The $N_f = 12$ step scaling function with Möbius Domain Wall fermions

Anna Hasenfratz[1], Claudio Rebbi[2], and Oliver Witzel[1]

[1]Department of Physics, University of Colorado, Boulder

[2]Department of Physics and Center for Computational Science, Boston University

University of Colorado Boulder

BOSTON UNIVERSITY

NUMERICAL SET-UP

Simulation details

- Symanzik gauge action
- 3 × stout smeared Möbius DW fermions
- L^4 volumes with $L = 8$, 10, 12, 14, 16, 20, 24, 28, and 32
- Antiperiodic BC in all four directions
- $m_f = 0$, L_s grows from 12 to 24 keeping $m_{\text{res}} < 10^{-5}$
- `Grid`[9] code fully optimized for KNL

Advantages of Domain Wall Fermions

- Preserves full SU(N)×SU(N) flavor symmetry; even at finite gauge coupling
- Effective gauge term generated by fermions and smearing is very small, hence
  - reduced cut-off effects
  - increased region of perturbative improvement

Gradient flow coupling

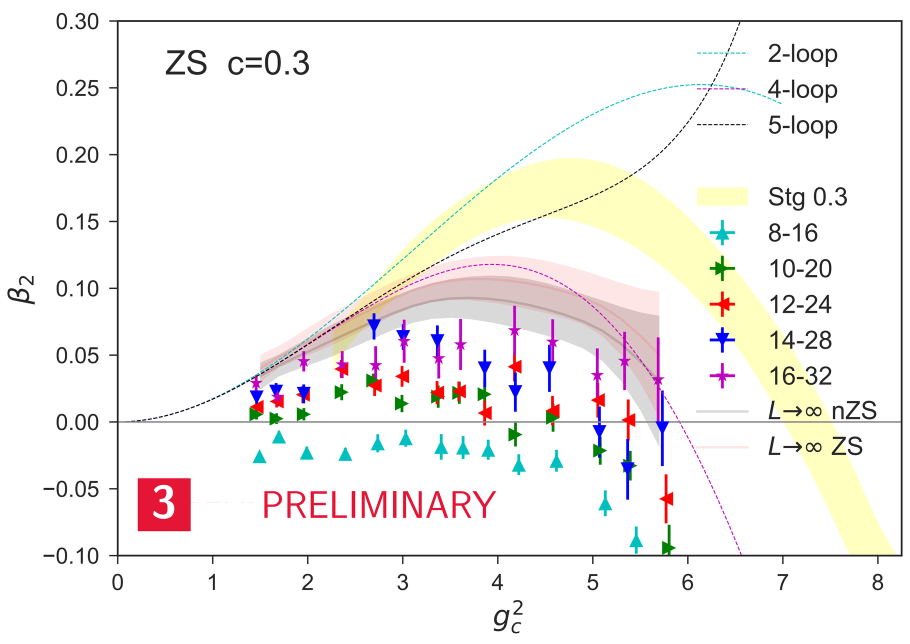
- Fully $\mathcal{O}(a^2)$ Symanzik improved setup
  - Symanzik (S) gauge action
  - Zeuthen (Z) flow [5]
  - Symanzik (S) operator
- Consistency checked by comparing different gradient flows Wilson (W), Symanzik (S), Zeuthen (Z) [5] and/or operators Wilson-plaquette (W), Symanzik (S) and clover (C)
- Include tree-level normalization (tln) [4]

