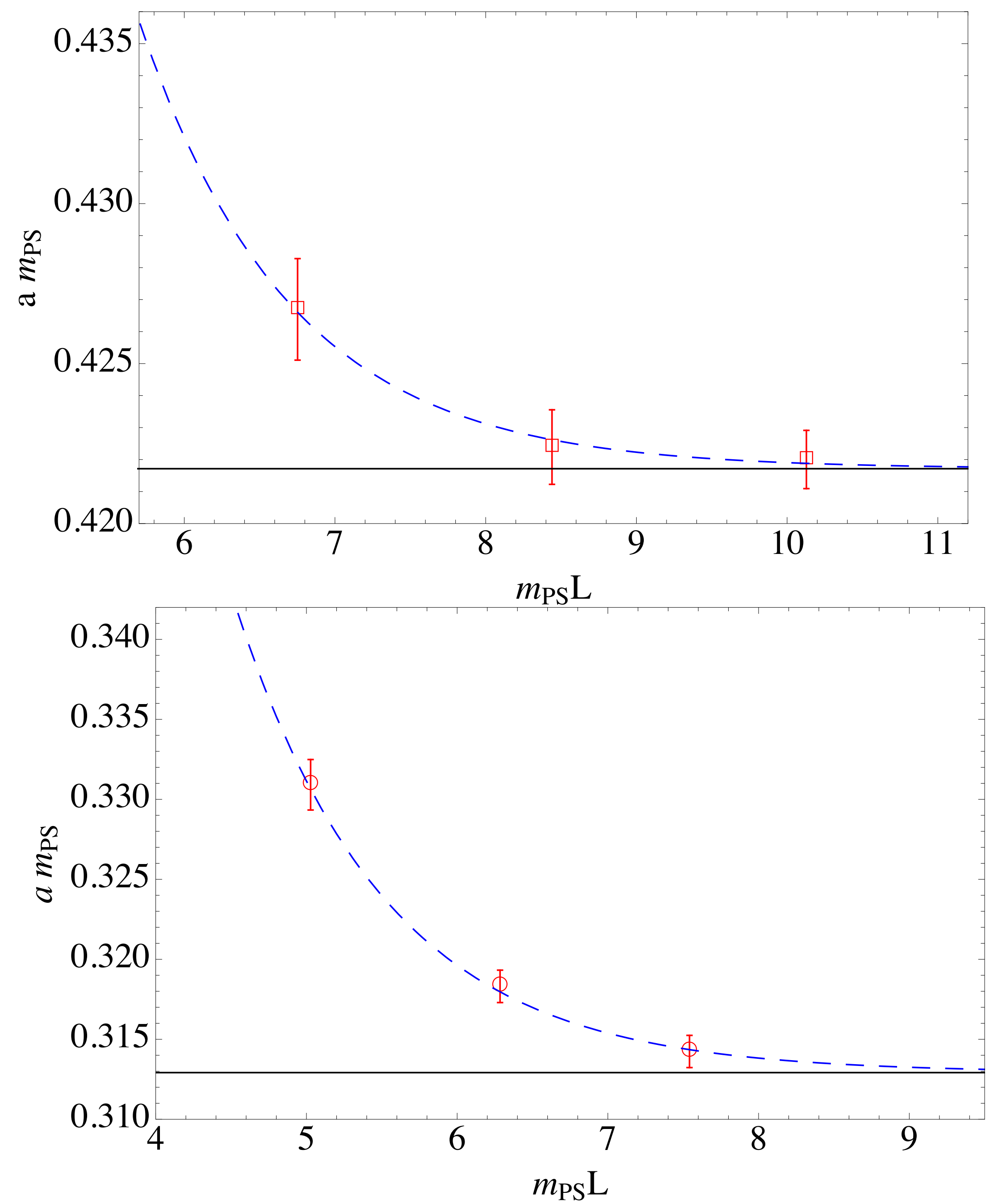
✤ Lattice systematics: Finite volume

- Finite volume effects are under control for $m_{PS} L \gtrsim 8$.

