Computation of hybrid static potentials from optimized trial states in SU(3) lattice gauge theory

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East Lansing, July 24, 2018

36th International Symposium on Lattice Field Theory

References

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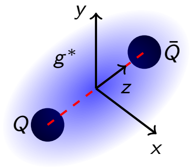
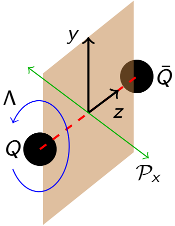
Outline

Quantum numbers

Hybrid meson: meson with excitations in the gluon fields
$\rightarrow$ exotic quantum numbers possible



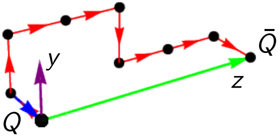
- $\Lambda = 0, 1, 2, \ldots$, absolute angular momentum w.r.t separation axis
- $\epsilon = +, -$, eigenvalue of operator $\mathcal{P}_x$, corresponding to reflection on the y-z-plane
- $\eta = g, u$, eigenvalue of operator $\mathcal{P} \circ \mathcal{C}$, the combination of parity and charge conjugation



Lattice trial states

Construction of trial states on the lattice:

- choose some non-trivial spatial path S for a Wilson loop



- the state $|\Psi_{\text{Hybrid}}\rangle_{S,\Lambda} = \sum_{k=0}^{3} \exp(i\Lambda k\pi/2)\mathcal{O}(k\pi/2)\,|\Omega\rangle$ has defined angular momentum Λ
- use projectors

$$\mathbb{P}_{\mathcal{P}_x,\epsilon} = \frac{1+\epsilon\mathcal{P}_x}{2} \qquad \mathbb{P}_{\mathcal{P}\circ\mathcal{C},\eta} = \frac{1+\eta\mathcal{P}\circ\mathcal{C}}{2}$$

to project $|\Psi_{\text{Hybrid}}\rangle_{S,\Lambda}$ onto the subspace of eigenstates to $\mathcal{P}_x, \mathcal{P}\circ\mathcal{C}$

...

Creation operators

- the state

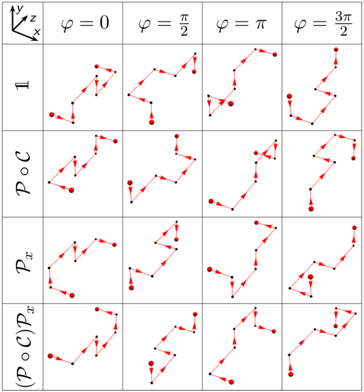
$$\left|\Psi_{\text{Hybrid}}\right\rangle_{S;\Lambda_\eta^\epsilon} = \mathbb{P}_{\mathcal{P}_x,\epsilon}\mathbb{P}_{\mathcal{P}\circ\mathcal{C},\eta}\left|\Psi_{\text{Hybrid}}\right\rangle_{S,\Lambda} = \bar{q}(-\frac{r}{2})a_{S;\Lambda_\eta^\epsilon}(-\frac{r}{2},\frac{r}{2})q(\frac{r}{2})\left|\Omega\right\rangle$$

has defined quantum numbers Λ_η^ϵ

- $\rightarrow$ generate creation operators

$$a_{S;\Lambda_\eta^\epsilon} = (1/4)\sum_{k=0}^{3}\exp(i\Lambda k\pi/2)\hat{R}(k\pi/2) \times(S+\eta S_{\mathcal{P}}+\epsilon S_{\mathcal{P}_x}+\epsilon\eta S_{\mathcal{P}\mathcal{P}_x})$$

- ... which shape to choose?



Hybrid static potentials

- Compute hybrid static potentials from correlation functions

$$W_{S,S';\Lambda_\eta^\epsilon}(r,t) = \langle \Psi_{\text{Hybrid}}(r,t)_{S;\Lambda_\eta^\epsilon} | \Psi_{\text{Hybrid}}(r,0)_{S;\Lambda_\eta^\epsilon} \rangle$$
$$\sim_{t\to\infty} \exp(-V_{\Lambda_\eta^\epsilon}(r)t)$$

- Usual problem: signal-to-noise ratio decreases exponentially for increasing t
  $\rightarrow$ find shapes which generate trial states with large ground state overlap
  $\rightarrow$ extract hybrid static potentials at region with larger signal-to-noise ratio
- to identify suitable shapes, we compute the effective mass

