Quasi-PDFs from twisted mass fermions at the physical point

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based on: arXiv: 1803.02685, arXiv: 1807.00232

In collaboration with:
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Overview of the talk

- Remarks on the theoretical and numerical setup
- Results for parton distributions (PDFs)
  - Unpolarized PDF
  - Helicity PDF
  - Transversity PDF
- Summary and Outlook

Quasi-PDF approach [X.Ji, Phys.Rev.Lett. 110, 262002 (2013)]

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(covered in the previous talk!)

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Computation of matrix elements between two proton states at finite momentum → Quasi-PDFs, $\tilde{q}(x, P, \mu)$

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Computation of matrix elements between two proton states at finite momentum

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→ Quasi-PDFs, $\tilde{q}(x, P, \mu)$

↓

At sufficiently large momenta, LaMET relates quasi-PDFs to light-front PDFs $q(x, \mu)$

- The contact with physical PDFs is made in three crucial steps:
  1. Non-perturbative renormalization of the matrix elements
  2. Matching procedure
  3. Target Mass Corrections (TMCs) to eliminate residual m_N/P effects.

Numerical setup

Lattice setup

- Gauge ensemble produced by using $N_f = 2$ light quarks

  ETM Collaboration, Phys. Rev. D 95 (2017), no. 9 094515

β=2.10,	c_{SW}=1.57751,	$a \simeq 0.093 fm$
$48^3 \times 96$	$a\mu = 0.0009$	$m_N \simeq 0.932\ GeV$
$L = 4.5\ fm$	$m_\pi \simeq 0.130 GeV$	$m_\pi L \simeq 2.98$

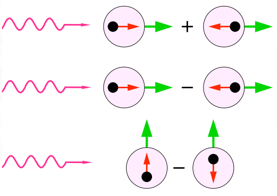
Statistics

$P = \frac{6\pi}{L}$			$P = \frac{8\pi}{L}$			$P = \frac{10\pi}{L}$		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_0	50	4800	γ_0	425	38250	γ_0	811	72990
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	811	72990
σ_{3i}	100	9600	σ_{3i}	425	38250	σ_{3i}	811	72990

For a detailed discussion about the setup see previous talk by K. Cichy!

Remarks on the numerical setup

We extract 3 kinds of PDFs:



matrix element

Unpolarized PDF $\langle N(\vec{P})|\overline{\psi}(0)\gamma_0 W(0,z)\psi(z)|N(\vec{P})\rangle$

Helicity PDF $\langle N(\vec{P})|\overline{\psi}(0)\gamma_5\gamma_3 W(0,z)\psi(z)|N(\vec{P})\rangle$

Transversity PDF $\langle N(\vec{P})|\overline{\psi}(0)\sigma_{3i} W(0,z)\psi(z)|N(\vec{P})\rangle$

★ For our choices of Dirac matrix no mixing with other operators occurs

- ▶ The renormalization is only multiplicative

★ The renormalization functions, Z-factors, have the same power-like divergence of the matrix elements

- ▶ Z-factors assume large values increasing the length of the Wilson line

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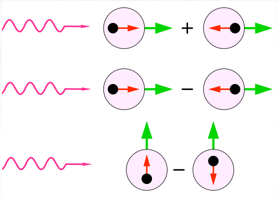

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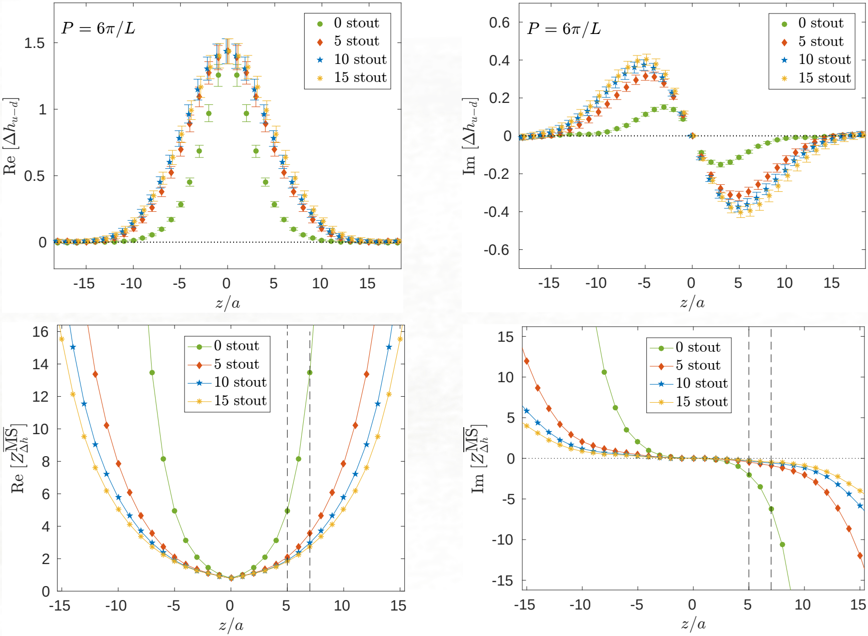
★ For our choices of Dirac matrix no mixing with other operators occurs