Notation

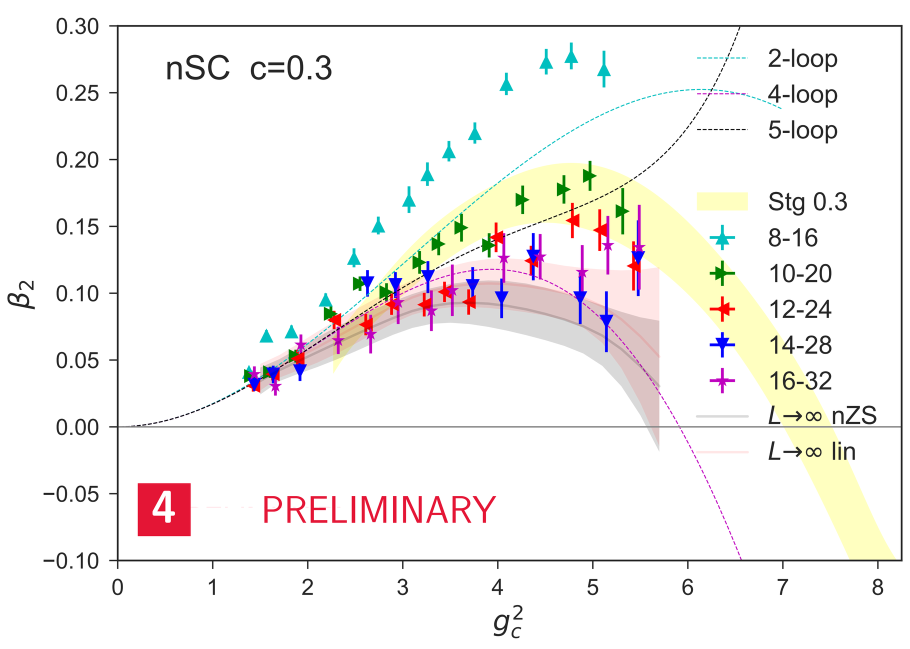
nZS refers to data obtained with tree-level normalization using Zeuthen flow (Z) and the Symanzik operator (S)

Perturbative improvement

Reduce finite volume cut-off corrections by applying tree-level normalization [4]



- Red band: $1/L^2 \to 0$ continuum limit of ZS without tln
- Gray band: $1/L^2 \to 0$ continuum limit of nZS with tln (Fig. 1)
- Both continuum limits are consistent
- nZS has smaller cut-off effects



- Effect of tnl is even larger - compare to Fig. 5
- Red band: $1/L^2 \to 0$ continuum limit of nSC with tln
- Gray band: $1/L^2 \to 0$ continuum limit of nZS (Fig. 1)
- $1/L^2 \to 0$ continuum limit of SC is blue band in Fig. 7
- Again all continuum limits are consistent

GRADIENT FLOW STEP SCALING

Universality

- The Gradient Flow (GF) renormalized step scaling function in the $L \to \infty$ continuum limit is independent of the lattice discretization
- Disagreement between results using the same renormalization scheme (fixed c) may
  - indicate different renormalized trajectories/fixed points or
  - be due to incorrect extrapolations to the continuum limit
- Agreement is necessary for the entire function, not only at the fixed point [1]

Gradient Flow step scaling function

We investigate the finite volume step scaling function [2,3]

