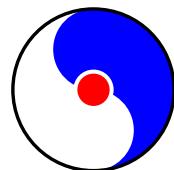


# Nucleon Form Factor calculation using DWQCD

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Research Center

**BROOKHAVEN**  
NATIONAL LABORATORY

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Theory (LATTICE2018), Lansing, MI

# Collaborators, Resources

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[ Nucleon structure ]

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- Also thanks to

RBC and UKQCD Collaborations  
  
USQCD, ALCC @ ANL  
BNL BG/Q  
RIKEN HOKUSAI and BG/Q

# The RBC & UKQCD collaborations

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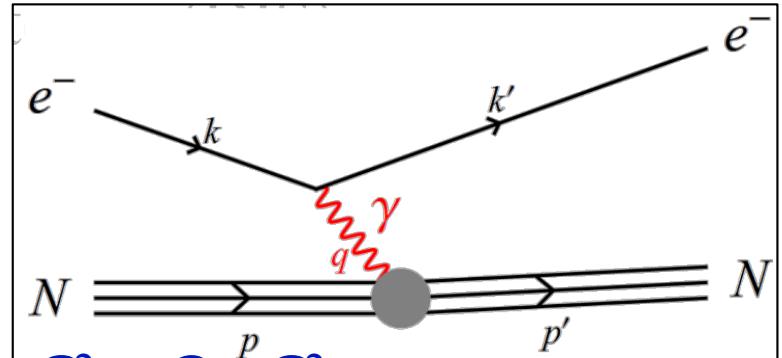
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Renwick Hudspith

# EM Form Factors of Nucleon

## ■ e-N Elastic Scattering

- Dirac Form Factor  $F_1(Q^2)$
- Pauli Form Factor  $F_2(Q^2)$



$$\langle \mathcal{N}(P') | J_{EM}^\mu | \mathcal{N}(P) \rangle = \bar{u}(P') \left[ \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} F_2(Q^2) \right] u(P)$$

- $F_1(Q^2=0)$  is EM charge of Nucleon
- $F_2(Q^2=0)=\kappa$  is Anomalous Magnetic Moment
- Mean square radii from slope of  $F_i(Q^2)$  at  $Q^2=0$ 
  - Dirac radius :  $r_1$
  - Pauli radius :  $r_2$

$$\langle r_i^2 \rangle = -\frac{6}{F_i(0)} \frac{\partial F_i(Q^2)}{\partial Q^2} \Big|_{Q^2=0}$$