- ▶ The renormalization is only multiplicative

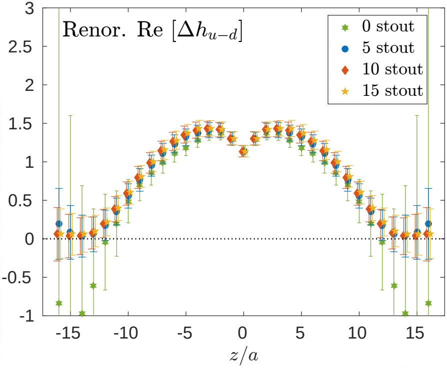
★ The renormalization functions, Z-factors, have the same power-like divergence of the matrix elements

- ▶ Z-factors assume large values increasing the length of the Wilson line
- ▶ To smooth the divergence we apply 3-D stout smearing only to the Wilson line entering the matrix elements and vertex functions
  - We test $\{0, 5, 10, 15\}$ levels of smearing

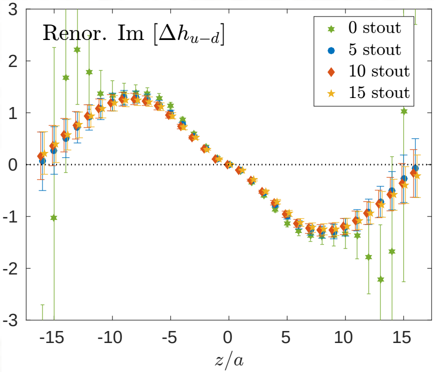
Effect of the stout smearing



Renormalized matrix elements for helicity PDFs



$Re[\Delta h] \cdot Re[Z_{\Delta h}] - Im[\Delta h] \cdot Im[Z_{\Delta h}]$



$Re[\Delta h] \cdot Im[Z_{\Delta h}] + Im[\Delta h] \cdot Re[Z_{\Delta h}]$

★ Renormalized ME with and without smearing are compatible

★ Absence of stout smearing leads to increased noise

NOTES:

1. The renormalized ME are not yet physical observables
2. The renormalized ME go to zero slower than the bare ME
   → *unphysical oscillations in the quasi and physical PDFs*

Towards the physical PDFs

After having renormalized the matrix elements, we proceed with:

1. computing quasi-PDFs

$$\widetilde{q}(x,P,\mu) = 2P \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP} h(z,\mu)$$

2. applying a matching procedure

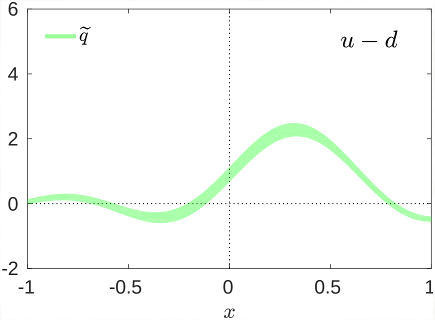
$$q(x,\mu) = \int \frac{d\xi}{|\xi|} \mathcal{K}\left(\frac{\mu}{xP},\xi\right) \underbrace{\tilde{q}(x,P,\mu)}_{\text{quasi-PDFs}}$$

   where for the *matching kernel* $\mathcal{K}$ we use the expression of Refs.[C. Alexandrou et al., arXiv:1803.02685[hep-lat]] and [C. Alexandrou et al., arXiv:1807.00232[hep-lat]].

