Is SU(3) gauge theory with 13 massless flavors conformal?

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on behalf of the Lattice Higgs Collaboration:

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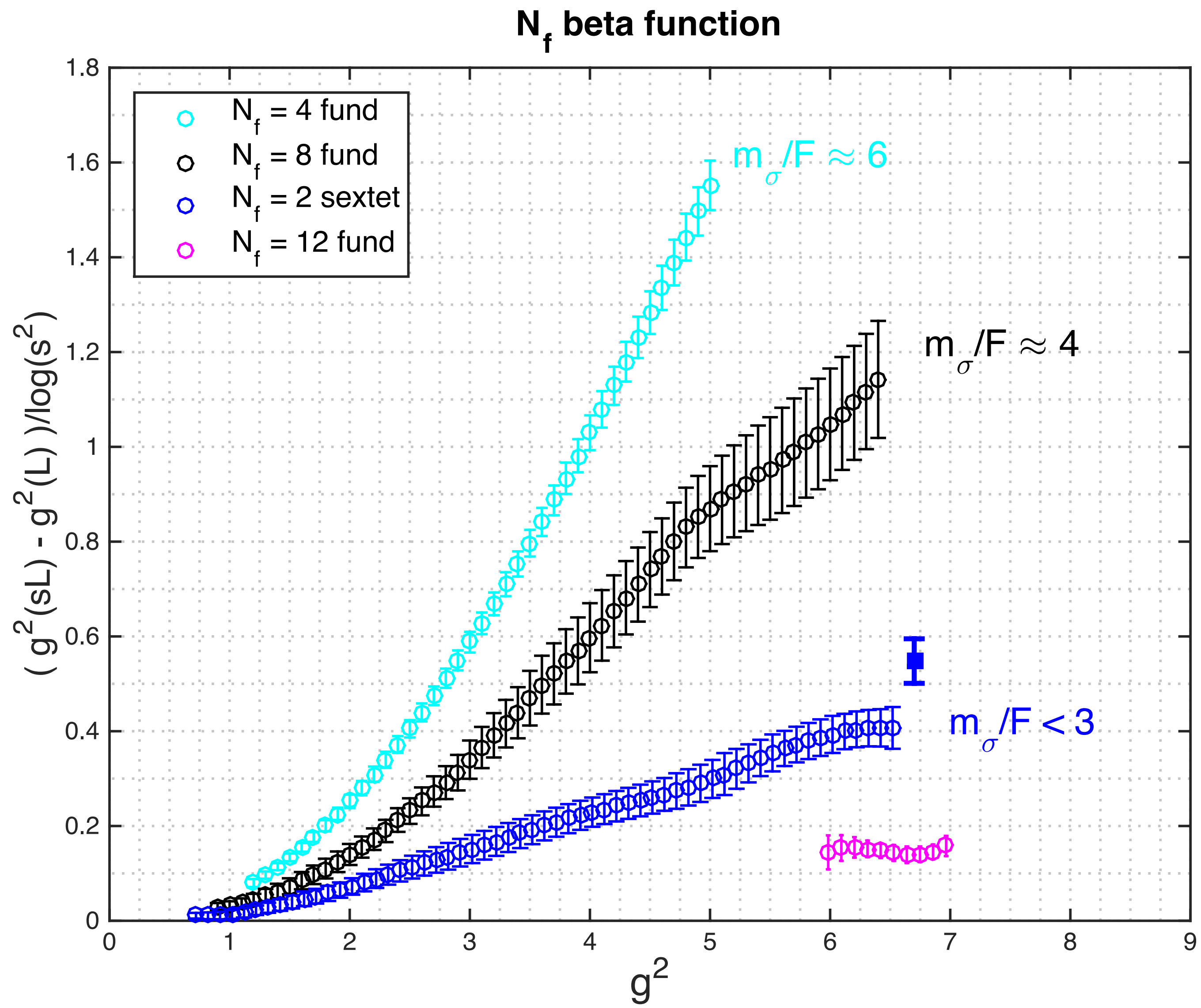
Context



LatHC studies of many SU(3) gauge theories shows trend of continuing decrease in beta function with increasing Nf and a steadily lighter scalar

Extensive simulations with a range of lattice spacings, especially for Nf = 12

Consistent picture that Nf = 12 is close to being conformal, paradigm of walking model

Recent critiques:
1. "staggered fermions are in the wrong universality class" - Hasenfratz, Rebbi, Witzel
2. "Nf = 10 with domain wall fermions has an infrared fixed point" - TW Chiu

Our response:
1. you can't add relevant operators to staggered fermions: spin models with extra fixed points not applicable
2. holding the renormalized gauge coupling fixed in some physical volume controls taste symmetry recovery
3. the Dirac operator eigenvalue spectrum from MC simulations shows the correct structure
4. we have simulated Nf = 10 with staggered fermions with a range of lattice spacings and find no IRFP

One role of an Nf = 13 study: can staggered fermions find an IRFP and a conformal theory?

Why Nf =13?

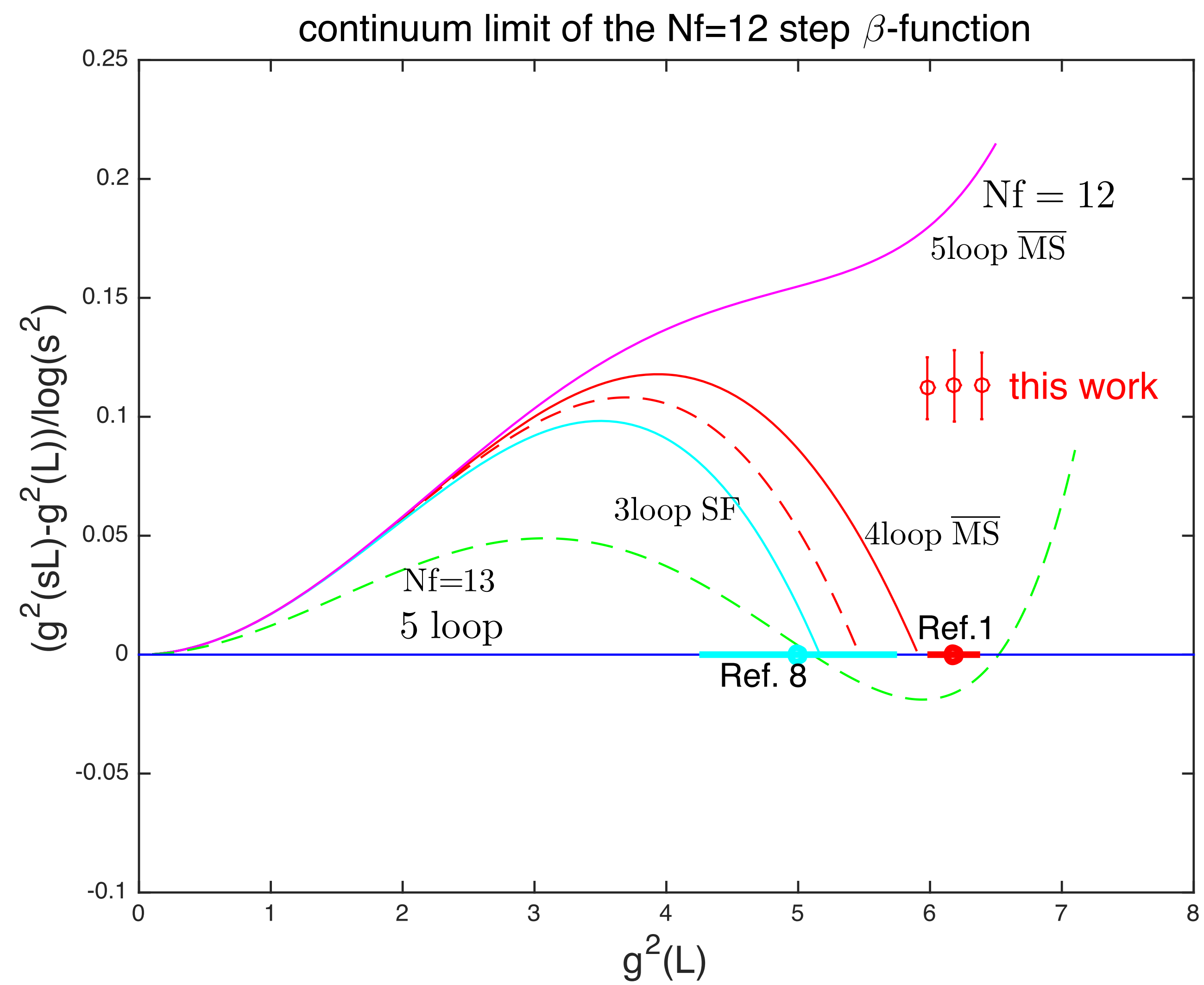
5-loop MS-bar calculation (20 year project)
of gauge coupling beta function

SU(3) Nf = 12 has an infrared fixed point (IRFP) at 2, 3, and 4-loop in MS-bar, which disappears at 5-loop

SU(3) Nf = 13 has an intriguing structure at 5-loop of an IRFP and a non-trivial UV fixed point

Nf = 13 IRFP also present at 2, 3 and 4-loop

suggestive of merger of IRFP and UVFP at lower edge of conformal window (Kaplan-Son-Stephanov, Vecchi)



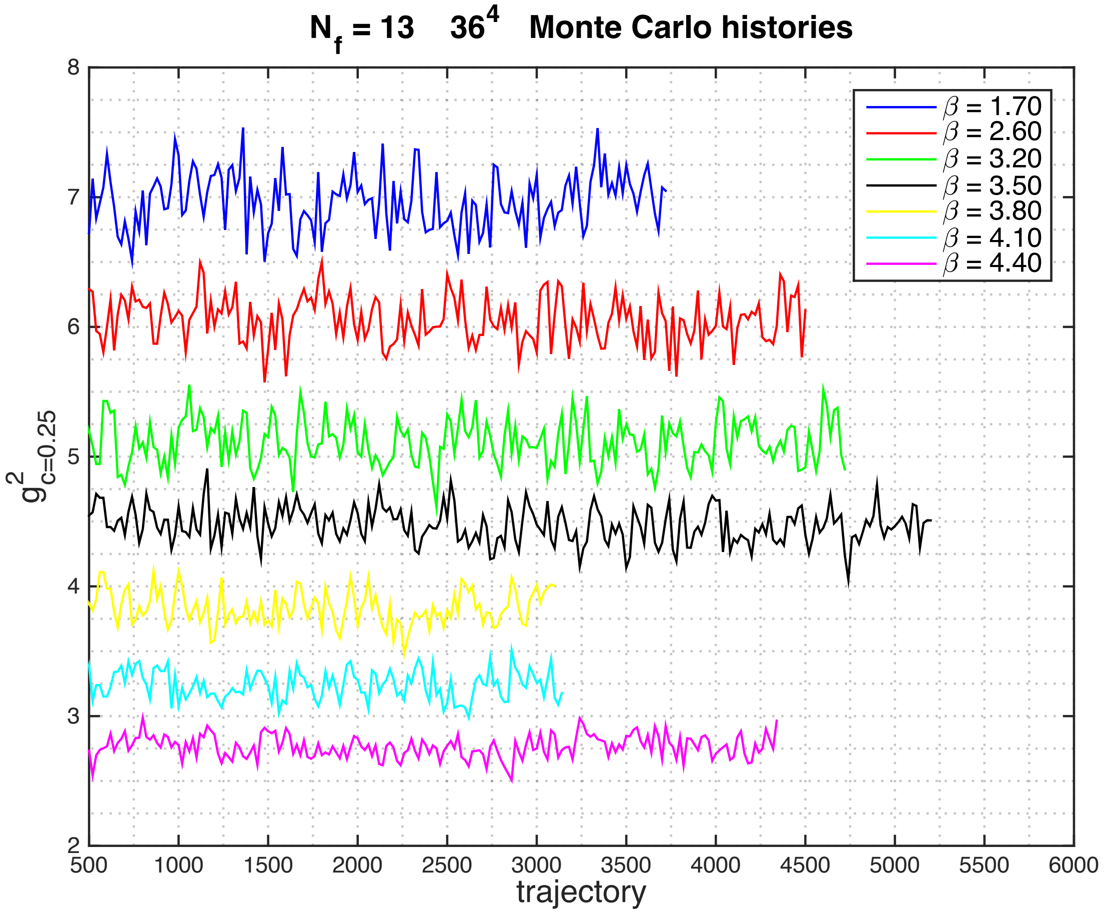
Fodor et al LatHC PRD94 091501R (2016)