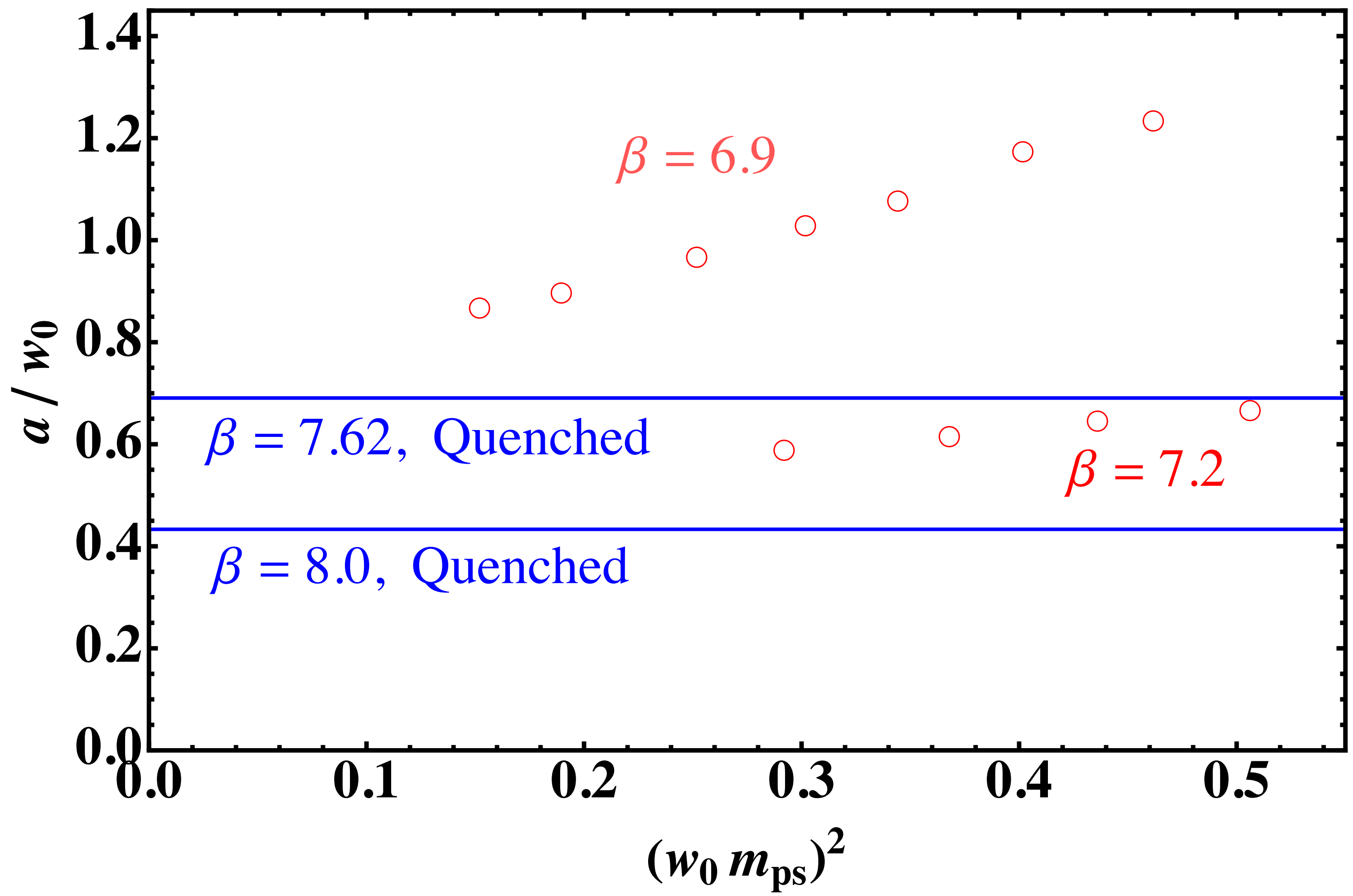
$\beta = 7.2$



c.f. Lattice QCD $m_\pi L \gtrsim 4$



✤ Lattice systematics: Finite lattice spacing



- rapid change of w_0/a

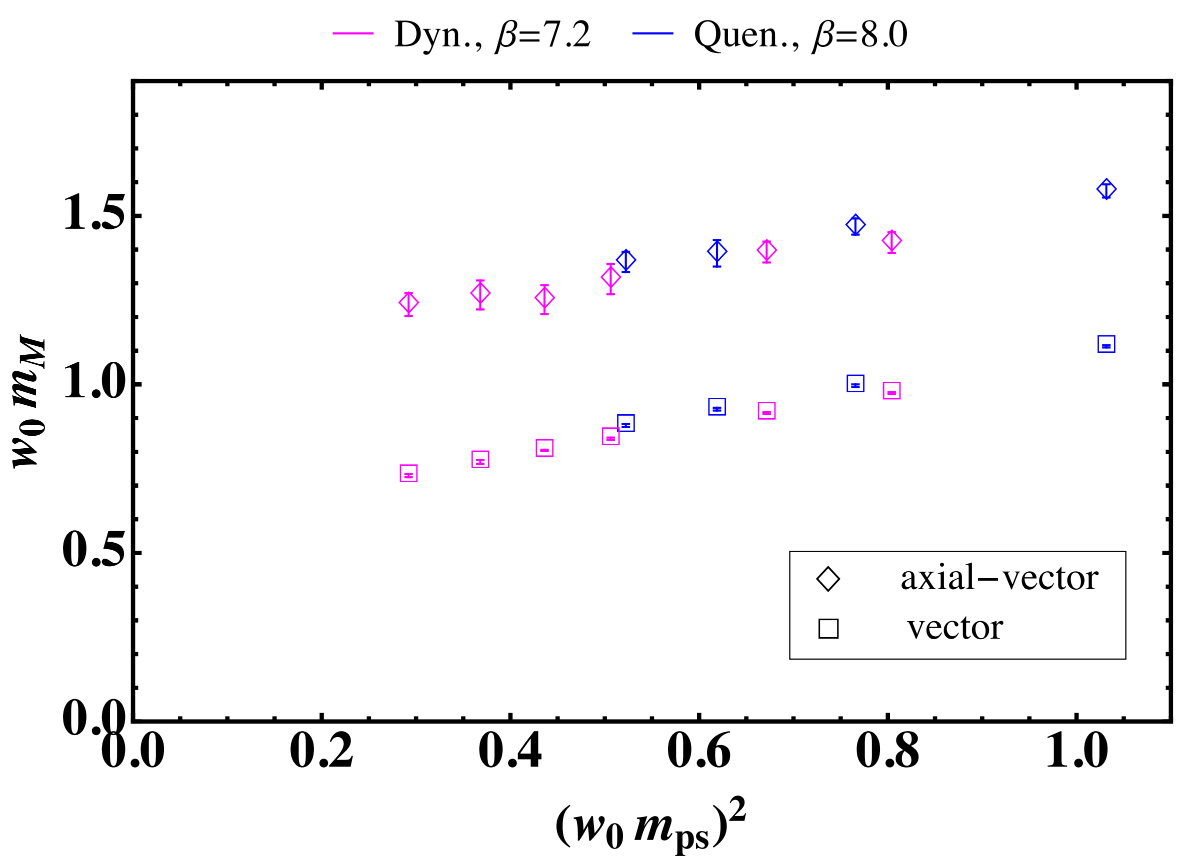
Nf=2 SU(2) *R. Arthur et al (2016)*

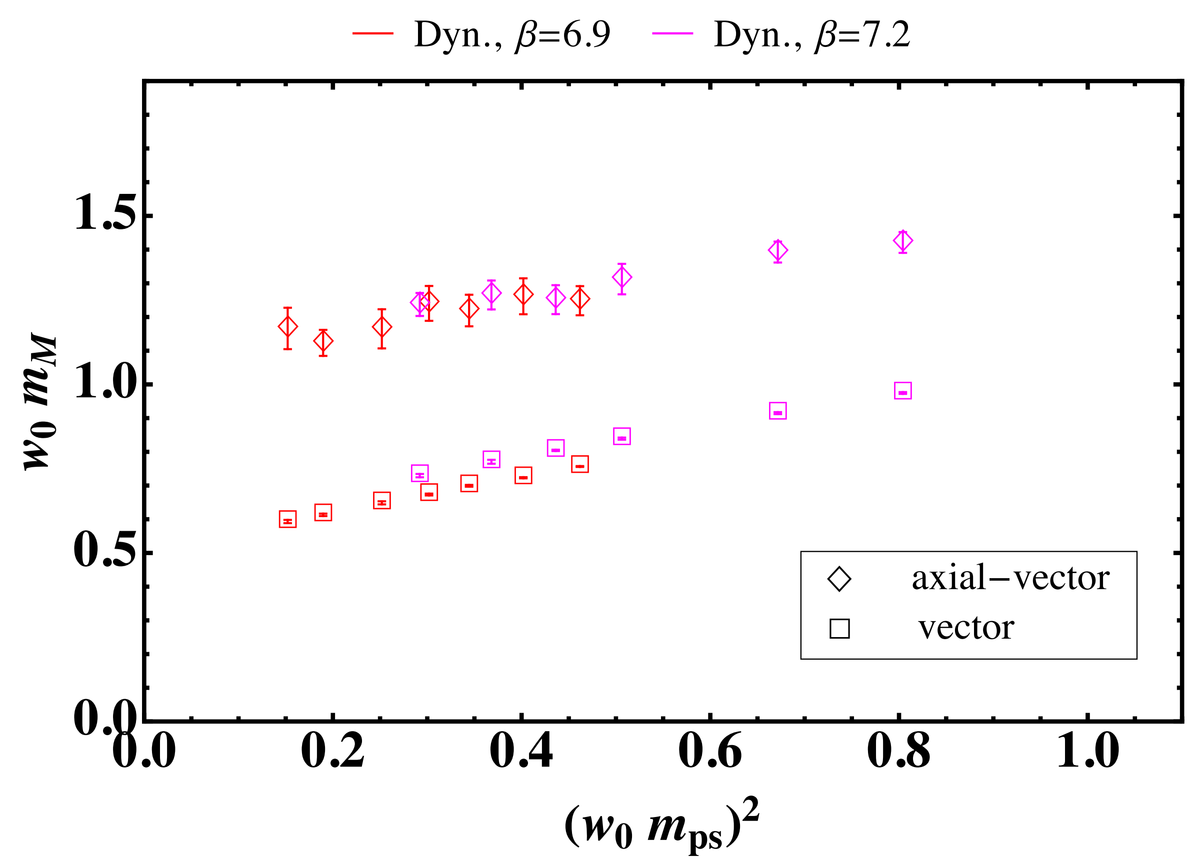
mixed SU(4) *V. Ayyar et al (TACo) (2017)*

c.f.) mild quark-mass dependence in QCD

- Special care is required to take continuum extrapolation.
  i.e. mass-dependent scale setting