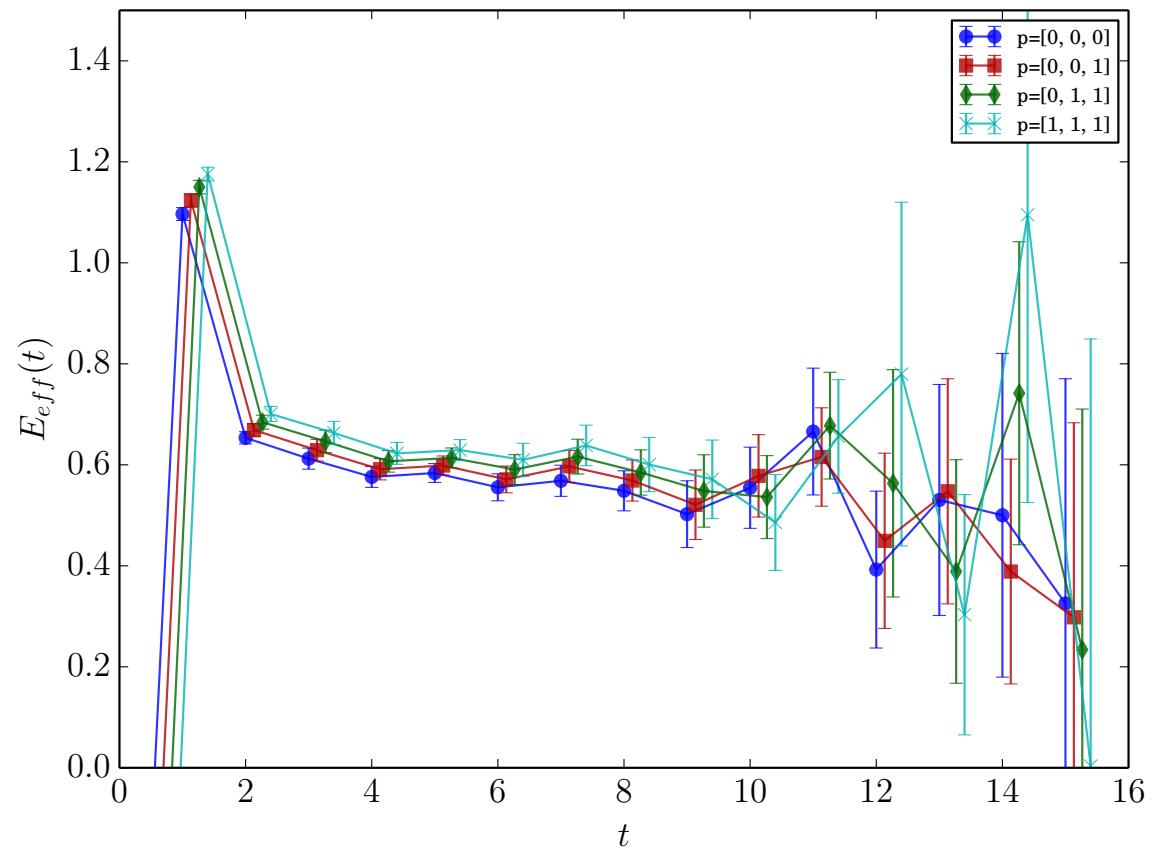
# Lattice ensemble

## ■ QCD ensemble (48I-ensemble)

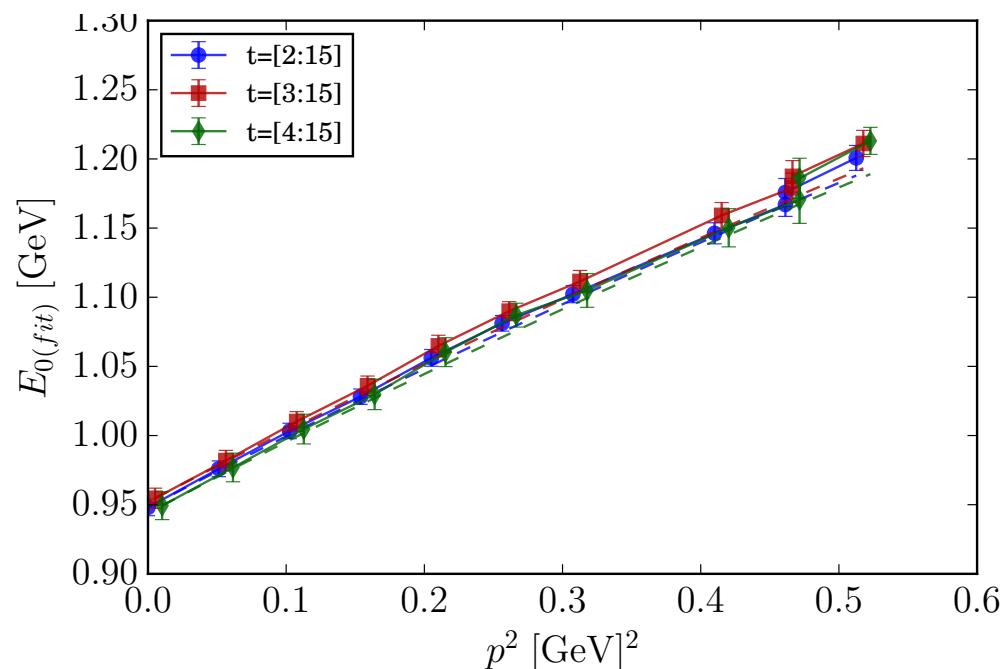
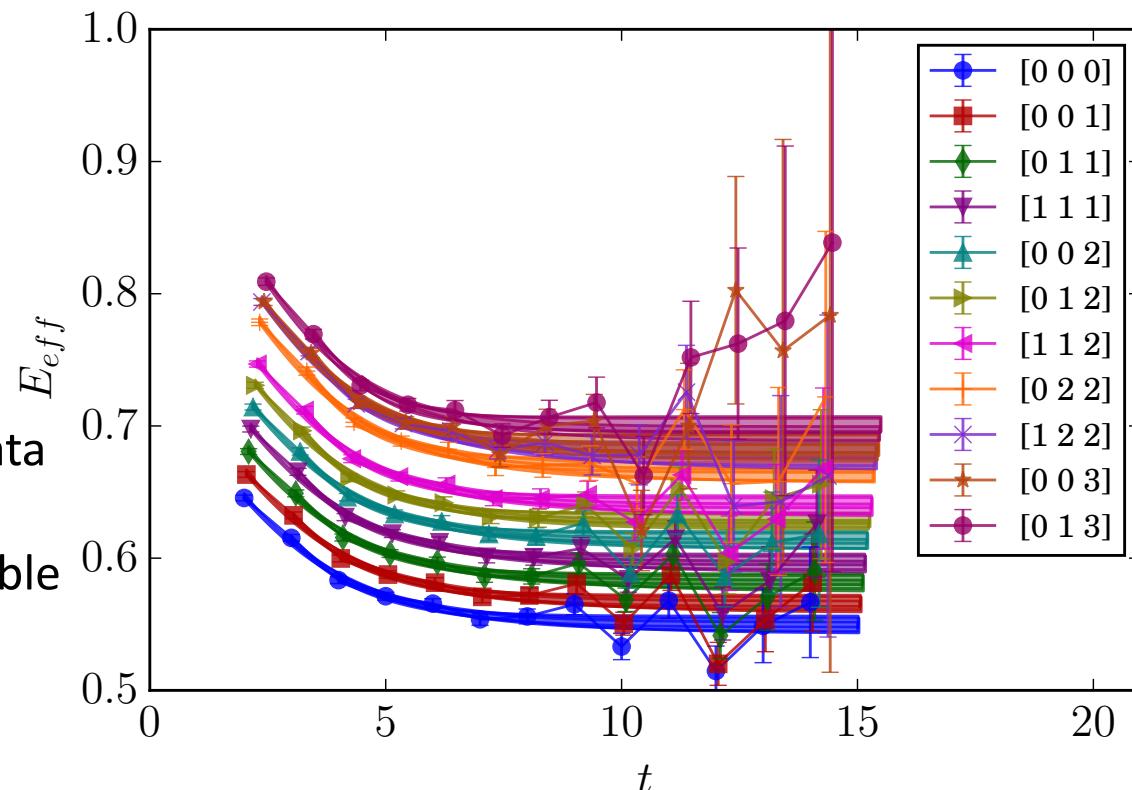
- $N_f=2+1$  (up, down, strange dynamical quark)  
Moebius Domain Wall Fermion, generated by RBC/UKQCD Collaboration
- Iwasaki RG-improved gauge, beta = 2.13
- Use masses of Pion, Kaon, Omega to set scale and quark masses
- Lattice cut-off  $1/a = 1.73 \text{ GeV}$ ,  $a=0.114 \text{ fm}$
- Volume =  $48^3 \times 96$  ,  $(5.5 \text{ fm})^3$ ,  $M_\pi L = 3.9$
- Five dim. length  $L_s = 24$ ,  $b+c=2$  (equiv  $L_s=48$ , Shamir DWF)  
→ residual symmetry breaking  $m_{\text{res}} \sim 0.45 m_{ud}$
- Physical Pion mass  $M_\pi = 139.2(4) \text{ MeV}$
- 130 configuration, with 34 k (256 sloppy + 4 exact)  
measurements AMA, eigV compression [ Jung, Lehner ]

# Plateau of 2pt functions

- $E_{eff}(t) = \log C(t) / C(t+1)$  to see excited state contaminations.
- Gaussian-smeared sources for quark propagators optimized to create state weakly coupled to the excited states.
- 147 momentum, upto  $p \sim 0.72 \text{ GeV}^2$



Two exponential fit  
 Smeared-Smeared data  
 Chi2/dof are reasonable  
 Dispersion relation OK  
 $m_N \sim 948(6) \text{ MeV}$   
 (sub percent stat error)

