LATTICE 18
East Lansing, MI, USA

Non-perturbative renormalization of operators in near-conformal systems using gradient flow

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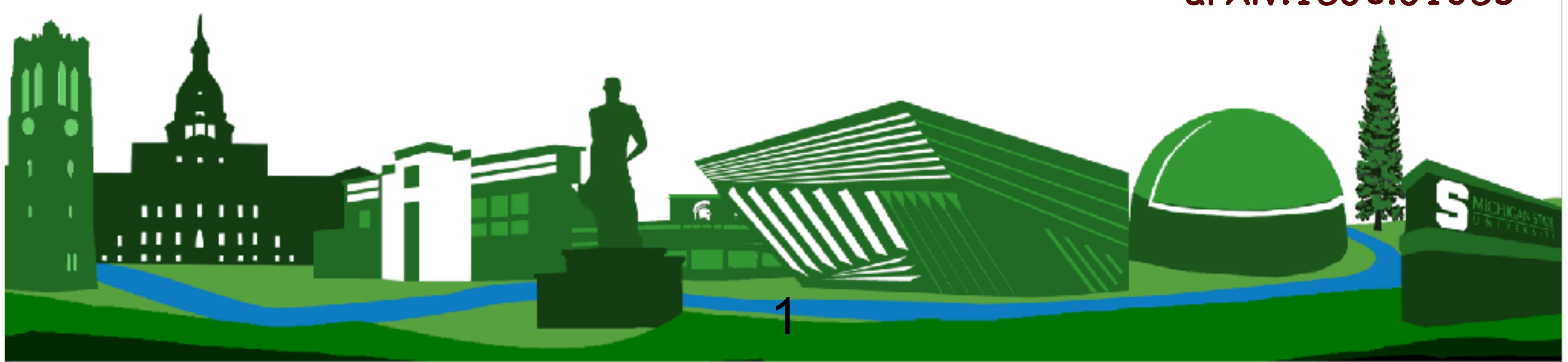
with Andrea Carosso and Ethan Neil

arXiv:1806.01385



1) Is gradient flow a renormalization group transformation?
2) Can we use GF to calculate anomalous dimensions?

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2) Can we use GF to calculate anomalous dimensions?

1) It is not, but it can be tricked:
- normalize correctly
- calculate appropriate quantities

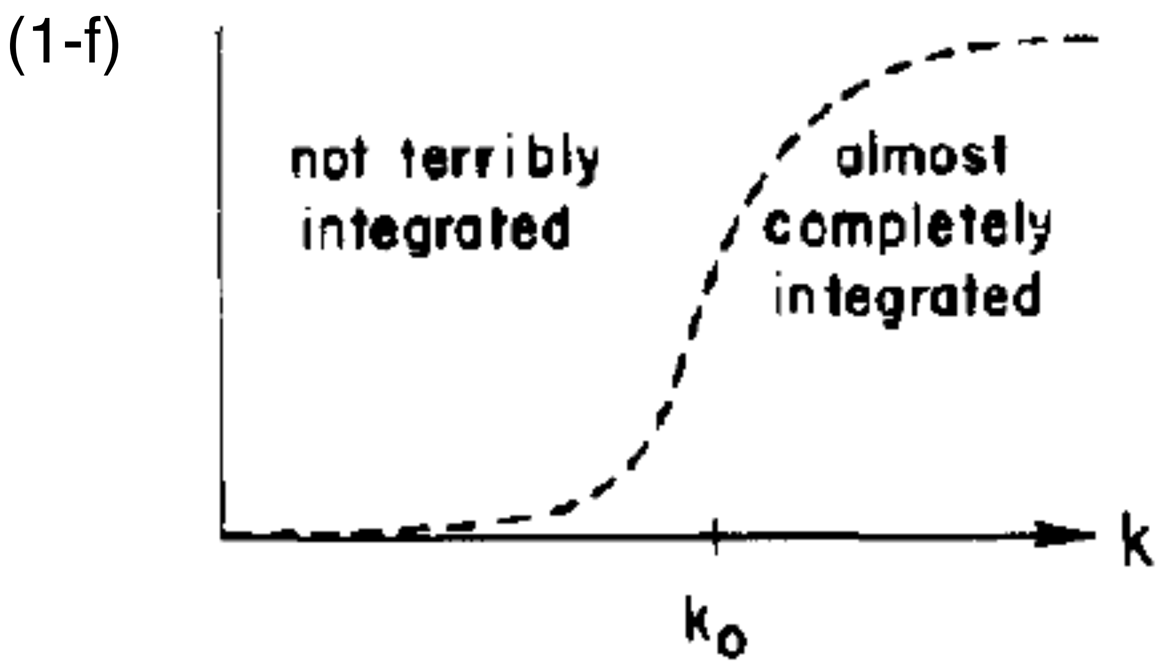
→ GF acts like RG blocking with **continuous** scale change

2) Pilot study: N_f=12 flavor SU(3), determine anomalous dimension of mass and baryon operators

next talk: Andrea Carosso, Φ^4 model

Wilson RG in a nutshell:

Step 1: Introduce “blocked” fields and integrate out the original ones
Step 2: rescale $\Lambda_{cutoff} \rightarrow \Lambda_{cutoff}/b$ (or $a \rightarrow b\,a$)



Credit: Wilson-Kogut 1973,Ch.11

- The partition function is unchanged,
- The action changes $S(\phi, g_0) \rightarrow S(\phi, g')$
- The RG flow runs along the renormalized trajectory either to the ξ=0 trivial or ξ=∞ UVFP

Correlation function of $\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle_{g,m}$

An RG transformation of scale change b:

$$S(\phi, g) \rightarrow S(\phi, g')$$

$$\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle_{g,m} = b^{-2\Delta_O}\langle \mathcal{O}(0)\mathcal{O}(x_b = x_0 / b)\rangle_{g',m'}$$

$\Delta_O = d_O + \gamma_O$ scaling dimension and $x_0 >> b$

We do not need to simulate with $S(\phi, g')$
— just use the principle of MCRG

Monte Carlo Renormalization Group

Swendsen PhysRevLett.42.859,1979

Action — Configuration ensemble

$$\begin{tikzcd}[column sep=huge, row sep=large]
S(\phi, g_0) \arrow[r, "\text{MC}"] \arrow[d, "\text{RG}"] & \{\phi\} \arrow[d, "\text{block}"] \\
S(\phi, g') \arrow[r, "\text{MC}"] & \{\Phi_b\}
\end{tikzcd}$$