✤ Numerical results: mass

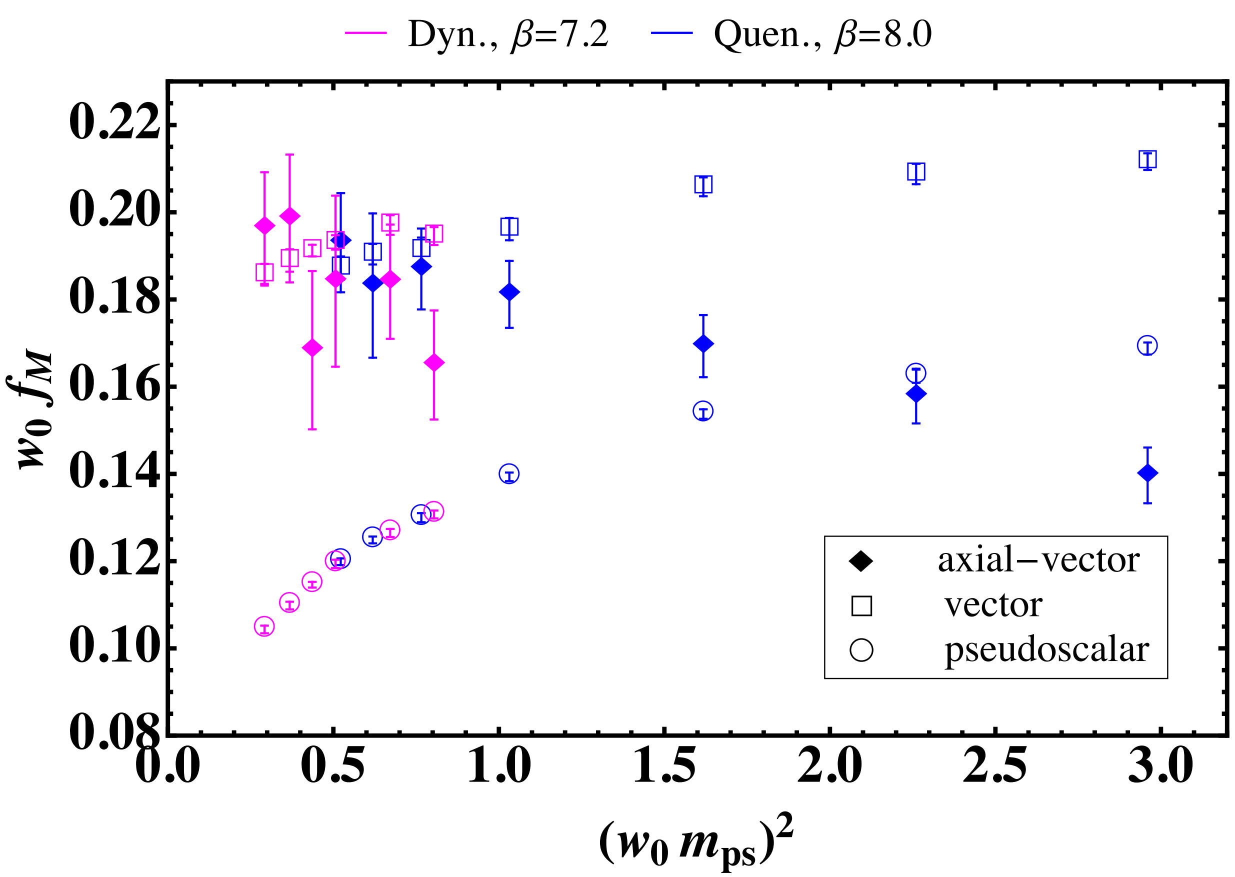


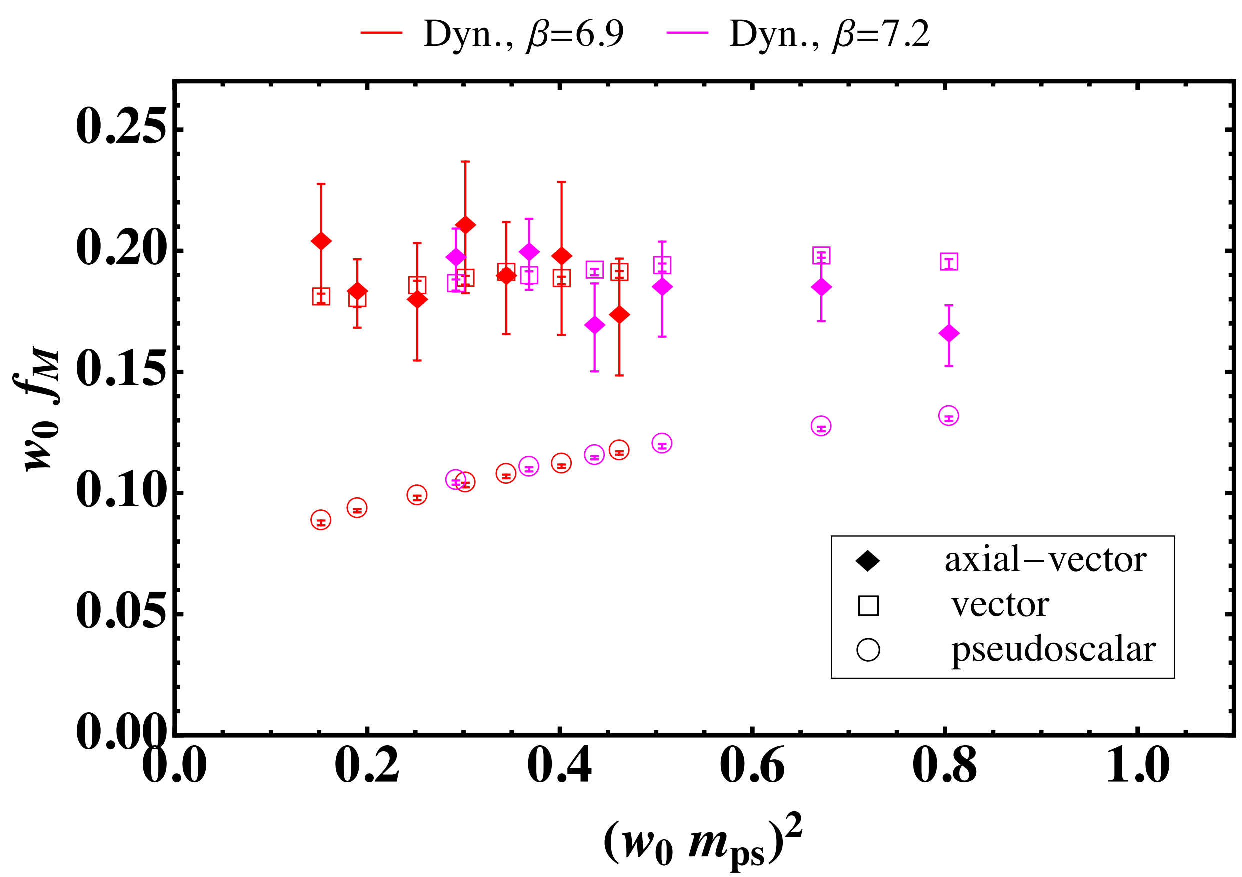


- Quenching effects are small: still far from chiral limit?

- Lattice spacing artifact is sizable for vector meson masses.

✤ Numerical results: decay constant





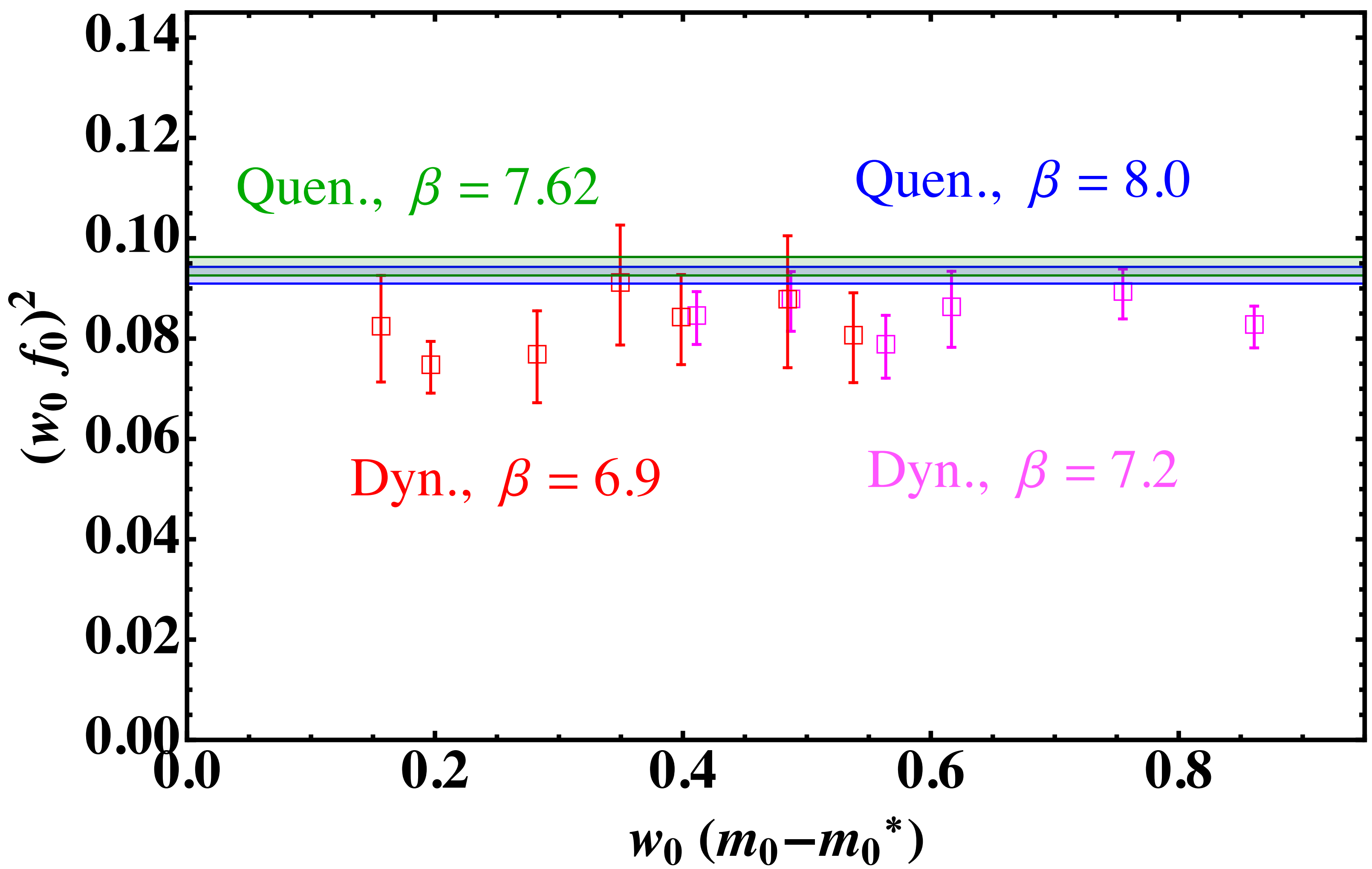
- Quenching effects are small: still far from chiral limit?

- Lattice spacing artifacts are also small for decay constants.

✤ Numerical results: f_O^2

- Sum of squared decay constants: $f_\pi^2(0) = \lim_{q^2 \to 0} \Sigma(q^2) = f_0^2 - f_\rho^2 - f_{a_1}^2$

Bennett et. al. (2017)



- Dynamical results also show that f_O^2 has small mass dependence.

- Consistent with NLO EFT results. $f_0^2 = F^2 + (b + 2c) f^2$

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✤ Anti-symmetric reps in Sp(2N)

- The anti-symmetric reps of Sp(2N) group requires to subtract the omega-trace term which is a singlet.

$$T^{ij} = u^i v^j - u^j v^i - \frac{1}{N}\Omega^{ij} u^k \Omega_{k\ell} v^\ell, \text{ which is traceless satisfying } \Omega_{ij} T^{ij} = 0.$$

$$\Omega = \begin{bmatrix} 0 & \mathbb{I}_N \\ -\mathbb{I}_N & 0 \end{bmatrix}$$

- Implementation to Hirep code

$$(R^A U)_{(ij)(lk)} = U^A_{(ij)(lk)} = \mathrm{tr}\left[(e_A^{(ij)})^\dagger U e_A^{(lk)} U^T\right] \qquad i < j, l < k$$

Del Debbio, Patella, Pica (2010)