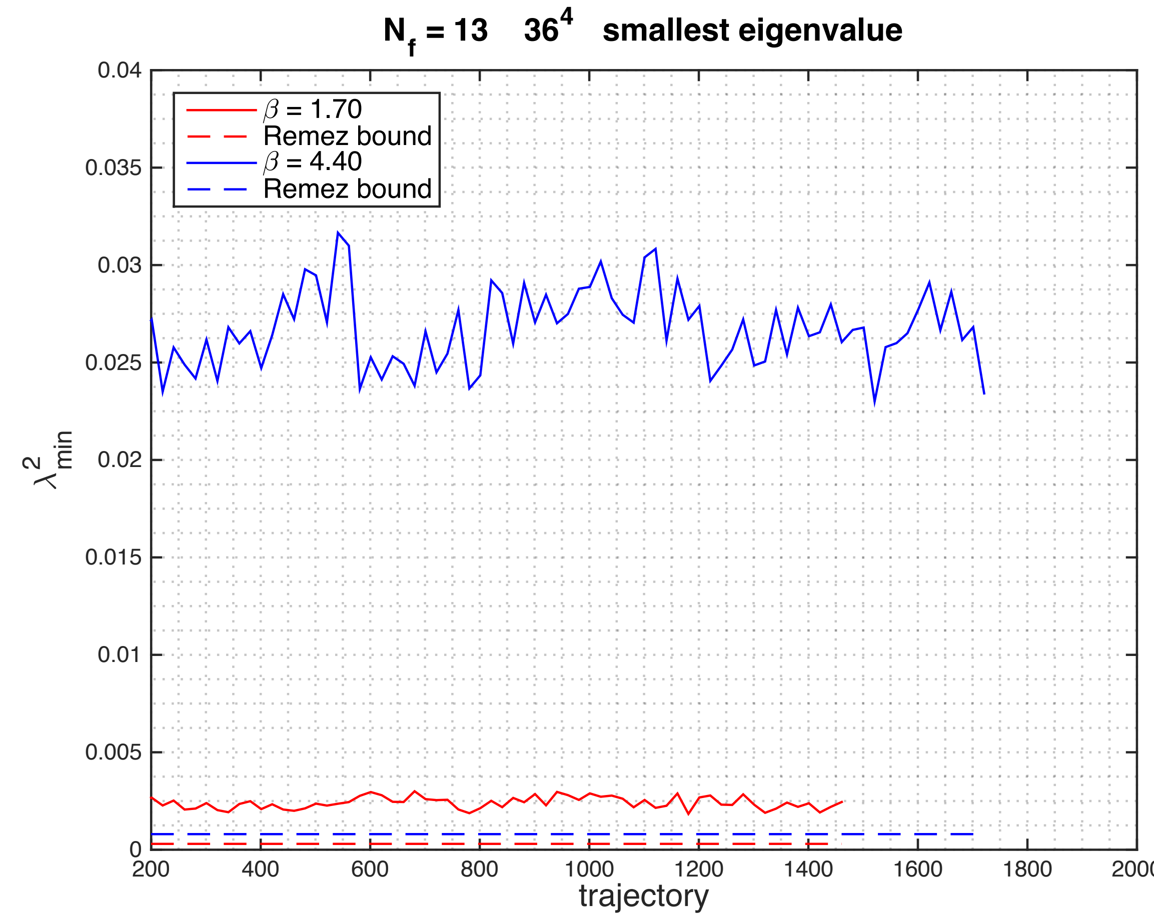
Can we test with staggered fermions if Nf = 13 is conformal?
Interesting setting to test new predictions for conformal behavior (5-loop MS-bar, Ryttov-Shrock delta)

Lattice 2017: Daniel Nogradi presented results for SU(3) Nf = 14 fund rep and Nf = 3 sextet rep
perturbative beta function much smaller: < 0.014 (Nf = 14), < 0.001 (Nf = 3 sextet)
continuum extrapolation difficult

EPJ Web Conf. 175 (2018) 08028

Nf = 13 might be less difficult for continuum extrapolation





stout-smeared staggered fermions, Symanzik gauge action, RHMC algorithm: standard technology

for beta function with gradient flow and step-scaling, need a range of volumes and couplings:
L = 12, 16, 18, 20, 24, 30, 32, 36, 48 (with some 54)
bare coupling beta from 1.7 to 4.4 in steps of 0.3 (gives renormalized coupling in range 2 — 9)
large set of ensembles (9 volumes x 10 couplings) which allows various tests e.g.

step-scaling s = 3/2: $12 \to 18, \quad 16 \to 24, \quad 20 \to 30, \quad 24 \to 36, \quad 32 \to 48$

5 lattice spacings for continuum extrapolation — crucial

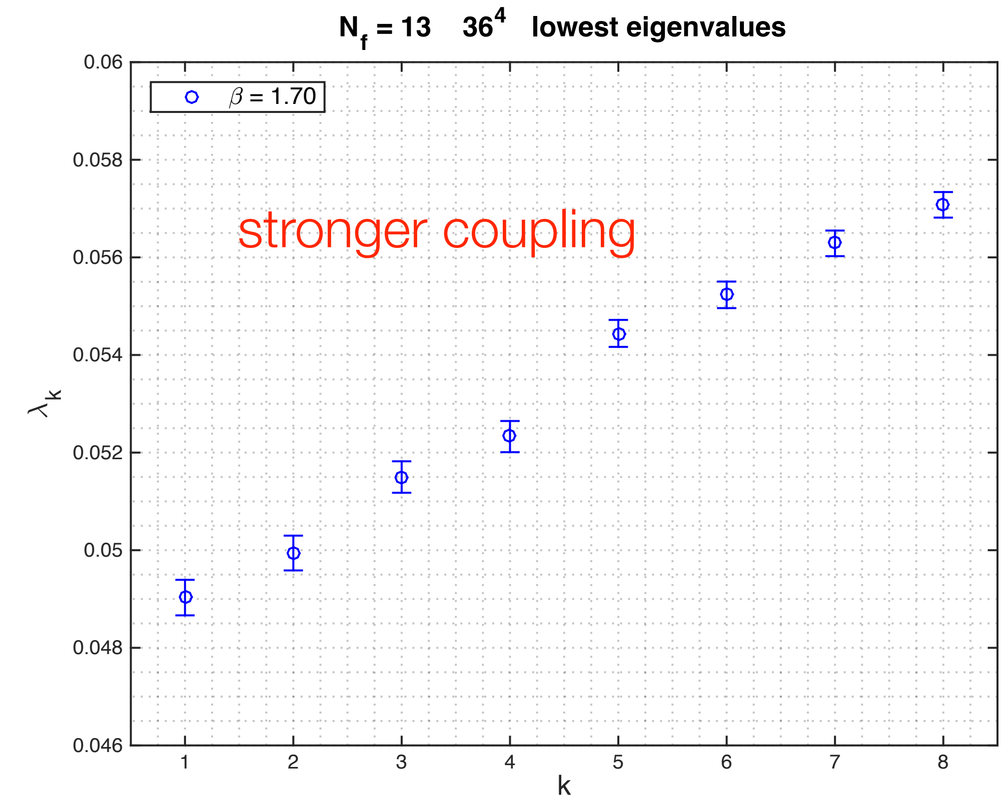
several thousand trajectories — goal few per mille accuracy in renormalized couplings

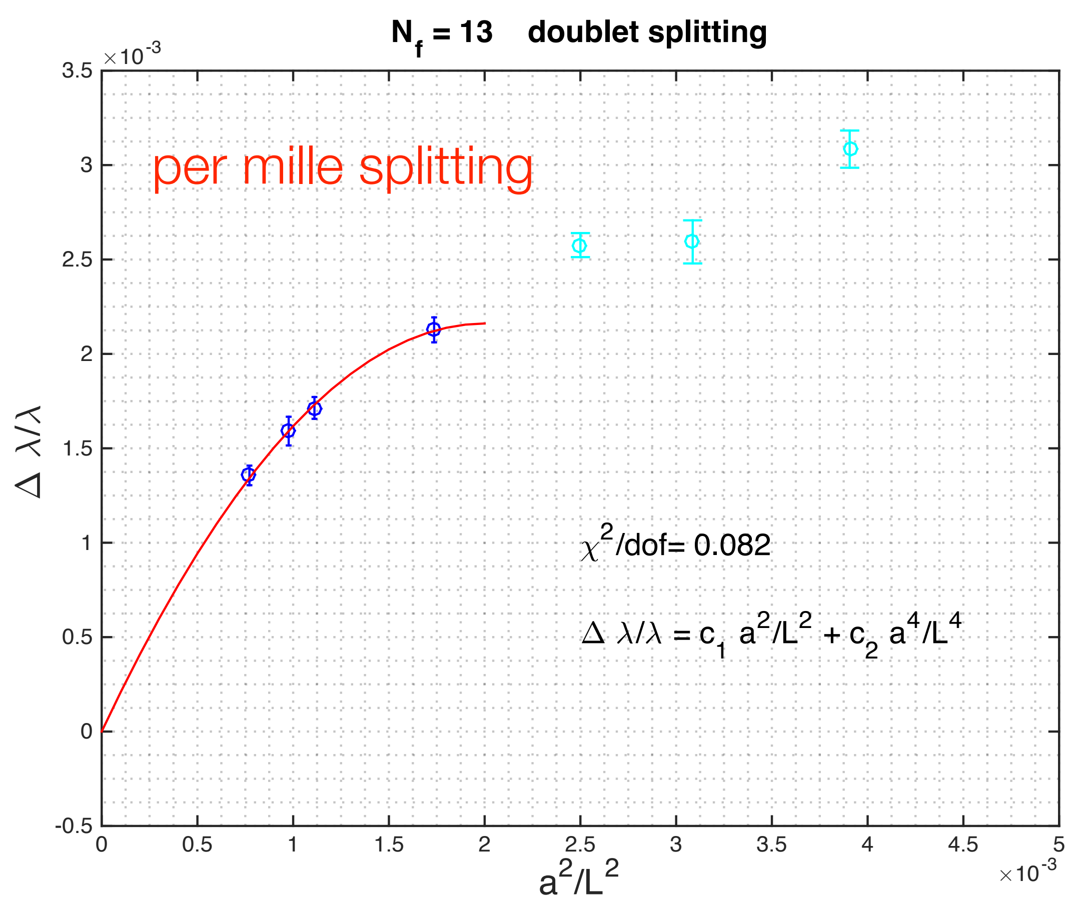
monitor RHMC algorithm throughout: lowest Dirac eigenvalue well above Remez bounds is taste symmetry of staggered fermions recovered?

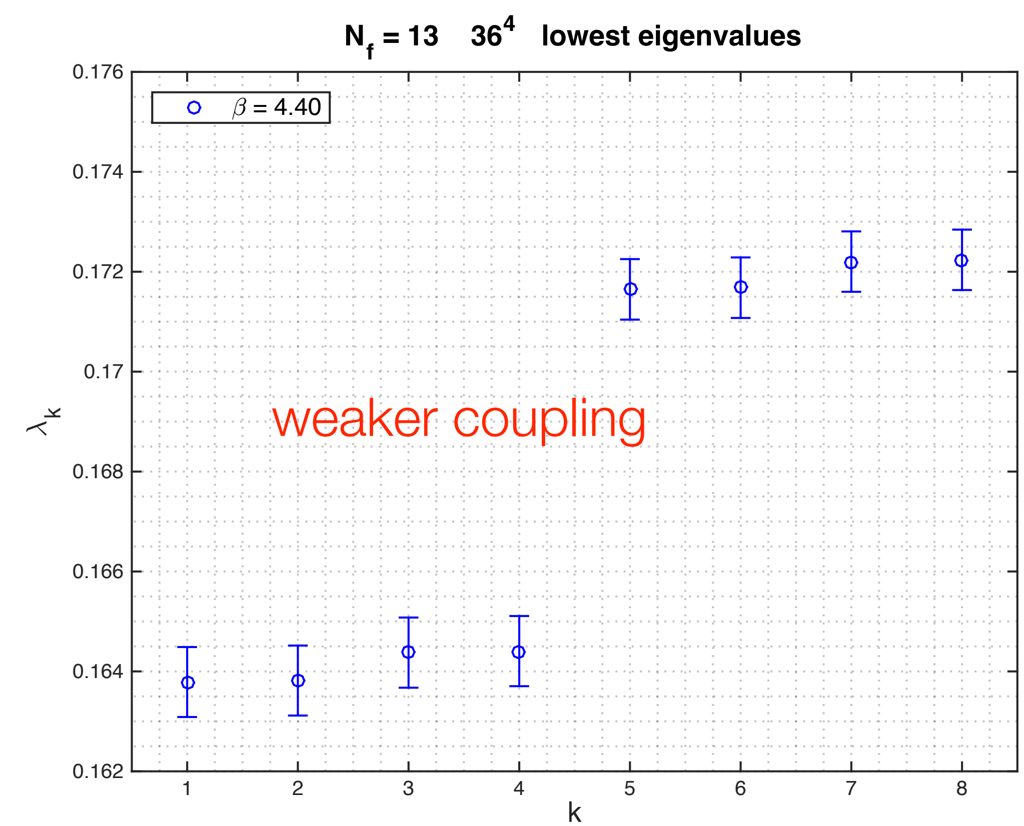
evident in Dirac operator eigenvalues

from strong to weak coupling: split eigenvalues first gather in pairs, then in quartets

eigenvalue splitting vanishes in the continuum limit, reaching the correct continuum theory: 1 of 3 arguments in response to claim staggered fermions are in wrong universality class







gradient flow: many possible discretizations e.g **SSC**

Symanzik gauge action for Monte Carlo generation and gradient flow

Clover operator for $E = 1/2 \text{ tr } G_{\mu\nu}^2$

first look: step function at fixed bare coupling

polynomial interpolation of step in g^2

data in groups of 5 for the possible steps L -> sL with s = 3/2

interesting structure:

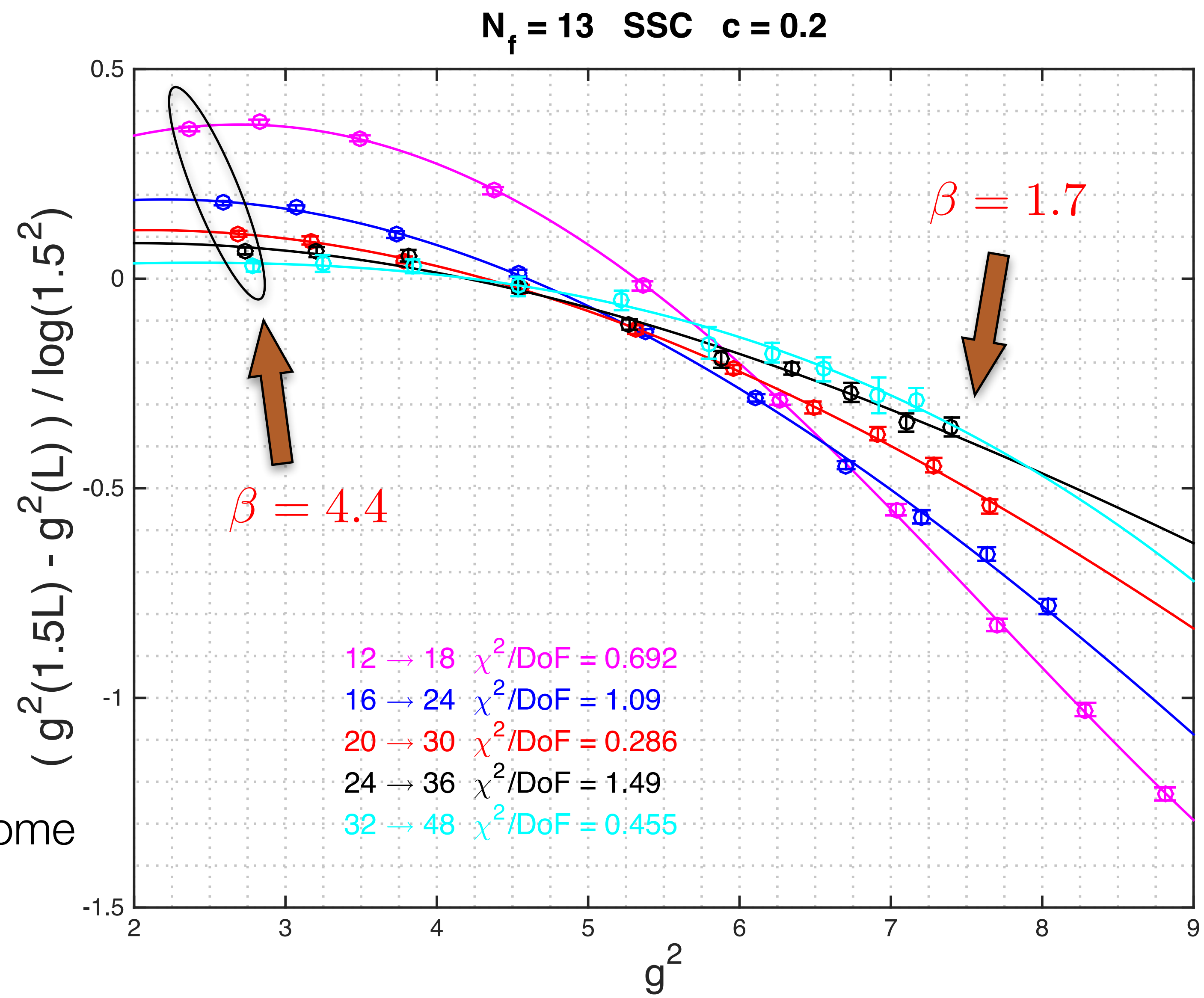
for each L -> sL pair, step function crosses zero at some renormalized coupling

cutoff effects appear to change sign as renormalized coupling increases i.e. approach continuum step function from above at weak coupling and from below at strong coupling

indication that continuum step function has a zero at an IRFP

gradient flow finite-volume scheme: $c = \sqrt{8t}/L = 0.20$

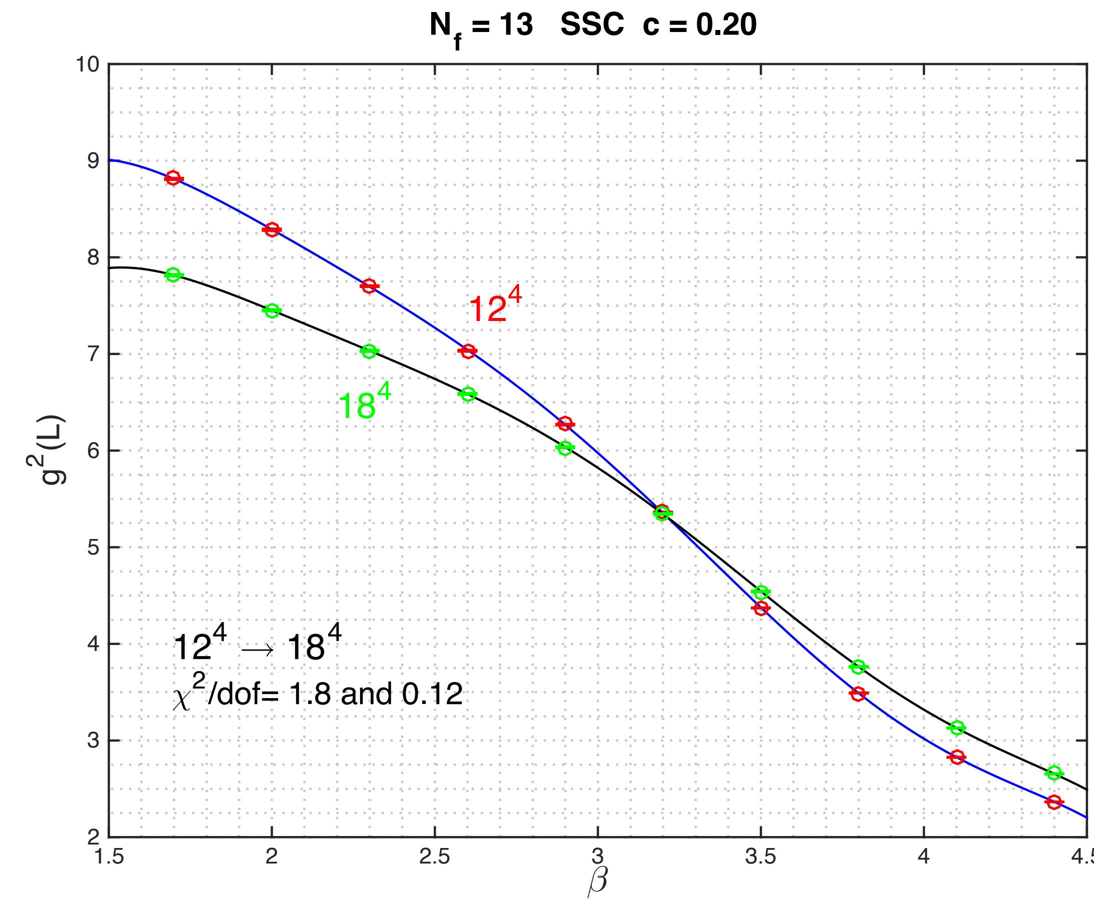
balance between good statistical accuracy (smaller c) and small cutoff effects (larger c)

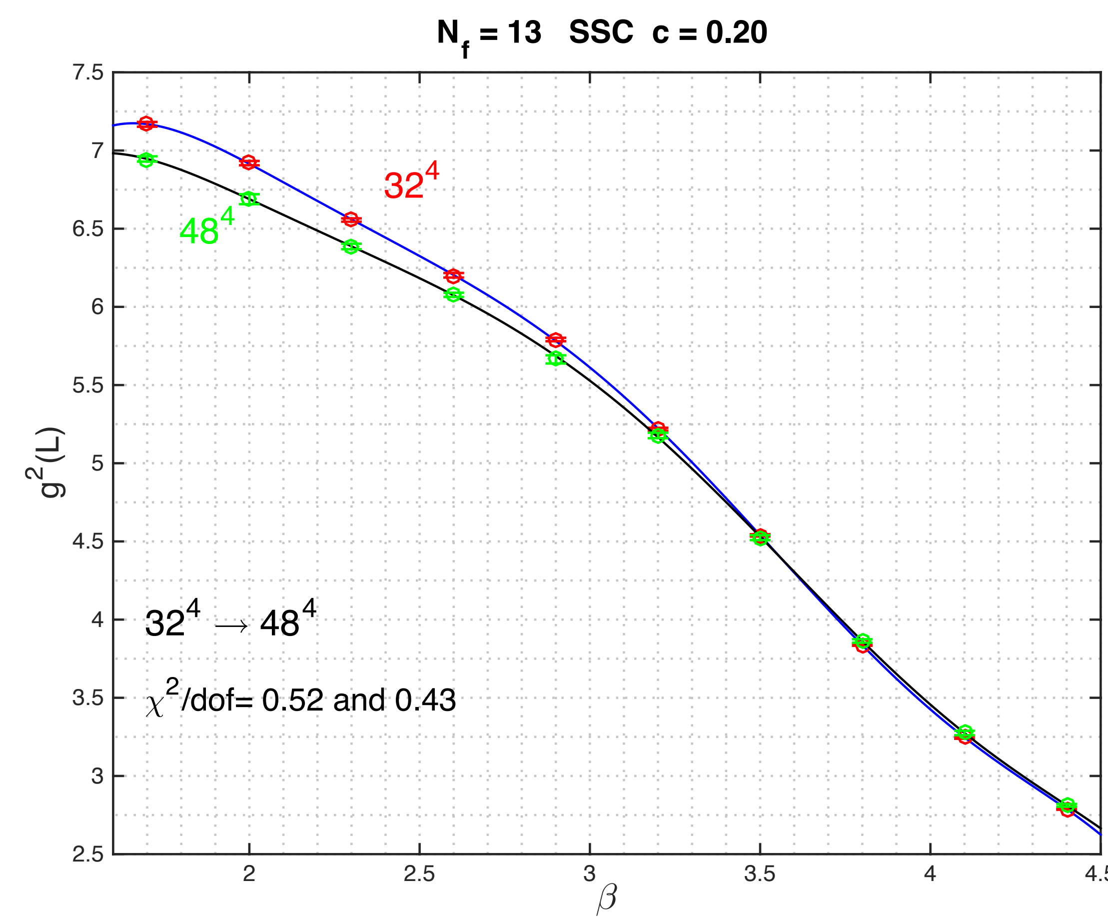


further analysis: polynomial interpolation of $g^2(L)$ in β

zero in step-function where curves cross

use interpolation to tune to chosen $g^2(L)$





continuum extrapolation at tuned coupling: $g^2(L) = 4.5$

leading cutoff effects expected to be $\mathcal{O}(a^2)$
with next order terms $\mathcal{O}(a^4)$

blue curve: quadratic fit in a^2

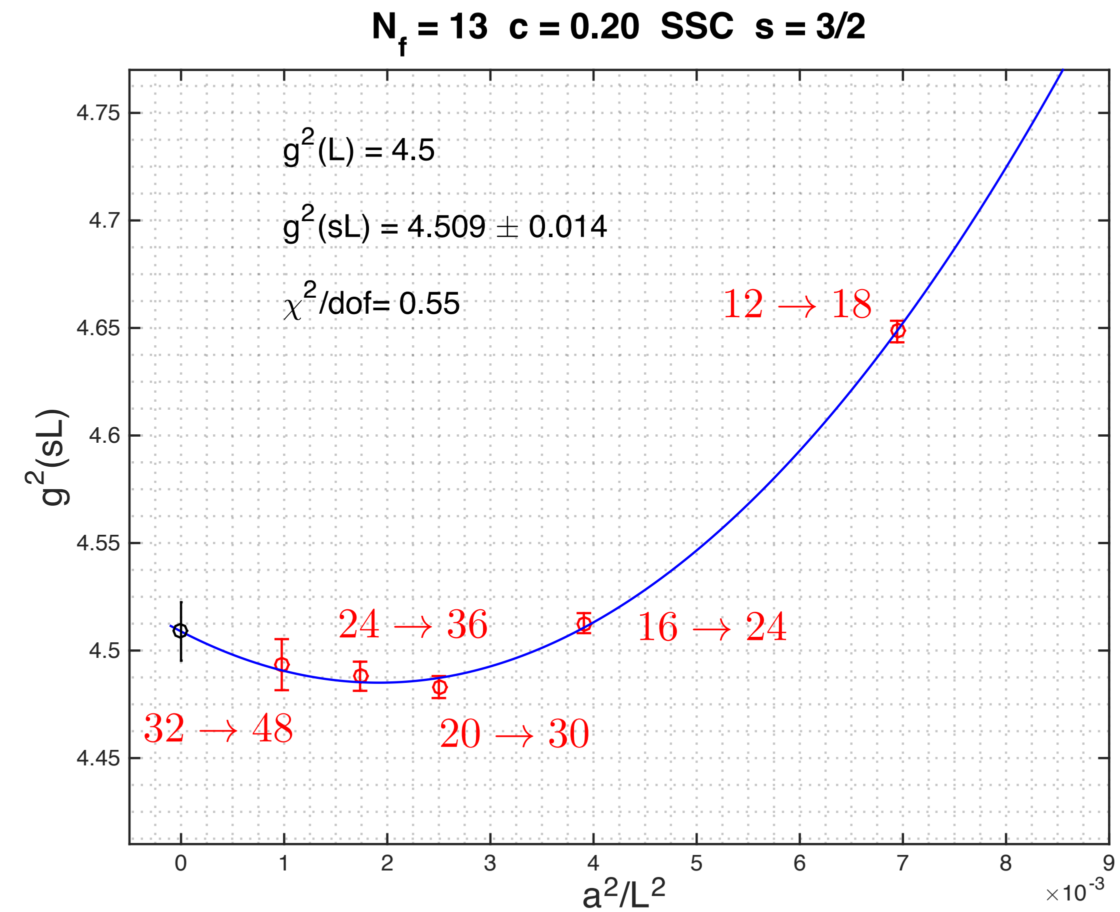
few per mille accuracy for individual data points and continuum result