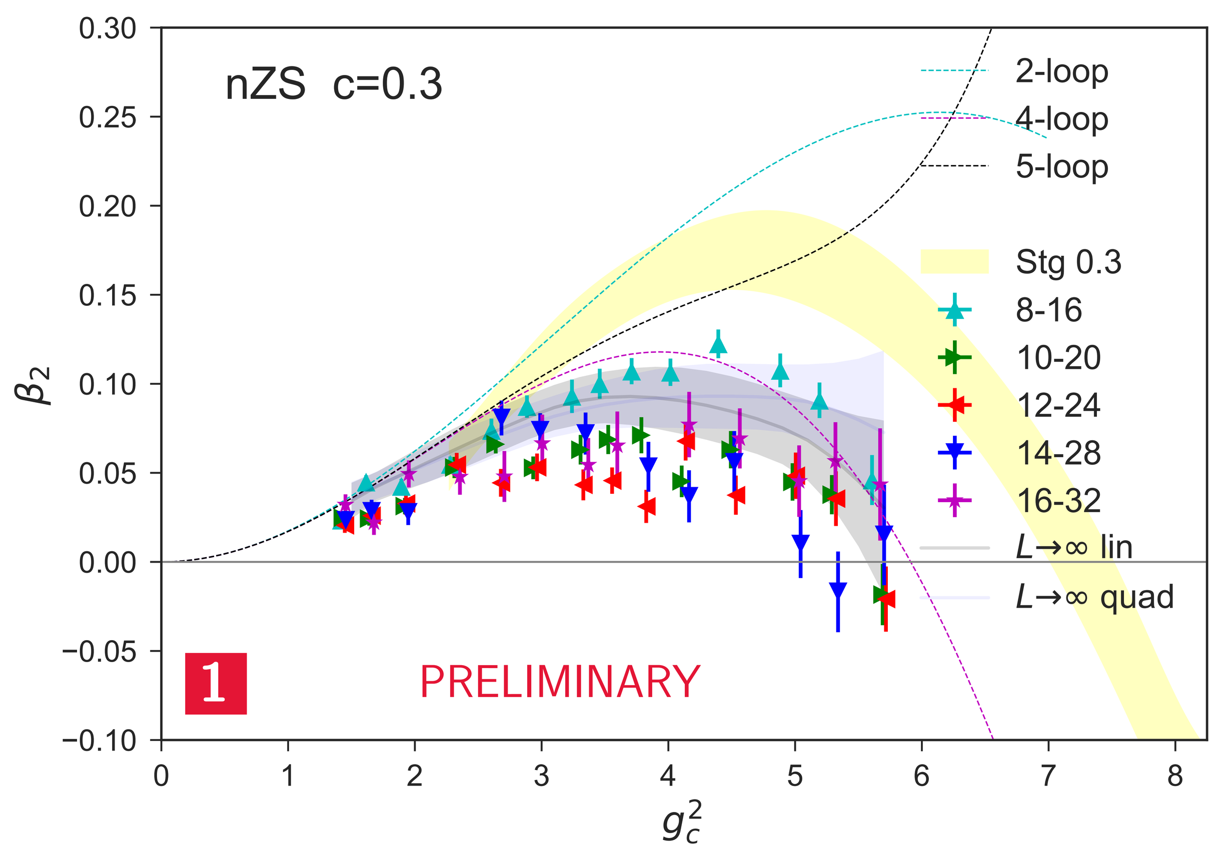
$$\beta_s^c(g_c^2; L) = \frac{g_c^2(sL) - g^2(L)}{\log(s^2)},$$

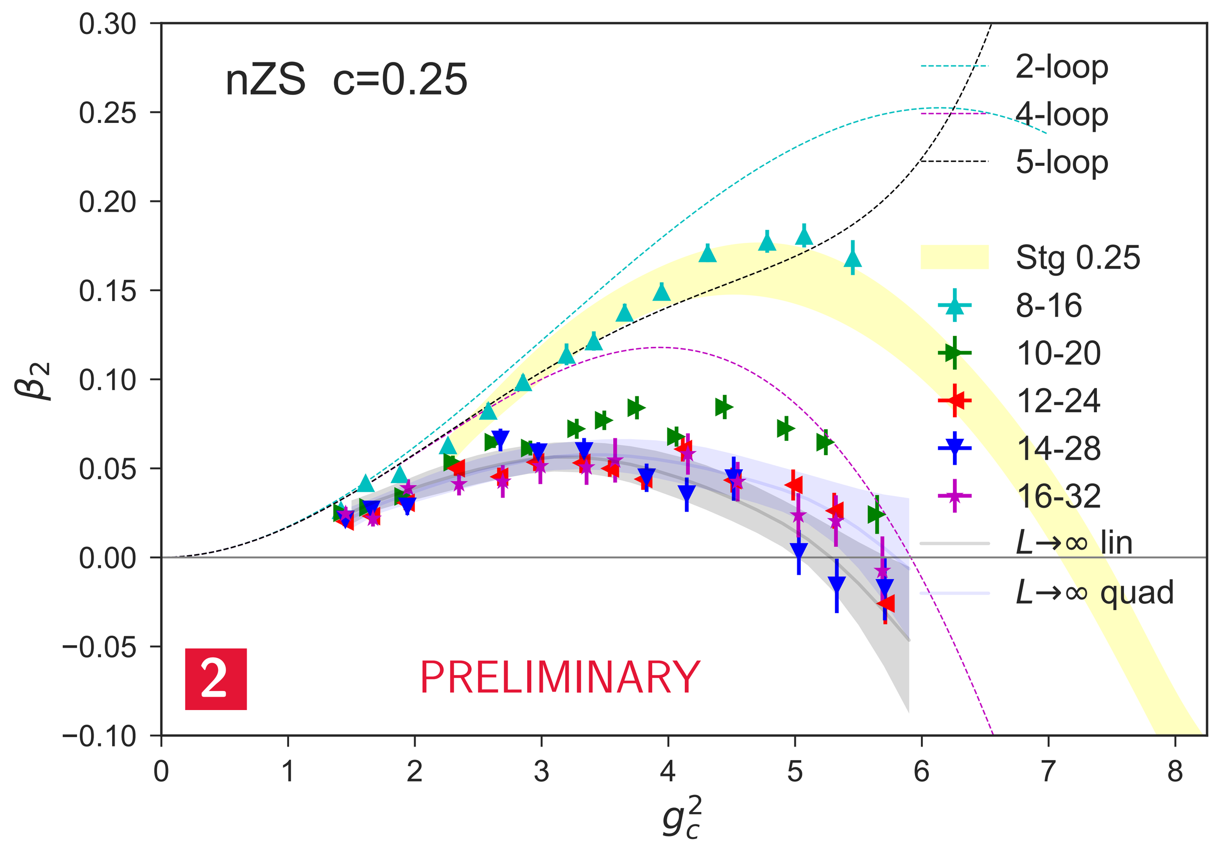
with $s = 2$, using tree-level normalized GF coupling