3. applying Target Mass Corrections (TMCs) to correct for $m_N/P \neq 0$ in a finite momentum frame [J.W. Chen et al., Nucl.Phys. B911 (2016) 246-273, arXiv:1603.06664 [hep-ph]].

Results for unpolarized and helicity PDFs

$P = 1.38 GeV$



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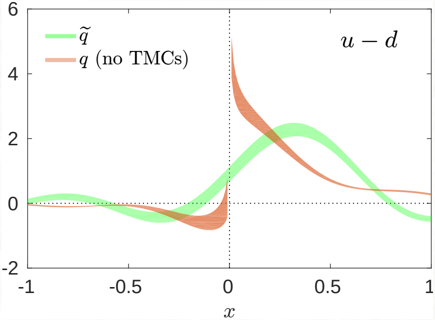

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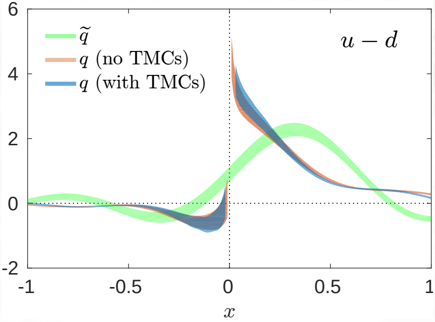

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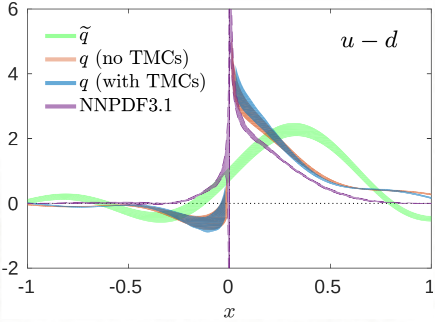

[NNPDF3.1: Eur.Phys.J. C77, 663 (2017), 1706.00428]

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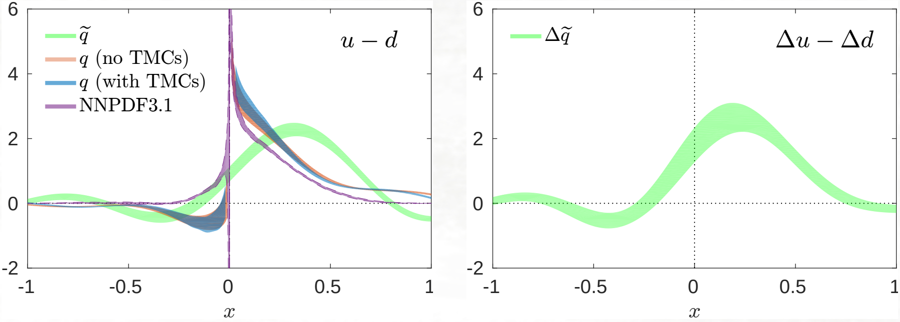

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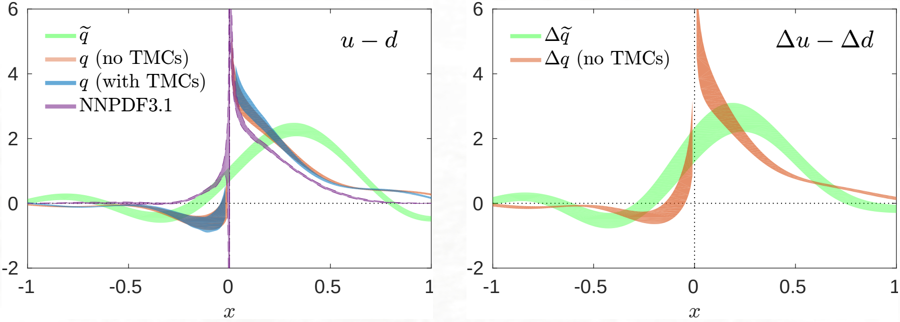

