Topological Susceptibility in N_f=2 QCD at Finite Temperature -- Volume Study

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July 24, 2018

Preliminaries

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  - Kei Suzuki: $U_A(1)$
  - YA: this talk about the topological susceptibility
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  - Christian Rohrhofer (Fri 17:50): meson correlation functions
- All results in this talk are still preliminary…
- Computer use:
  - Blue Gene/Q at KEK under Large Scale Simulation Program
  - Oakforest-PACS through
    - Post-K priority issues #9
    - HPCI System Research project
    - Multidisciplinary Cooperative Research Program @ CCS, Tsukuba

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    - N_f=2+1 phase diagram
- This report focuses on T≃220 MeV
    - two lattice spacings (last year)
    - finer lattice: one volume (last year) → three volumes

Method

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- Q_t measurements
  - global sum of the gluonic charge density (clover) after Wilson Flow ($t \simeq t_0$)
  - Overlap Index

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

susceptibility

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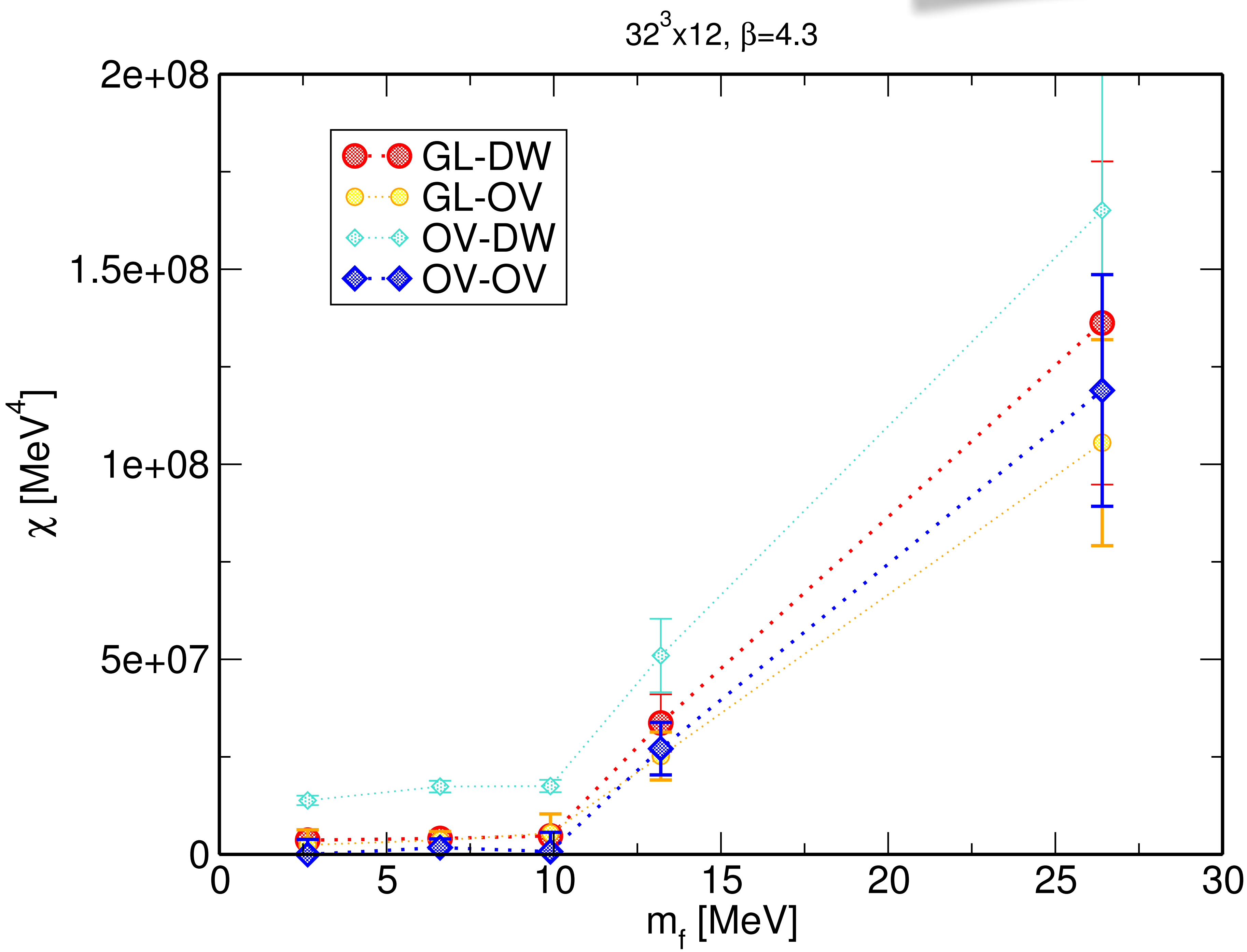