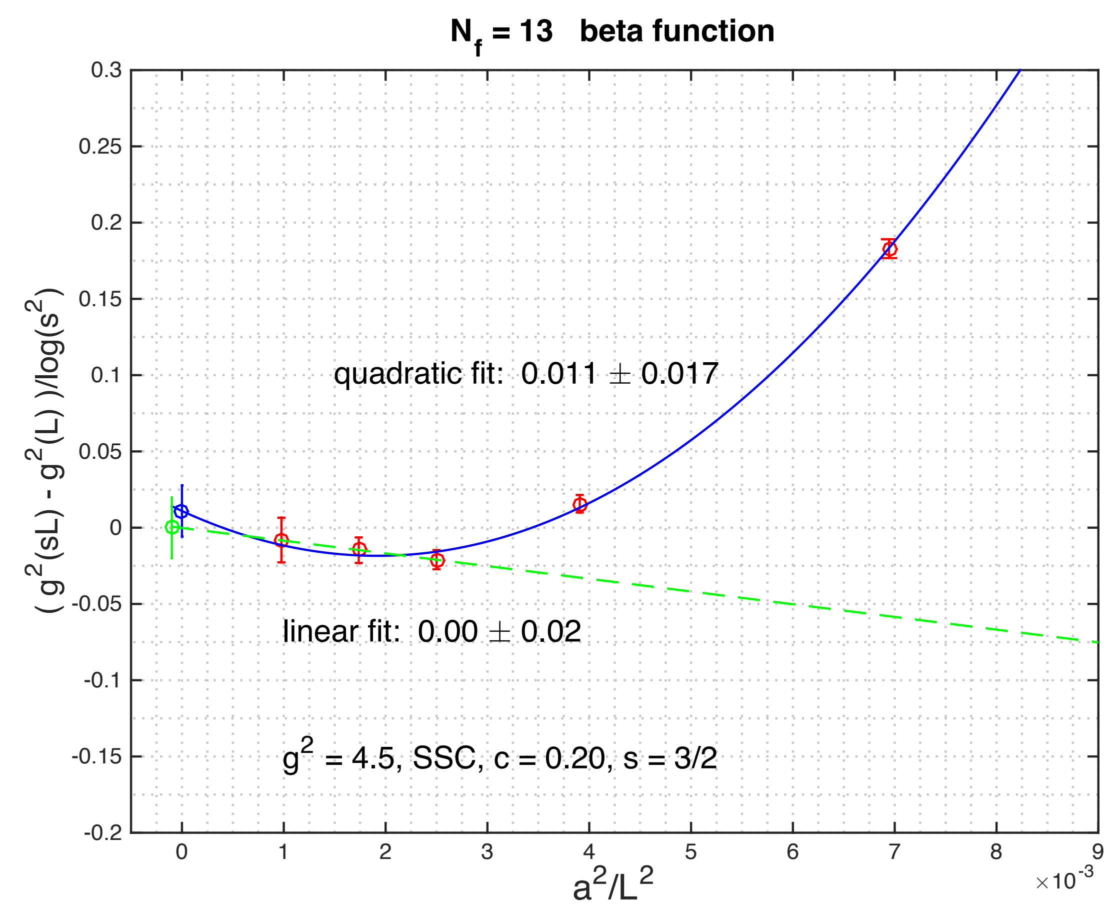
based on fit, coarser lattice spacing $8 \rightarrow 12$
would be significantly removed from continuum value

discrete step function $\dfrac{g^2(sL) - g^2(L)}{\log(s^2)}, \quad s = 3/2$

step function at this coupling is zero within error

linear fit in a^2 using 3 finest lattice spacings only gives consistent result (**green curve**)





repeat procedure for range of tuned renormalized couplings from ~ 3 to 4.5

shown here: result of quadratic continuum extrapolation

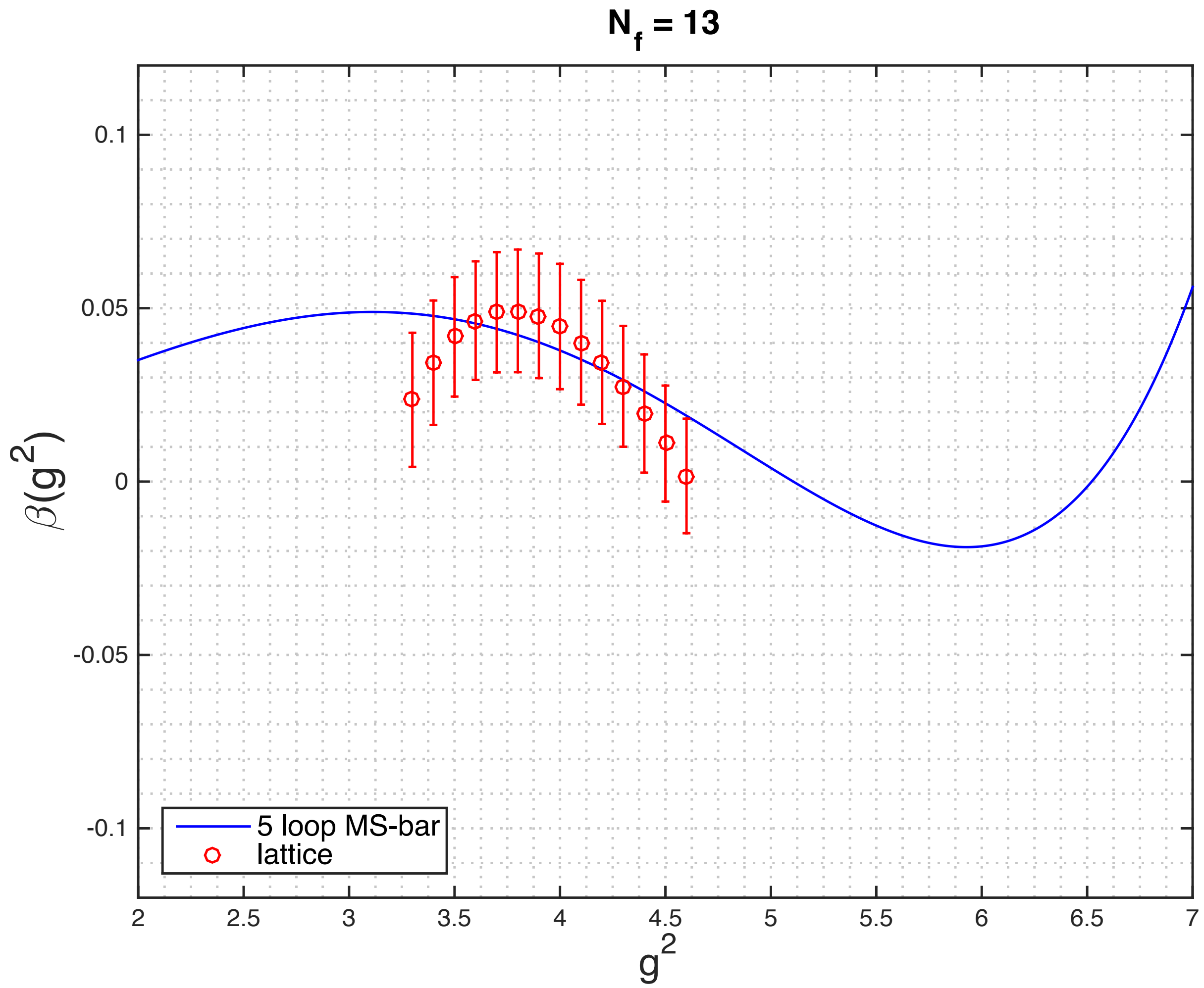
small but non-zero discrete step function

with an IRFP at $g^2 \sim 4.5$

for comparison: 5-loop MS-bar beta function

has local max ~ 0.05 and IRFP at $g^2 \sim 5$

non-perturbative results have similar qualitative and even quantitative behavior



at lower end of coupling expect beta function is too small to distinguish from zero at this level of accuracy

if we think beta function has an IRFP, can we demonstrate if it has conformal behavior?

additional properties: anomalous mass dimension γ^*

derivative of beta function at IRFP $\beta' = d\beta/dg^2|_{g_*^2}$

new technique presented by LatHC group at Lattice 2015:
Fodor et al PoS Lattice 2015 (2016) 310

full reconstruction of Dirac operator eigenvalue spectrum using Chebyshev polynomials to high order

$$\rho(t) = \frac{1}{\sqrt{1-t^2}} \sum_{k=0}^{\infty} c_k T_k(t)$$

uses recursive properties of Chebyshev polynomials and stochastic measurement of traces to extract c_k

example:

20 stochastic noises

maximum polynomial order 8000

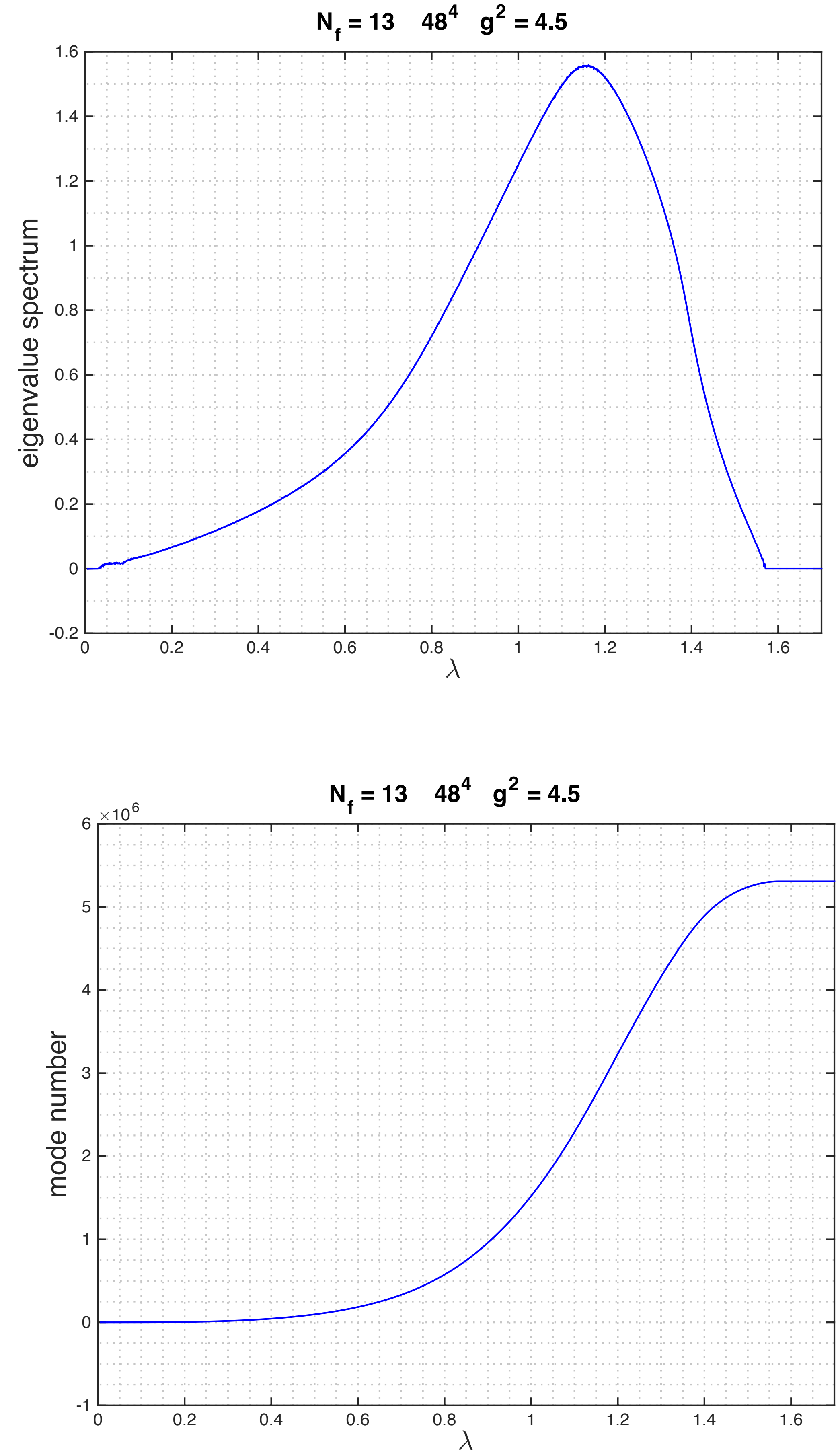
(bonus: all lower orders automatically generated)

