Non perturbative physics from NSPT: renormalons, the gluon condensate and all that

F. Di Renzo
University of Parma and INFN

in collaboration with L. Del Debbio e G. Filaci (Edinburgh)

arXiv:1807.09518

LATTICE 2018

MSU, 26-07-2018

Snapshots of Hadrons

or the Story of How the Vacuum Medium Determines the Properties of the Classical Mesons Which Are Produced, Live and Die in the QCD Vacuum

M. Shifman

Theoretical Physics Institute, Univ. of Minnesota, Minneapolis, MN 55455

...

It would be more accurate to say "the method of expansion of the correlation functions in the vacuum condensates with the subsequent matching via the dispersion relations". This is evidently far too long a string to put into circulation. Therefore, for clarity I will refer to the Shifman-Vainshtein-Zakharov (SVZ) sum rules. Sometimes, I will resort to abbreviations such as "the condensate expansion".

Twenty years ago, next to nothing was known about nonperturbative aspects of QCD. The condensate expansion was the first quantitative approach which proved to be successful in dozens of problems. Since then, many things changed. Various new ideas and models were suggested concerning the peculiar infrared behavior in Quantum Chromodynamics. Lattice QCD grew into a powerful computational scheme which promises, with time, to produce the most accurate results, if not for the whole set of the hadronic parameters, at least, for a significant part.

NH ELSEVIER

Nuclear Physics B 457 (1995) 202–216

NUCLEAR PHYSICS B

Renormalons from eight-loop expansion of the gluon condensate in lattice gauge theory ⋆

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PRL **108,** 242002 (2012) PHYSICAL REVIEW LETTERS week ending 15 JUNE 2012

Compelling Evidence of Renormalons in QCD from High Order Perturbative Expansions

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(Received 16 November 2011; published 12 June 2012)

We compute the static self-energy of SU(3) gauge theory in four spacetime dimensions to order α^{20} in the strong coupling constant α. We employ lattice regularization to enable a numerical simulation within the framework of stochastic perturbation theory. We find perfect agreement with the factorial growth of high order coefficients predicted by the conjectured renormalon picture based on the operator product expansion.

DOI: 10.1103/PhysRevLett.108.242002 PACS numbers: 12.38.Cy, 11.10.Jj, 11.15.Bt, 12.38.Bx

Agenda

- Getting the GLUON CONDENSATE from the OPE for the PLAQUETTE

- IR RENORMALONS

- Numerical Stochastic Perturbation Theory

- Numerical results

- Conclusions and prospects

The GLUON CONDENSATE and the OPE for the PLAQUETTE

One would like to compute the GLUON CONDENSATE

$$O_G = -\frac{2}{\beta_0}\frac{\beta(\alpha)}{\alpha}\sum_{a,\mu,\nu} G^a_{\mu\nu}G^a_{\mu\nu}$$

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In the naive continuum limit, the PLAQUETTE does the job for you …

$$a^{-4}P \xrightarrow{a\to 0} \frac{\pi^2}{12N_c}O_G = \frac{\pi^2}{12N_c}\left(\frac{\alpha}{\pi}G^2\right)$$

$$O_G = \frac{\alpha}{\pi}G^2\left[1 + O(\alpha)\right] \; .$$

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… but then you have MIXING with LOWER DIMENSIONAL OPERATORS, int the case at hand the IDENTITY, which results in POWER DIVERGENCE!

$$a^{-4}P = a^{-4}Z(\beta)\mathbb{1} + \frac{\pi^2}{12N_c}C_G(\beta)O_G + O(a^2\Lambda^6_{\mathrm{QCD}})$$

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$$a^{-4}P = a^{-4}Z(\beta)\mathbb{1} + \frac{\pi^2}{12N_c}C_G(\beta)O_G + O(a^2\Lambda^6_{\rm QCD})$$

This also means that if take Monte Carlo MEASUREMENTS you can read

$$\langle P\rangle_{\rm MC} = Z(\beta) + \frac{\pi^2}{12N_c}C_G(\beta)a^4\langle O_G\rangle + O(a^6\Lambda^6_{\rm QCD})$$

… in which you recognise an OPE: in a given regime you separate scales and you know that Wilson coefficients are computable in Perturbation Theory

