Phase diagram of strongly interacting Higgs-Yukawa theory

Nouman Tariq

Lattice 2018

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- Generalizes recently studied four-fermion model*
- Latter exhibits a strong coupling phase where fermions acquire mass without breaking symmetries.This is separated from weak coupling phase by a narrow broken symmetry phase
- Motivation for this work: Elimination of broken phase by tuning an additional parameter in the expanded phase diagram to achieve a direct transition from massless to massive phases without breaking symmetries.

* Shailesh et.al JHEP 1610 (2016) 058 , S.Catterall et.al Phys.Rev. D96 (2017) no.3, 034506

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Massless phase $G < G_c^1$,narrow broken phase $G_c^1 < G < G_c^2$, Strong coupling: *massive* phase $G > G_c^2$ $SO(4)$ symmetry restored

Fermion/Bosonic propagator at strong coupling

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Momentum space

$$F(p) = \frac{i\sqrt{6G^2}\sum_\mu sinp_\mu}{\sum_\mu sin^2 p_\mu + m_F^2} \qquad B(p) = \frac{8(6G^2)}{4\sum_\mu sin^2 p_\mu + m_B^2}$$

where $m_F^2 = 4(6G^2) - 2$ and $m_B^2 = 4(6G^2) - 8$

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In the limit $G \to \infty$ $\;G^2 < \epsilon_{abcd}\psi^a\psi^b\psi^c\psi^d >$ tends to a constant
Fermions are massive at strong coupling.This corresponds to pairing of elementary fermion ψ with composite fermion $\Psi_a = \epsilon_{abcd}\psi^b\psi^c\psi^d$
Four fermion condensate can be thought of as a bilinear formed from Ψ_a and ψ^a

Phase structure from numerical simulation

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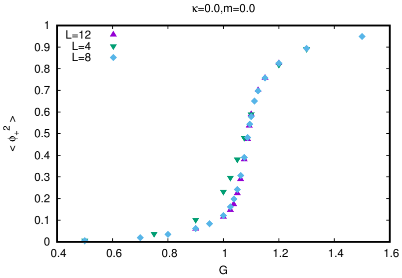

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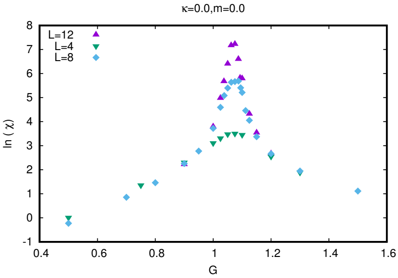

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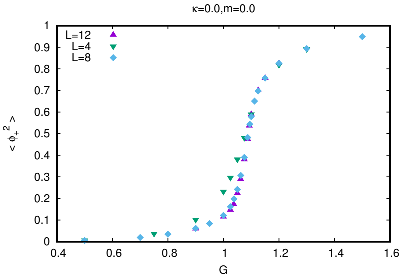

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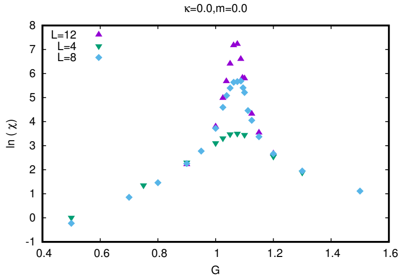

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Connected susceptibility $\chi_{conn} = \frac{1}{V}\sum_x < \psi^a(x)\psi^b(x)\psi^a(0)\psi^b(0) >$
$\chi_{conn} \sim L^4$ in the transition region $G = 1.05$

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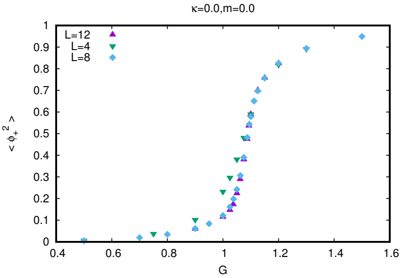

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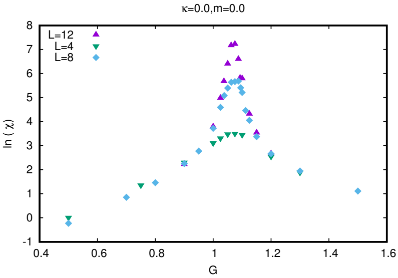

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Existence of single peak suggests single phase transition however closer examination shows two phase transitions which bound $SO(4)$ symmetry breaking phase with non-zero fermion bilinear condensate.