statistical error is small, not visible on this scale

can reconstruct not only eigenvalue density but also e.g. mode number

$$\nu(\lambda) = \int_0^{\lambda} \rho(\lambda') d\lambda'$$

i.e. number of eigenvalues below some cut



new method presented by J Kuti for LatHC group at Simons Center Conformal Gauge Theories Workshop January 2018

anomalous dimension γ^* via step-scaling of mode number

continuum limit similar to step-function

mode number is renormalized:

$$\nu_R(\lambda_R) = \nu(\lambda), \quad \lambda_R = Z_p^{-1} \cdot \lambda$$

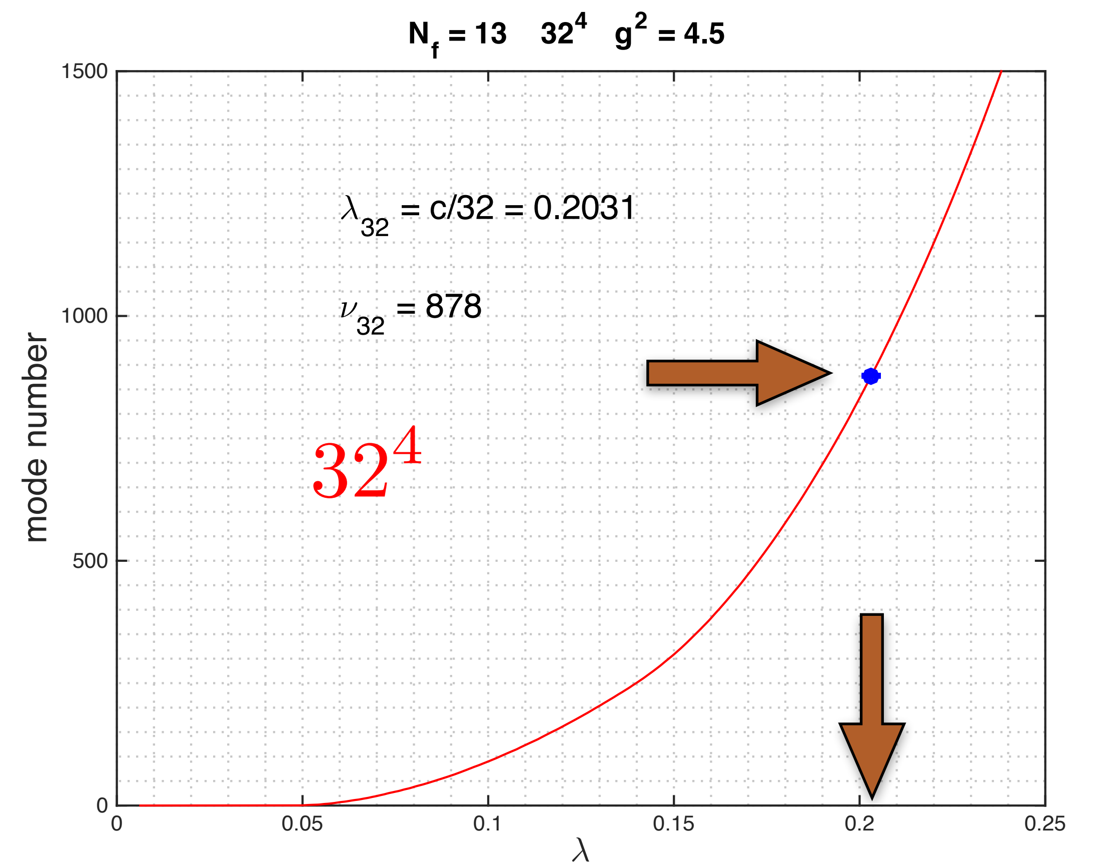
measure mode number for eigenvalue cut set via finite volume $\lambda_L = \dfrac{c}{L}$

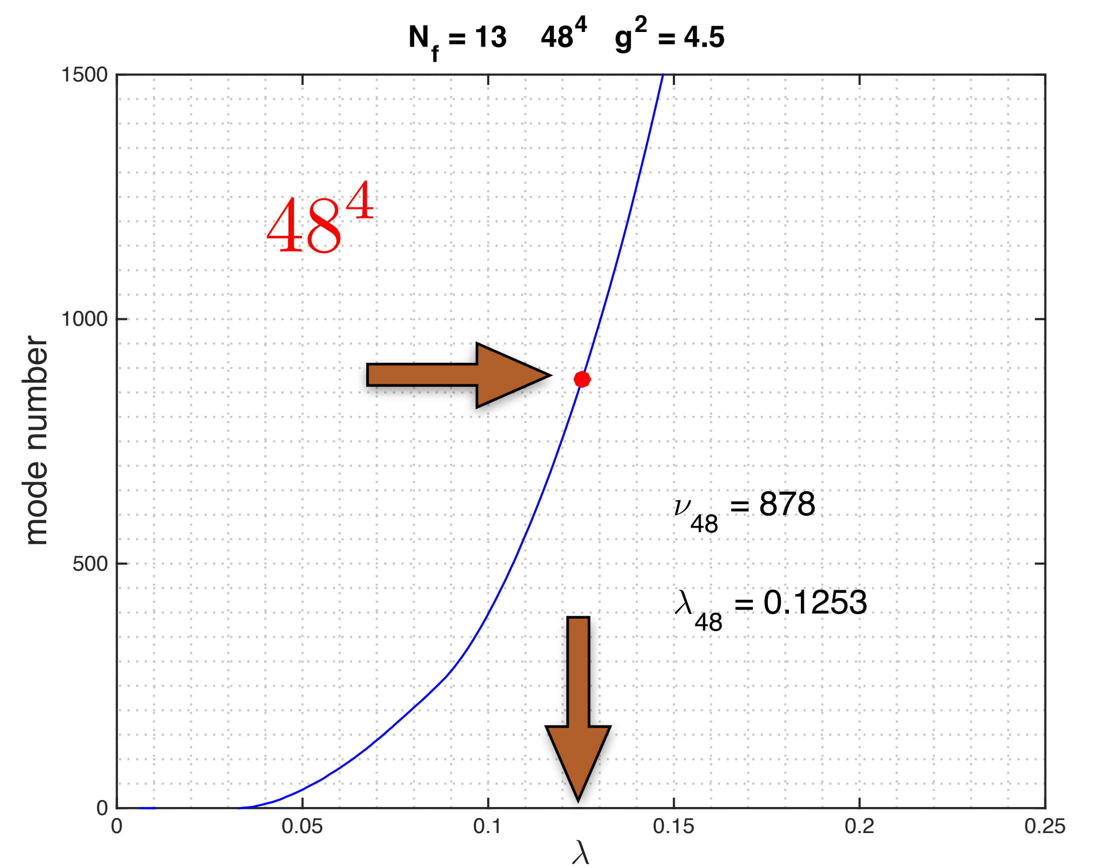
example upper figure: $L = 32, \quad c = 6.5$

match mode number with eigenvalue cut λ_{sL} for larger volume sL e.g. $s = 3/2, \quad sL = 48$

renormalization factors via eigenvalue ratio

$$\frac{\lambda_L}{\lambda_{sL}} = \frac{sZ_p(g_0, L/a)}{Z_p(g_0, sL/a)}$$





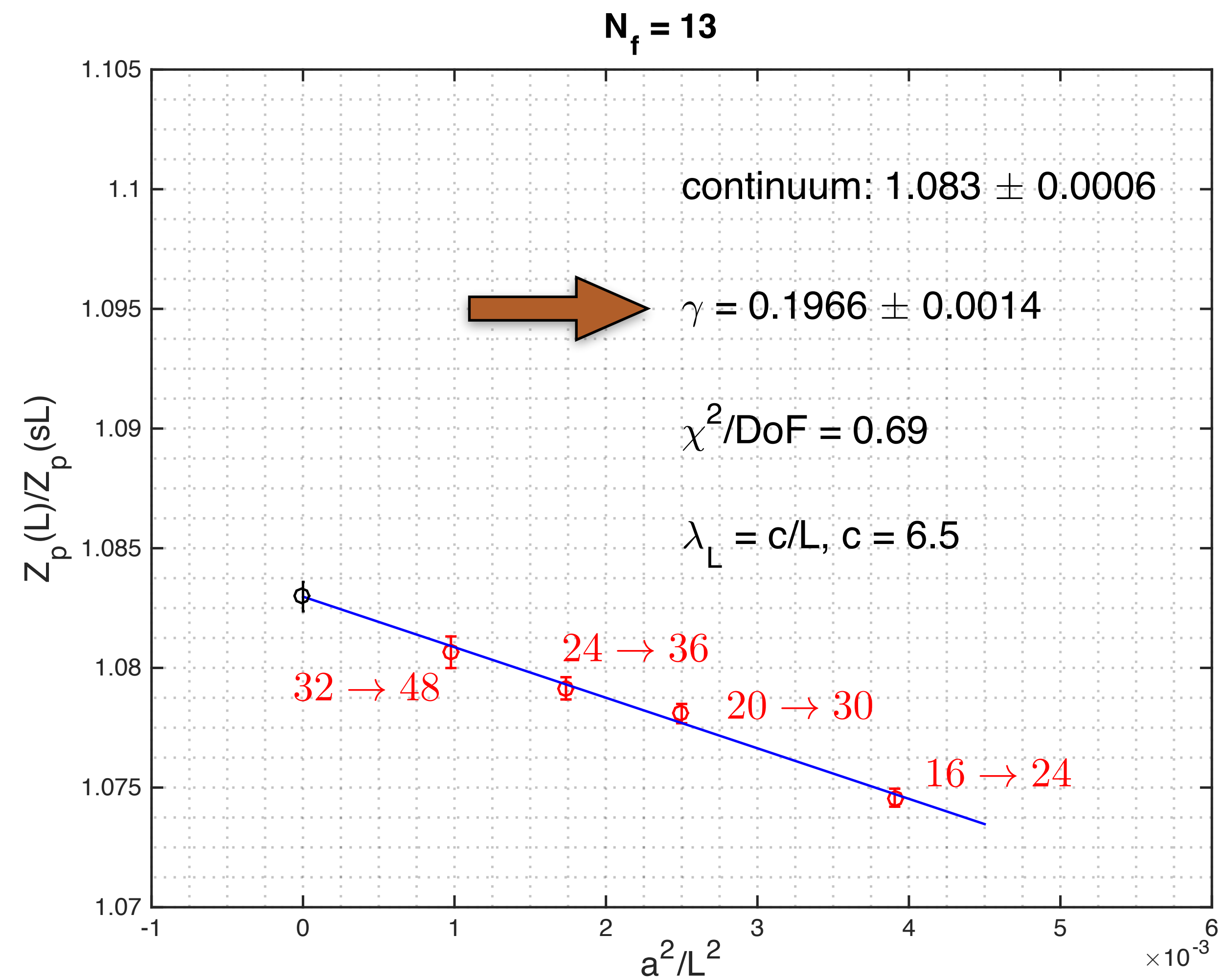
repeat for sequence of bare couplings and paired volumes $L \to sL$ at tuned renormalized coupling

cutoff effects removed as $a^2/L^2 \to 0$

blue curve: linear fit in a^2

anomalous dimension

$$\gamma = \frac{\log Z_p(L)/Z_p(sL)}{\log(s)}$$



at renormalized coupling ~ 4.5, anomalous dimension 0.1966(14)