RG transformed expectation values can be calculated without explicit knowledge of the blocked action

$$\langle \mathcal{O}(0)\mathcal{O}(x_b)\rangle_{g',m'} = \langle \mathcal{O}_b(0)\mathcal{O}_b(x_b)\rangle_{g,m}$$

$\mathcal{O}_b = \mathcal{O}(\Phi_b)$ is the operator of the blocked fields

Gradient flow could be "blocking"

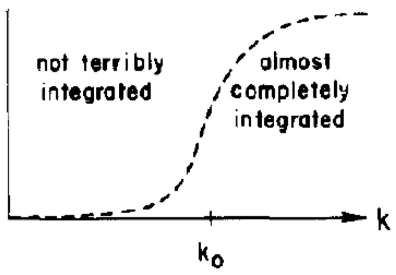
Luscher Comm.Math Phys 293, 899 (2010)

GF is a continuous smoothing that removes short distance fluctuations

Gauge flow: $\partial_t V_t = -(\partial S_{\mathrm{W}}[V_t])V_t, \quad V_0 = U$

Fermions evolve on the gauge background:

$$\partial_t \chi_t = \Delta[V_t]\chi_t, \quad \chi_0 = \psi$$



Luscher JHEP 04 123 (2013)

(The flow action does not have to match the model)

GF misses two important attributes of an RG transformation:

- there is no rescaling $\Lambda_{\mathrm{cut}} \rightarrow \Lambda_{\mathrm{cut}}/b$ or coarse graining
- linear transformation does not have the correct normalization (wave function renormalization or $Z_\phi = b^{-\eta/2}$)

Both issues can be solved

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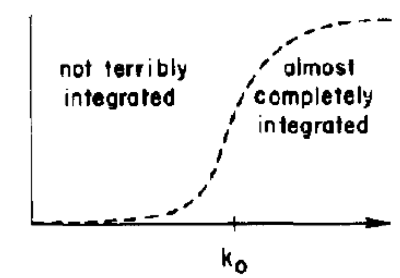

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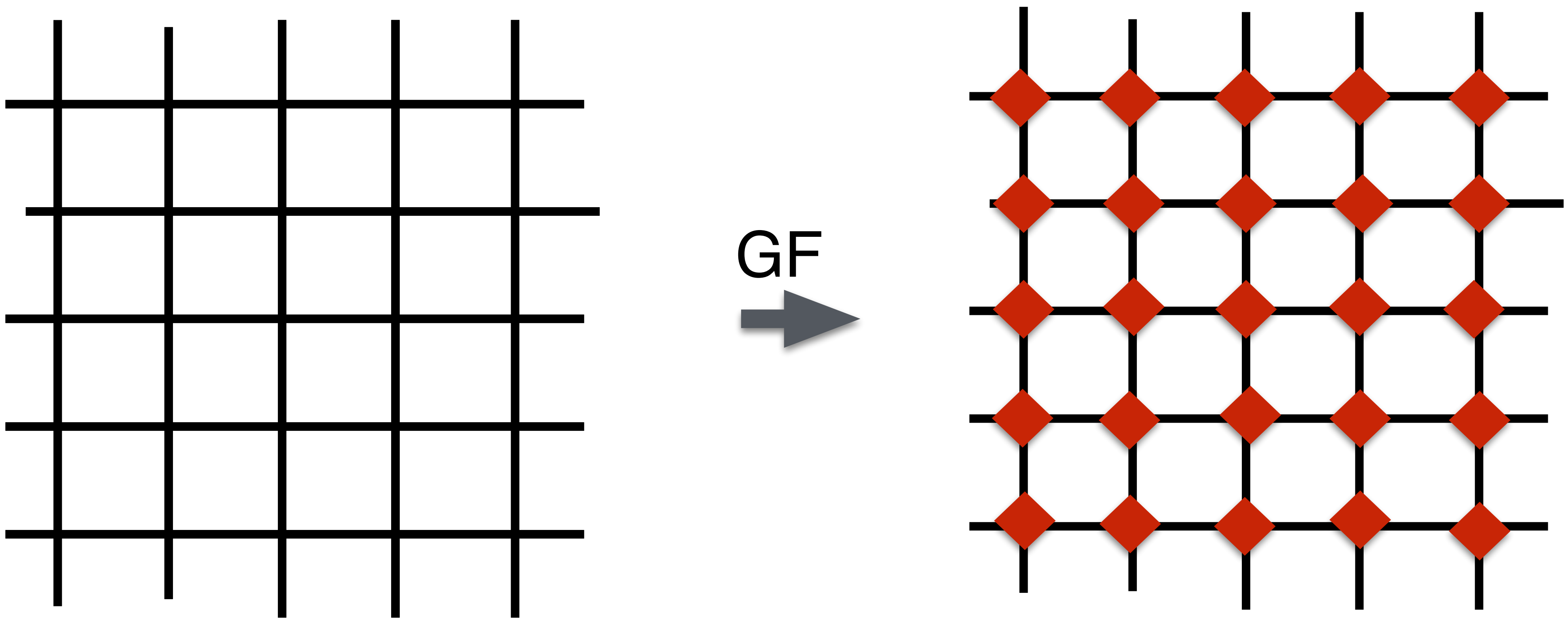
GF does not flow to FP