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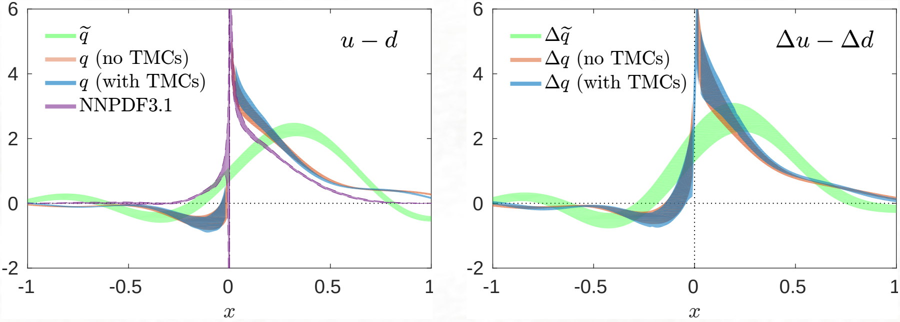

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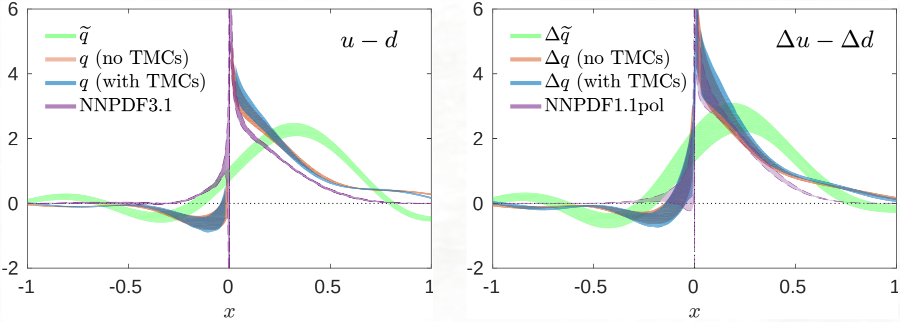

[NNPDF3.1: Eur.Phys.J. C77, 663 (2017), 1706.00428]

[NNPDF1.1pol: Nucl.Phys.B887, 276 (2014), 1406.5539]

Phenomenological PDFs: determined from a global fit to DIS and SIDIS data.

Comparison of lattice with phenomenological PDFs

★ Results presented at 2 GeV in $\overline{\text{MS}}$-scheme

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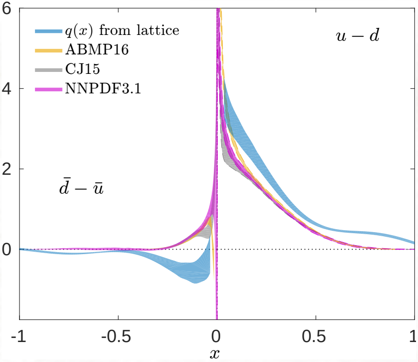
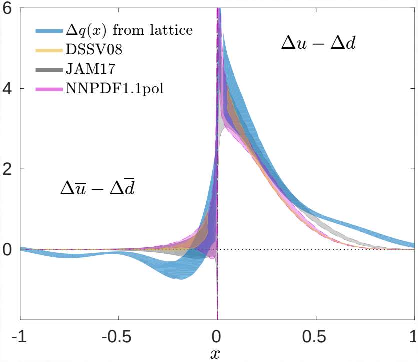




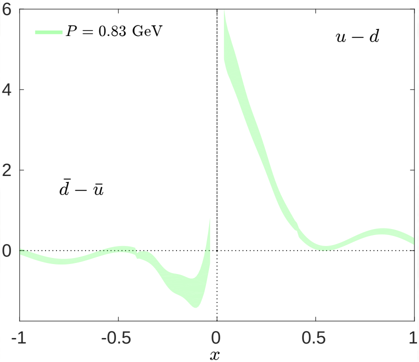
[ABMP16: Phys.Rev.D96, 014011 (2017)]
[CJ15: Phys.Rev.D93, 114017 (2016)]

[DDSV08: Phys.Rev.D80, 034030 (2009)]
[JAM17: Phys.Rev. Lett. 119, 132001 (2017)]

★ Similar behavior of lattice data as compared to phenomenological PDFs

★ Significant overlap for the polarized PDF with phenomenology for $0 < x < 0.5$.