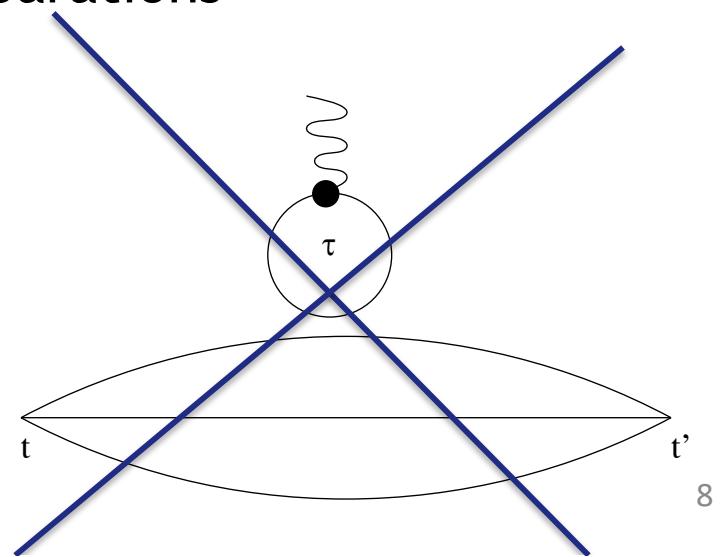
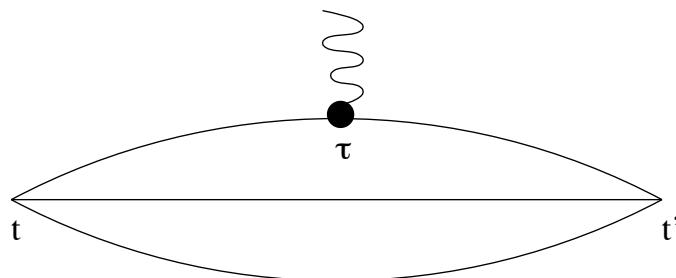


# 3pt functions

- Smeared-smeared 2 pt
- Smeared-Smeared 3pt , local vector currents for up and down separately, zero-momentum sink

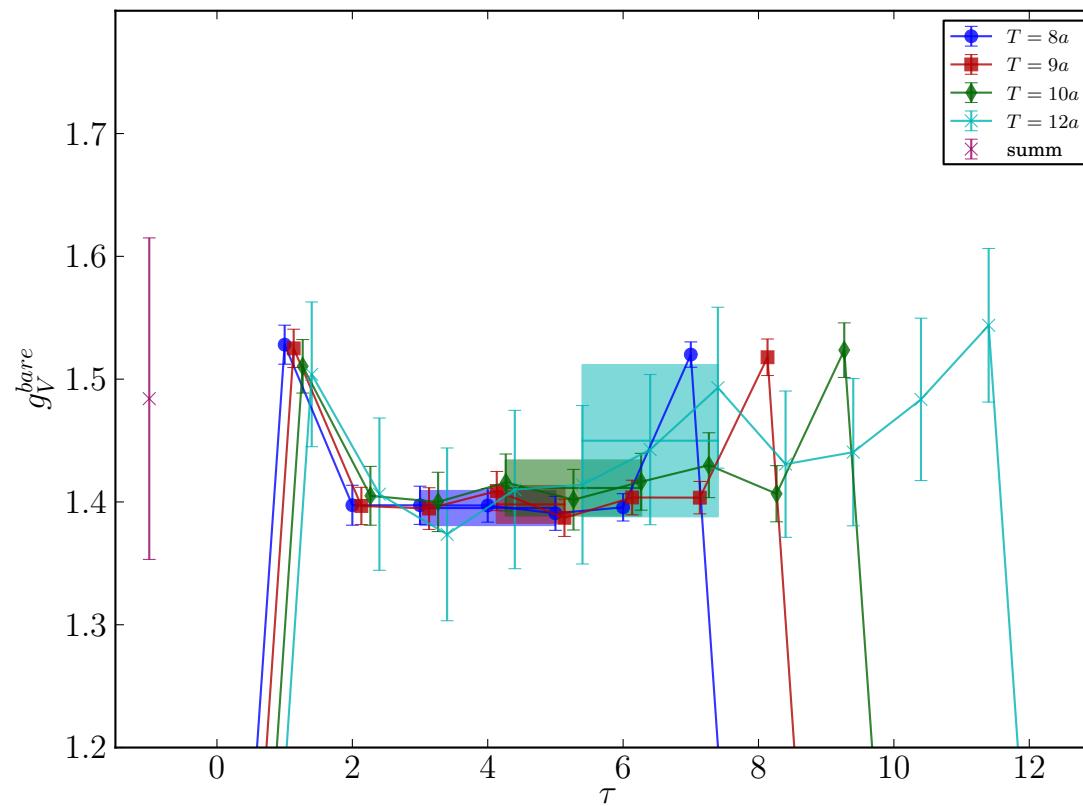
$$\begin{aligned}
 &= \frac{C_{\text{3pt}}^{\mathcal{O}}(\tau, T; P, P')}{\sqrt{C_{\text{2pt}}(T, P)C_{\text{2pt}}(T, P')}} \sqrt{\frac{C_{\text{2pt}}(T - \tau, P)C_{\text{2pt}}(\tau, P')}{C_{\text{2pt}}(T - \tau, P')C_{\text{2pt}}(\tau, P)}} \\
 &\xrightarrow{\{T, \tau, T - \tau\} \rightarrow \infty} \frac{\sum_{S, S'} (\bar{U}(P, S)\Gamma_{\text{pol}}U(P', S')) \cdot \langle P', S' | \mathcal{O} | P, S \rangle}{\sqrt{2E(E + M_N) \cdot 2E'(E' + M_N)}}.
 \end{aligned}$$

- Project appropriate spinor matrix to extract the matrix elements for the ground state from large time separations
- Iso-vector component,  
connected diagrams