$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2-1)} \frac{1}{C(c,L)} t^2 \langle E(t) \rangle$$

- The flow time is set by the volume, $\sqrt{8t} = cL$
- $C(c, L)$ is the tree-level normalization (tln) factor introduced in [4]

Nonperturbative results





Summary

- Full $O(a^2)$ improved setup [5] has small volume dependence
- Linear (gray) and quadratic (blue) $1/L^2$ extrapolations are consistent for nZS
- Other discretizations are consistent, unless their cut-off effects are very large (SC)
- DW simulations are very different from staggered where the strong bare gauge coupling leads to coarse gauge fields, destroying perturbative improvements
- Staggered result [6,7] are inconsistent with DW over a wide g_c^2 range

Comments and Outlook

- Step scaling functions for both $c = 0.25$ and 0.3 are at or below 4-loop PT
- Such a behavior is familiar from 2+1 flavors [8]
- Both $c = 0.3$ and 0.25 suggest an IRFP for $N_f = 12$ around $g_c^2 \sim 6$; additional simulations at stronger bare coupling are needed for verification
- Our findings for the $N_f = 10$ step-scaling function [1] show a similar robustness
- We do not address here the controversy between Refs. [6] and [7]. Our DW simulations are at weaker coupling where different staggered simulations appear consistent.