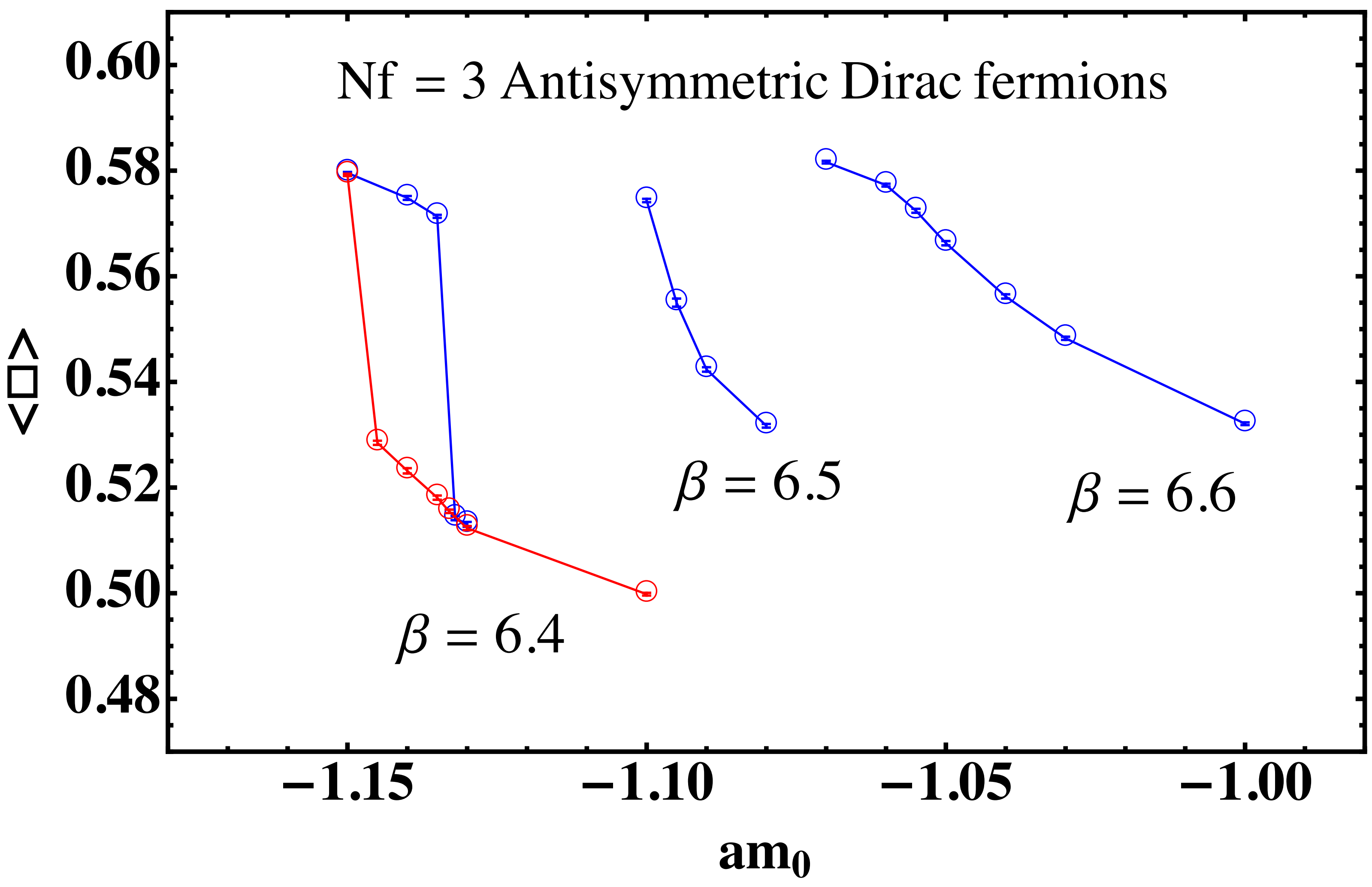
for $j = N + i,\ i \neq 1$ and $i \leq N$,

$$(e^{ij}_{AS})_{k\,N+k} = -(e^{ij}_{AS})_{N+k\,k} \equiv \begin{cases} \frac{1}{\sqrt{2\,i\,(i-1)}}, & \text{for } k < i \\ \frac{-(i-1)}{\sqrt{2\,i\,(i-1)}}, & \text{for } k = i \end{cases}$$

otherwise,

$$(e^{ij}_{AS})_{k\ell} \equiv \frac{1}{\sqrt{2}}(\delta_{ik}\delta_{j\ell} - \delta_{jk}\delta_{i\ell})$$

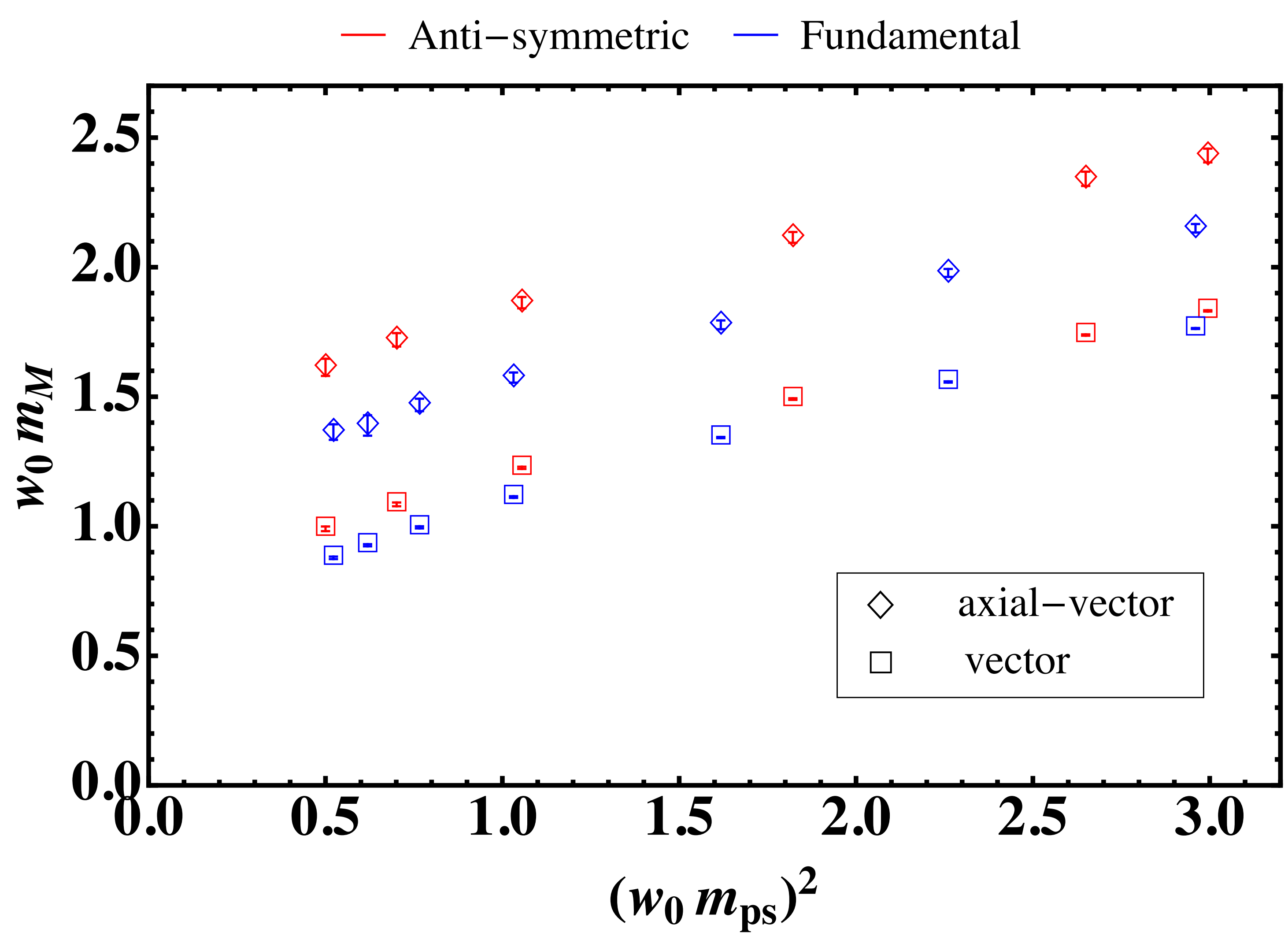
✤ Phase space of bare parameters



$T \times L = 8 \times 8^3$

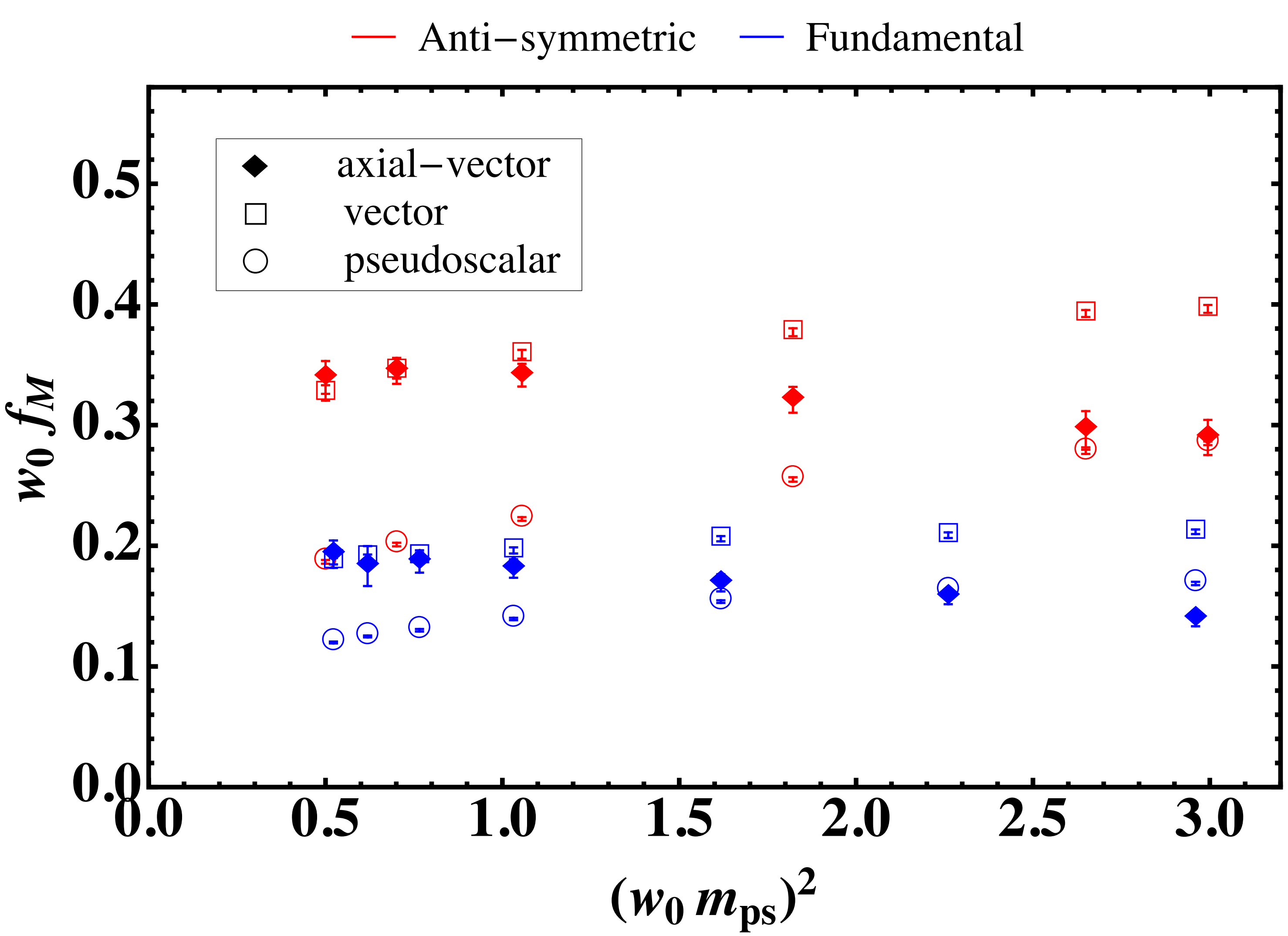
- Strong hysteresis in the plaquette values (cold and hot) indicates the existence of a first order bulk phase transition for $\beta \lesssim 6.5$.