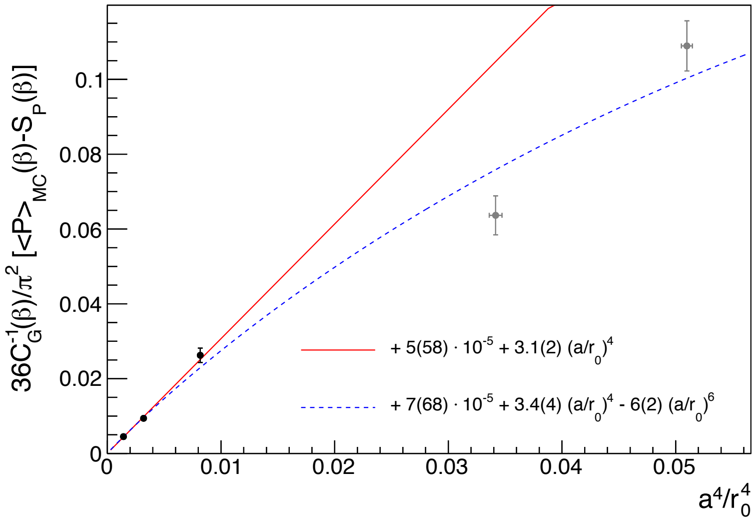
$$a^{-1} \gg \Lambda_{\rm QCD}$$

$$Z(\beta) = \sum_{n=0} p_n\beta^{-(n+1)}\,, \quad C_G(\beta) = 1+\sum_{n=0} c_n\beta^{-(n+1)}$$

$$\langle P \rangle_{\mathrm{MC}} = Z(\beta) + \frac{\pi^2}{12 N_c} C_G(\beta) a^4 \langle O_G \rangle + O(a^6 \Lambda^6_{\mathrm{QCD}})$$

has been the starting point for LATTICE DETERMINATIONS of the GLUON CONDENSATE

- COMPUTE THE PLAQUETTE BY MONTE CARLO
- COMPUTE THE PLAQUETTE IN PERTURBATION THEORY
- SUBTRACT THE LATTER FROM THE FORMER
- LOOK FOR ASYSMPTOTIC SCALING
- READ THE GLUON CONDENSATE



This goes back to work by the PISA GROUP in the (late) eighties and nineties.

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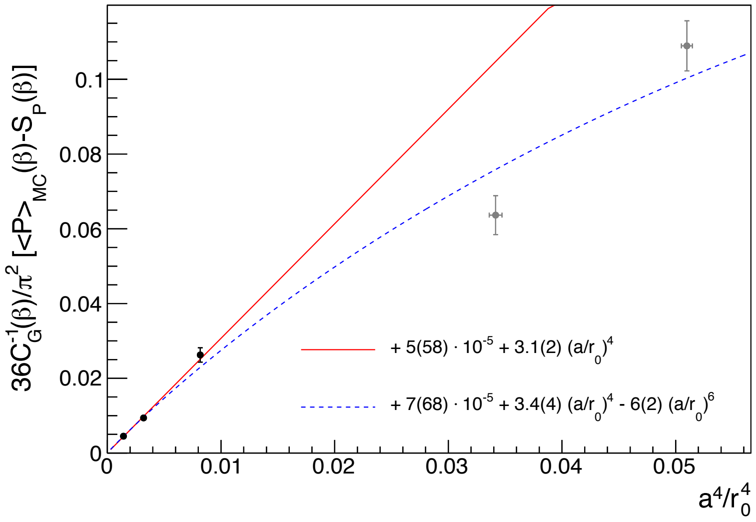

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But there is a problem:
the OPE separates the scales, but not PERTURBATIVE AND NON-PERTURBATIVE PHYSICS

There is a contribution in the perturbative tail attached to the identity which scales exactly as the gluon condensate. This has to do with the fact that PERTURBATIVE SERIES IN FIElD THEORIES ARE ASYMPTOTIC, which in turns gives rise to ambiguities which in asymptotically free field theories are known to have to do with the beta function: this is famous/infamous story of the IR RENORMALON.

The IR RENORMALON enters the stage…

Expected form for a condensate of dim 4

$$W^{\rm ren} = C \int_0^{Q^2} \frac{k^2\, dk^2}{Q^4}\, \alpha_s(k^2)$$

Change variable

$$z \equiv z_0 \left(1 - \alpha_s(Q^2)/\alpha_s(k^2)\right), \qquad z_0 \equiv \frac{1}{3b_0} \qquad 4\pi\alpha_s(Q^2) \equiv 6/\beta\,, \qquad \gamma \equiv 2\frac{b_1}{b_0^2}$$

You end up with a new integral representation
(BOREL INTEGRAL)

$$W^{\rm ren} = \mathcal{N} \int_0^{\infty} dz\, e^{-\beta z}\, (z_0 - z)^{-1-\gamma}$$

This directly encodes the perturbative behaviour

$$W^{\rm ren} = \sum_{\ell=1} \beta^{-\ell} \left\{c_\ell^{\rm ren} + \mathcal{O}(e^{-z_0\beta})\right\}, \qquad c_\ell^{\rm ren} = \mathcal{N}'\, \Gamma(\ell+\gamma)\, z_0^{-\ell}$$