References

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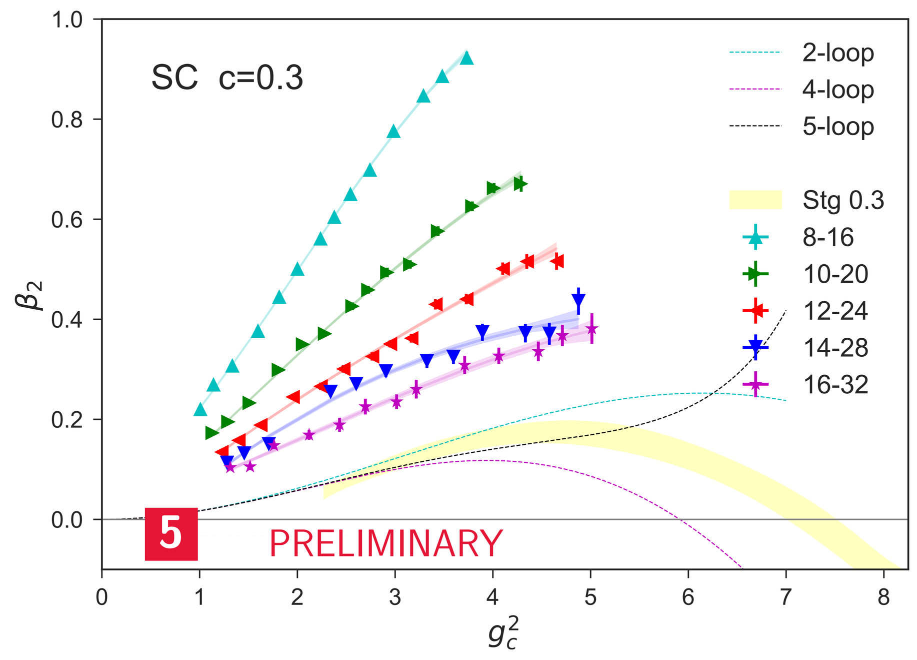
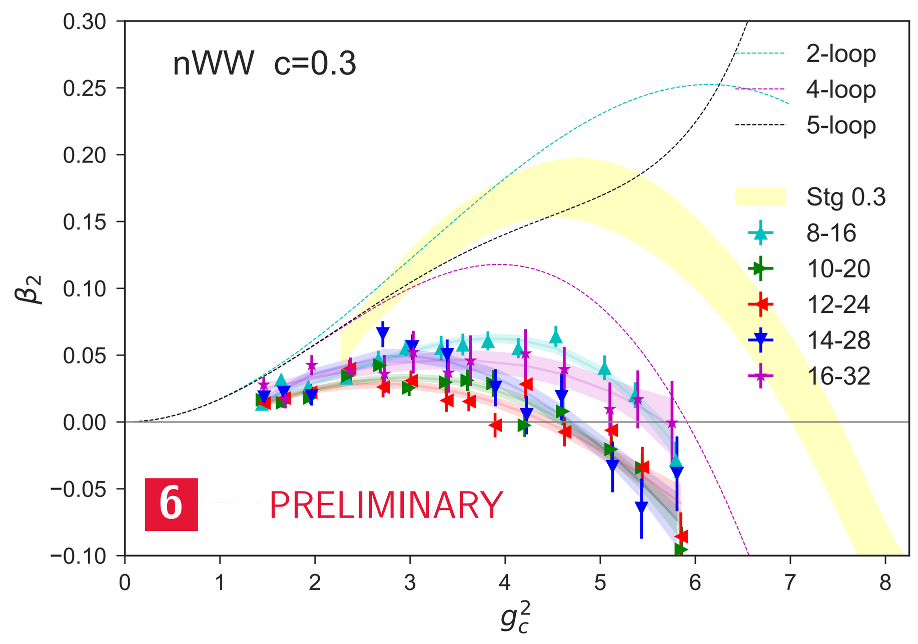
ANALYSIS

