Lattice Calculation of Neutron Electric Dipole Moment with Overlap Fermions

{J. Liang, T. Draper, K.-F. Liu and Y.-B. Yang}   χQCD collaboration

Introduction

The baryon asymmetry in our universe suggests that there are other sources of CP-violation besides the weak CP phase in the CKM matrix. One possibility is the "strong CP-violation" coming from the θ term of QCD. However, its direct consequence, the neutron electric dipole moment (nEDM), has not yet been discovered by experiments. *Ab initio* calculation of lattice QCD can provide a quantitative relation between the nEDM and the parameter θ, and will be most helpful to understand the strong CP-violation.

However, it is pointed out in [1] that all the previous lattice results of nEDM need to be corrected and upon the correction all the previous lattice results are reduced to one-sigma signal or less, making the calculation more challenging. The noisy behavior of the $\not{C}\not{P}$ form factor F_3 is understandable: the distribution of the topological charge Q involved in the calculation is very extended and therefore F_3 fluctuates heavily configuration by configuration. Furthermore, the fluctuation of topological charge is proportional to $\sqrt{V}$ (V is the lattice volume), so F_3 gets noisier when volume increases.

To solve this problem, The Cluster Decomposition Error Reduce (CDER) algorithm [2] is used to remove the large $\sqrt{V}$ fluctuation. We can then have results on a large lattice 32ID ($32^3 \times 64$, $L \sim 4.5$ fm, $m_\pi \sim 171$ MeV) as well as a smaller one 24I ($24^3 \times 64$, $L \sim 2.6$ fm, $m_\pi \sim 337$ MeV). Both of them are domain-wall lattices generated by the RBC/UKQCD collaboration [3] and we use overlap fermions as valence quarks.

The CDER algorithm

As illustrated in Fig. 1, the 3-point and 4-point functions with local topological charge can be written as follows to utilize the CDER algorithm:

$$C_3^Q(t_f, R) = \sum_{\vec{x}} \left\langle \sum_{r}^{|r|\le R} q(x+r)\chi(x)\bar{\chi}(0) \right\rangle, \quad (1)$$

$$C_4^Q(t_f, \tau, R) = \sum_{\vec{x}\vec{y}} e^{-i\vec{p}\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \left\langle \chi(x) \sum_{r}^{|r|\le R} q(y+r) J_\mu(y)\bar{\chi}(0) \right\rangle. \quad (2)$$

Since the correlation lengths between the local topological charge $q(x)$ and the nucleon interpolation field χ or the current J_μ are finite, an optimal cutoff can be found for lattices with size larger than the correlation length which keeps the physics unchanged but reduces the error. Numerically, we shall use a constant fit starting from the largest R down to smaller ones until the $\chi^2/d.o.f.$ is large.

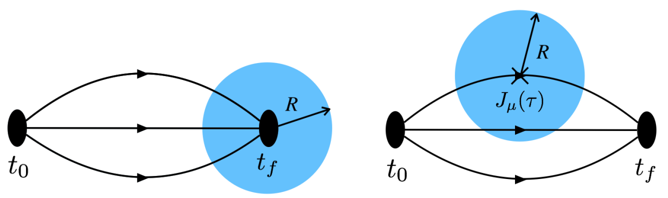

Figure 1: Illustration of CDER in this calculation. The blue area denotes the summation range of the local topological charge with cutoff R. For 3-point functions (left) the topological charge is bound to the sink while for 4-point functions (right) the topological charge is bound to the current.

The $\not{C}\not{P}$ angle α_1

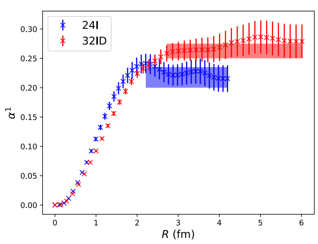

Figure 2: The $\not{C}\not{P}$ angle α_1 as a function of cutoff R for both the 24I and 32ID lattices at their unitary points.

The $\not{C}\not{P}$ angle α_1 can be extracted from the ratio of $\frac{\mathrm{Tr}[\gamma_5 C_3^Q]}{\mathrm{Tr}[\Gamma_e C_2]}$, where C_2 is the usual nucleon 2-point function. For the 24I lattice, as is shown by the blue points in Fig. 2, the central value of α_1 starts to saturate at cutoff $R \sim 2$ fm, and the error of α_1 at $R \sim 2$ fm is just $\sim 10\%$ smaller than the one at the largest R since the size of this lattice is not much larger than the correlation length. For the 32ID lattice (red points in Fig. 2), since the pion mass is lighter, the central value of α_1 starts to saturate at cutoff $R \sim 2.7$ fm. But this time the error of α_1 at $R \sim 2.7$ fm is only half of the one at the largest R due to the large volume of this lattice. The blue and red bands in the figure are the results of constant fits which show the improvement of CDER more clearly. We can obtain more significant improvement when we work on larger lattices.

The $\not{C}\not{P}$ form factor F3

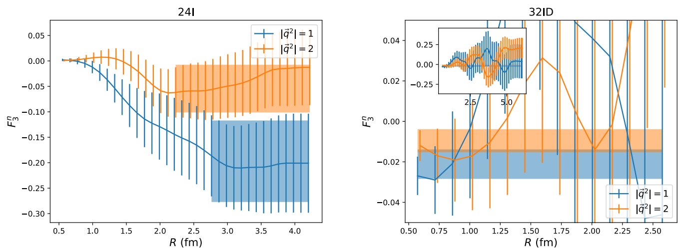

Figure 3: The nuetron $\not{C}\not{P}$ form factor F_3^n as a function of cutoff R for both the 24I and 32ID lattices at their unitary points. The two momentum transfers are included. The bands show the corresponding results of constant fits. For the 32ID lattice, a subplot which shows the whole R range is added on the upper left corner.

The $\not{C}\not{P}$ form factor of neutron F_3^n is extracted by taking a ratio of C_4^Q and C_2. Preliminary results are shown in Fig. 3. For the 24I lattice, we cannot go to too aggressive cutoffs and the constant fits are done from the largest cutoff down to around 2.5 fm. For the smallest momentum transfer, we have $F_3^n(24\mathrm{I}) = -0.197(80)$. For the 32ID lattice, the noise is very large at large cutoffs; constant fits can be done all the way down to ~ 0.6 fm. For the smallest momentum transfer, we have $F_3^n(32\mathrm{ID}) = -0.0211(73)$. The difference between these two results can be interpreted as mainly due to the pion mass difference.

Summary

In summary, we used overlap fermions on two domain-wall ensembles to calculate the nEDM. The CDER algorithm is used such that we can obtain signals on the large volume lattice. We observe very different results on the two lattices which is possibly due to the sea pion mass dependence. We will also study the effects of the valence pion mass. With the CDER technique, we also plan to carry out similar calculations on other lattices with large volume and physical pion mass in the near future.

References

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