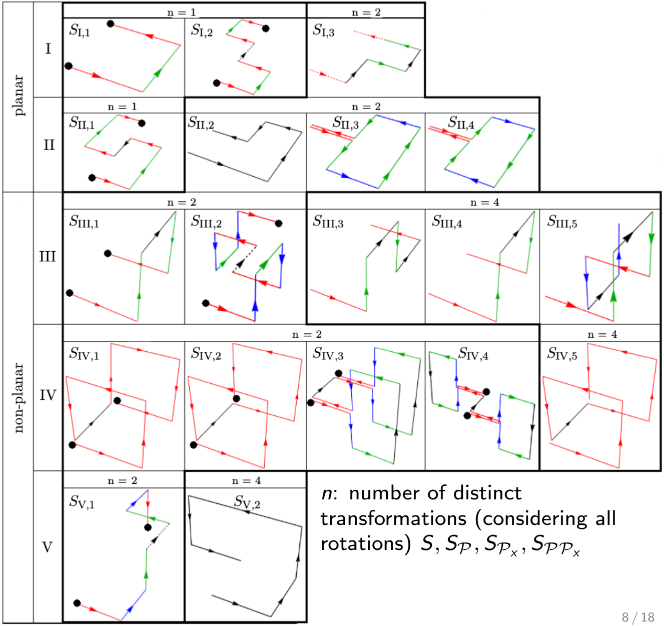
$$V_{\text{eff},S;\Lambda_\eta^\epsilon}(r,t)a = \ln\left(\frac{W_{S,S';\Lambda_\eta^\epsilon}(r,t)}{W_{S,S';\Lambda_\eta^\epsilon}(r,t+a)}\right)$$

  at small separations $t/a = 1, 2$ and 100 gauge configurations
  for a large set of operators S

Operator set

Starting set of operators

- arrows represent straight lines of links
  - solid: length ≥ 1
  - dotted: length ≥ 0
- we vary
  - length of any straight line
  - placement on the separation axis
- colors mark paths of same length



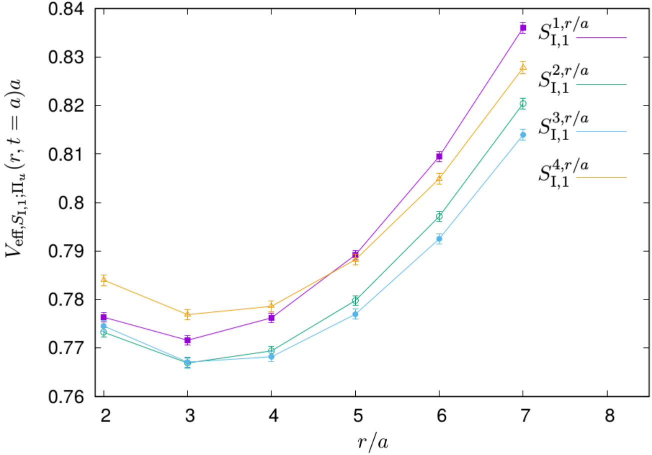
n: number of distinct transformations (considering all rotations) $S, S_{\mathcal{P}}, S_{\mathcal{P}_x}, S_{\mathcal{P}\mathcal{P}_x}$

