Lattice Computation of the Ghost Propagator in Linear Covariant Gauges

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QCD Green's functions

- In a Quantum Field Theory, knowledge of all Green's functions allows a complete description of the theory
- In QCD, propagators of fundamental fields (e.g. quark, gluon and ghost propagators) encode information about non-perturbative phenomena
- In particular, gluon and ghost propagators encode information about confinement/deconfinement
- Since gluon/ghost propagators are gauge dependent quantities, we need to choose a gauge
  - Landau gauge $\partial_\mu A_\mu = 0$
  - Linear covariant gauge $\partial_\mu A_\mu = \Lambda$

Landau gauge fixing on the lattice

- gauge fixing functional

$$F^{Landau}(U^g) = -\operatorname{Re}\operatorname{tr}\sum_{x,\mu}\left[\, g(x)\, U_\mu(x)\, g^\dagger(x+\hat{\mu})\right]$$

- First variation: Landau gauge condition $\partial_\mu A^a_\mu = 0$
- Second variation: defines a symmetric matrix

$$\begin{aligned} M^{ab}_{x,y} &= \sum_\mu \Big(\operatorname{Re}\operatorname{tr}\left[\left\{t^a, t^b\right\}(U_\mu(x) + U_\mu(x-\hat{\mu}))\right]\delta_{xy} \\ &- 2\operatorname{Re}\operatorname{tr}\left[t^b t^a U_\mu(x)\right]\delta_{x+\hat{\mu},y} - 2\operatorname{Re}\operatorname{tr}\left[t^a t^b U_\mu(x-\hat{\mu})\right]\delta_{x-\hat{\mu},y}\Big) \end{aligned}$$

- continuum limit $-\frac{1}{2}\left(\partial_\mu D^{ab}_\mu + D^{ab}_\mu \partial_\mu\right)$
- in Landau gauge: $= -\partial_\mu D^{ab}_\mu$

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Linear covariant gauges (LCG) on the lattice

- gauge fixing functional

$$F^{LCG}(U^g;g) = F^{Landau}(U^g) + \mathrm{Re}\,\mathrm{tr}\sum_x [ig(x)\Lambda(x)]$$

- First variation: LCG condition $\partial_\mu A^a_\mu = \Lambda^a(x)$
- Second variation defines the same symmetric matrix as in Landau gauge

$$\begin{aligned} M^{ab}_{x,y} &= \sum_\mu \Big(\mathrm{Re}\,\mathrm{tr}\left[\left\{t^a,t^b\right\}(U_\mu(x) + U_\mu(x-\hat\mu))\right]\delta_{xy} \\ &- 2\,\mathrm{Re}\,\mathrm{tr}\left[t^b t^a U_\mu(x)\right]\delta_{x+\hat\mu,y} - 2\,\mathrm{Re}\,\mathrm{tr}\left[t^a t^b U_\mu(x-\hat\mu)\right]\delta_{x-\hat\mu,y} \Big) \end{aligned}$$

- continuum limit $-\frac{1}{2}\left(\partial_\mu D^{ab}_\mu + D^{ab}_\mu \partial_\mu\right)$
- not the correct one for LCG

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- A simple solution: correct $M^{ab}(x,y)$

$$[\Delta M]^{ab}_{xy} = \mathrm{Re}\,\mathrm{tr}\sum_{\mu}\left[\left[t^a, t^b\right]\left(U_\mu(x) - U_\mu(x-\hat{\mu})\right)\right]\delta_{xy}$$

$$\begin{aligned} \left[M^{+}\right]^{ab}_{xy} &= M^{ab}_{xy} + [\Delta M]^{ab}_{xy} \to -[\partial_\mu D_\mu]^{ab}_{xy} \\ \left[M^{-}\right]^{ab}_{xy} &= M^{ab}_{xy} - [\Delta M]^{ab}_{xy} \to -[D_\mu \partial_\mu]^{ab}_{xy} \end{aligned}$$

- M, M^{+}, M^{-} can not be distinguished as quadratic forms:

$$\omega^a(x)\,[\Delta M]^{ab}_{xy}\,\omega^b(y) = \omega^a(x) f_{abc}\,\mathrm{Re}\,\mathrm{tr}\left[it^c \ldots\right]\omega^b(y) = 0$$

- the minimizing condition can only define a symmetric quadratic form

- M^+ is a real non-symmetric matrix; can not be inverted using Conjugate Gradient (as one does in Landau gauge)
  - Generalized Conjugate Residual