# Bare gV

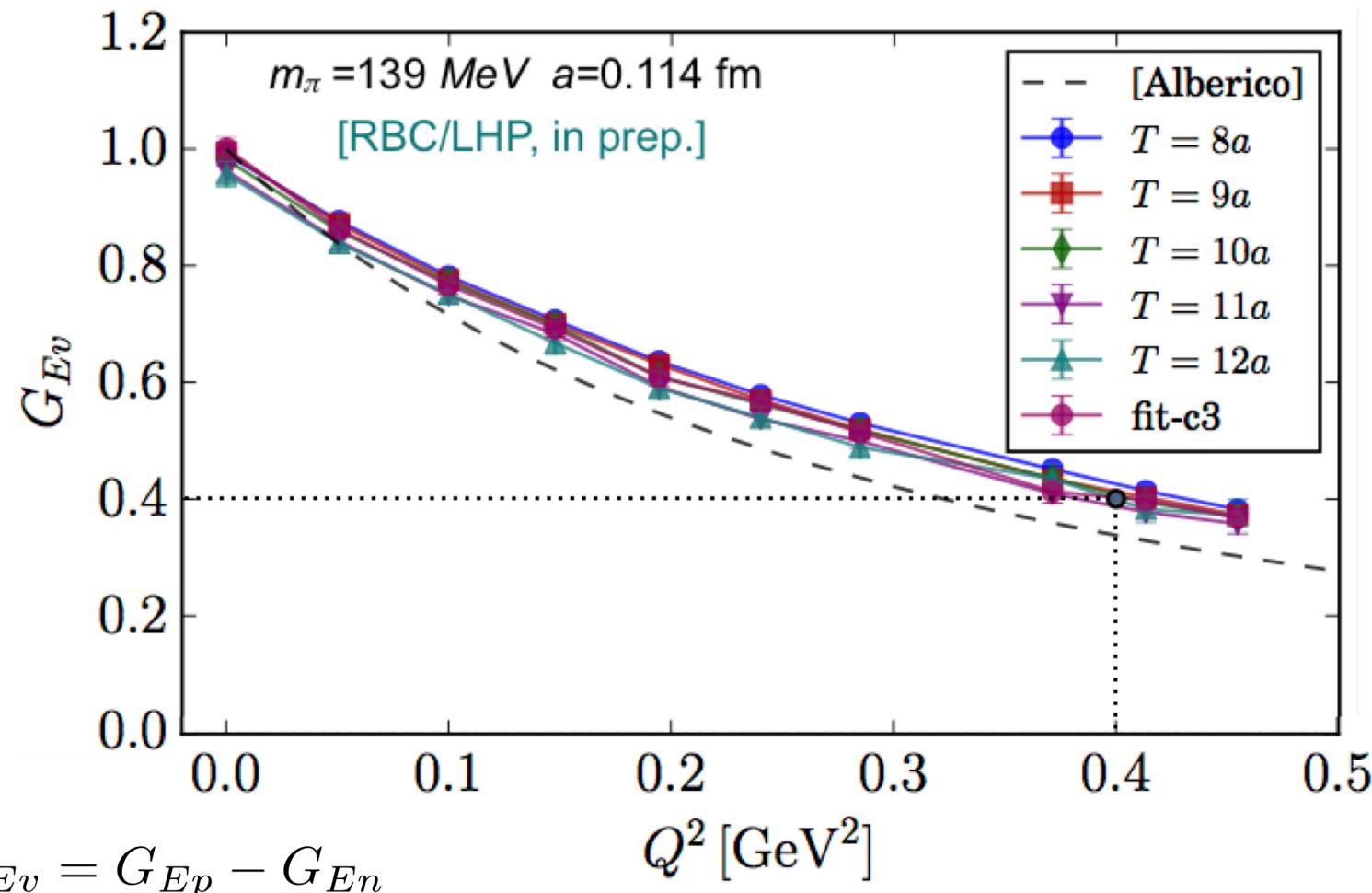
- time separations = 8,9,10,12 or 0.91 - 1.37 fm to check effects from exited states
- Renormalization factors for V and A :  $Z(V,A) = 1 / gV(T=8)$



# Electric From Factor $G_E$

- Statistical error is small
- Dash curve is Kelly's fit to experimental data
- Small T separation deviates from curve, while large T agrees within large stat. error

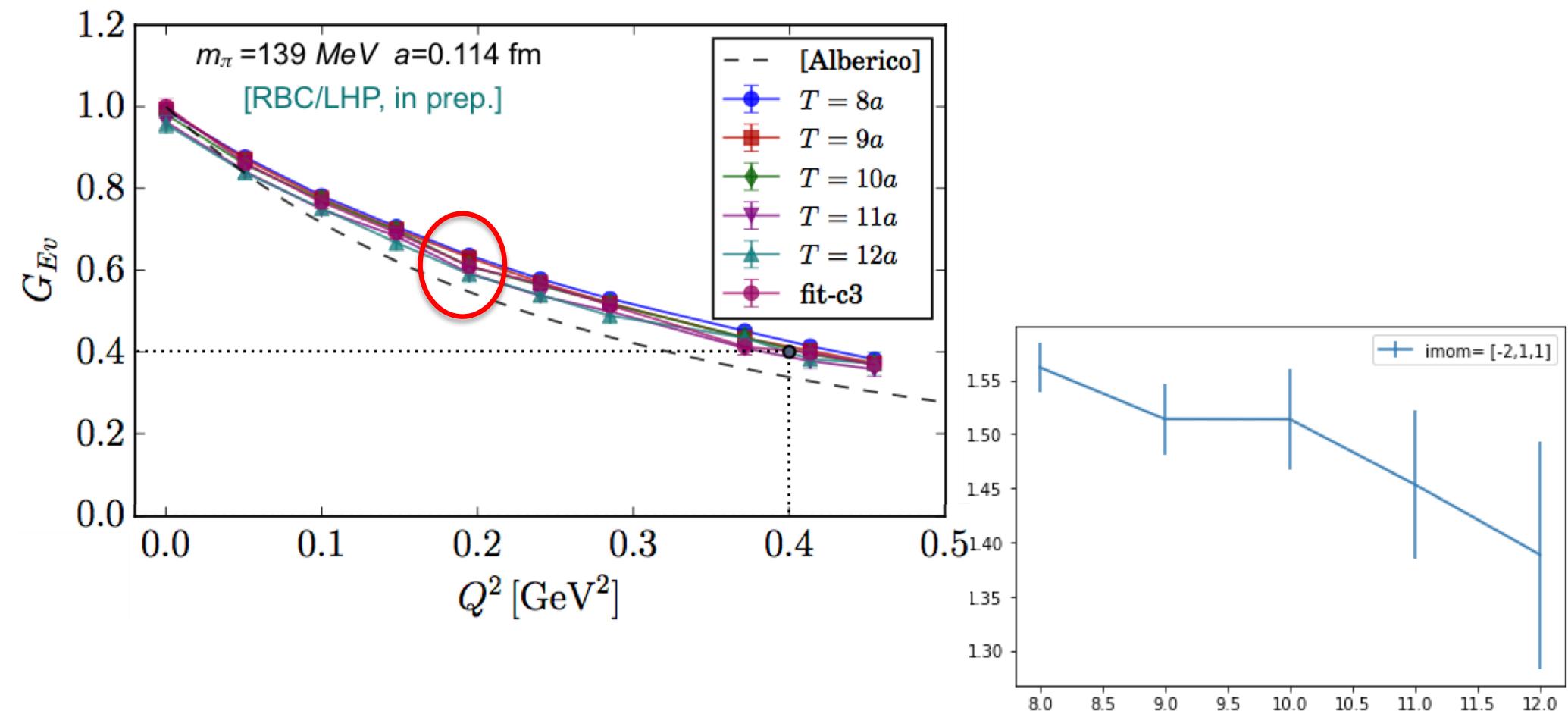
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$$