✤ Quenched results: Mass



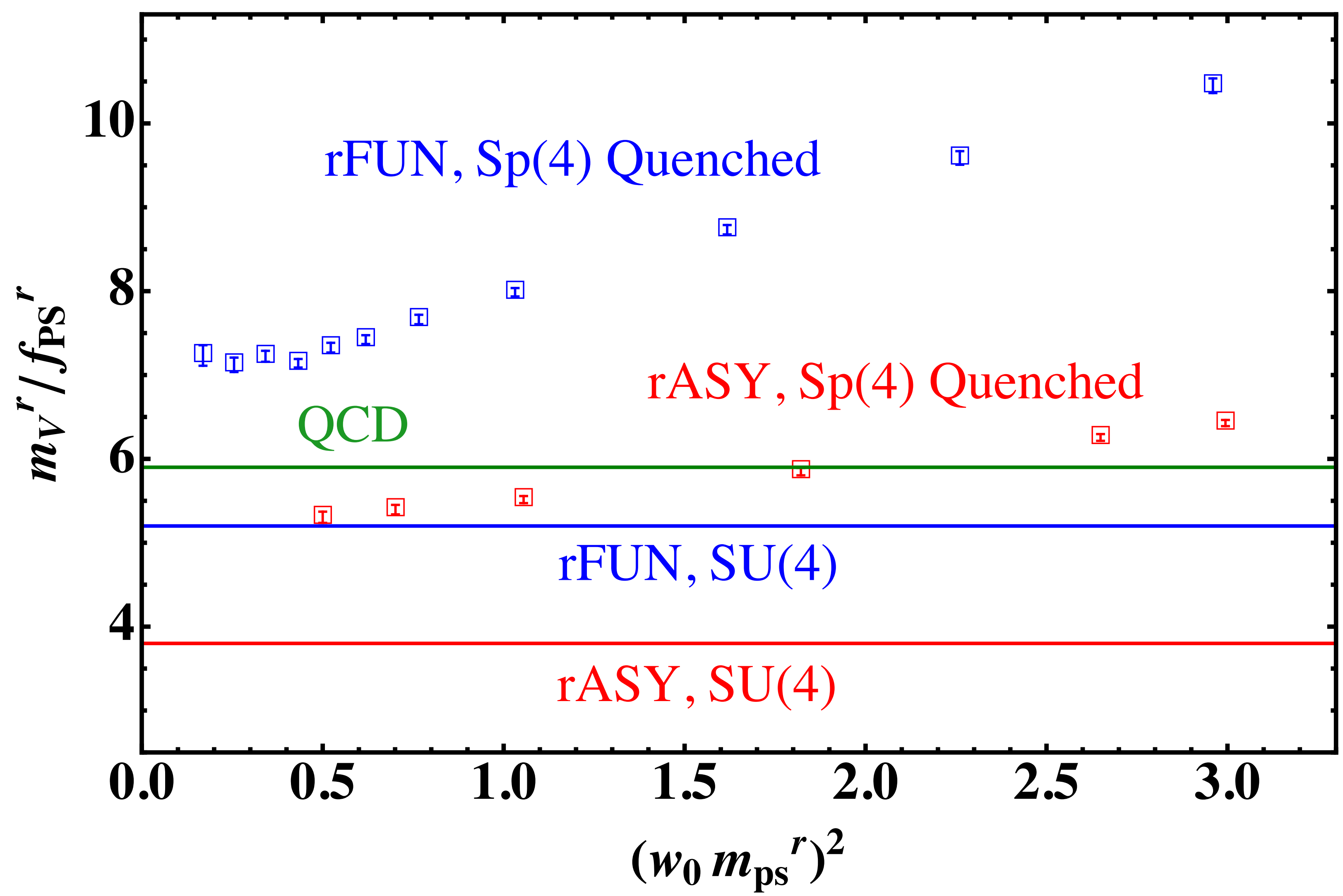
- Vector and axial-vector: The mass of anti-sym. reps. is larger than that of fund. reps. for a given pseudoscalar mass.

✤ Quenched results: Decay constant



- The decay constant of anti-sym. reps. is larger than that of fund. reps. for a given pseudoscalar mass in each case.

✤ Vector meson mass in units of f_{ps}



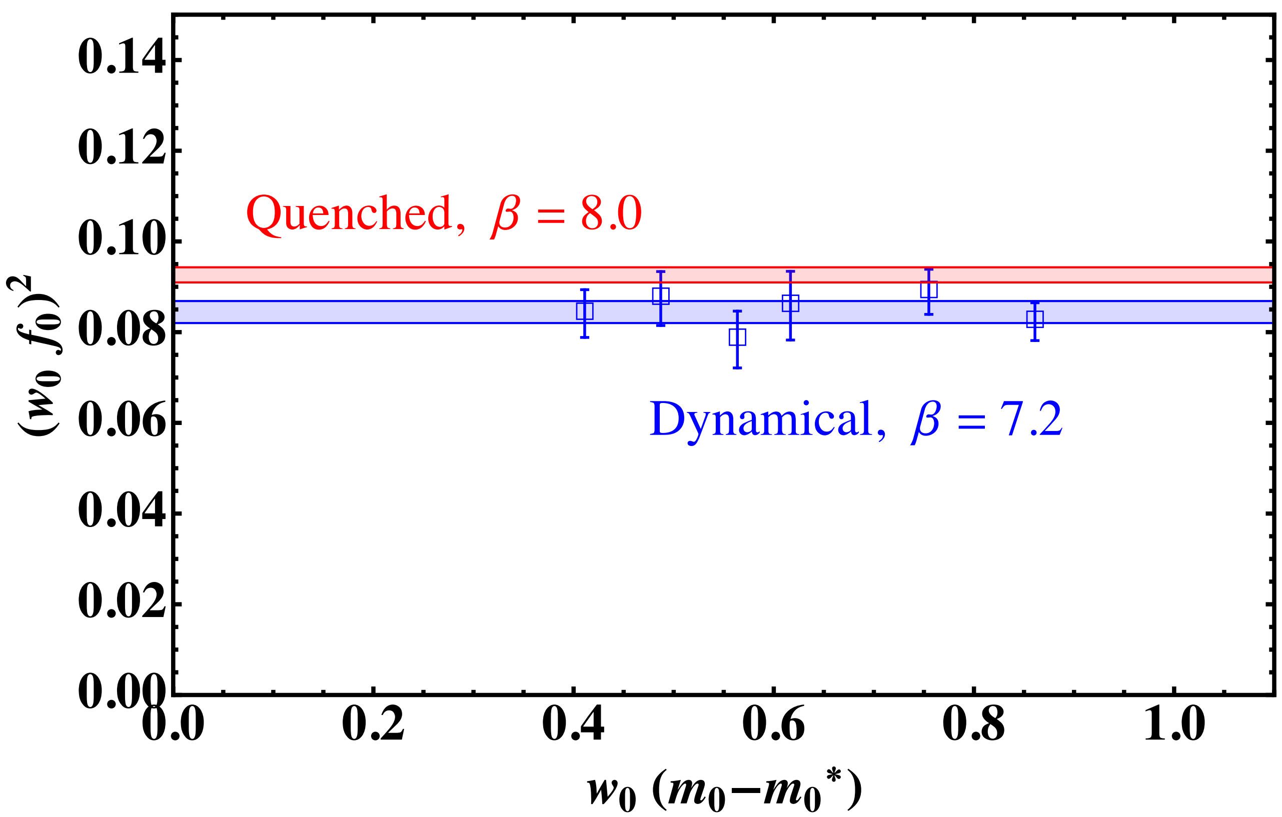
TACoS (2017)