GF vs RG

Original Φ fields

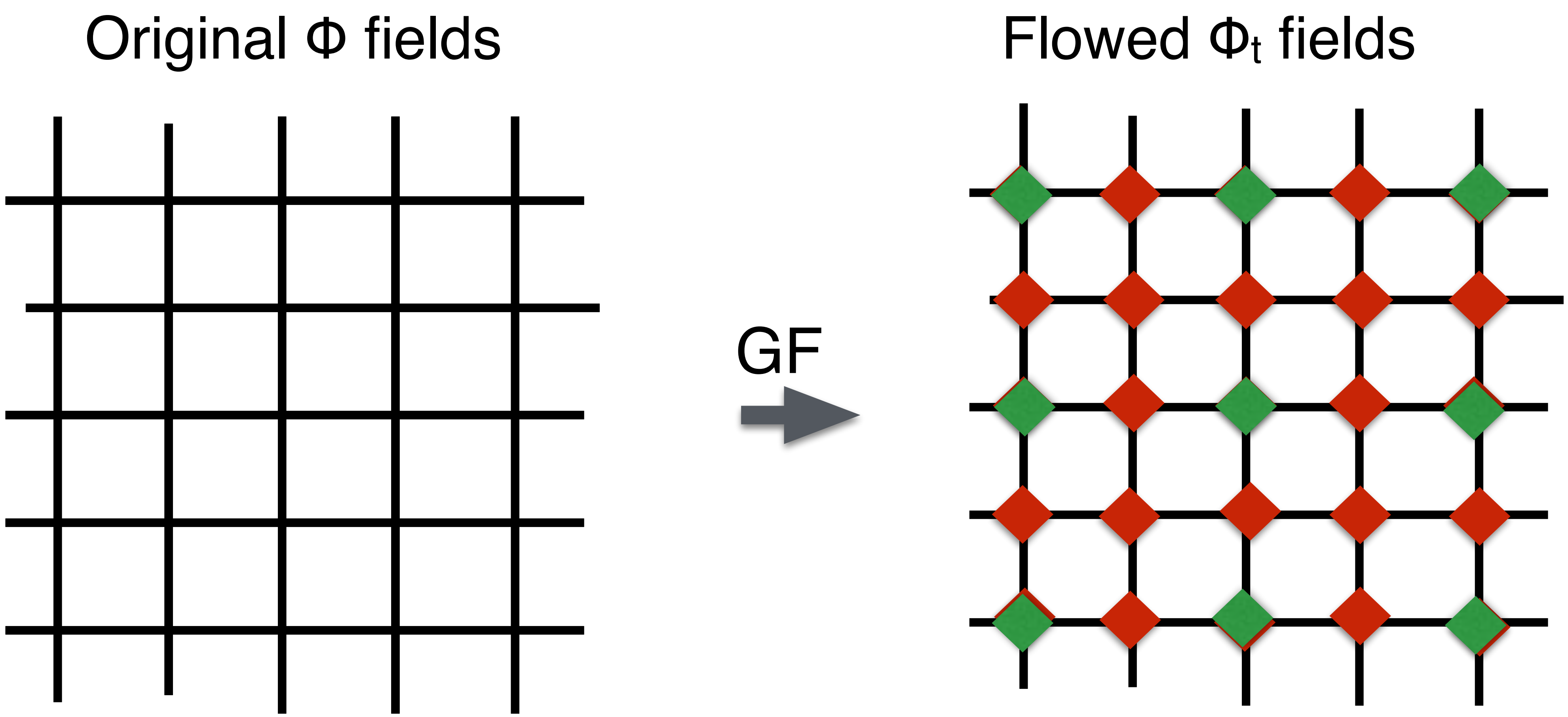
Flowed Φ_t fields



RG transformation (b=2)

- gradient flow : $\phi_t(\phi)$
- blocked fields: $\Phi_b = Z_b \phi_t = b^{-\eta/2} \phi_t$
- Coarse grain and rescale with *b : x* $\rightarrow$ *x/b*

GF vs RG



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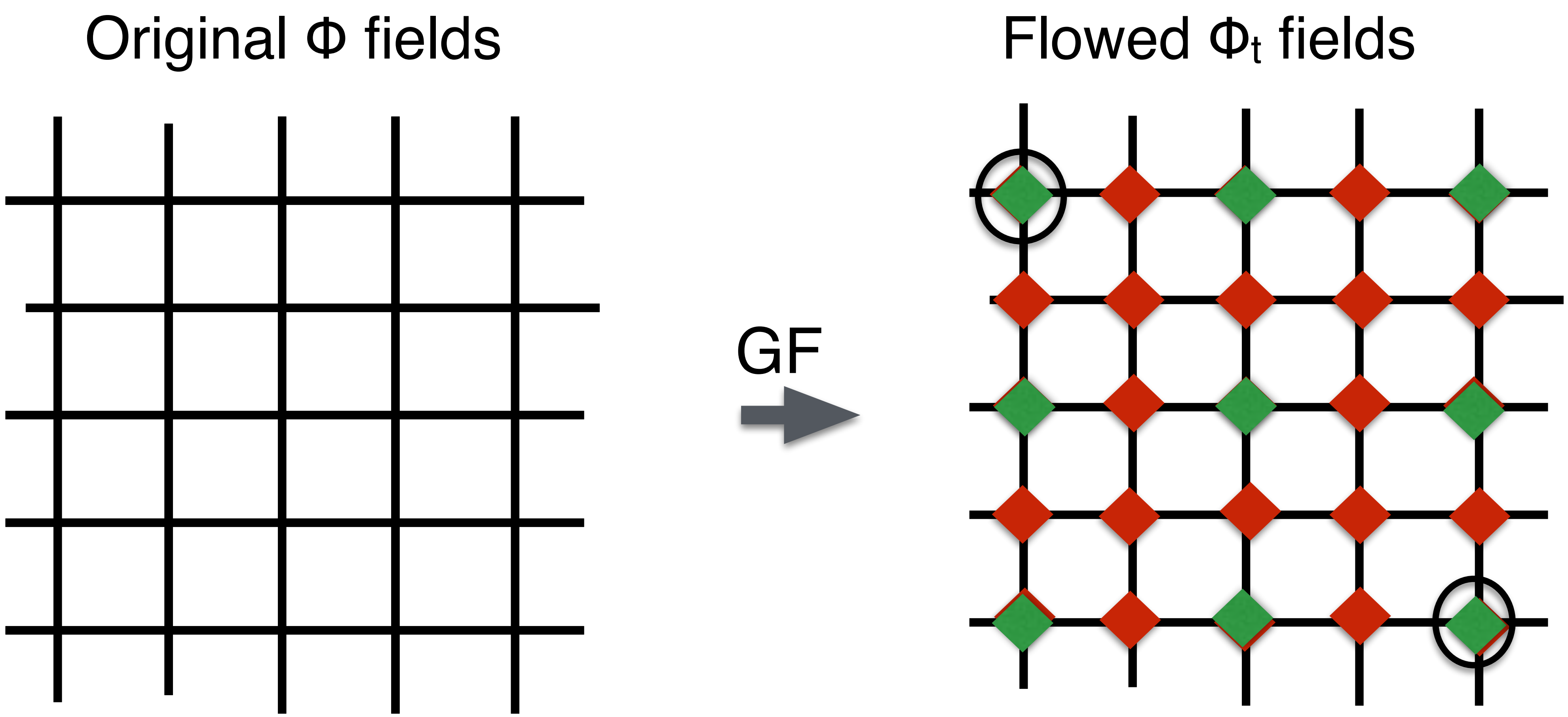