$$G_{Ev} = G_{Ep} - G_{En}$$

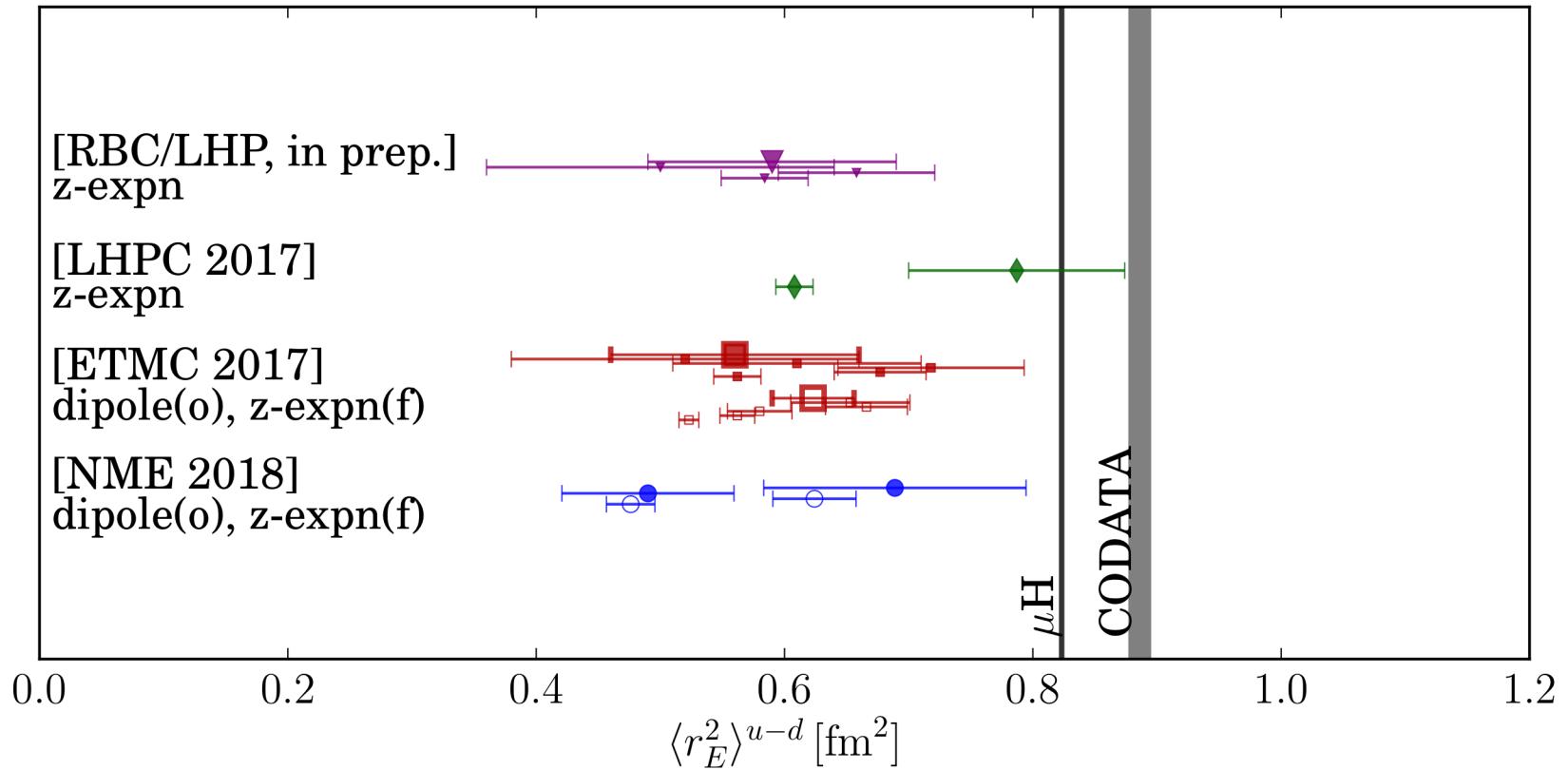
# Electric Form Factor $G_E$

## ■ Excited state



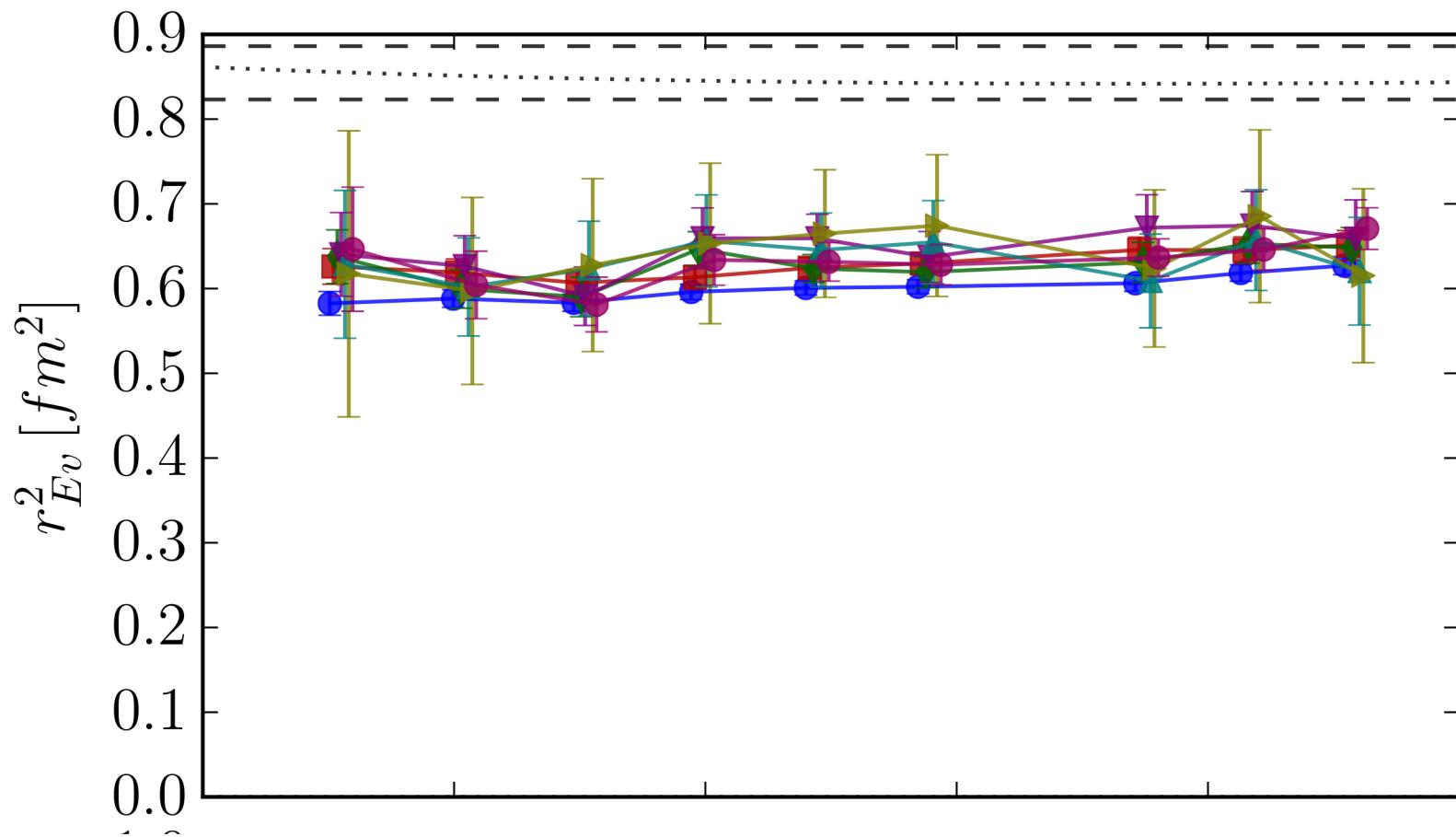
# Electric charge radius

- z-expansion fit for  $Q^2$  dependence



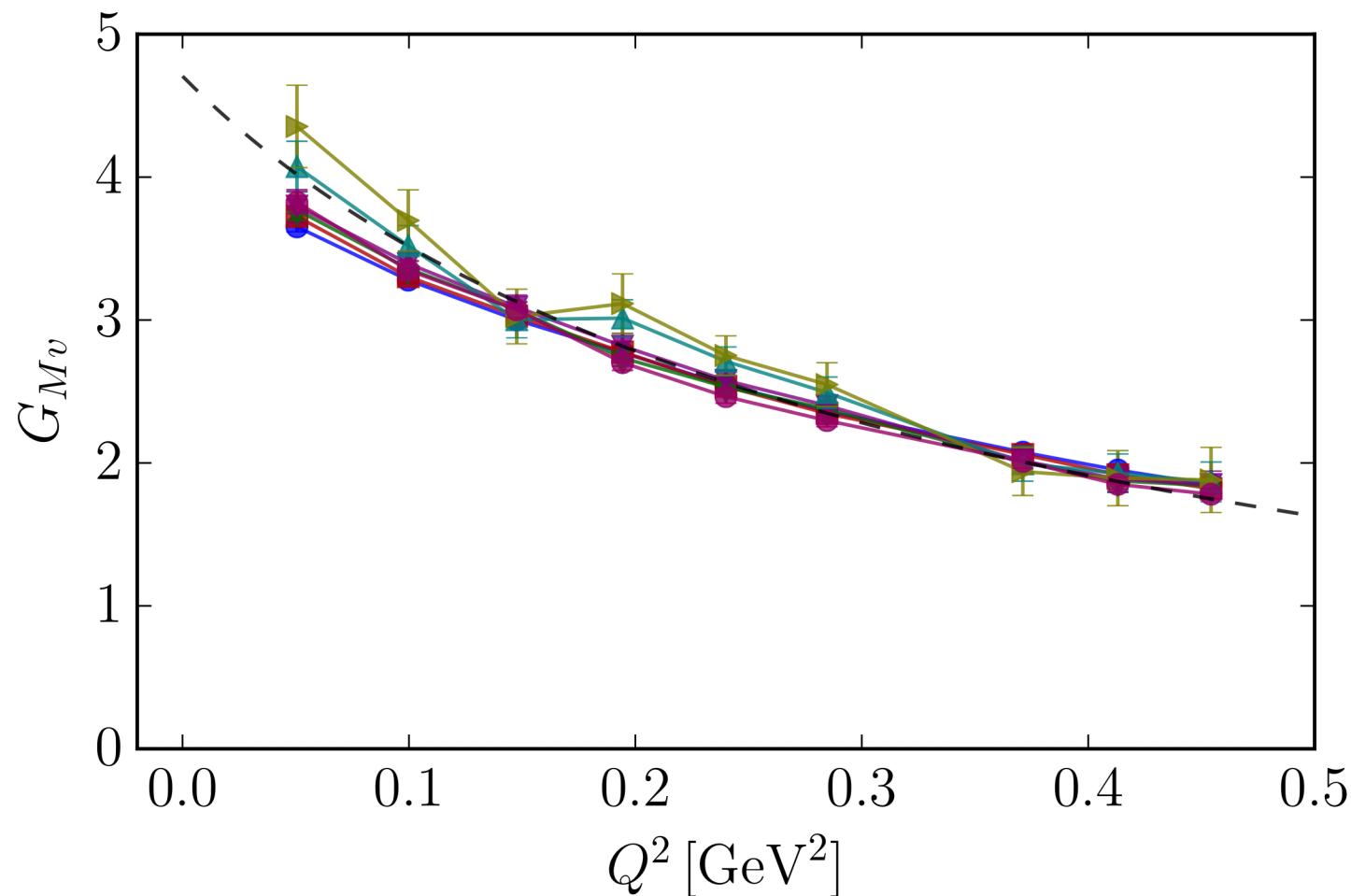
$$\langle r_i^2 \rangle = -\frac{6}{F_i(0)} \frac{\partial F_i(Q^2)}{\partial Q^2} \Big|_{Q^2=0}$$

- $T(\text{sep})$  dependence
- Not strong enough trends toward experimental value



# Magnetic Form Factor $G_M$

- Larger stat. error
- Better agreement with experiments
- ( less visible excited state effects )