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All in all

- The coefficients grow factorially
- In order to compute the integral you pick up an imaginary part proportional to $e^{-\beta z_0}$
- In order to sum the series you need a prescription, with an ambiguity which turns out to be just of the same order
- The ambiguity at hand scales just as the GC!

$$e^{-\beta z_0} \sim \frac{\Lambda^4}{Q^4}$$

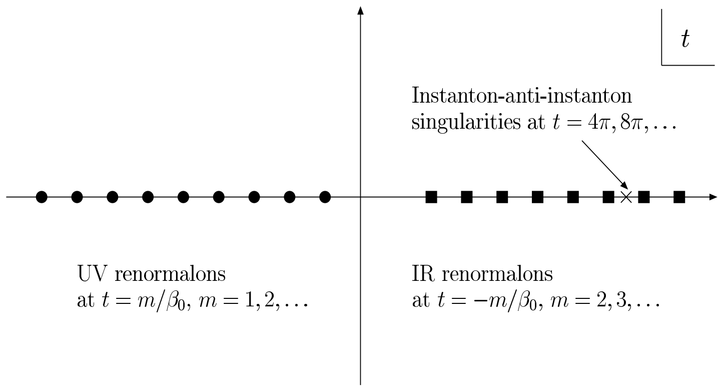

Figure 3: Singularities in the Borel plane of $\Pi(Q^2)$, the current-current correlation function in QCD. Shown are the singular points, but not the cuts attached to each of them. Recall that $\beta_0 < 0$ according to (2.18).

NSPT (directly in the LGT case) Di Renzo, Marchesini, Onofri 94

YM theory: start with the Wilson action

$$S_G = -\frac{\beta}{2N_c} \sum_P \mathrm{Tr} \left(U_P + U_P^\dagger \right)$$

Langevin equation:

$$\frac{\partial}{\partial t} U_{x\mu}(t;\eta) = \left(-i \nabla_{x\mu} S_G[U] - i \eta_{x\mu}(t) \right) U_{x\mu}(t;\eta)$$

$$\langle \eta_{i,k}(z)\ \eta_{l,m}(w) \rangle_\eta = \left[\delta_{il}\, \delta_{km} - \frac{1}{N_c}\, \delta_{ik}\, \delta_{lm} \right] \delta_{zw}$$

Asymptotically in stochastic time

$$\lim_{t\to\infty} \langle O[U(t;\eta)] \rangle_\eta = \frac{1}{Z} \int DU\, e^{-S_G[U]}\, O[U]$$

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Now we look for a solution in the form of a perturbative expansion

$$U_{x\mu}(t;\eta) \to 1 + \sum_{k=1} \beta^{-k/2} U_{x\mu}^{(k)}(t;\eta)$$

… which you can plug e.g. in an Euler scheme

$$U_{x\mu}(n+1;\eta) = e^{-F_{x\mu}[U,\eta]}\; U_{x\mu}(n;\eta) \qquad F_{x\mu}[U,\eta] = \epsilon \nabla_{x\mu} S_G[U] + \sqrt{\epsilon}\, \eta_{x\mu}$$

Batrouni et al (Cornell group) PRD 32 (1985)

not the end of the story: STOCHASTIC GAUGE FIXING

To have gauge degrees of freedom under control interleave a gauge fixing step to the Langevin evolution

$$U'_{x\mu} = e^{-F_{x\mu}[U,\eta]}\, U_{x\mu}(n)$$

$$U_{x\mu}(n+1) = e^{w_x[U']}\, U'_{x\mu}\, e^{-w_{x+\hat{\mu}}[U']}$$

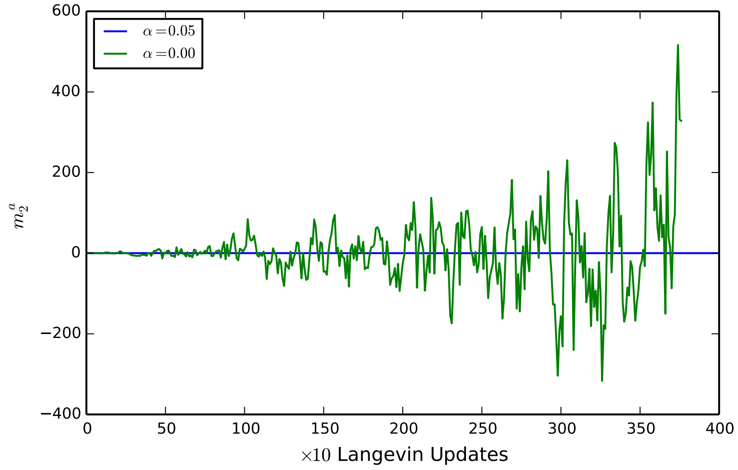

Figure 1. The effect of stochastic gauge fixing.