Possible symmetry breaking bilinears

There are two possible choices of bilinears corresponding to anti-ferromagnetic $m\sum_x \varepsilon(x)\psi^a\psi^b$ and ferromagnetic bilinear $m\sum_x \psi^a\psi^b$ bilinears.Fermions naturally induce anti-ferromagnetic ordering.

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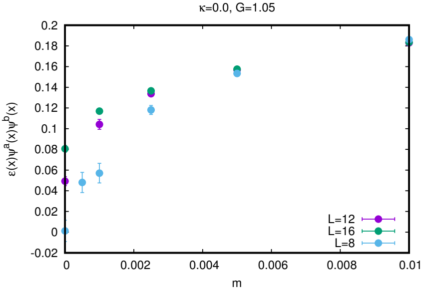
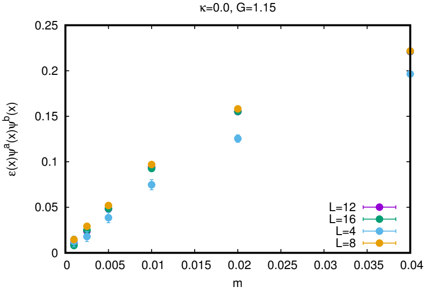
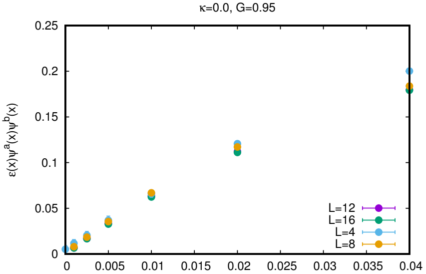
This property will manifest itself in the phase diagram.

Narrow broken phase at $\kappa = 0.0$







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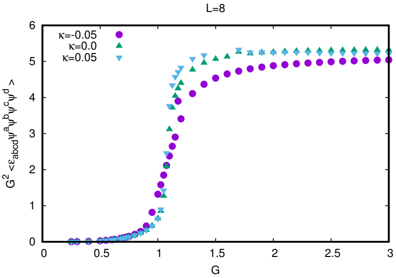

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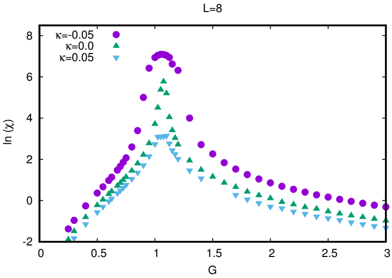

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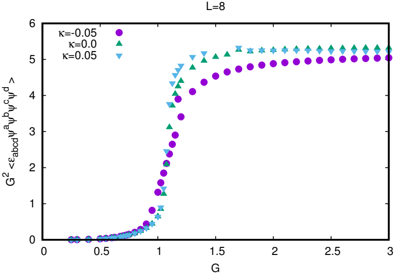

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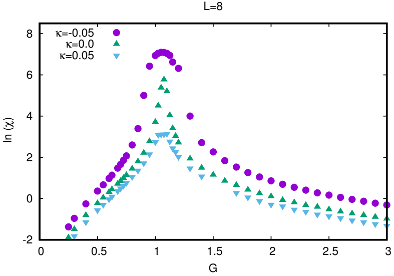

$$\frac{1}{V}\sum_x < \psi^a(x)\psi^b(x)\psi^a(0)\psi^b(0) >$$

The susceptibility peak widens as you move towards negative κ and it shrinks towards positive κ.