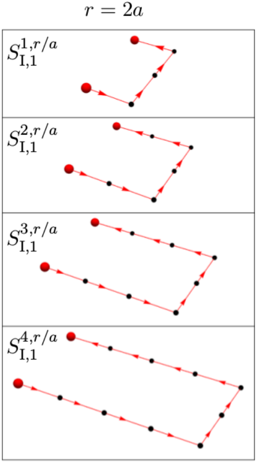
Optimizing operators

Example: optimizing $S_{\text{I},1}$ with x, z extensions E_x, E_z for state Π_u

$\rightarrow$ variations $S_{\text{I},1}^{E_x,E_z}$

- variation of E_x



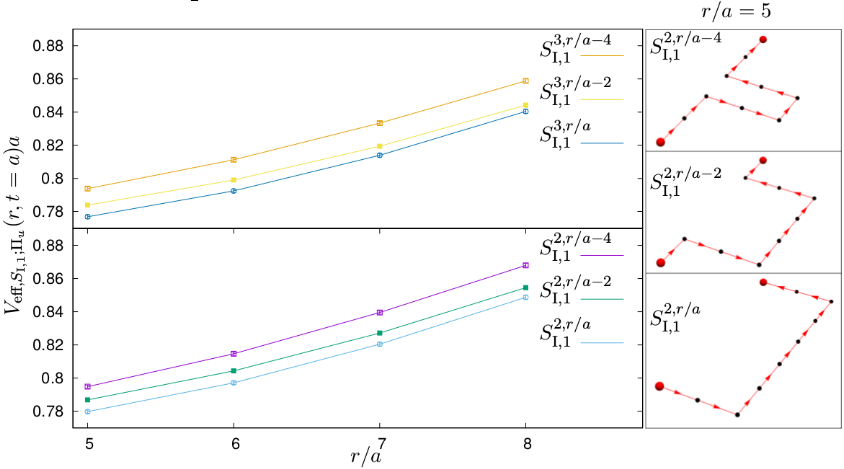


Optimizing operators

Example: optimizing $S_{\text{I},1}$ with x, z extensions E_x, E_z for state Π_u

$\rightarrow$ variations $S_{\text{I},1}^{E_x,E_z}$

- variation of E_z

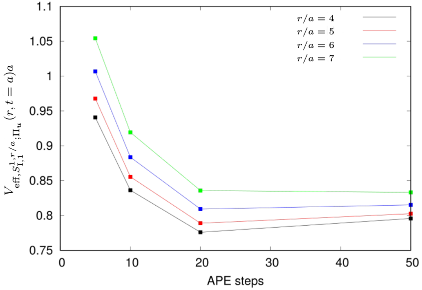


Lattice setup

- Wilson plaquette action

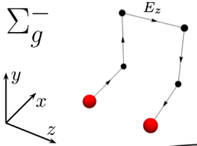
$$S_g[U] = \frac{\beta}{3} \sum_n \sum_{\mu<\nu} \mathrm{Re}\{\mathrm{Tr}\,[\mathbb{1} - U_{\mu\nu}(n)]\}$$

- Lattice dimensions: $24^3 \times 48$
- $\beta = 6.0 \rightarrow a \approx 0.093\,fm$
- 5500 gauge configurations, generated using Chroma QCD
- check for autocorrelation using binning
- APE smearing of spatial links
  - $\alpha_{\mathrm{APE}} = 0.5$
  - optimized $N_{\mathrm{APE}} \approx 20$



Sets of optimized trial states

- Optimization of linear combinations too expensive
- Each operator is optimized independently for each sector Λ_η^ϵ and quark separation r/a
- Compute the correlation matrix $C(t)$ using a subset of 3 best operators for each sector



$S_{\mathrm{III},1} = U_x^2 U_y^2 U_z^{E_z} U_{-y}^2 U_{-x}^2$

r/a	2	3	4	5	6	7	8
E_z	2	3	4	5	6	7	8

- Perform variational analysis by solving a generalized eigenvalue problem

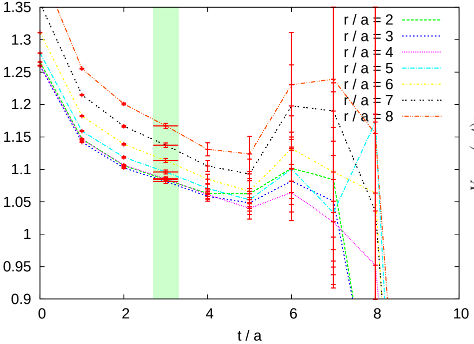
$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

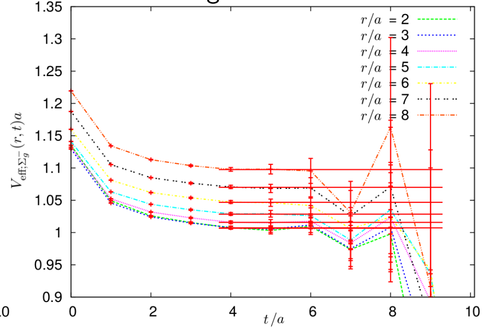
Effective potentials

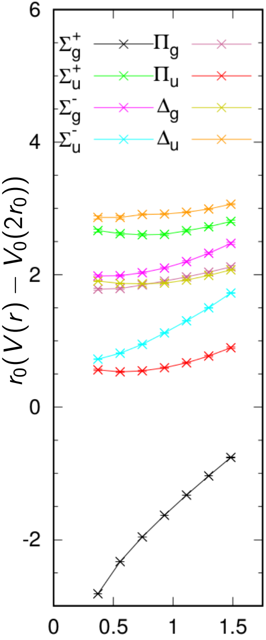
Exemplary effective potentials for sector Σ_u^-

- previous results, non-optimized operators
- no real plateau visible, only crude guess possible