which has by the way an obvious interpretation

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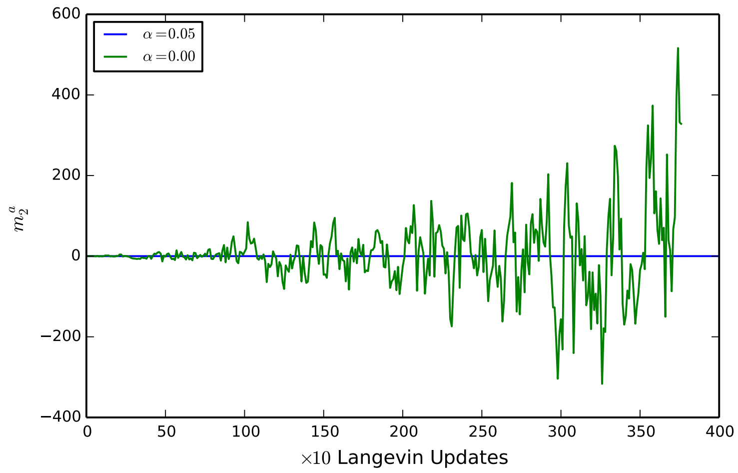

Figure 1. The effect of stochastic gauge fixing.

not the end of the story: FERMIONS, i.e. QCD Di Renzo, Scorzato 2001

From the point of view of the functional integral measure $\quad e^{-S_G} \det M = e^{-S_{eff}} = e^{-(S_G - Tr \ln M)}$

and in turns $\quad \nabla^a_{x\mu} S_G \mapsto \nabla^a_{x\mu} S_{eff} = \nabla^a_{x\mu} S_G - \nabla^a_{x\mu} \mathrm{Tr}\, \ln M = \nabla^a_{x\mu} S_G - \mathrm{Tr}\, ((\nabla^a_{x\mu} M) M^{-1})$

Batrouni et al (Cornell group) PRD 32 (1985)

In $\; U_{x\mu}(n+1;\eta) = e^{-F_{x\mu}[U,\eta]}\, U_{x\mu}(n;\eta) \;$ we now write

$$F = T^a(\epsilon \Phi^a + \sqrt{\epsilon}\eta^a) \qquad \Phi^a = \left[\nabla^a_{x\mu} S_G - \mathrm{Re}\left({\xi_k}^\dagger (\nabla^a_{x\mu} M)_{kl} (M^{-1})_{ln} \xi_n\right)\right]$$

where $\; \langle \xi_i \xi_j \rangle_\xi = \delta_{ij} \;$ or (this is what we always do)

$$\Phi^a = \left[\nabla^a_{x\mu} S_G - \mathrm{Re}\left({\xi_l}^\dagger (\nabla^a_{x\mu} M)_{ln} \psi_n\right)\right] \qquad M_{kl}\psi_l = \xi_k$$

A first high order computation in LQCD

We use twisted BC (no zero modes!)

$$U_\mu(x+L\hat{\nu}) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger\,, \qquad \Omega_\nu\Omega_\mu = z_{\mu\nu}\Omega_\mu\Omega_\nu\,, \qquad z_{\mu\nu} \in Z_{N_c}$$

and consistently give fermions (fundamental representation) smell degrees of freedom
(copies which transform into each other according to the anti fundamental representation of the gauge group; physical observables are singlets!)

$$\psi(x+L\hat{\nu})_{ir} = \sum_{j,s} \left(\Omega_\nu\right)_{ij}\psi(x)_{js}\left(\Lambda_\nu^\dagger\right)_{sr}\,, \qquad \Lambda_\nu \in \mathrm{SU}(N_c)$$

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We tried both Wilson fermions
(exploratory study of critical mass)

n	$-m_c^{(n)}$ on 16^4	$-m_c^{(n)}$ in infinite volume
1	$2.61083\ldots$	$2.60571\ldots$
2	$4.32(3)$	$4.293(1)$ [34, 35]
3	$1.21(1)\cdot 10^1$	$1.178(5)\cdot 10^1$ [36, 37]
4	$3.9(2)\cdot 10^1$	$3.96(4)\cdot 10^1$ [37]
5	$1.7(2)\cdot 10^2$	-
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… and staggered fermions (for the first time in NSPT);
these are ultimately to prefer to go the really high order computations…

