N=4 3d lattice SYM

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Motivations (3d)

- Mirror symmetry
- Dimensionful coupling, yet CFTs
- Holographic cosmology (Skenderis et al.)
  - Has exactly the quartic adjoint scalar potential terms that are needed
  - Can do a better job than $\Lambda\mathrm{CDM}$ at modeling small angle $\ell < 30$ CMB
  - Lattice simulations can potentially predict large angle statistics and CMB anomalies
- Holography and $\mathrm{AdS}_4 \times \mathrm{X}_6$

Parent theory

- Dimensional reduction of N=1 6d SYM (all in adjoint).

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}F_{\mu\nu} + \frac{i}{g}\mathrm{Tr}\Psi^T C\Gamma_\mu D_\mu \Psi$$

$$A_\mu \to A_i, \quad \phi_\alpha, \quad i = 0,1,2; \quad \alpha = 1,2,3$$

$$\Psi_p, \quad p = 1,\ldots,8 \to \psi_a^I, \quad a = 1,2; \quad I = 1,2,3,4$$

- Hence “N=4.”

Continuum (twisted)

- Spacetime group

$$SO(4) \simeq SU(2)_l \times SU(2)_r$$

- R-symmetry group

$$SU(2)_R \times U(1)_R$$

- Twisted rotation group

$$SU(2)' = \mathrm{diag}(SU(2)_r \times SU(2)_R)$$

Continuum (twisted, cont.)

- Donaldson-Witten twist

$$g^2\mathcal{L}_{4d}^{\mathcal{N}=2} = \mathrm{Tr}\Big(\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}\mathcal{D}_\mu\bar{\phi}\mathcal{D}^\mu\phi - \alpha\left[\phi,\bar{\phi}\right]^2 - \frac{i}{2}\eta\mathcal{D}_\mu\psi^\mu + i\alpha\phi\left\{\eta,\eta\right\} - \frac{i}{2}\bar{\phi}\left\{\psi_\mu,\psi^\mu\right\} + \mathcal{L}_\chi\Big),$$

$$\mathcal{L}_\chi = \mathrm{Tr}\left(\frac{i}{8}\phi\left\{\chi_{\mu\nu},\chi^{\mu\nu}\right\} - i\chi^{\mu\nu}\mathcal{D}_\mu\psi_\nu\right)$$

- Dimensionally reduce to 3d. $\mathcal{D}_2 \to [\phi_3, \cdot]$

Previous work

- Anosh Joseph [1307.3281]
- Blau-Thompson twist [hep-th/9612143]

$$SU(2)' = \mathrm{diag}(SU(2)_E \times SU(2)_N)$$

- From the dimensional reduction

$$SO(6) \to SO(3) \times SO(3) \simeq SU(2)_E \times SU(2)_N$$

- Earlier Q=8 formulation by orbifold method [Cohen, Kaplan, Katz & Unsal 2003]

Our twisted lattice

- Must complexify everything.
- Dynamical lattice spacing, as usual in these twisted/orbifold lattices.
- Lift additional fields with generic mass terms.

$$\mathcal{L} = \mathrm{Tr}\left(\frac{1}{4}\bar{\mathcal{F}}_{\mu\nu}(n)\mathcal{F}_{\mu\nu}(n) + \frac{1}{2}\bar{\mathcal{D}}^{+}_{\mu}\bar{\phi}(n)\mathcal{D}^{+}_{\mu}\phi(n) - \alpha\left[\phi(n),\bar{\phi}(n)\right]^2\right.$$

$$\left.+\frac{i}{2}\bar{\mathcal{D}}^{+}_{\mu}\eta(n)\psi_{\mu}(n) + i\alpha\phi(n)\left\{\eta(n),\eta(n)\right\} - \frac{i}{2}\bar{\phi}(n)\left(\psi_{\mu}(n)\bar{\psi}_{\mu}(n) + \bar{\psi}_{\mu}\left(n-e_{\mu}\right)\psi_{\mu}\left(n-e_{\mu}\right)\right)\right) + \mathcal{L}_{\chi},$$