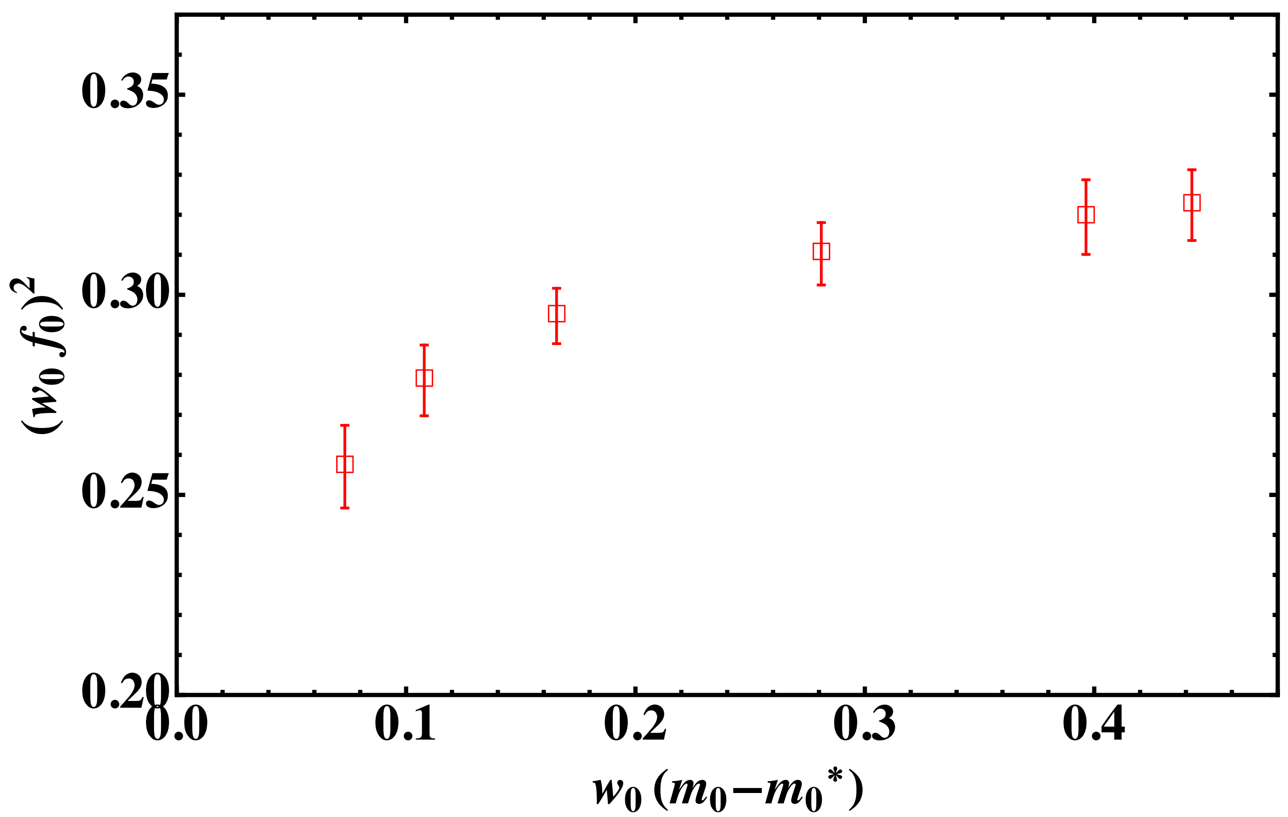
- Vector meson masses are relatively larger than those of SU(4) theories.

- Most simulation points seem to be far from chiral regime.

- With some caveats, KSRF relation indicates that the value of g_{VPP} is big.

✤ Numerical results: f_0^2





- Quench results f_0^2 for anti-sym. reps show strong quark mass dependence.
- NLO EFT for anti-sym. reps is under development.
- Not like the case of fund. reps, quenching effect could be substantial.

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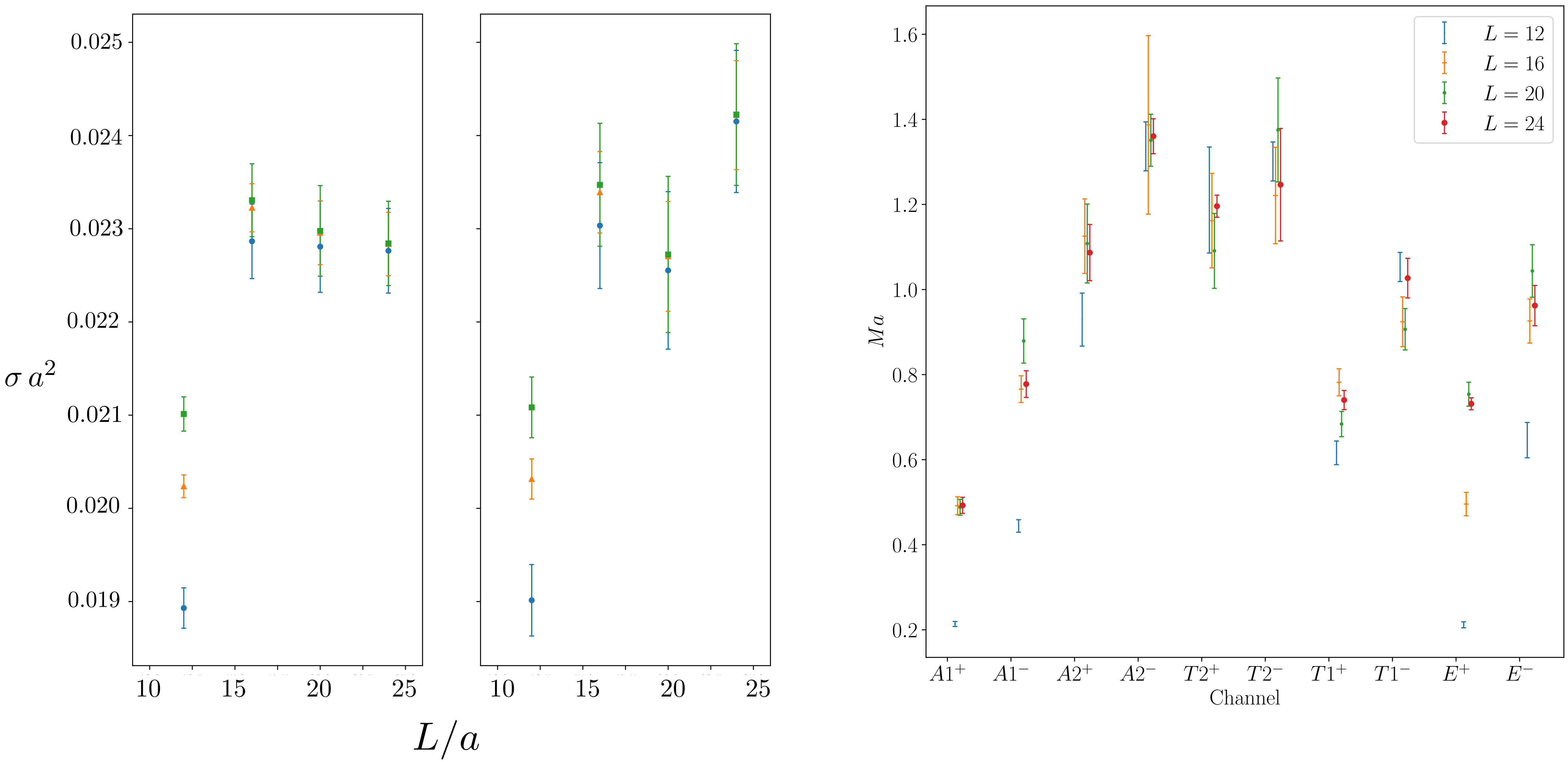
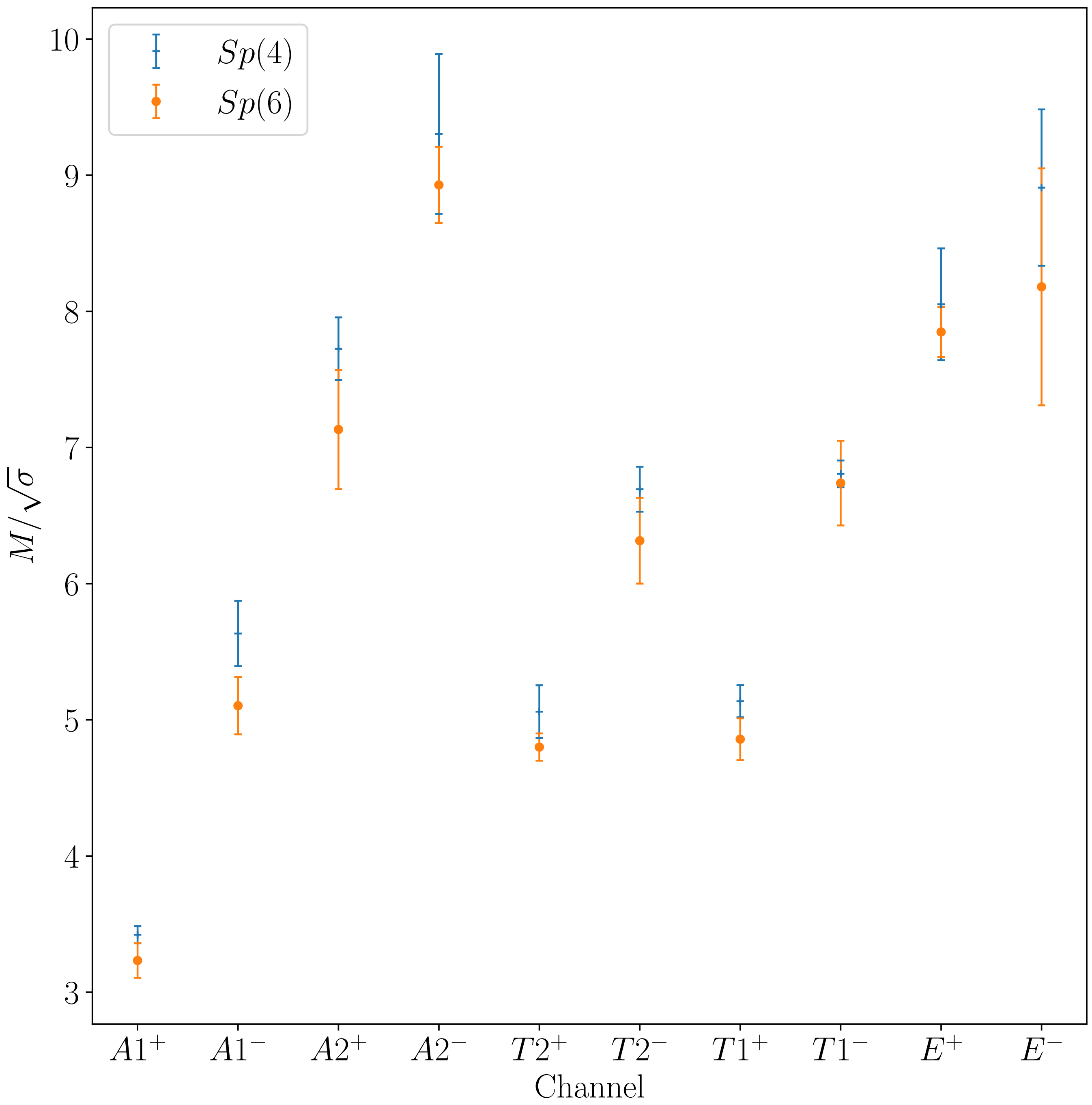
✤ Glueball spectrum in Sp(6)