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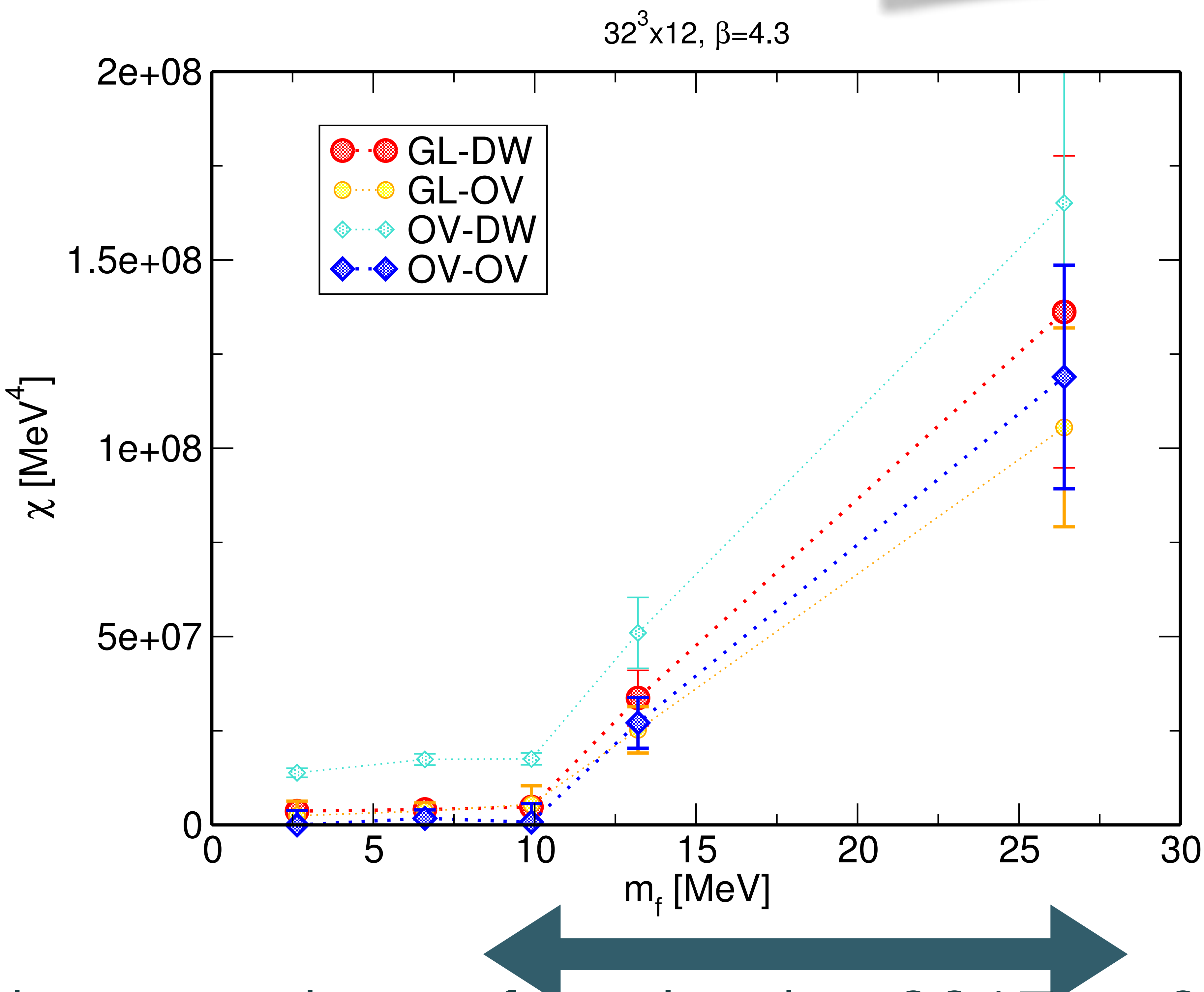