2-point functions do not care about decimation:

$$\langle O_b(\Phi_b(0))O_b(\Phi_b(x_b))\rangle_{g,m} = b^{-\eta}\langle O(\phi_t(0))O(\phi_t(x_b))\rangle_{g,m}$$

At the level of expectation values GF is a proper RG transformation

GF as RG

Put it together

$$\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle_{g,m} = b^{-2\Delta_O}\langle \mathcal{O}(0)\mathcal{O}(x_b = x_0/b)\rangle_{g',m'}$$ RG

$$\langle \mathcal{O}(0)\mathcal{O}(x_b)\rangle_{g',m'} = \langle \mathcal{O}_b(0)\mathcal{O}_b(x_b)\rangle_{g,m}$$ MCRG

$$\langle O_b(\Phi_b(0))O_b(\Phi_b(x_b))\rangle_{g,m} = b^{-\eta}\langle O(\phi_t(0))O(\phi_t(x_b))\rangle_{g,m}$$ GF

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Ratio of flowed & unplowed correlators predict the anomalous dimension

$$\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle} = b^{2\Delta_O - 2n_O\Delta_\phi}$$

$$x_0 \gg b$$
$$\Delta_O = d_O + \gamma_O$$
$$\Delta_\phi = d_\phi + \eta/2$$

Anomalous dimensions

Calculate η by an operator that does not have an anomalous dimension:

— vector or axial charge (A(x))

The super-ratio

$$R(t,x_0)=\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A_t(0)A_t(x_0)\rangle}\right)^{n_O/n_A}=b^{\gamma_O}$$

independent of $x_0 >> b$ and predicts γ

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- t and b are still independent!
  - Natural choice : $b^2 \sim t$
- it is advantageous to flow only the source, not the sink
- γ is universal at the FP only : set fermion mass to zero
- t has to be large enough, and

Anomalous dimensions

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- t and b are still independent!
  - Natural choice : $b^2 \sim t$
- it is advantageous to flow only the source, not the sink
- γ is universal at the FP only : set fermion mass to zero
- t has to be large enough, and $x_0 \gg \sqrt{8t}$

Pilot study: N_f=12

Low statistics study with staggered fermions

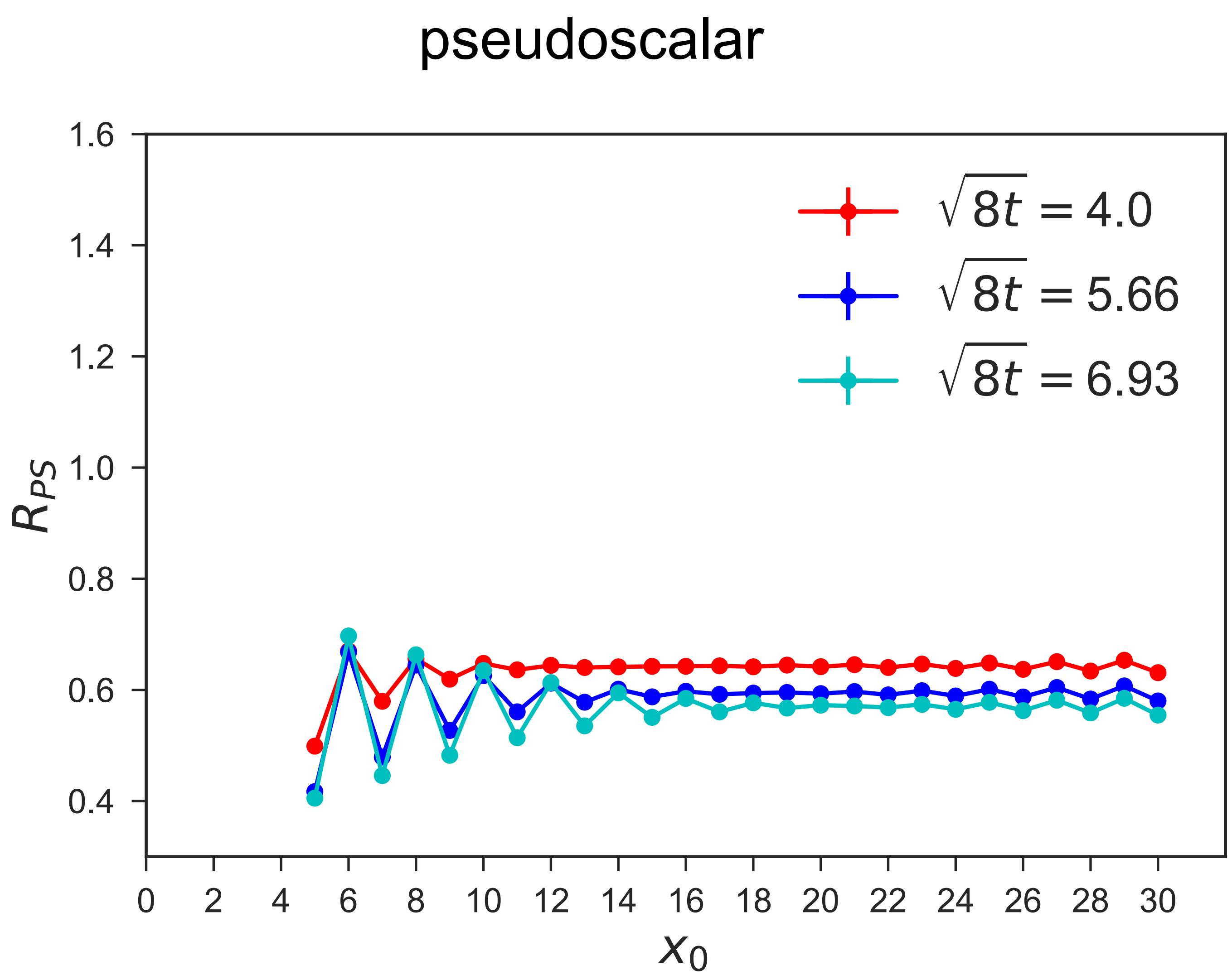
- 24^3x48 , 32^3x64 volumes, m=0.0025

– mass anomalous dimension γ_m =0.23-0.25 from perturbation theory, FSS numerical studies, Dirac eigenmodes
– the gauge coupling walks very slow - substantial scaling violation effects are expected
– baryon and tensor anomalous dimensions would be interesting where no non-perturbative prediction exists