Momentum dependence of lattice PDFs



★ As the momentum increases, the data approach phenomenological results

★ The oscillations are smoothened out as the momentum increases

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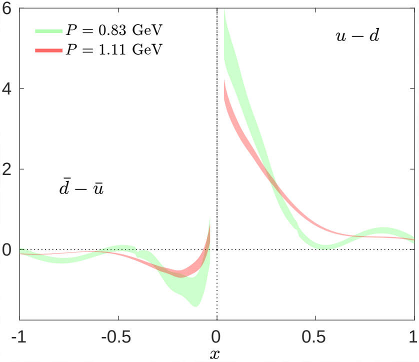

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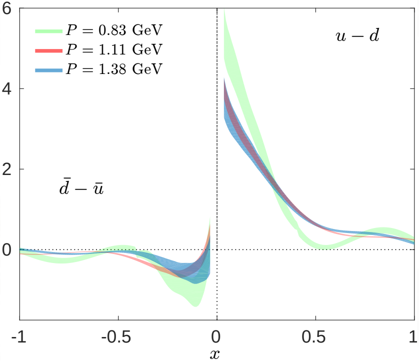

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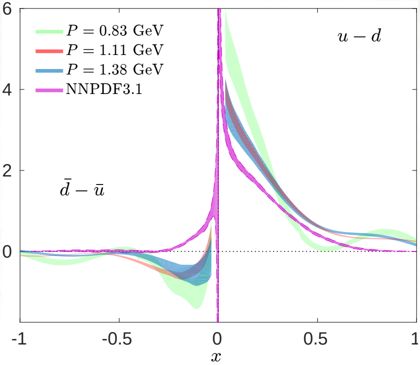

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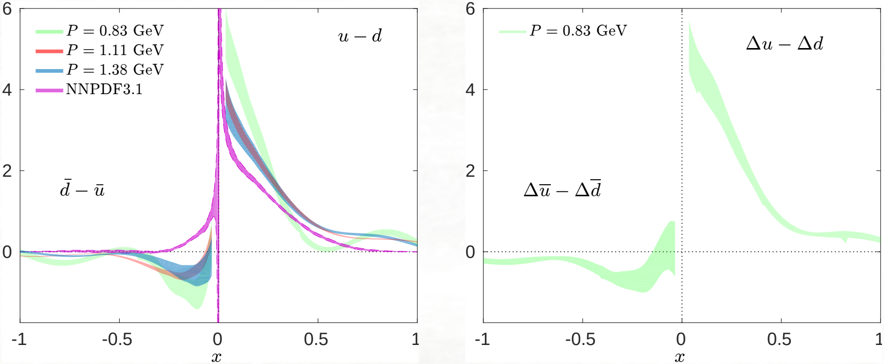

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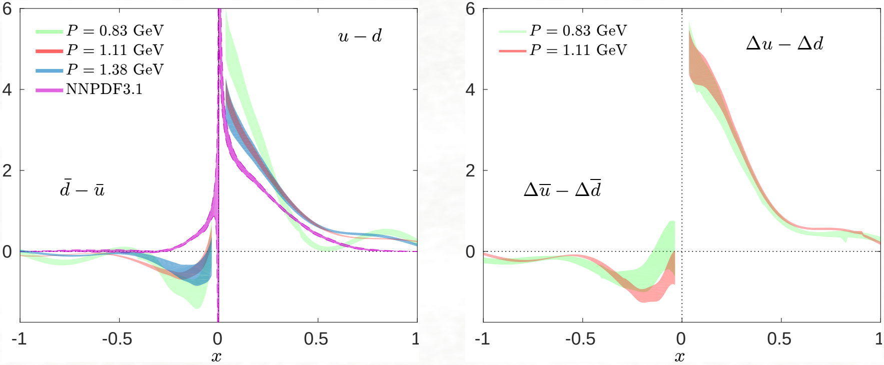

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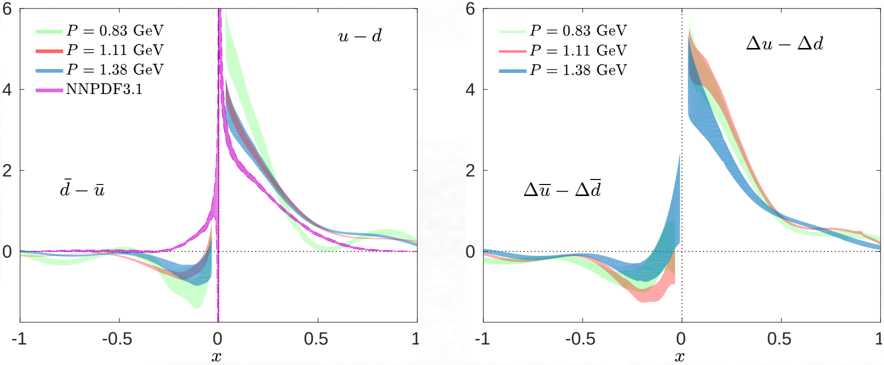