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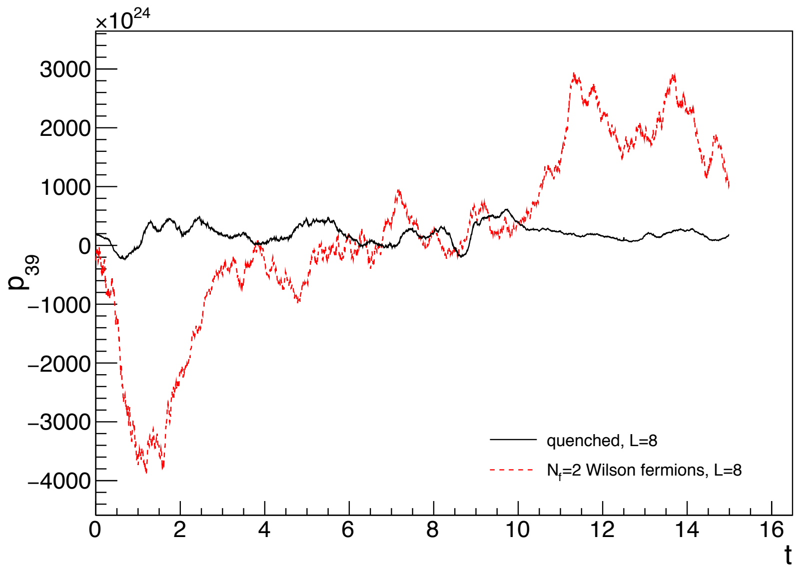
There is now a GRID version of NSPT! (thanks to P. Boyle and G. Cossu for cooperation)

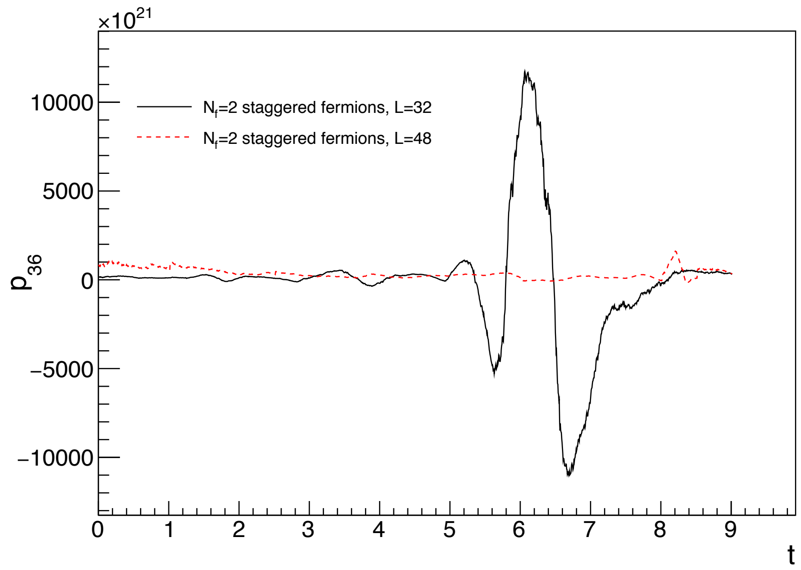
We experienced some NUMERICAL INSTABILITIES

Very high orders for toy models are indeed known to have numerical instabilities.

Not yet found in (quenched) high orders NSOT simulations

Now we found them with fermions in…





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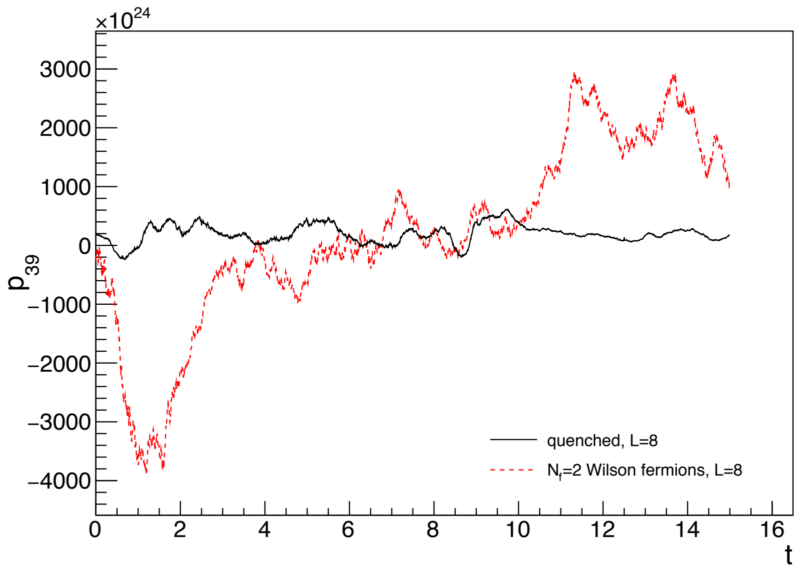