Ratio of ratios - pseudo scalar

$$\mathcal{R}_t^O(x_0) = \frac{\langle O(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A(0)A_t(x_0)\rangle}\right)^{n_O/n_A} = t^{\gamma_O}$$

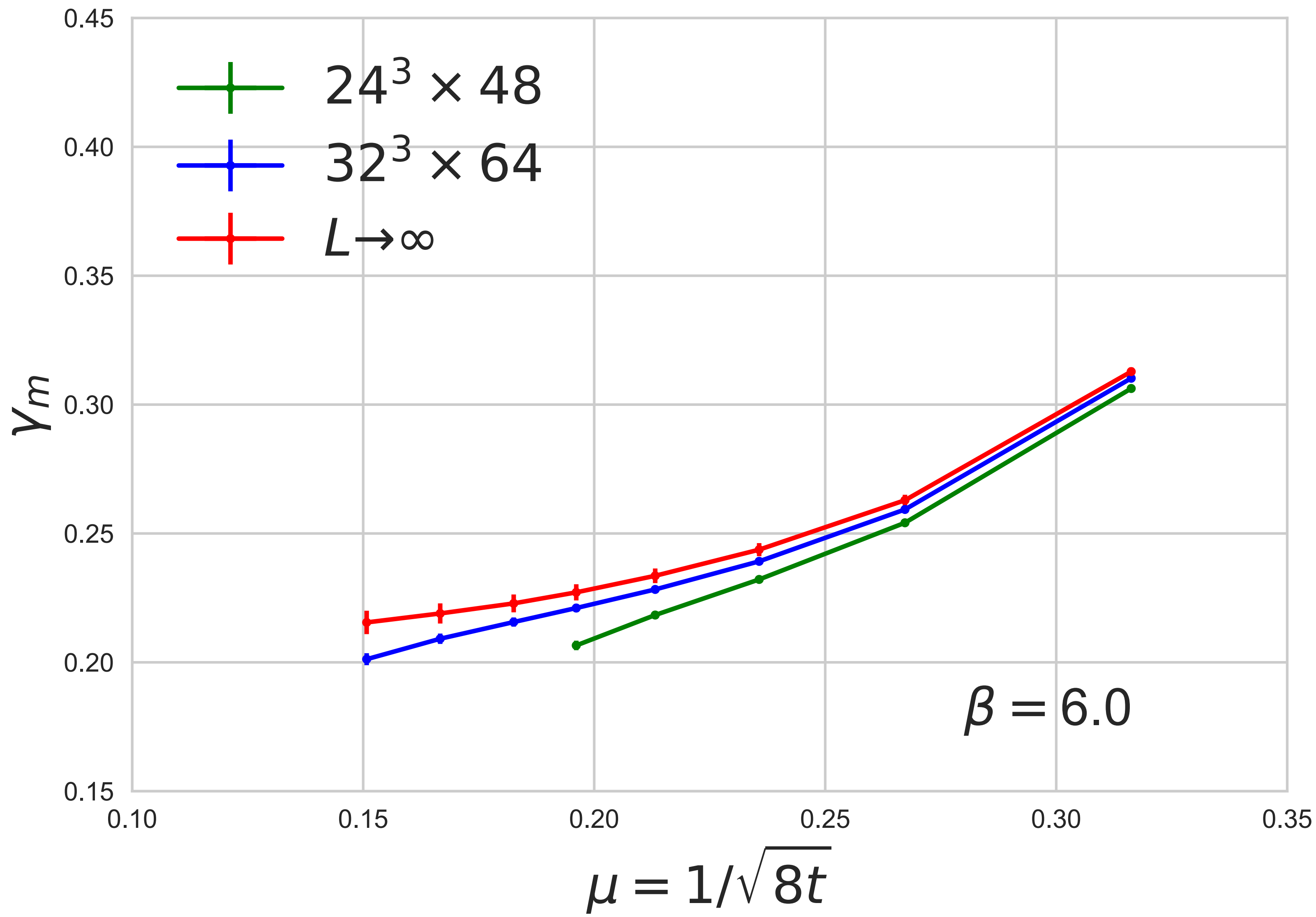
has no x_0 dependence if $x_0 >> b$



Oscillation is due to operator overlap $\propto 2\sqrt{8t}$ —> limits max t

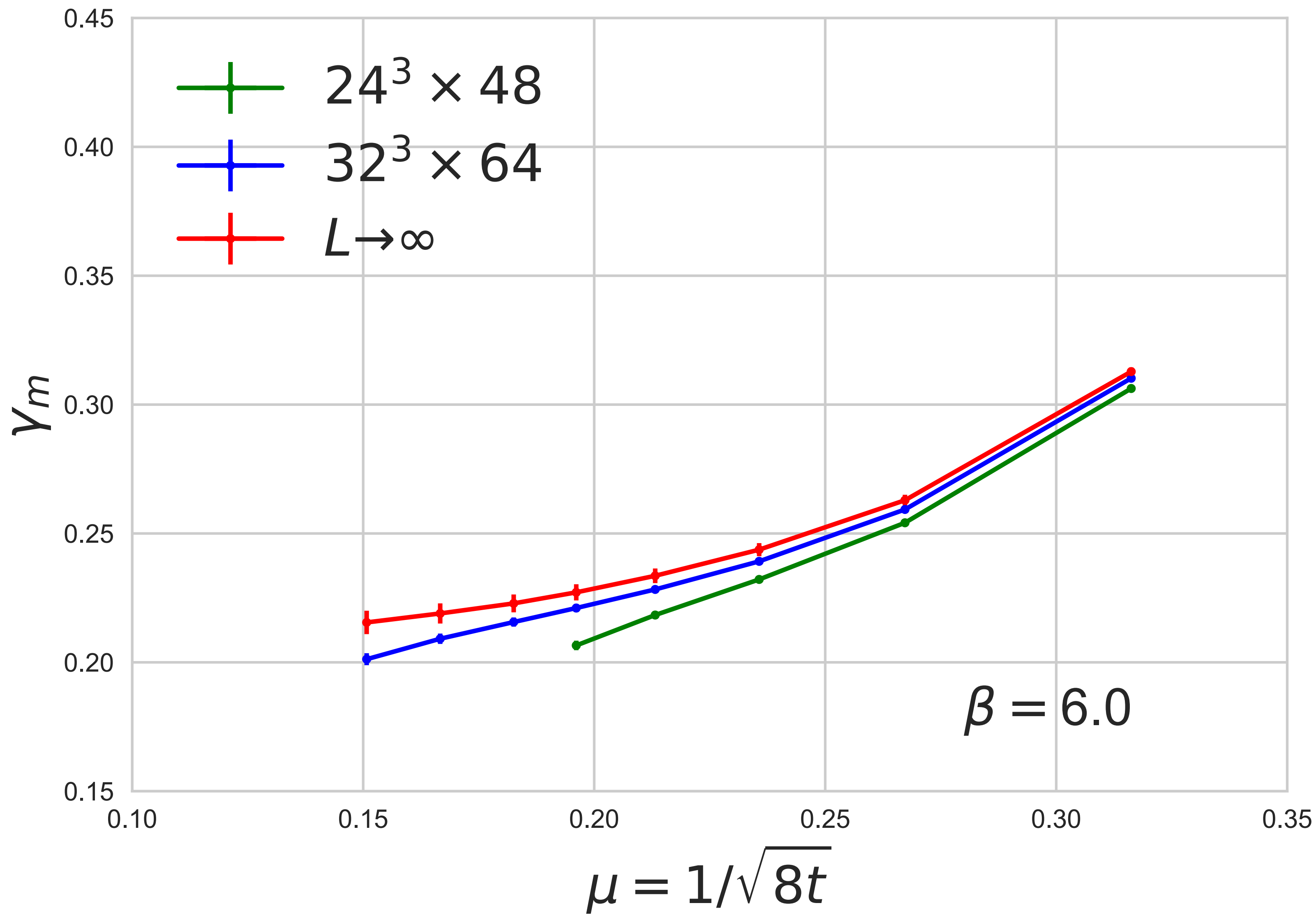