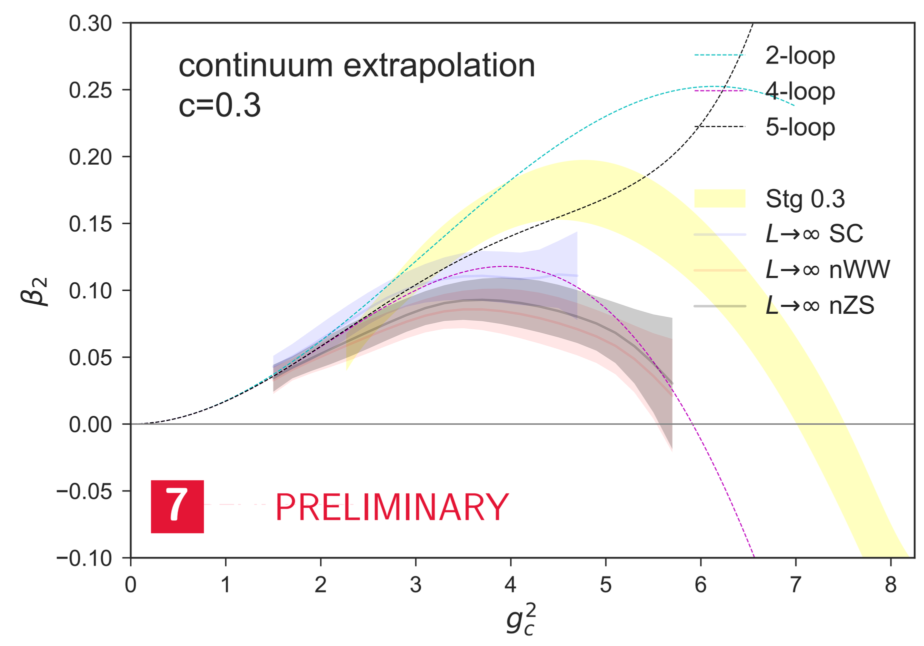
Use alternative flow, operator

- nZS data show very small volume dependence (Fig. 1)
- SC data even with tln have larger cut-off effects (Fig. 4)
- SC data without tln are much worse (Fig. 5)
- WW data improve significantly with tln (Fig. 6)
- We do not observe the breakdown of tln found in [10]



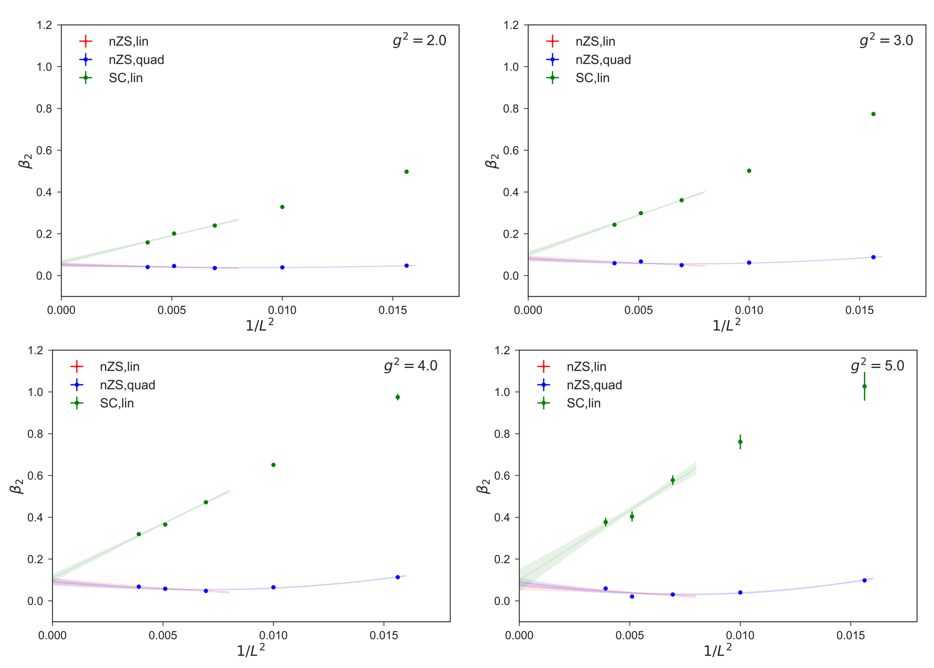


Nevertheless continuum limits do agree



$L \to \infty$ extrapolations

- Interpolate $\beta_2(L)$ vs g^2 with a low(3rd) order polynomial
- Alternatively, interpolate $g^2(L)$ vs β to predict $\beta_2(L)$; both approaches are consistent
- Extrapolate in $1/L^2$ using the three largest volume pairs
- Alternatively, include $1/L^4$ and use all 5 volume pairs; both approaches are consistent
- Different discretization agree, too



- $L \to \infty$ extrapolations of data at $c = 0.3$
- Blue: quadratic extrapolations, all 5 volume pairs, nZS
- Red: linear extrapolations, 3 largest volume pairs, nZS
- Green: linear extrapolations, 3 largest volume pairs, SC

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