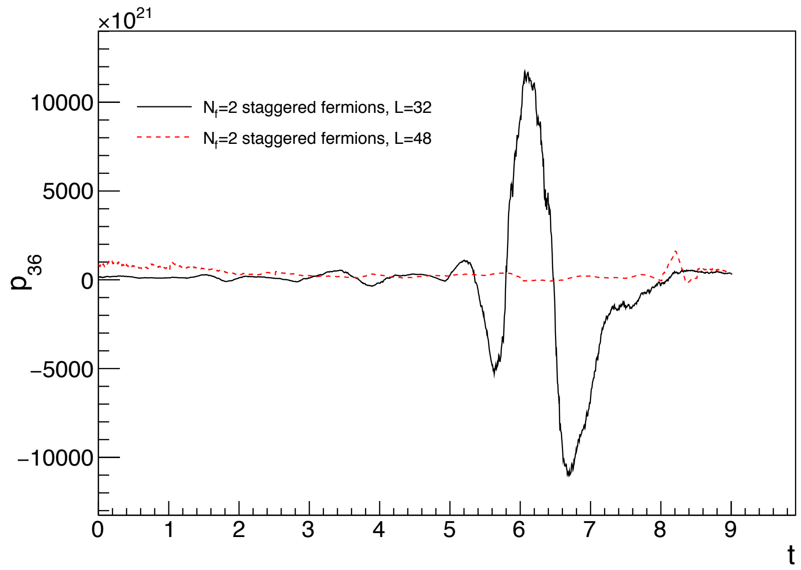

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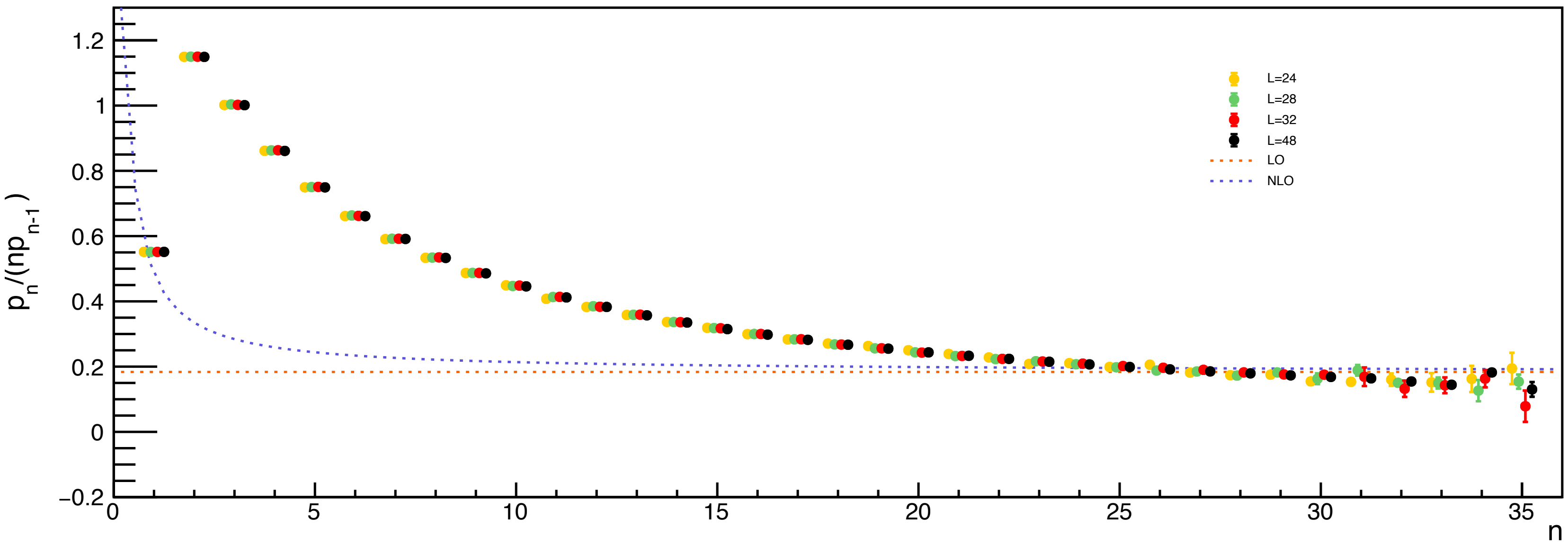
… despite this …

Can you directly INSPECT the IR RENORMALON?
YES!