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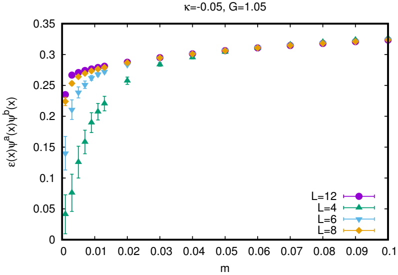

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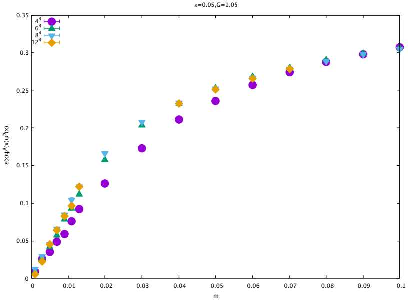

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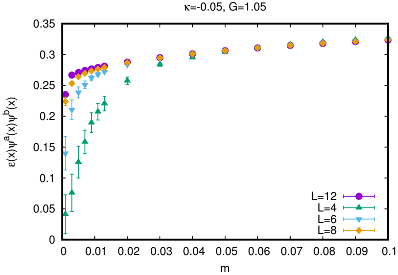

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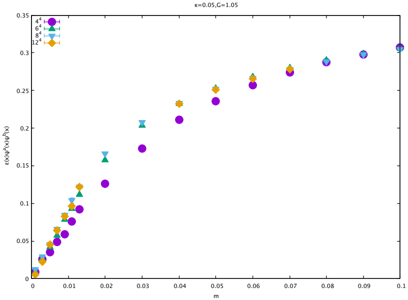

$\kappa = 0.05$

No spontaneous symmetry breaking with small positive κ
At $\kappa = 0.1$ we find evidence in favor of ferromagnetic phase

$\kappa > 0.05$

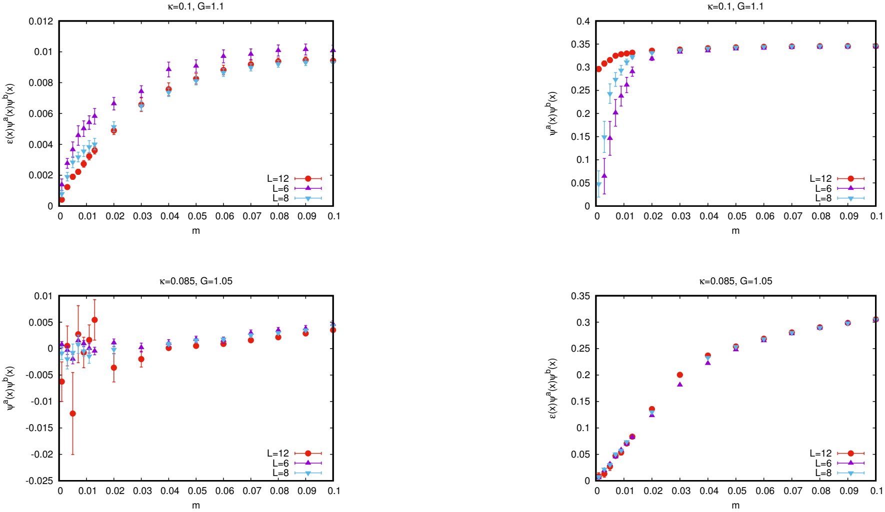

Figure: Left : $G = 1.1, \kappa = 0.1$, Right : $G = 1.1, \kappa = 0.1$, Bottom left: $G = 1.05, \kappa = 0.085$, Bottom right: $G = 1.05, \kappa = 0.085$

Volume scaling of susceptibility

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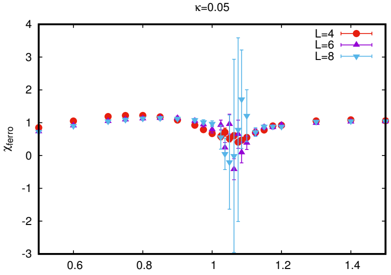