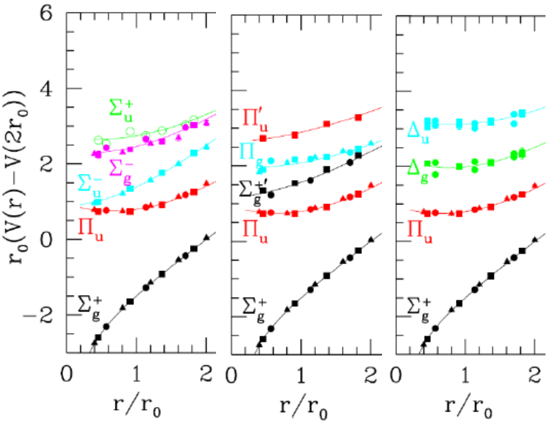
- effective potentials from optimized operators
- plateau reached much earlier, allowing for fit in region with favorable signal-to-noise ratio







Our results



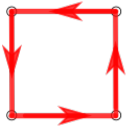
Previous results

[K.J.Juge, J. Kuti, C. Morningstar.
Gluon excitations of the static quark potential and the hybrid quarkonium spectrum.
Nucl.Phys.Proc.Suppl., 63:326-331, 1998 [arXiv:hep-lat/9709131]]

chromoelectric and chromomagnetic field

On the lattice field strength tensor corresponds to plaquette

$$P_{\mu\nu} = \mathrm{Tr}\left[e^{igaF_{\mu\nu}}\right] \Rightarrow \mathrm{Tr}\left(F_{\mu\nu}^2\right) \approx \frac{2}{g^2a^2}\left(2 - P_{\mu\nu}\right)$$



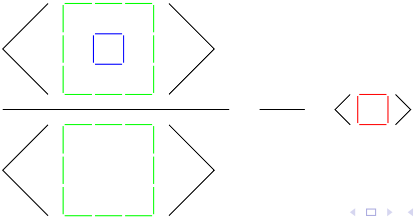
plaquette

$\Rightarrow E^2$ and B^2 are gauge invariant quantities

$$\Delta B_j^2 \equiv \langle B_j(\vec{x})^2\rangle_{Q\bar{Q}} - \langle B_j^2\rangle_{vac} = \frac{2}{g^2a^2}\left[\langle P_{kl}\rangle - \frac{\langle W \cdot P_{kl}(T/2,\vec{x})\rangle}{\langle W\rangle}\right]$$

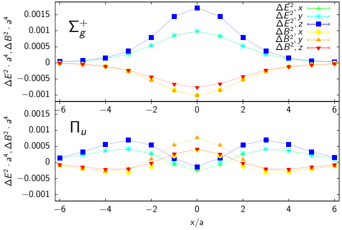
$$\Delta E_j^2 \equiv \langle E_j(\vec{x})^2\rangle_{Q\bar{Q}} - \langle E_j^2\rangle_{vac} = \frac{2}{g^2a^2}\left[\frac{\langle W \cdot P_{0j}(T/2,\vec{x})\rangle}{\langle W\rangle} - \langle P_{0j}\rangle\right]$$

$\langle E^2\rangle$ in presence of $Q\bar{Q}$

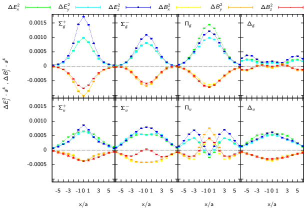


$\langle E^2\rangle$ in vacuum

hybrid static potential flux tube profile on mediator axis



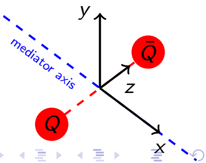
Computed in SU(2) lattice gauge theory with lattice spacing $a \approx 0.073 fm$.
This was done for all quantum numbers:



Very recently flux tubes for hybrid static potentials have been investigated for the first time in
[M. Cardoso, N. Cardoso, P. Bicudo: "Colour fields of the quark-antiquark excited flux tube", arXiv:1803.04569 [hep-lat]]
with discrepancies to our work.
The only major difference in the computations were SU(3) rather than SU(2) lattice gauge theory.

In contrary to this work, our studies involved treating the E- and B-field components separately which led to results consistent with analytic approaches in
[N. Brambilla, A. Pineda, J. Soto and A. Vairo: "Effective field theories for heavy quarkonium" (2005) [hep-ph/0410047]].

The next step will be to compute chromoelectric and chromomagnetic field strength components in SU(3) Lattice gauge theory



Outlook

- Use the obtained hybrid static potentials to solve Schrödinger equations and obtain hybrid meson spectra
- computations using a smaller lattice spacing
- QCD configurations

Thank you!