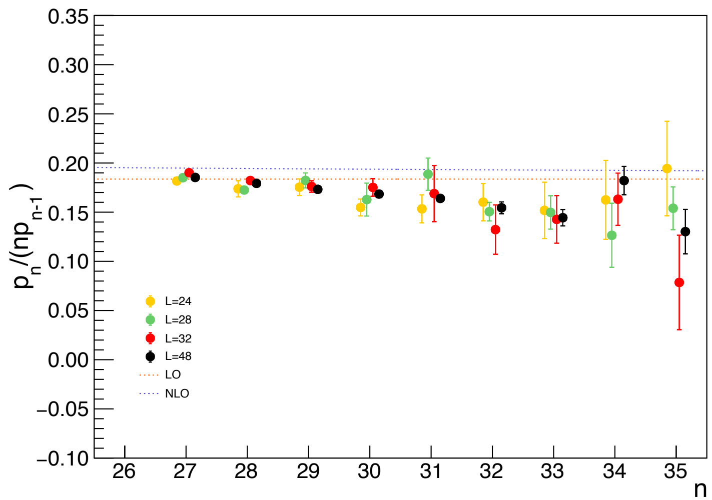
Expect

$$\frac{p_n}{n p_{n-1}} = \frac{3\beta_0}{16\pi^2}\left[1 + \frac{2\beta_1}{\beta_0^2}\frac{1}{n} + O\left(\frac{1}{n^2}\right)\right]$$

… an interesting exercise with COVARIANCE MATRIX…



Not (YET) ready to full control FINITE SIZE EFFECTS, but …



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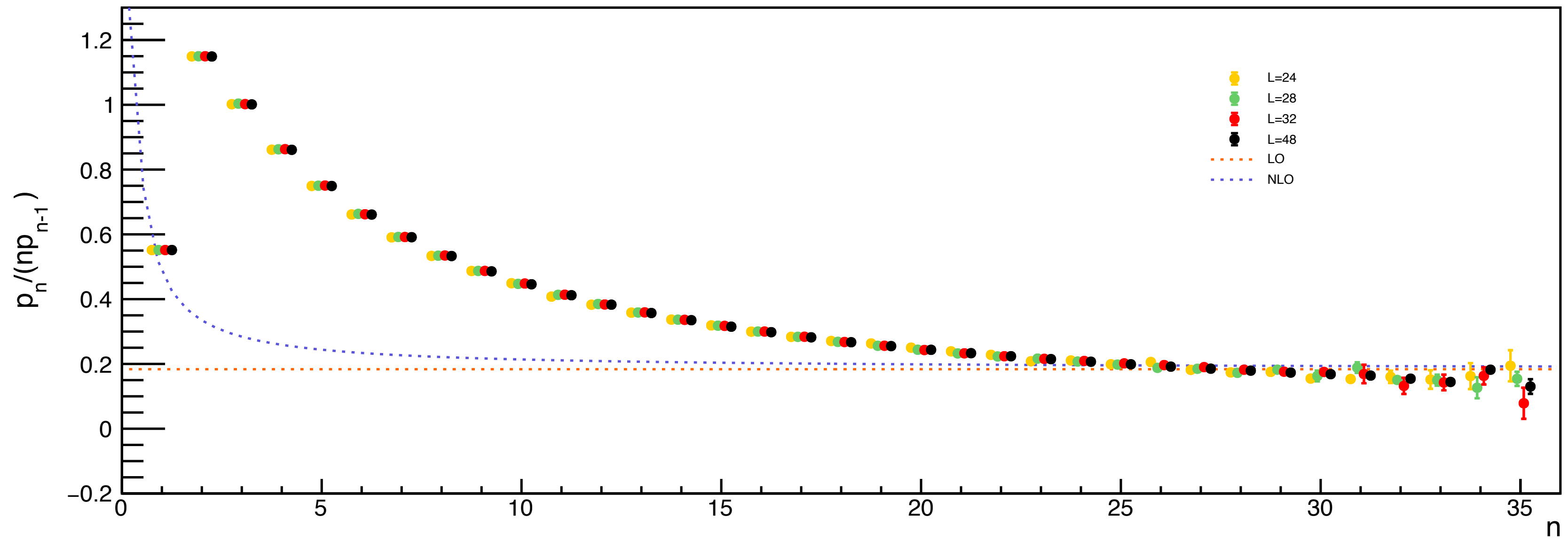