$$\mathcal{L}_{\chi} = \mathrm{tr}\left[\frac{i}{8}\left(\phi(n)\bar{\chi}_{\mu\nu}(n)\chi_{\mu\nu}(n) + \phi\left(n+e_{\mu}+e_{\nu}\right)\chi_{\mu\nu}(n)\bar{\chi}_{\mu\nu}(n)\right)\right.$$

$$\left.-\frac{i}{2}\left(\bar{\chi}_{\mu\nu}(n)\bar{\mathcal{D}}^{+}_{\mu}\bar{\psi}_{\nu}(n) + \chi_{\mu\nu}(n)\mathcal{D}^{+}_{\mu}\psi_{\nu}(n)\right)\right].$$

- Lattice gauge invariance and Q invariance of

$$\chi_{\mu\nu}(n) = \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\bar{\chi}_{\rho\lambda}\left(n + e_\mu + e_\nu\right).$$

- Implies

$$\sum_{\mu=1}^{4} e_\mu = 0.$$

- So, the theory must be 3d.

- After using EOM,

$$\mathcal{L} = Q\,\mathrm{Tr}\left(\frac{1}{4}\chi_{\mu\nu}(n)\mathcal{F}_{\mu\nu}(n) + \frac{1}{2}\bar{\mathcal{D}}_{\mu}^{+}\bar{\phi}(n)\psi_{\mu}(n) + \alpha\eta(n)\left[\phi(n),\bar{\phi}(n)\right]\right)$$

$$-\frac{1}{8}\epsilon_{\mu\nu\rho\lambda}\mathrm{tr}\left(\mathcal{F}_{\mu\nu}(n)\mathcal{F}_{\rho\lambda}\left(n+e_{\mu}+e_{\nu}\right)\right),$$

- Last term is Q invariant using lattice Bianchi identity.

- Where Q, which is nilpotent, acts as:

$$
\begin{aligned}
&Q\,\phi(n) = 0, \quad Q\,\bar{\phi}(n) = i\eta(n), \\
&Q\,\eta(n) = \left[\bar{\phi}(n), \phi(n)\right], \\
&Q\,\mathcal{U}_\mu(n) = i\psi_\mu(n), \quad Q\,\bar{\mathcal{U}}_\mu(n) = -i\bar{\psi}_\mu(n) \\
&Q\,\psi_\mu(n) = \mathcal{D}^+_\mu \phi(n), \quad Q\,\bar{\psi}_\mu(n) = \bar{\mathcal{D}}^+_\mu \phi(n) \\
&Q\,\chi_{\mu\nu}(n) = \bar{\mathcal{F}}_{\mu\nu}(n) + \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\mathcal{F}_{\rho\lambda}\left(n + e_\mu + e_\nu\right).
\end{aligned}
$$

Counterterms

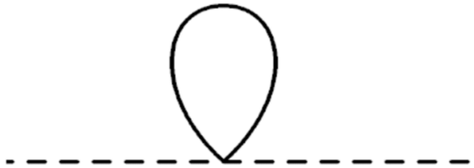
- Fine-tuning is calculable because the theory is super-renormalizable.
- As will see in arguments below, all counterterms are one-loop.