flow time dependence of the plateau gives anomalous dimension

Pseudo scalar



Flow time dependence indicates slowly running gauge coupling

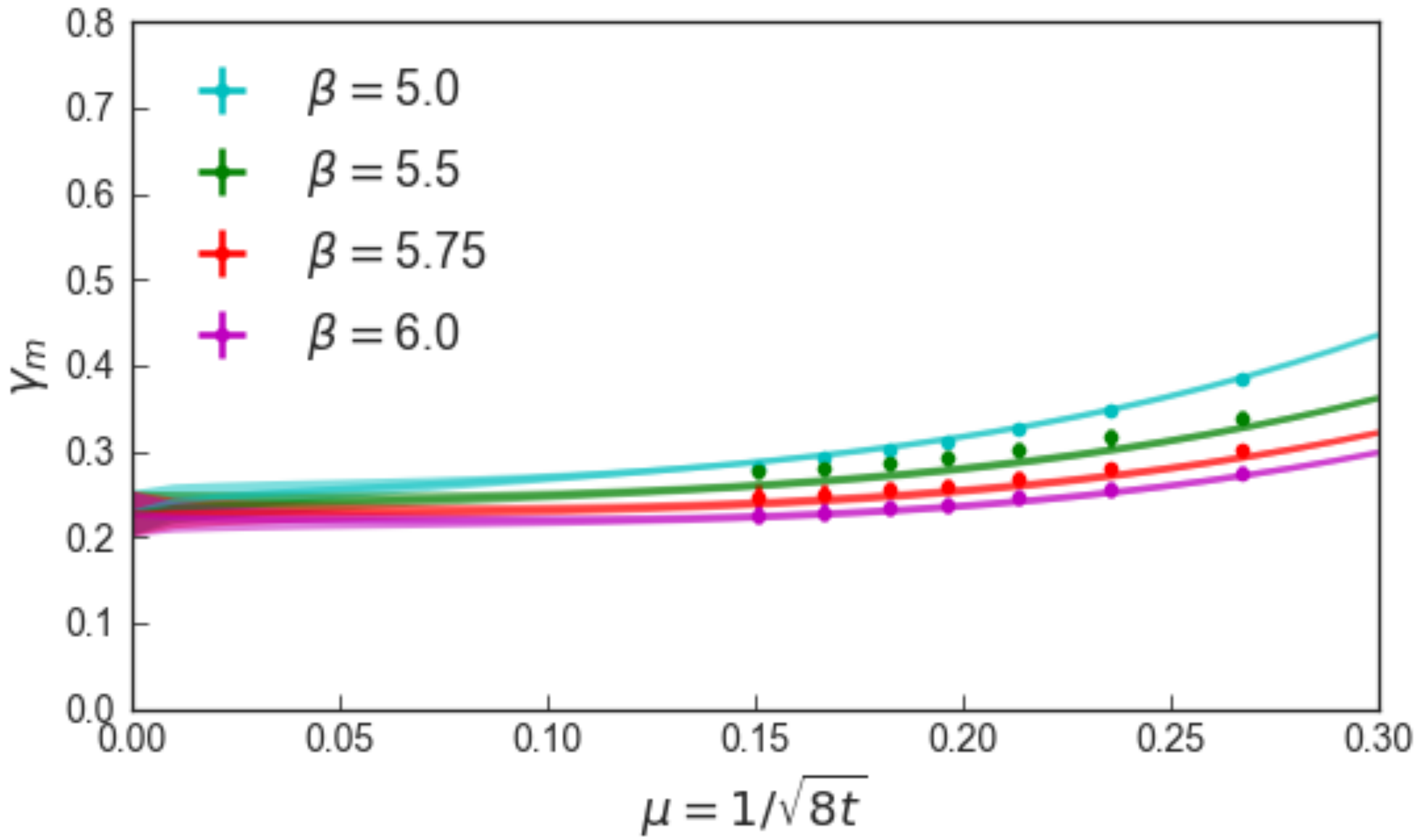
Finite volume corrections



$$R(g',t,L) = s^{-\gamma_0} R(g, s^2 t, sL)$$

$$R(g, s^2 t, s^2 L) = R(g, s^2 t, sL) + s^{-\gamma_0}\big(R(g,t,sL) - R(g,t,L)\big) + \text{h.o.}$$