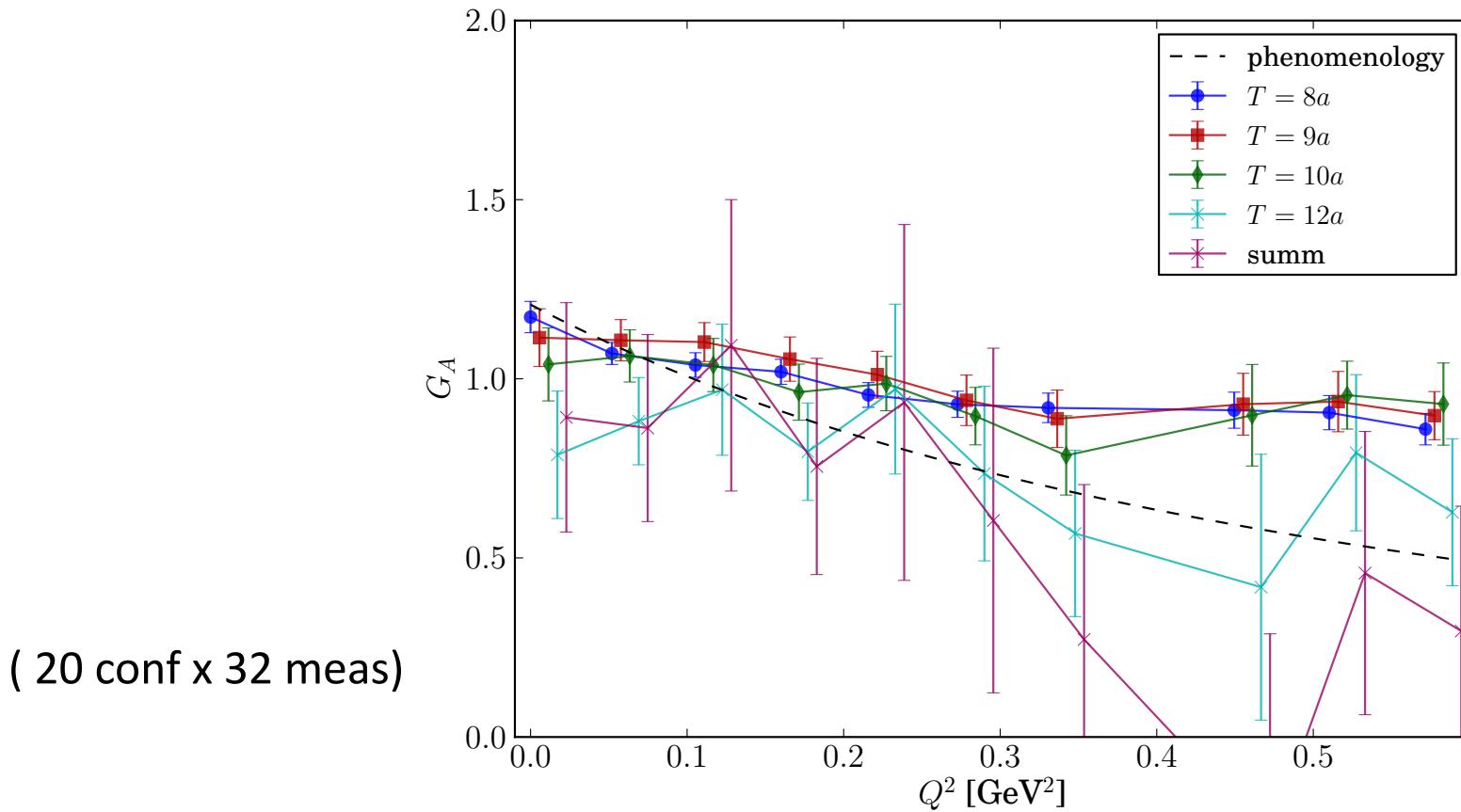


# Axial vector form factor $G_A$

- $G_A(0) = gA$

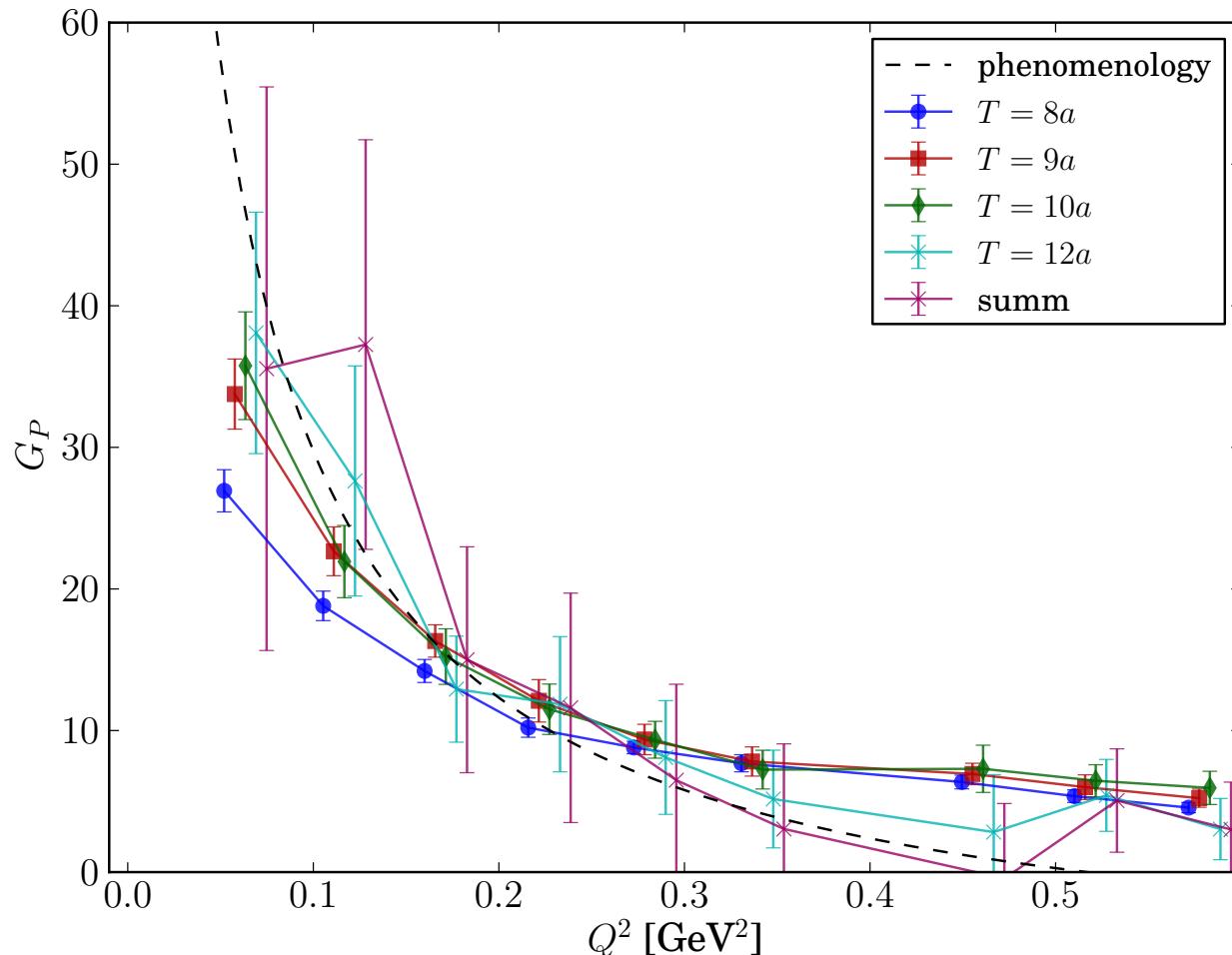
$$\langle p | A_\mu^+ | n \rangle = \bar{u}_p \left[ \gamma_\mu \gamma_5 G_A(q^2) + q_\mu \gamma_5 \frac{G_P(q^2)}{2M_N} \right] u_n$$