- Finite volume effects are under control for $\sqrt{\sigma}\, L \gtrsim 3$

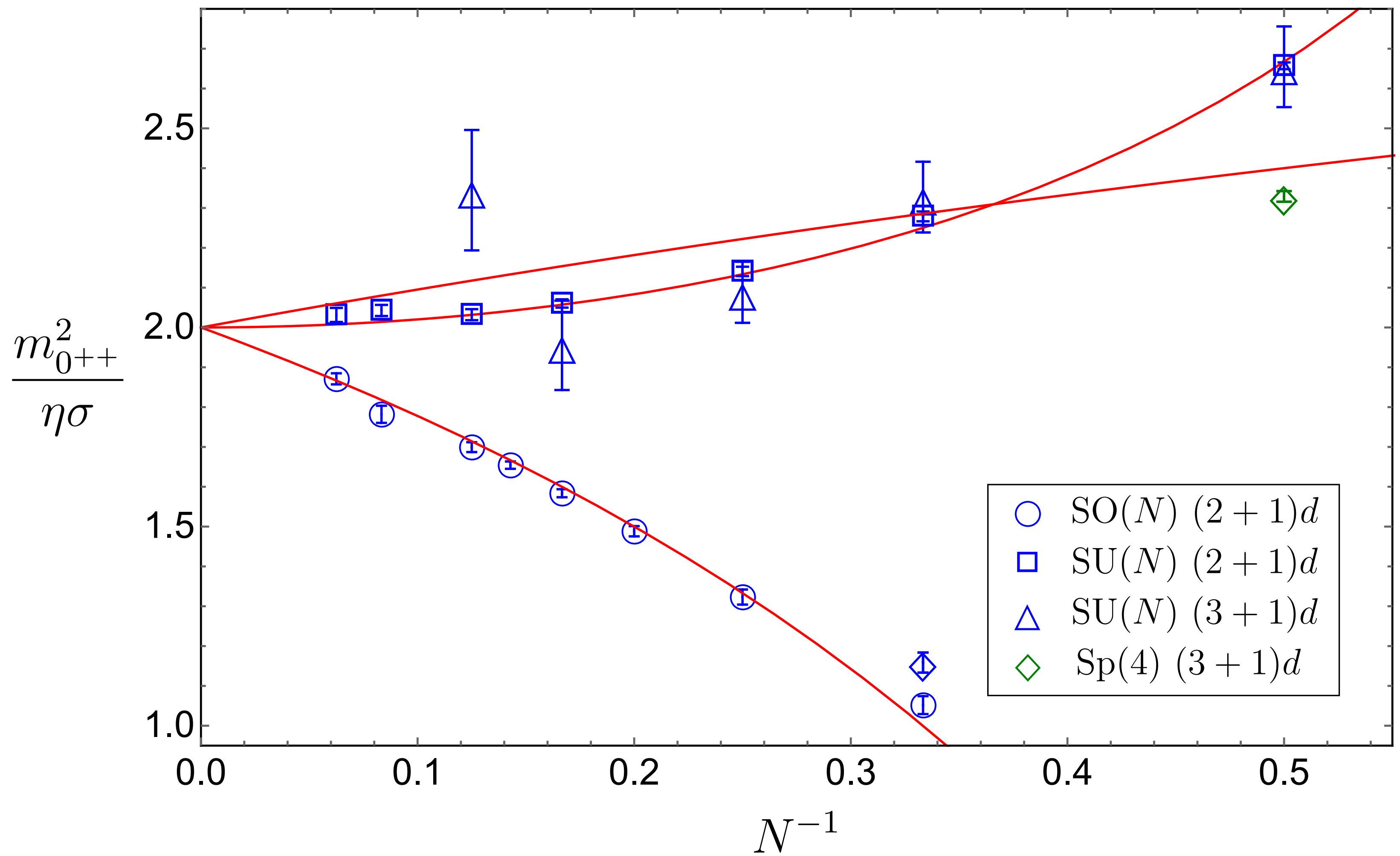
✤ Glueball spectrum in Sp(6)



- Glueball spectrum for Sp(6) at a single finite lattice spacing compared to that for Sp(4).

- Continuum extrapolation is required to make proper comparison and discuss the large N behavior.

✤ Casimir scaling and Sp(2N)



- The preliminary result at finite lattice spacing show that $m_{0^{++}}/\sqrt{\sigma}$ of Sp(6) is smaller than that of Sp(4).

➡ Qualitatively agree with the universal Casimir scaling.

Hong et. al. (2017)

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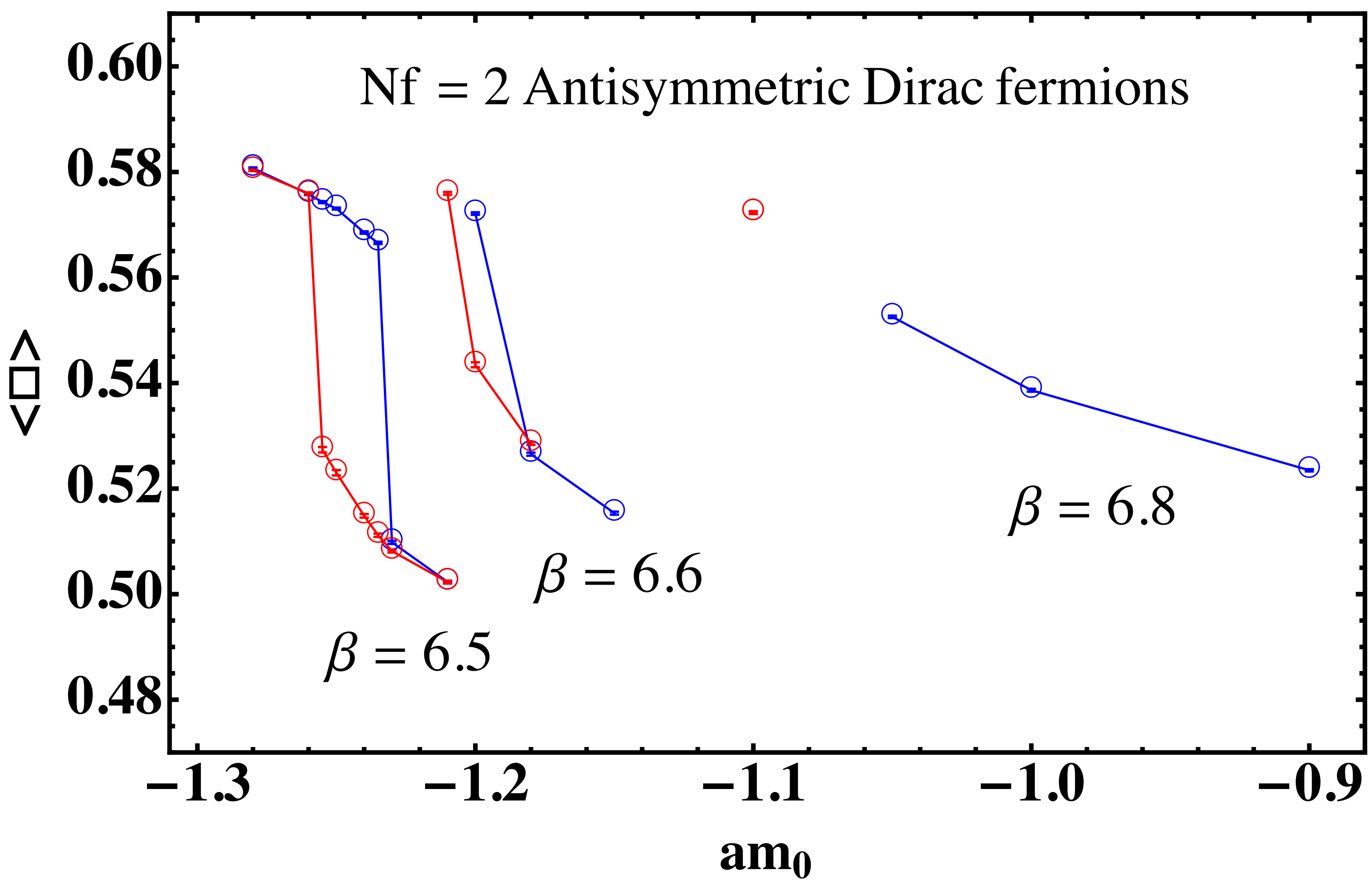
✤ Conclusion & Outlook

- Mesons in Sp(4) with two-flavor fund. fermions
  - Dynamical results share the features of quenched ones.
  - Lattice artifacts: Finite volume effects are under control for $m_{PS} L \gtrsim 8$.
    Lattice spacing effects are sizable for vector mesons.
  - Premature to discuss dark matter phenomenology.
- Mesons in Sp(4) with anti-symmetric. fermions
  - Numerical code is ready: modified Hirep code
  - Very preliminary quenched results show qualitatively different features compared to the case of fund. reps.
- Calculation of glueball spectrum in Sp(6) is ongoing.
- Future directions
  - Develop the code and EFT for mixed reps.
  - Measure baryon spectrum.