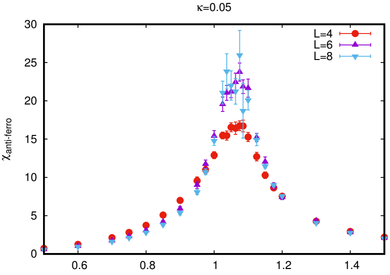

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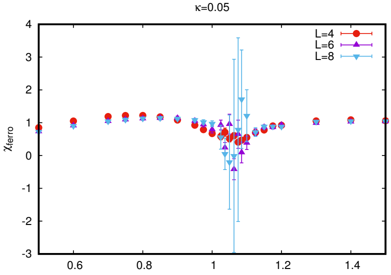

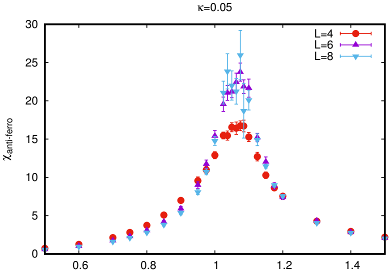

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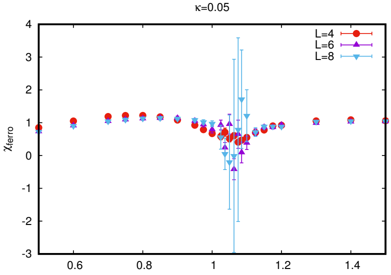

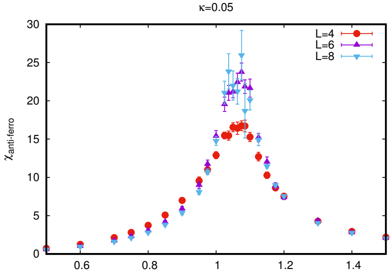

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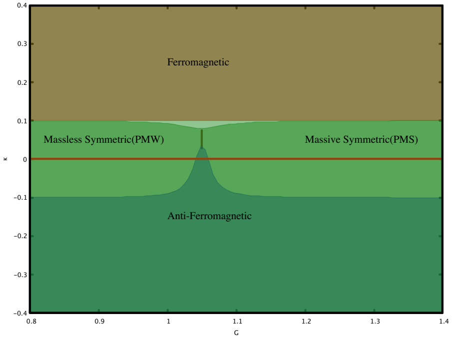
This ensures no symmetry broken phase $\kappa = 0.05$

Phase Diagram



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Questions

- Can the four-fermion phase be induced by strong gauge coupling? (work in progress)
- Can the four-fermion phase be explained through a continuum model?

Continuum model: S.Catterall, Nouman Butt Phys.Rev. D97 (2018) no.9, 094502

Thank you!

Collaborators

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Funding and computing resources

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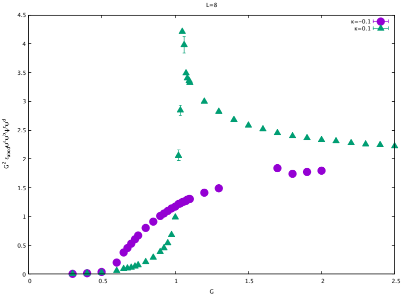
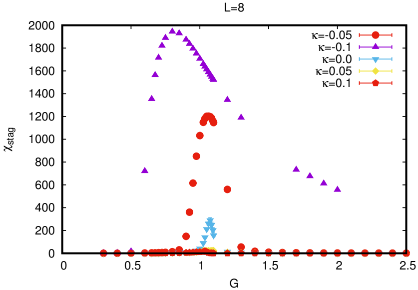
Backup



L=8
κ=-0.1
κ=0.1
$G^2 \epsilon_{abcd}\psi^a\psi^b\psi^c\psi^d$
G



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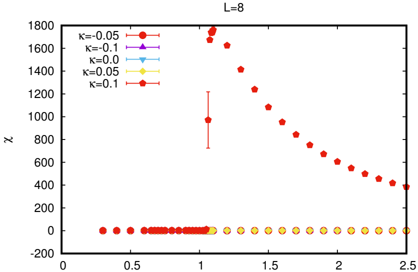

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