Y. Saad, Iterative Methods for Sparse Linear Systems

- What about zero modes?
  - one should work in the subspace orthogonal to the null space of M^+
  - constant vectors are *not* zero modes of M^+

$$M^+ M^+ x = M^+ b$$

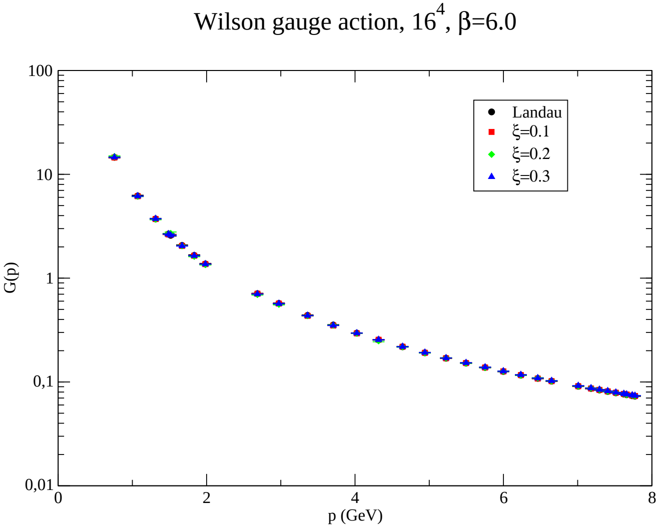
Suman, Schilling, PLB373(1996)314

  - can be solved in two steps

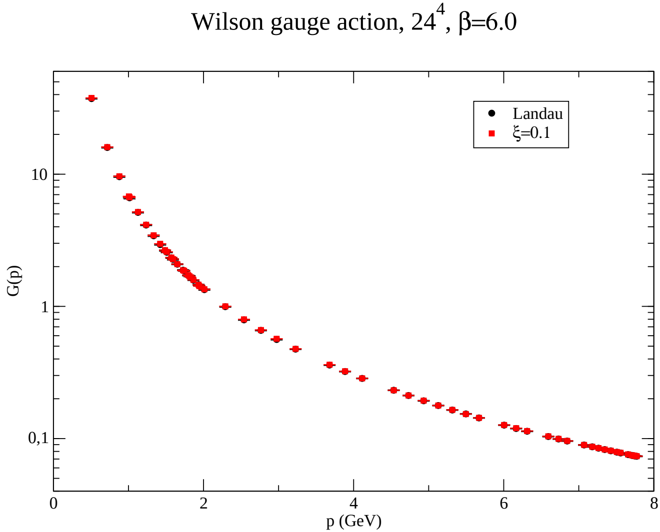
$$\begin{aligned} M^+ Y &= M^+ b \\ M^+ X &= Y \end{aligned}$$

Results

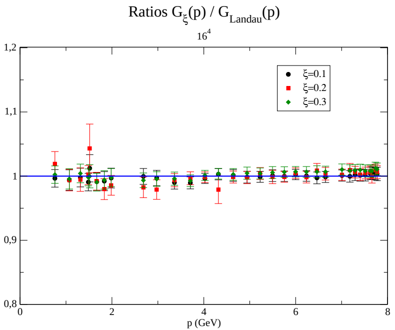
- 100 U's, 20 Λ's



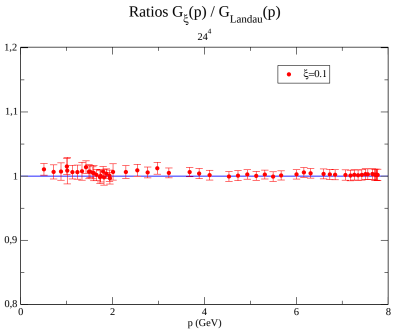
- 100 U's, 20 Λ's



Ratios



Ratios $G_\xi(p) / G_{Landau}(p)$
16^4
ξ=0.1
ξ=0.2
ξ=0.3
1,2
1,1
1
0,9
0,8
0
2
4
6
8
p (GeV)



Ratios $G_\xi(p) / G_{Landau}(p)$
24^4
ξ=0.1
1,2
1,1
1
0,9
0,8
0
2
4
6
8
p (GeV)

Outlook

- FP sector of the gauge fixed Lagrangian

$$\mathcal{L}_{\text{ghost}} = \overline{c}^a \partial_\mu D^{ab}_\mu c^b$$

- correct Hermiticity assignment for the ghost fields is

$$c^\dagger(x) = c(x), \qquad \overline{c}^\dagger(x) = -\overline{c}(x)$$

Alkofer, Von Smekal, Phys. Rep. 353 (2001) 281

- can be achieved by choosing two independent real Grassman ghost fields

$$c(x) \to u(x), \qquad \overline{c}(x) = iv(x)$$

- demanding $\mathcal{L}_{\text{ghost}} = \mathcal{L}^\dagger_{\text{ghost}}$

$$\mathcal{L}_{\text{ghost}} = \frac{1}{2} \begin{pmatrix} u & v \end{pmatrix} \begin{pmatrix} 0 & -iD\partial \\ i\partial D & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} := \frac{1}{2} \bar{\phi} \mathfrak{D} \phi$$

- operator $\mathfrak{D}$ is Hermitian
- seems a suitable candidate to use on the lattice

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Conclusions

- brief discussion of how to compute ghost propagator in LCG
- Very preliminary results for small lattice volumes
    - no differences with Landau gauge results
    - similar results in SU(2) gauge theory
      [Cucchieri's talk next week @ Confinement XIII]
- Outlook
    - comparison with the inversion of $\mathfrak{D}$
    - larger volumes to access infrared region

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