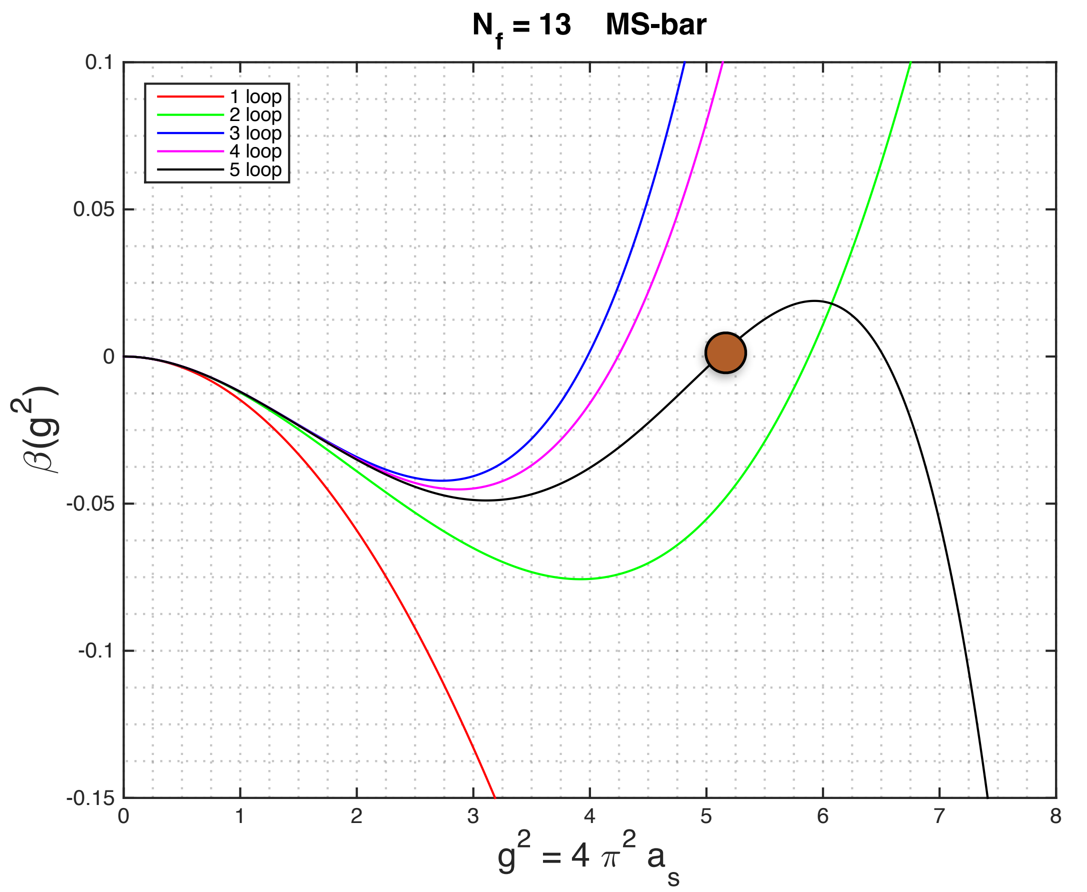
comparison with perturbation theory

MS-bar: Baikov, Chetyrkin, Kühn
JHEP 1410 (2014) 076

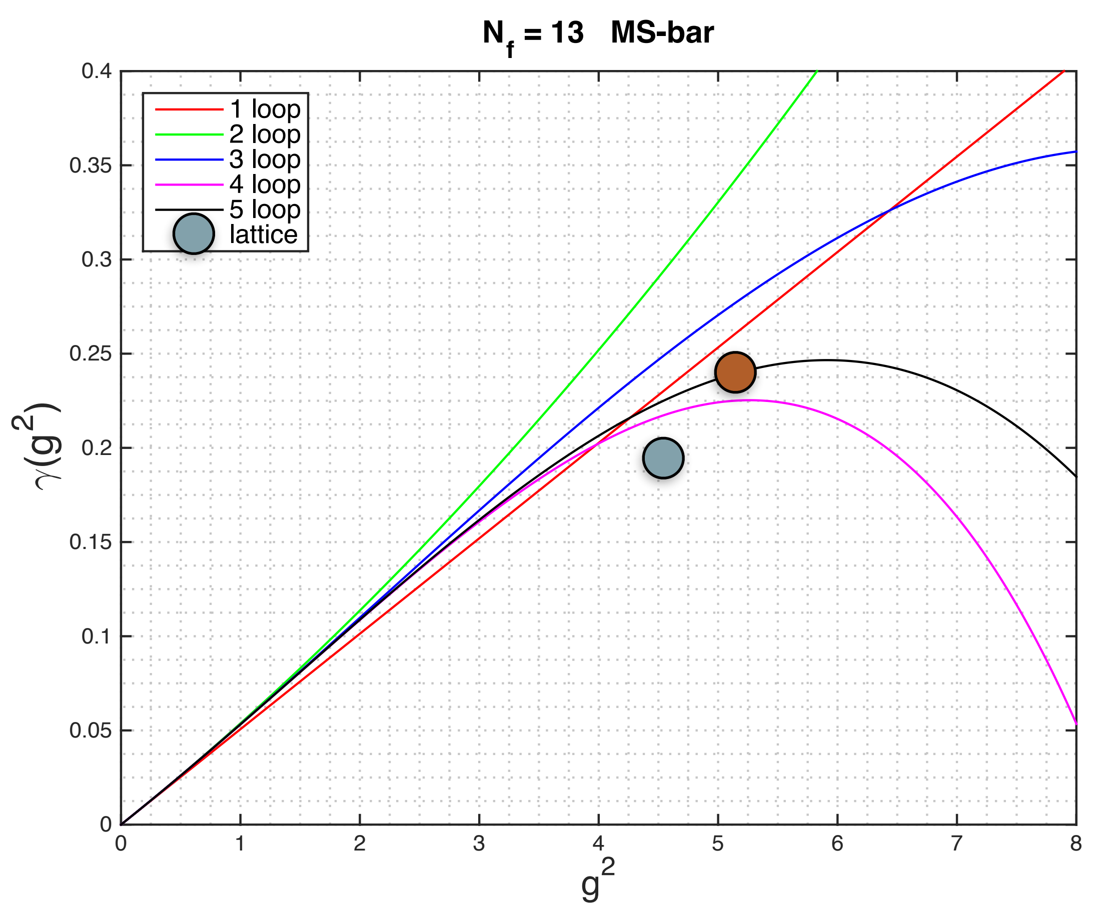
delta scheme: Ryttov, Shrock
PRD95 (2017) 105004

MS-bar loop order	γ^*	Ryttov-Shrock order	γ^*
2	0.404	1	0.174
3	0.220	2	0.221
4	0.210	3	0.231
5	0.239	4	0.237

perturbative predictions for anomalous dimension rather stable with increasing order



non-perturbative lattice result quite close to high loop order prediction



what happens at stronger coupling?

blue curve: quadratic fit in a^2

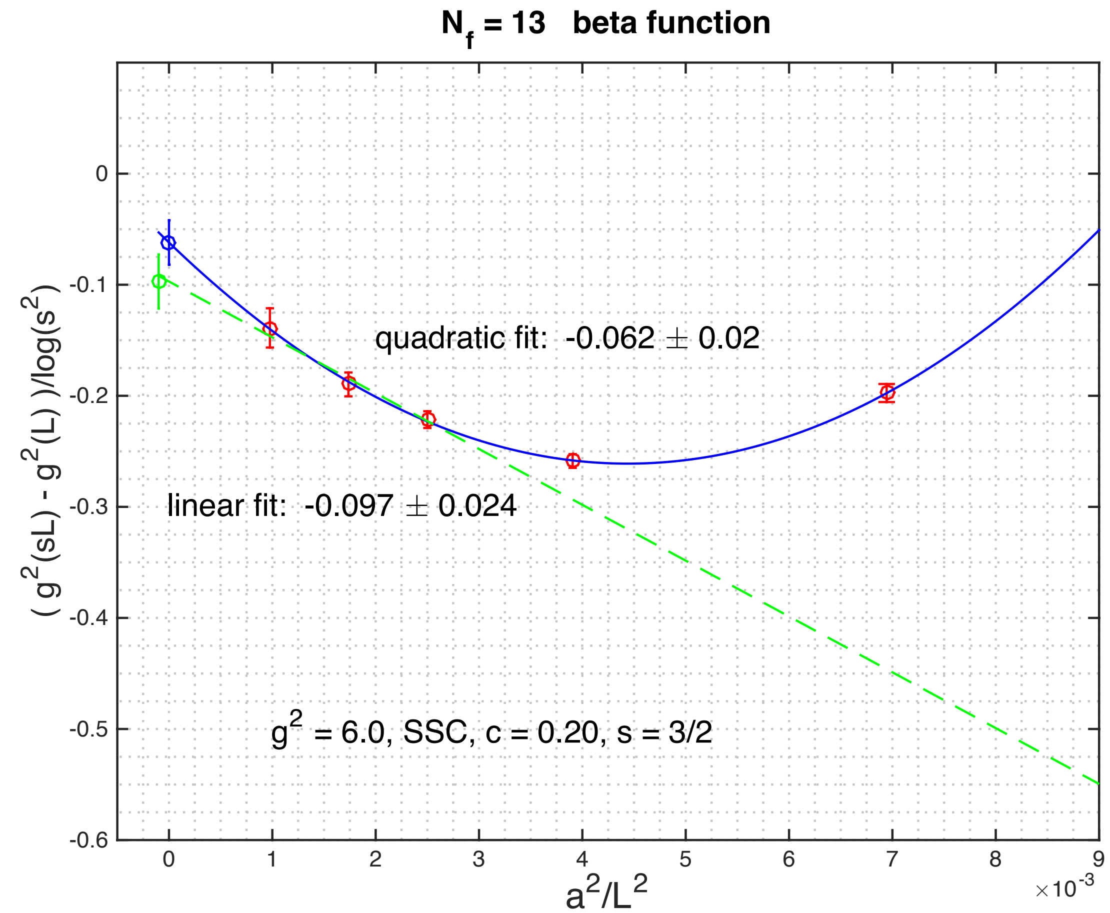
green curve: linear fit in a^2
of 3 smallest lattice spacings

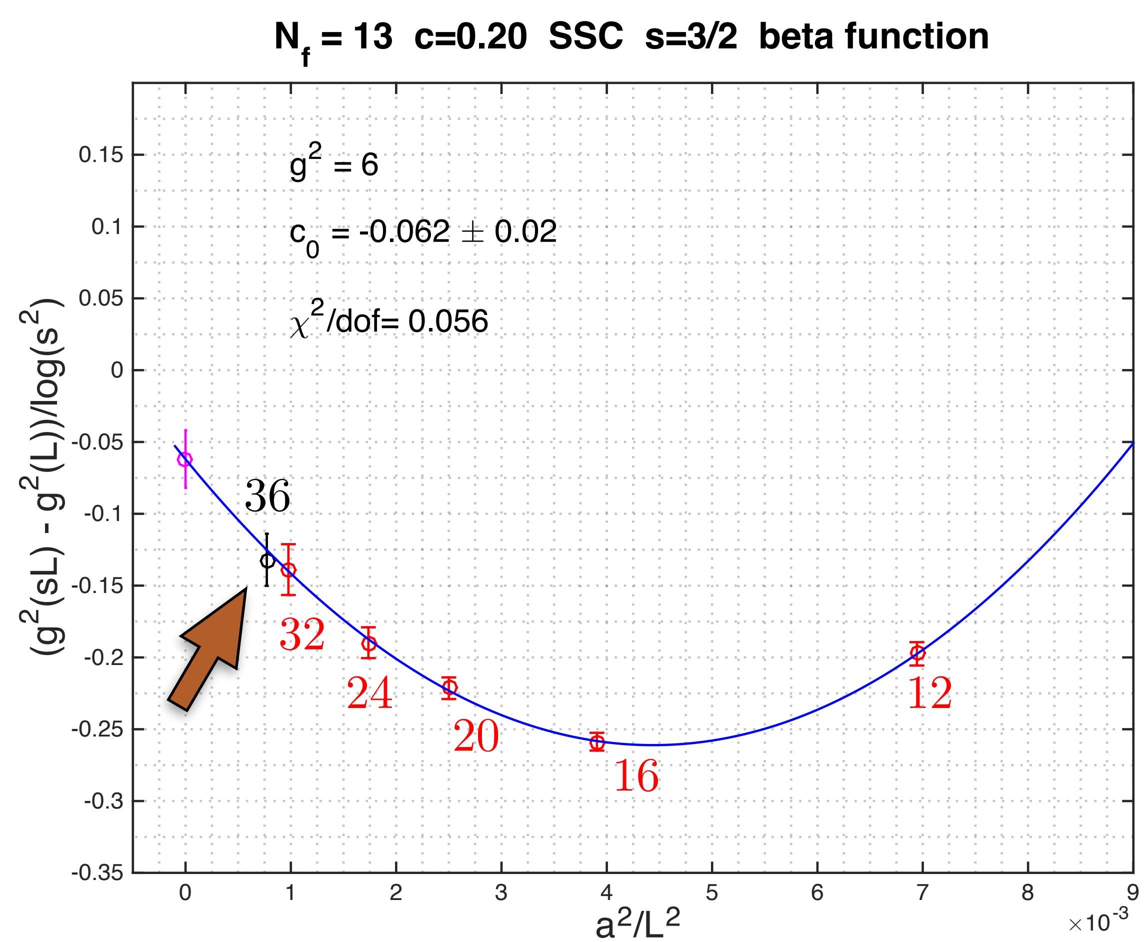
continuum step-function appears to change sign with increasing renormalized coupling