Pseudo scalar:



$\gamma_m = 0.24(3), \quad t \to \infty$

extrapolate to t → ∞ :

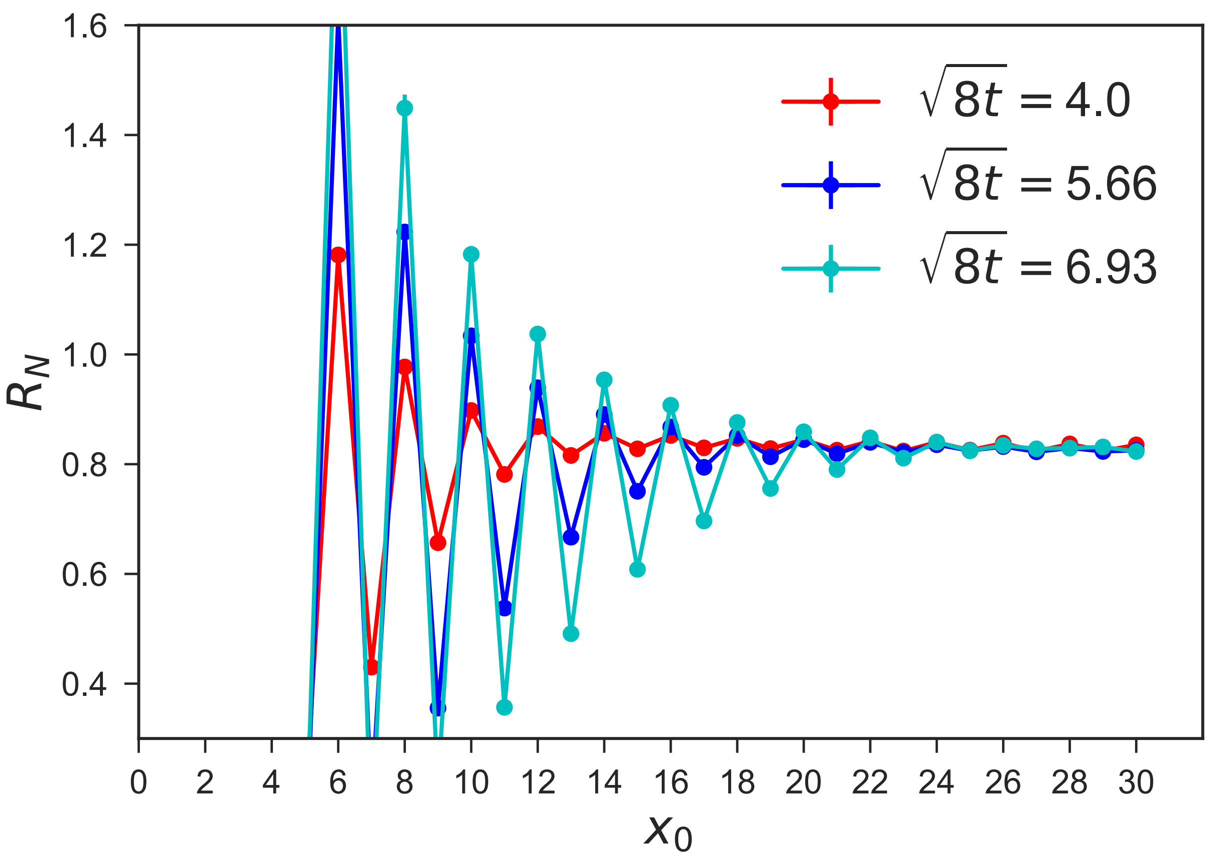
$$\gamma_m(\beta, t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