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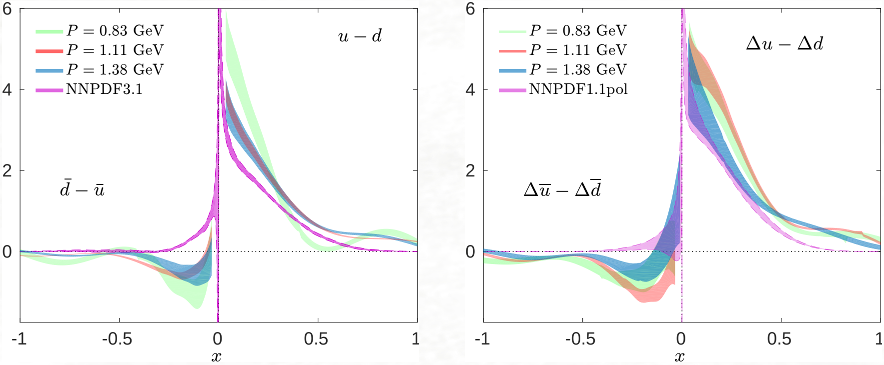

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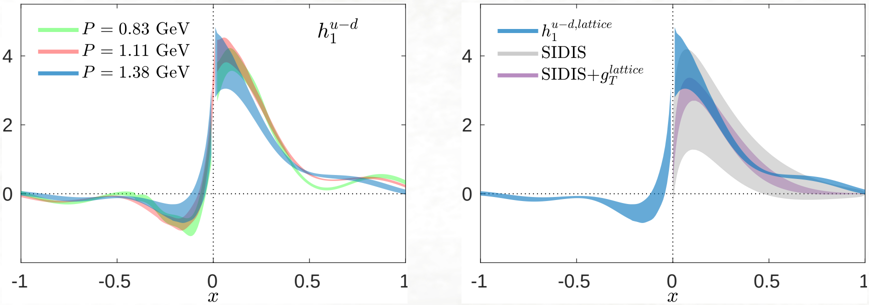
Results for transversity PDF



[C. Alexandrou et al., arXiv: 1807.00232]

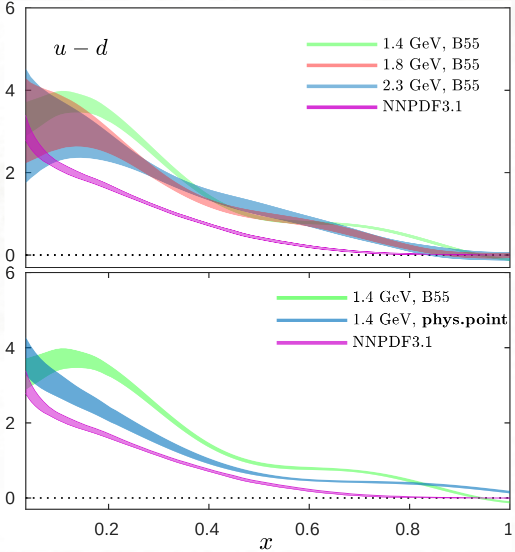
★ Small dependence of distributions on the nucleon momentum

★ Milder oscillatory behavior for the large momentum

★ The statistical uncertainties of the lattice PDFs are strikingly smaller than the phenomenological fits of the SIDIS data

★ At $P = 1.38$ GeV, $g_T = 1.10(34)$ by integrating over $x \in [-1, 1]$ and agreement with Mellin *moments* calculation [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)].