to check extrapolation: additional ensemble 54^4

one more step $36 \to 54$

black data point lies on curve predicted by fit of other lattice spacing data





quadratic continuum extrapolation across range of renormalized couplings

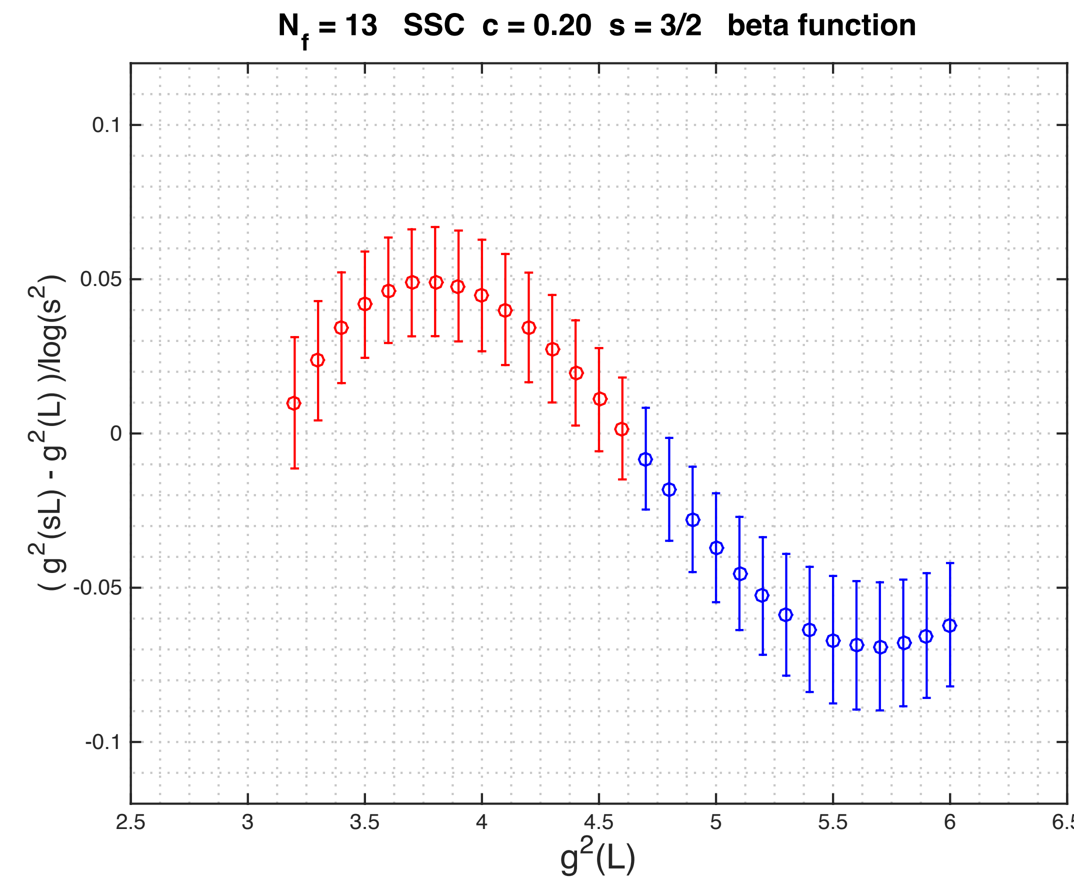
physical property: slope of beta function at IRFP

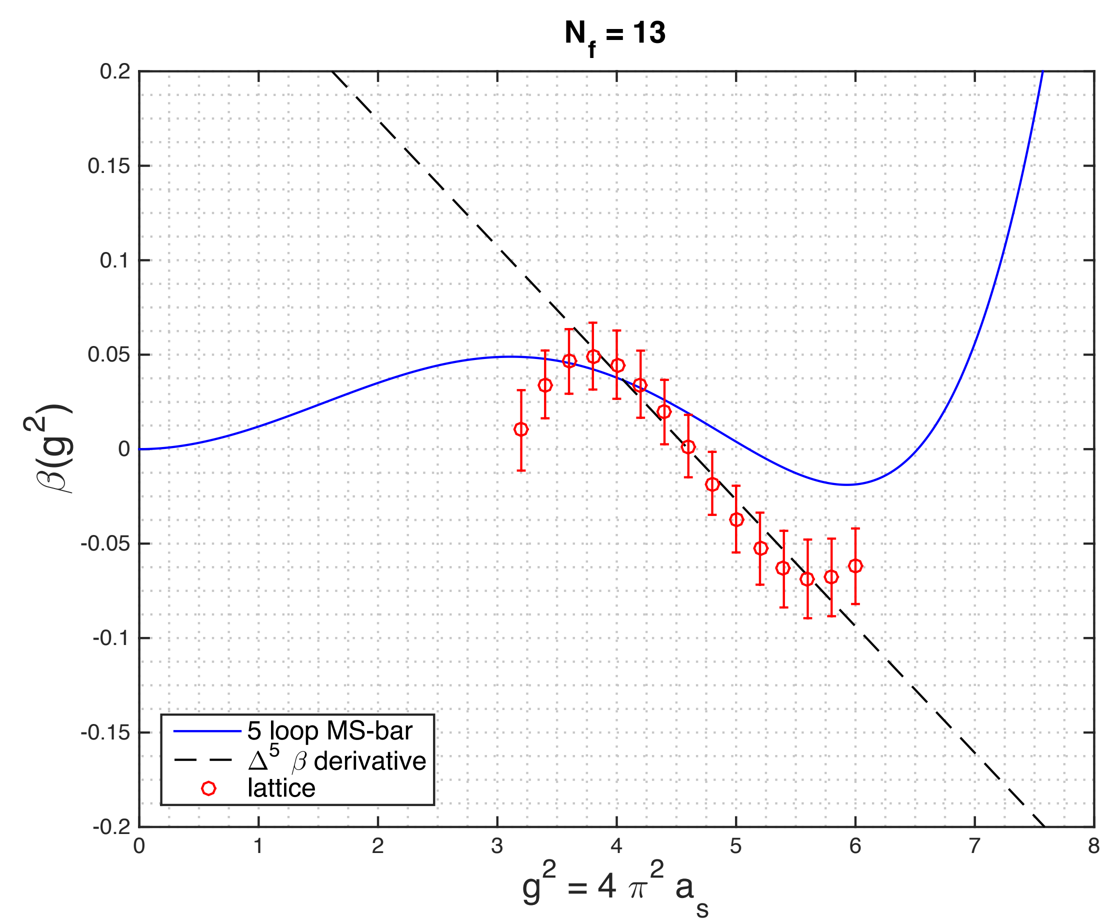
$$\beta' = d\beta/dg^2|_{g_*^2}$$

MS-bar loop order	β'	Ryttov-Shrock delta order	β'
2	0.087	2	0.051
3	0.078	3	0.072
4	0.075	4	0.069
5	0.037	5	0.067

perturbative results quite close to lattice data

5-loop MS-bar beta function less steep than lower orders





compare discretization choice

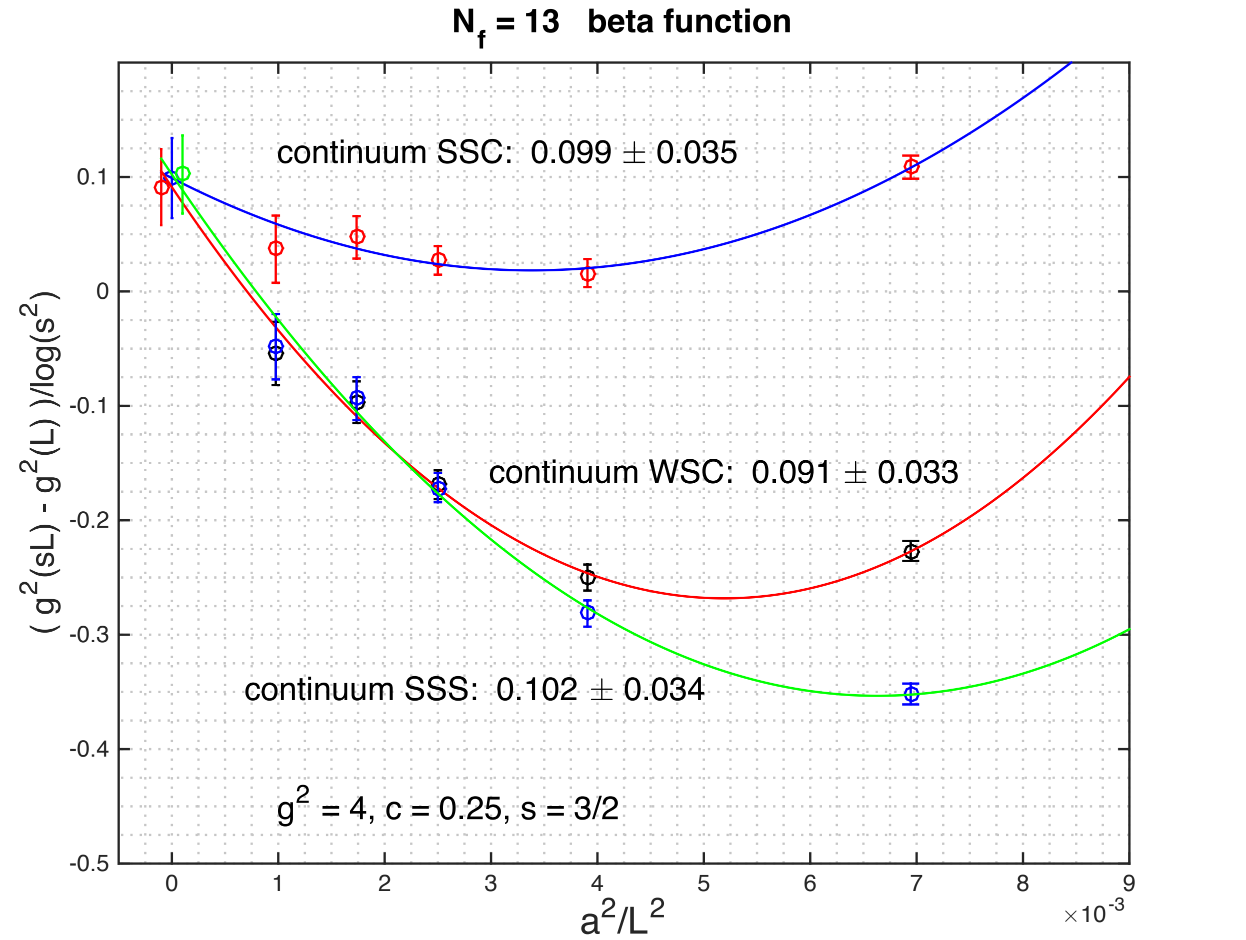
SSC: Symanzik for MC and Flow, Clover for E