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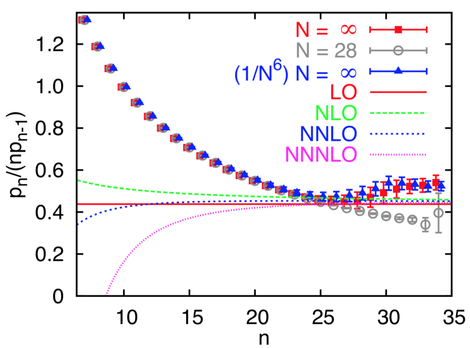
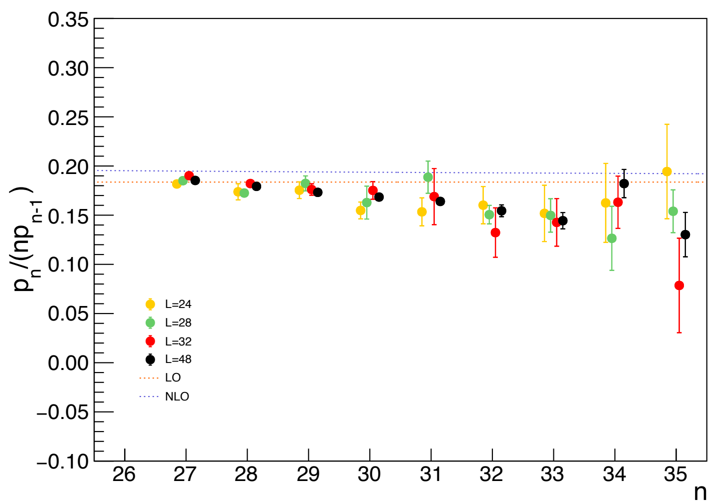


Bali Pineda 2014 (pure YM)



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You can now sum the series in a given prescription.

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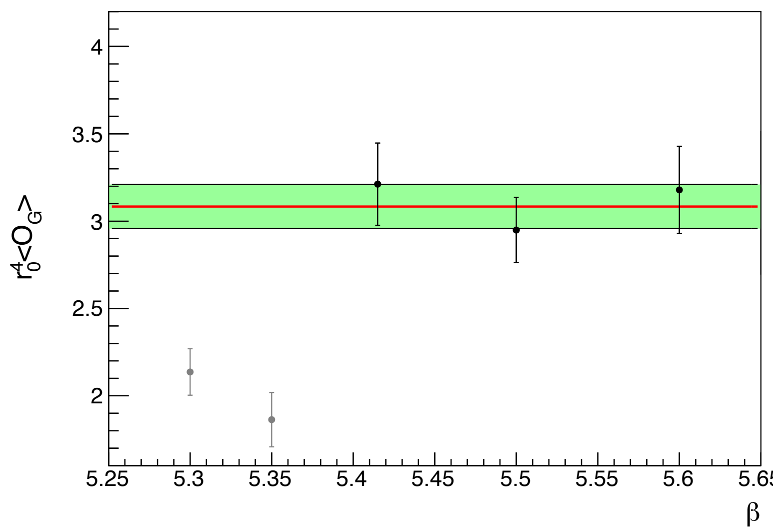
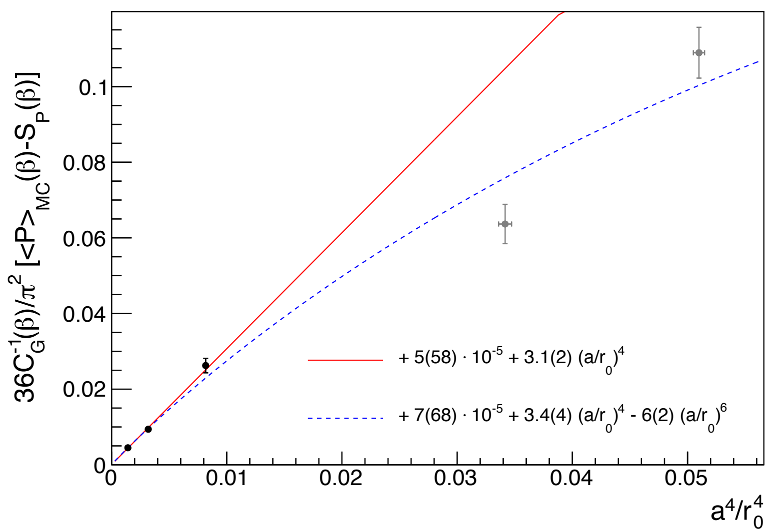
A natural prescription is summing UP TO THE MINIMAL TERM (inversion point)
(remember: finite volume still a systematic error at this stage)

$$S(\beta)_P = \sum_{n=0}^{\bar{n}} p_n \beta^{-(n+1)}$$

$$\langle O_G \rangle = \frac{36}{\pi^2} C_G^{-1}(\beta)\, a^{-4} \left[\langle P \rangle_{\text{MC}}(\beta) - S_P(\beta) \right]$$

$$\delta \langle O_G \rangle = \frac{36}{\pi^2} C_G^{-1}(\beta)\, a^{-4} \sqrt{\frac{\pi \bar{n}}{2}}\, p_{\bar{n}} \beta^{-\bar{n}-1}$$





CONCLUSIONS (and PROSPECTS!)

- NSPT can do a good job looking into renormalons, even including fermions

- Now new prospects in from of us (in progress):

    * try different fermionic representations

    * and different color/flavor content

    * with the idea that (possible quasi) conformal window should CHANGE THE PICTURE! (an alternative perspective on BSM candidates?)