Backup Slides

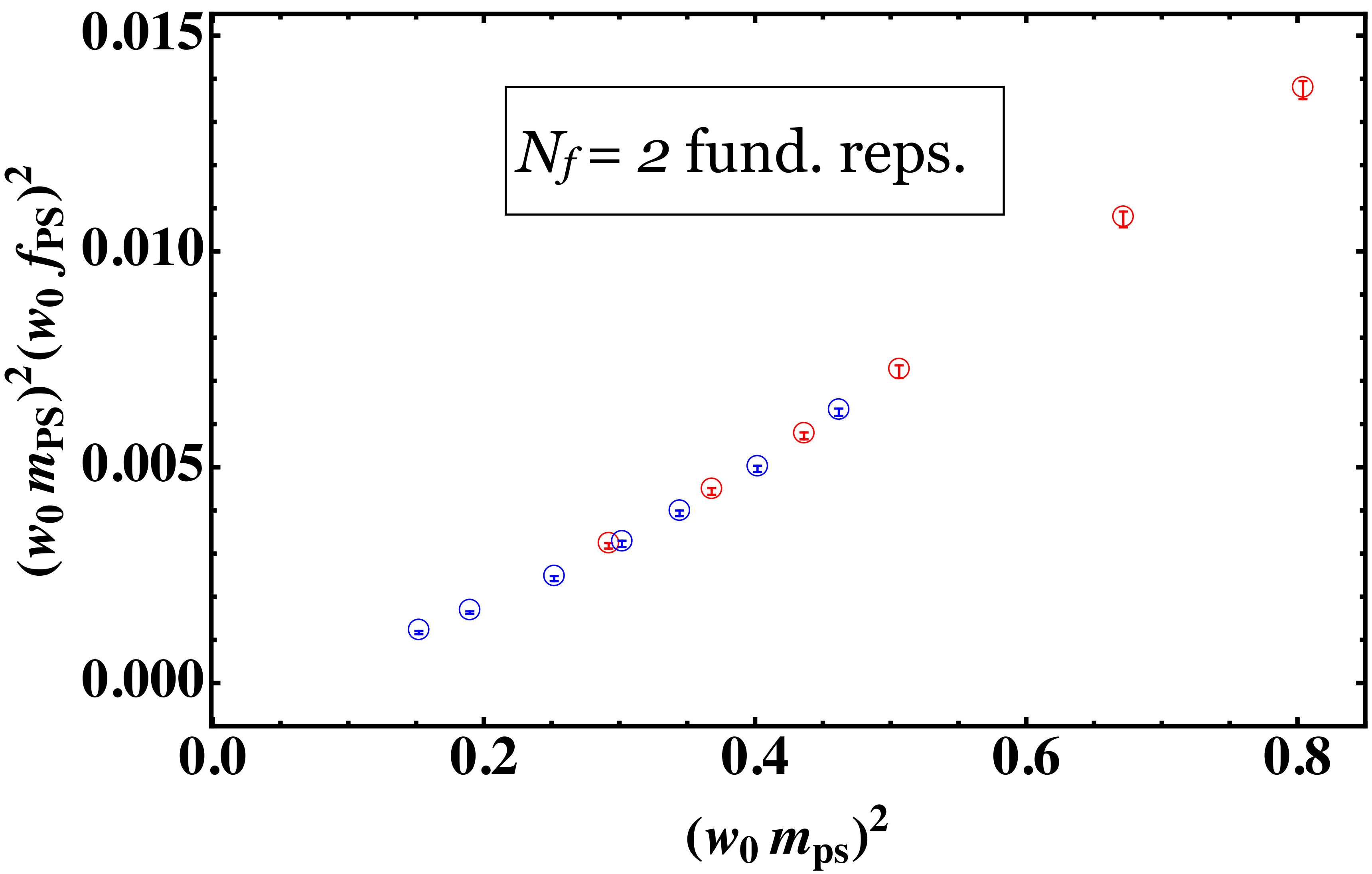
✤ Phase space of bare parameters



$T \times L = 8 \times 8^3$

- Strong hysteresis in the plaquette values (cold and hot) indicates the existence of a first order bulk phase transition for $\beta \lesssim 6.7$.

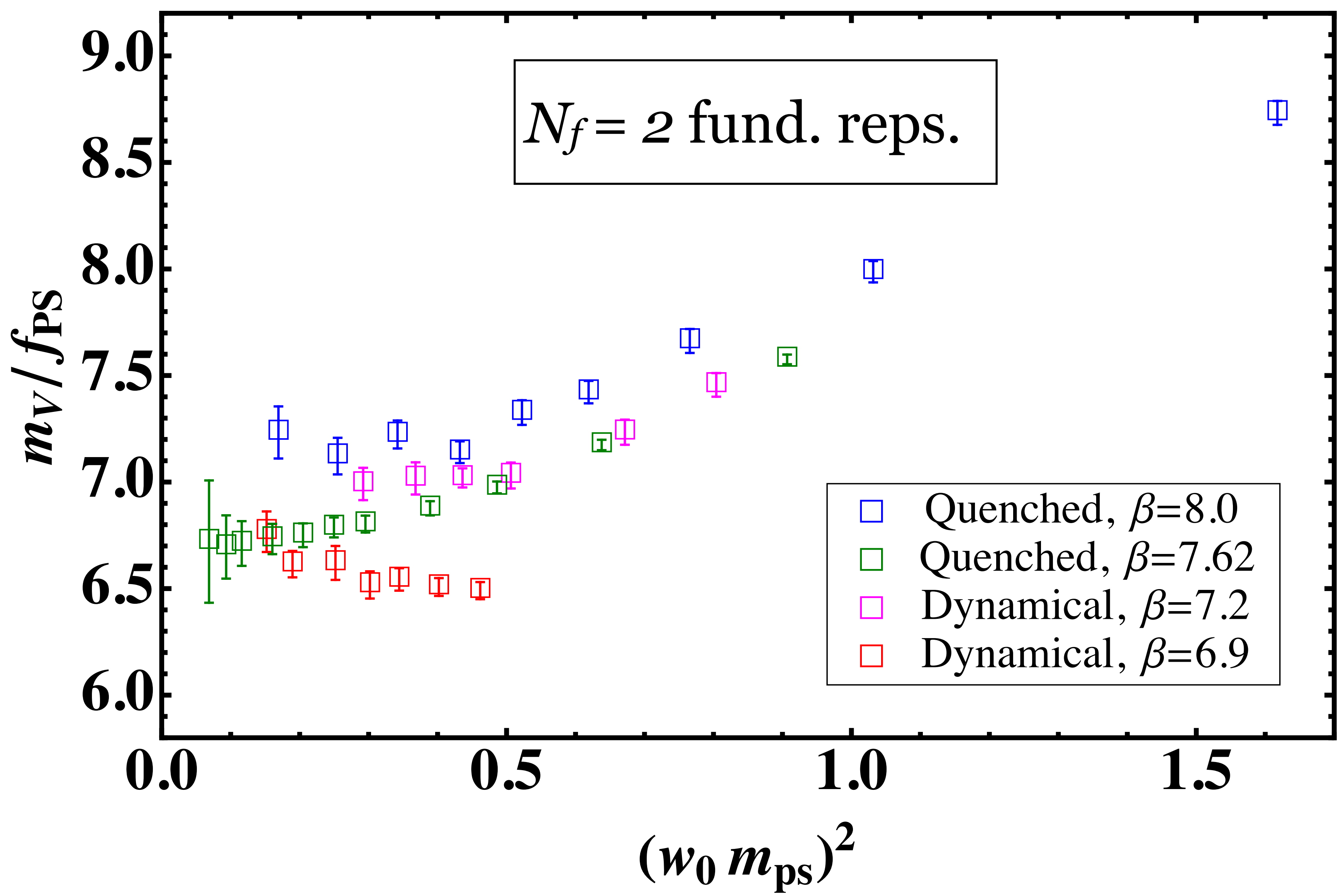
✤ GMOR relation



- Not reached to the linear regime, yet. $m_\pi^2 f_\pi^2 = m v^3 + m^2 v_5^2$

- Start to see finite lattice spacing artifact as we approach the massless limit.

✤ Vector meson mass in units of f_{ps}



- Caution!: Lightest four data points in Quenched results suffer from finite volume artifacts.