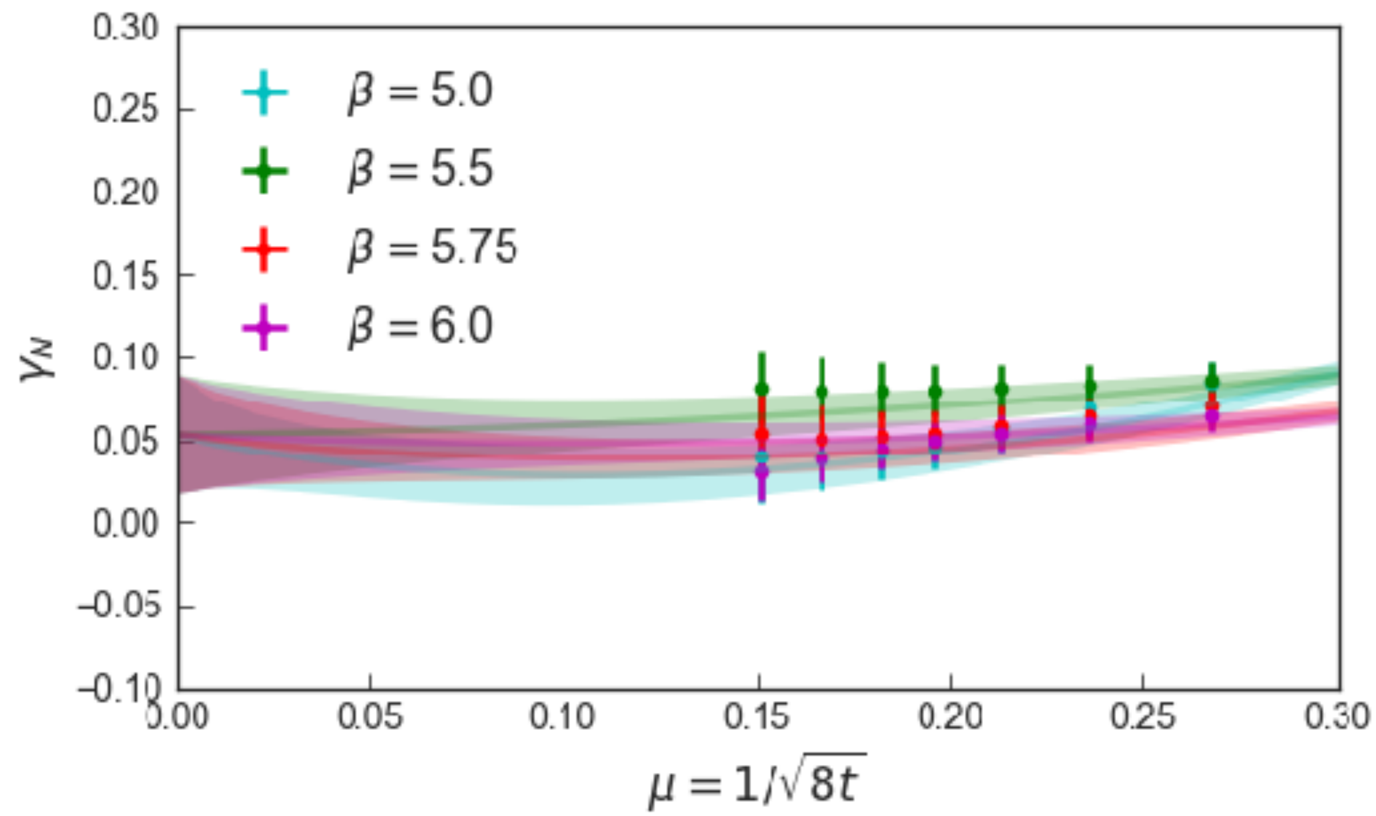
error: systematic + statistical

result consistent with other methods

Nucleon channel

nucleon Lambda





Minimal flow time dependence, but limited x_0 range

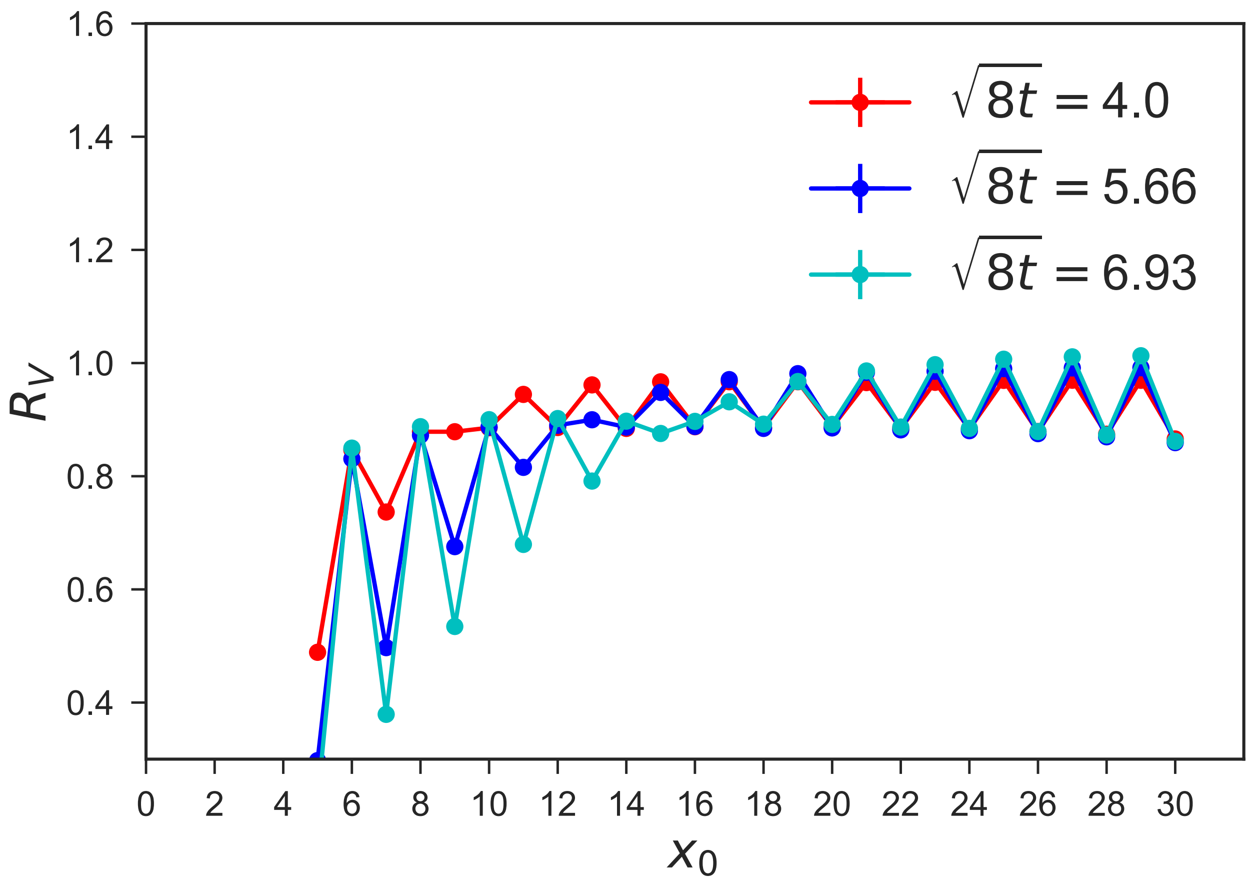
Anomalous dimension is small

$\gamma_N = 0.05(5)$

(perturbative: $\gamma_N = 0.09$)

Vector channel

vector - tensor



Oscillation pronounced but little flow time dependence

Fit as

$$\frac{A_t e^{-m1x_0} + B_t e - m2x_0}{Ae^{-m1x_0} + Be - m2x_0} = \frac{A_t}{A}\frac{1 + B_t / A_t e^{-\Delta m x_0}}{1 + B / Ae^{-\Delta m x_0}}$$

2 anomalous dimensions, from A_t/A and B_t/B both vanish within errors

Summary & outlook

- GF can describe an RG transformation
  - can aid our understanding of GF away from perturbation theory
  - determine anomalous dimension in conformal system (probably most promising method to get nucleon anomalous dim.)
  - determine renormalization factors in QCD (needs work)
- Finite volume effects deserve more attention
- Staggered fermions are a poor choice here (oscillations): DW is more promising
- Anyone with existing conformal configurations can try the method (but need massless or nearly massless configs)
- Beyond BSM:
  - Z factors in QCD need perturbative matching
  - 3D O(n) model: might not compete with FSS but can predict anomalous dimension of irrelevant operators

(A. Carosso, next talk)