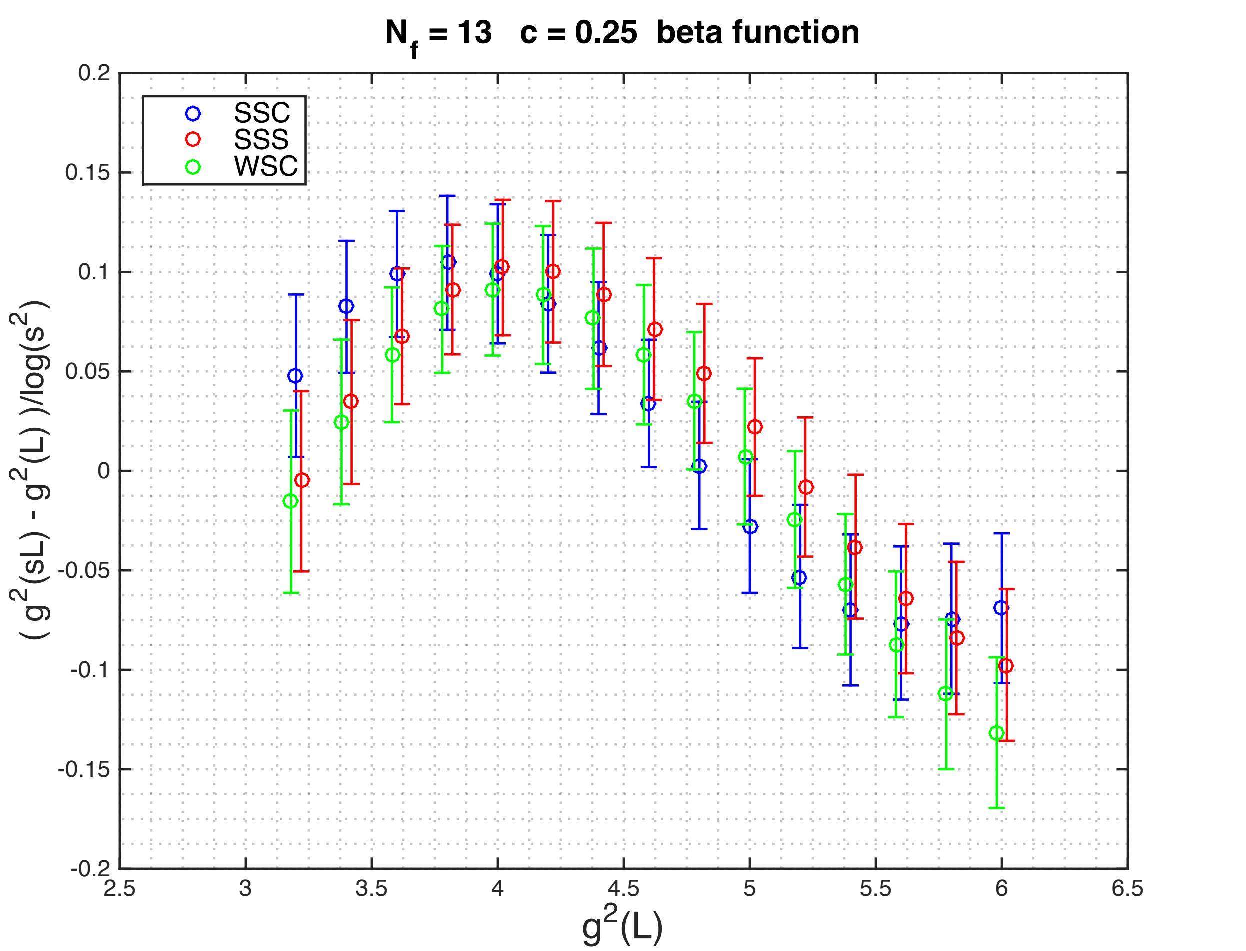
SSS: Symanzik for all 3

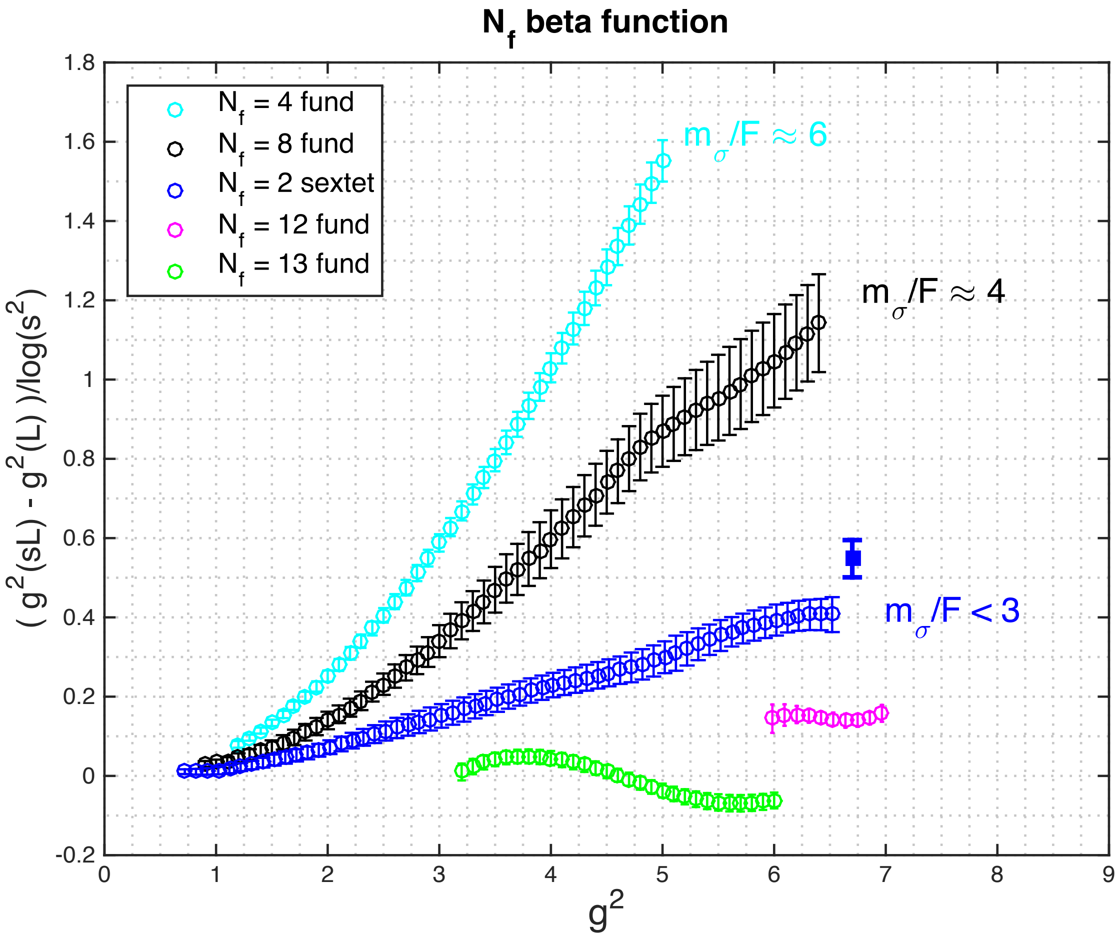
WSC: Wilson for Flow, Symanzik for MC, Clover for E

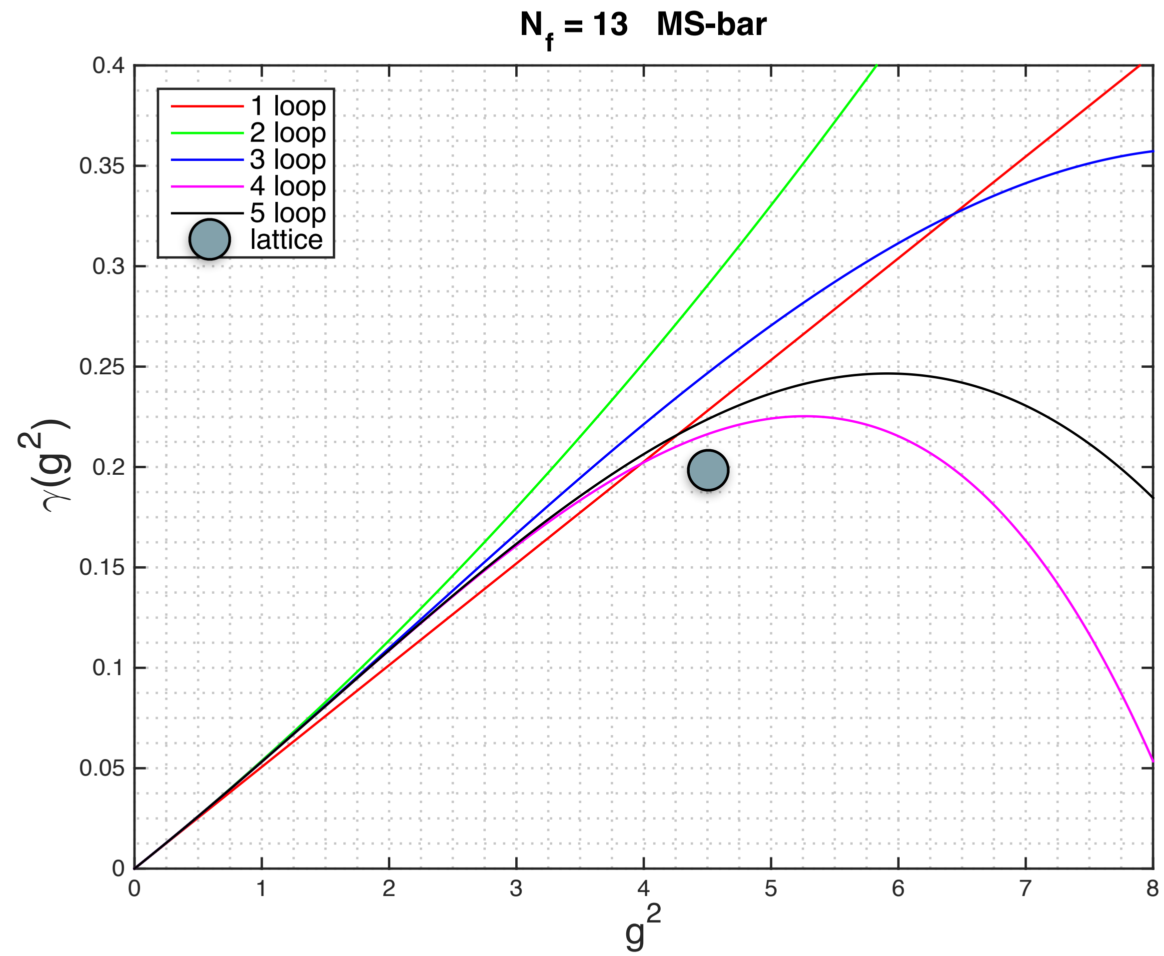
larger cutoff effects for WSC and SSS — similar to our experience in other Nf beta function studies

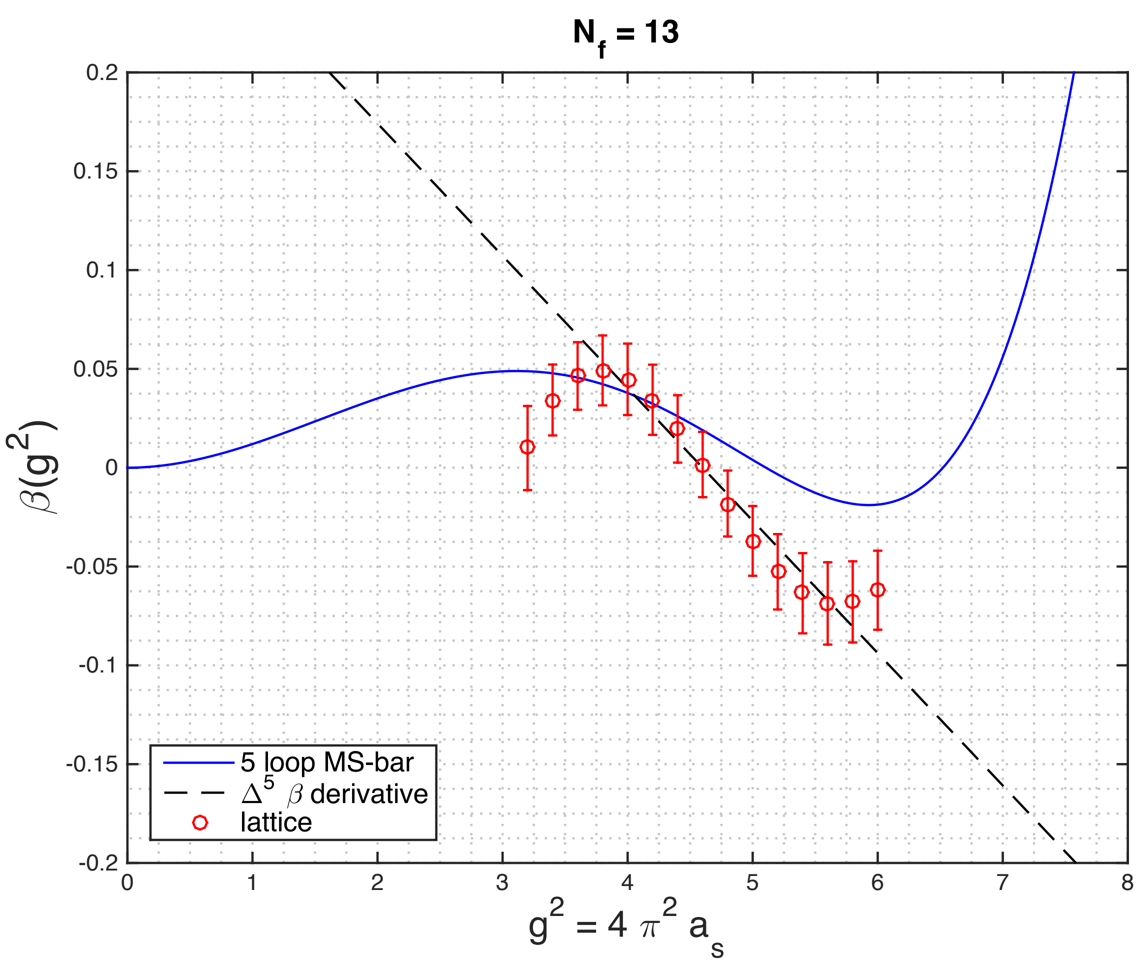
consistent results in the continuum limit from independent fits — a very useful crosscheck











Nf = 13 does appear conformal: not just a zero in the beta function, but also conformal like in physical properties, can test new pert thy results

to date, looks like Nf = 12 walking, but not inside the conformal window

where does Nf = 10 appear in this picture?

wait for more: Daniel Nogradi on Nf = 10 (next talk)

Lattice 2018 K Holland