- Relevant for neutrino scattering and meson productions
- Significant deviation from experiment fit, also seen in other lattice calculations



# Pseudoscalar form factor $G_P$

- Significant excited state effects
- Better agreement with experimental fit for larger T
- Physical pion mass is important for pion pole



# Summary

## On-going calculation (RBC-LHP)

- On physical DWF quark mass  $M(\pi) \sim 135$  MeV
- Two lattice spacings,  $a \sim 0.11$  fm (also 0.085 fm available)
- Volume  $\sim (5.5 \text{ fm})^3$ ,  $M(\pi) L \sim 3.8$
- 130 config, 34 k measurements
- Statistical error getting better, especially for  $G_E$
- Large  $G_E$  at large  $Q^2 > \sim 0.3 \text{ GeV}^2$  undershoots experiments
- Charge radius also turns out to be small
- Excited state effects and other systematic errors ?

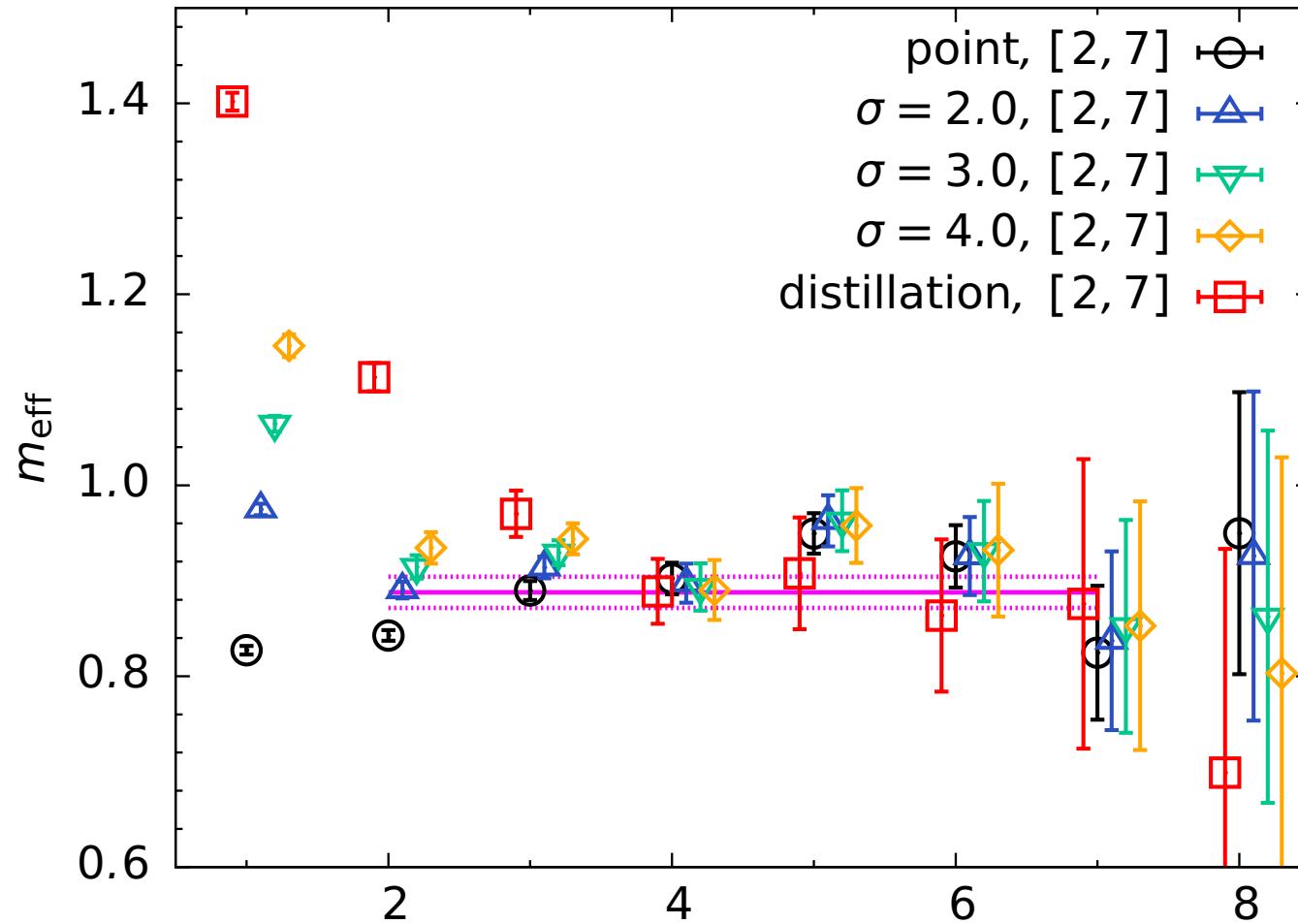
# Further study for interpolation operator and distillation

- Physical pion at coarse lattice spacing

( $1/a = 1\text{GeV}$ ,  $L \sim 5\text{ fm box}$ )  
60 eigenV of 3D lap

[ Yong-Chull Jang ]

Fit mass error goes up with distillation data



# Future plans

- More analysis,  $G_A$ ,  $G_P$  (violation of PCAC ?)
- More excited states studies including distillation program
  - Excited states seem more visible at high  $Q^2$
  - Is this related to smaller electric charge radius ?
- Finite Volume study on coarse ensembles
- Continuum limit on finer lattice 64 cube

# Isovector F1, F2