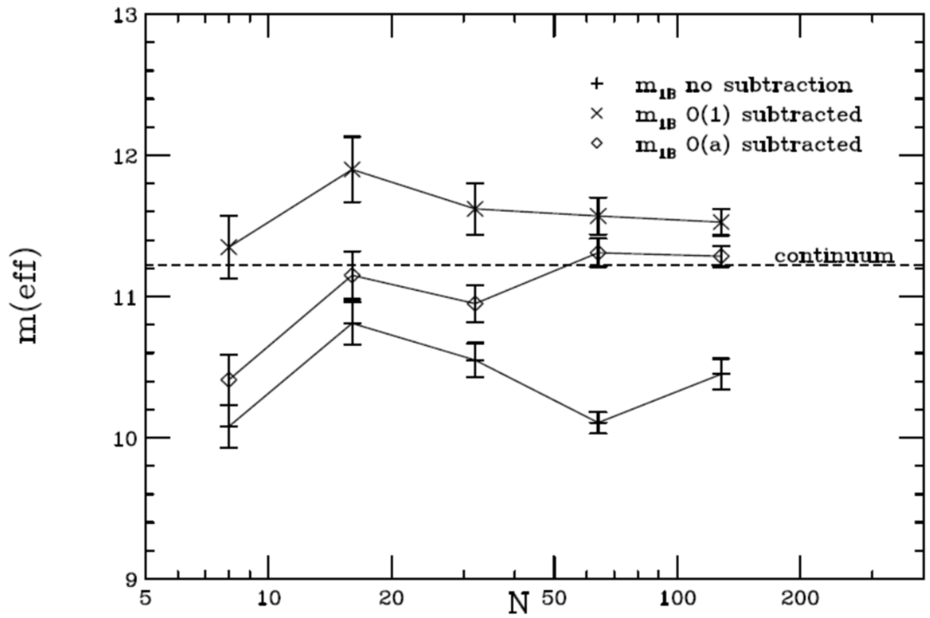
Power of super-renormalizability

- So, finite number of counterterms that can all be calculated at one loop in lattice perturbation theory.
- Includes the effects of the mass terms and scalar Q breaking.

Example from SUSY QM

- Finite CT in SUSY QM with naïve discretization [JG et al., 2004]
- Doublers appearing in this diagram (cancelled by scalar loop in the Q-exact case)
- Also shows power of Symanzik improvement





Prospects

- Currently coding up the Anosh Joseph discretization b/c it is much cleaner.
- Seems to have only one CT due to exact Q, point group, lattice gauge invariance.

$$\mathcal{L}(n) = \frac{1}{g^2} Q \mathrm{Tr}\left(\chi_{ab}(n)\mathcal{D}_a^{+}\mathcal{U}_b(n) + \eta(n)\bar{\mathcal{D}}_a^{-}\mathcal{U}_a(n) + \frac{1}{2}\eta(n)d(n) + B_{abc}(n)\bar{\mathcal{D}}_a^{+}\chi_{bc}(n)\right)$$

- For dimension counting it is best to have canonical normalization (otherwise everything is very confusing)

$$\Phi \to g\Phi, \quad \Psi \to g\Psi, \quad \mathcal{U}_m = \frac{1}{ag} e^{ag\mathcal{A}_m}$$

Renormalization in the Blau-Thompson twist

- [boson] = ½, [fermion] = 1, [d]=3/2, [F]=3/2, [Q] = ½
- So since loop is multiplied by $[g^2] = 1$
- Must have no more than dim=2 to be unsuppressed by lattice spacing.
- But also be fermionic if under Q.
- So restricting to Q-exact operators, must be of form

$$Q\mathrm{Tr}\Psi, \quad Q\mathrm{Tr}\Psi\Phi, \quad Q(\mathrm{Tr}\Psi\mathrm{Tr}\Phi)$$

- Point group, lattice gauge invariance, limit to [shifts d EOM by constant]

$$Q\mathrm{Tr}\eta = \mathrm{Tr}d$$

- Perturbative calculations underway, including Symanzik improvement.
- Must write down all dim=3 Q invariant fermionic operators.

Conclusions

- The holographic cosmology doesn't really require SUSY, but it will be interesting to see how it impacts large angle predictions.
- Concrete realization of Symanzik improvement vs. SUSY in 3d should be very enlightening.
- In the Blau-Thompson twist we only have one 1-loop CT to determine in order to get full N=4 SUSY. Symanzik improvement will require more.