Dependence of the unpolarized PDF on the quark masses



- ★ Results from B55 ensemble ($M_\pi \simeq 375$ MeV, $a \simeq 0.082$ fm) with setup of Ref.[C. Alexandrou et al., Phys.Rev.D96, 014513 (2017)]
- ★ As P increases, the results reach a universal curve

- ★ Comparison of unpolarized PDF at momentum $\simeq 1.4$ GeV between B55 and physical point ensemble
- ★ The shift to the right of the PDF for B55 is compatible with a larger value $\langle x \rangle_{u-d}$ observed for the same ensemble [A. Abdel-Rehim et al., Phys.Rev.D92, 114513 (2015)]

Conclusions

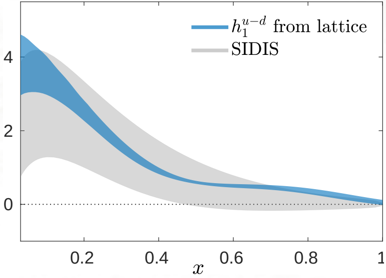
- We have presented a reconstruction of parton distribution functions at a physical pion mass ensemble, within the twisted-mass formulation, from first principles of QCD

- We have shown that lattice PDFs approach the phenomenological curves as the momentum increases

- Enormous progress in this field has allowed for the first time a *qualitative comparison* with PDFs extracted from scattering data

- Lattice QCD can be a powerful tool to determine PDFs very poorly constrained from phenomenology

  Example: transversity PDF
  $\rightarrow$ smaller statistical errors from lattice than from SIDIS experiments



Outlook

We are still at the beginning of a long way, with many systematics to control!

★ take the continuum limit $a \to 0$. The extrapolation requires at least 3 values of the lattice spacing

★ go to higher momenta with controlled excited states contamination

★ truncation effects in the matching and conversion from RI′ to $\overline{\mathrm{MS}}$

★ discretization effects in the renormalization functions

★ simulate $N_f = 2 + 1 + 1$ QCD at a physical pion mass ensemble

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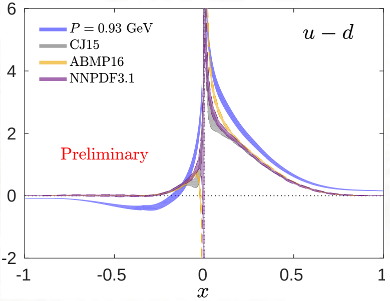
Preliminary result at one momentum for an ETMC ensemble:

[C. Alexandrou et al., arXiv: 1807.00495]

- $N_f = 2+1+1$ QCD
- $M_\pi \simeq 135$ MeV
- $a \simeq 0.081$ fm
- $V = 64^3 \times 128$, $M_\pi L \simeq 3.55$



Thank you for your attention

Standard vs. derivative Fourier transform

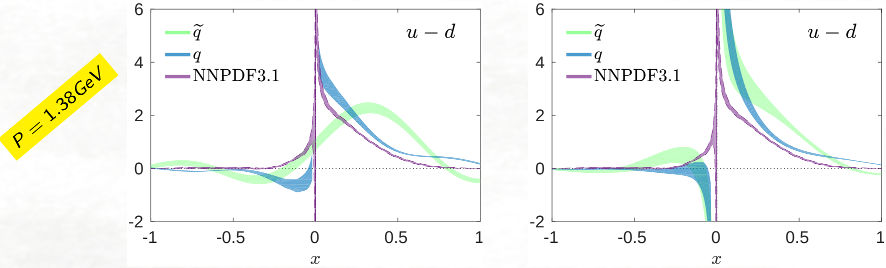
★ Standard Fourier transform defining qPDFs: $\tilde{q}(x) = 2P \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP} h(z)$
can be rewritten using integration by parts as:

$$\tilde{q}(x) = h(z) \frac{e^{ixzP}}{2\pi ix} \Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP}}{ix} h'(z) \,, \quad \text{where e.g. } h'(z) = \frac{h(z+1) - h(z-1)}{2}$$

★ Derivative Fourier transform defining qPDFs: [H.W. Lin et al., arXiv:1708.05301]

$$\tilde{q}(x) = -\int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP}}{ix} h'(z) \,, \quad \text{exact if} \quad h(z) \frac{e^{ixzP}}{2\pi ix} \Big|_{-z_{\max}}^{z_{\max}} = 0$$

★ Both standard and derivative method have systematic uncertainties



Standard method → oscillations

Derivative method: → small-x problematic