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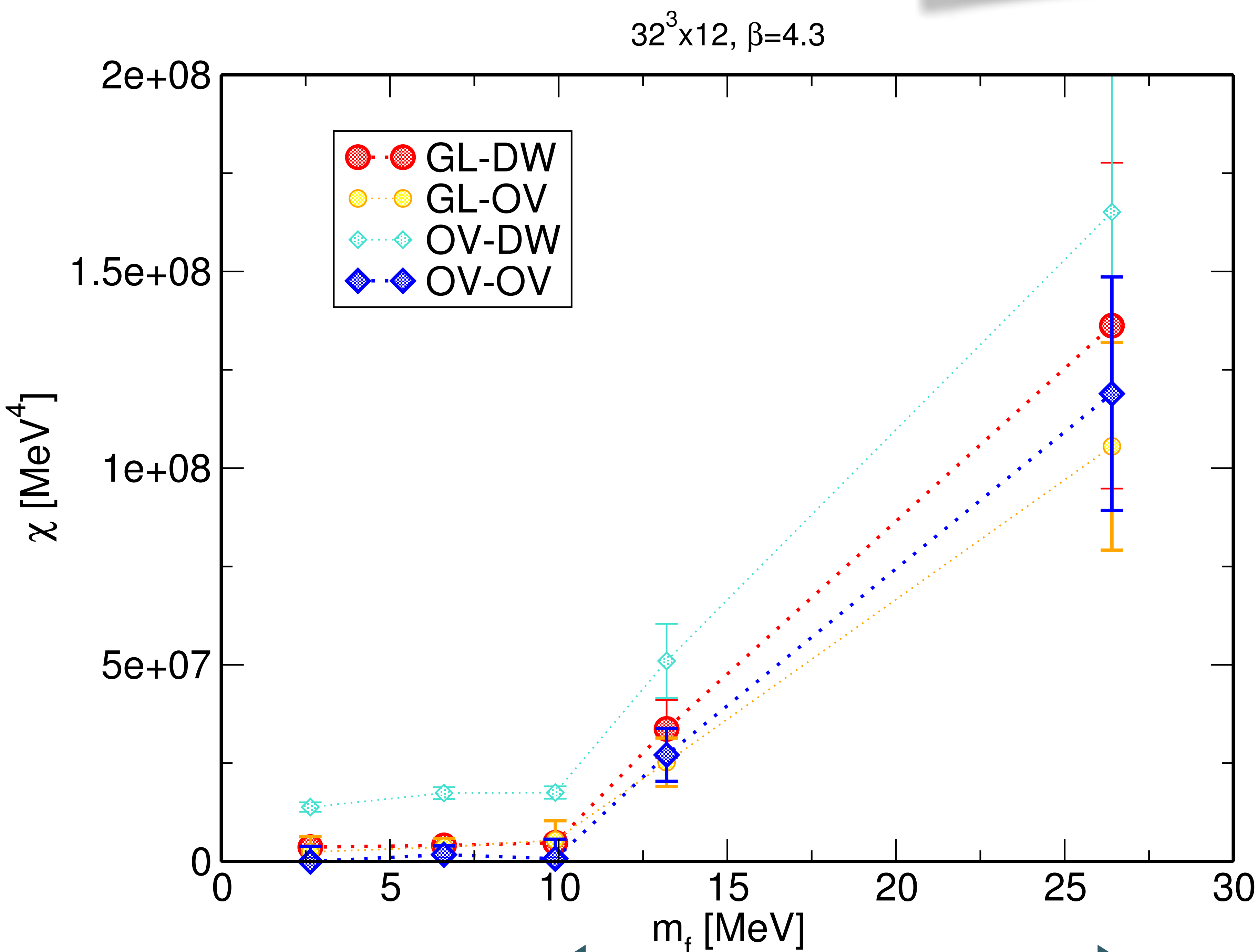

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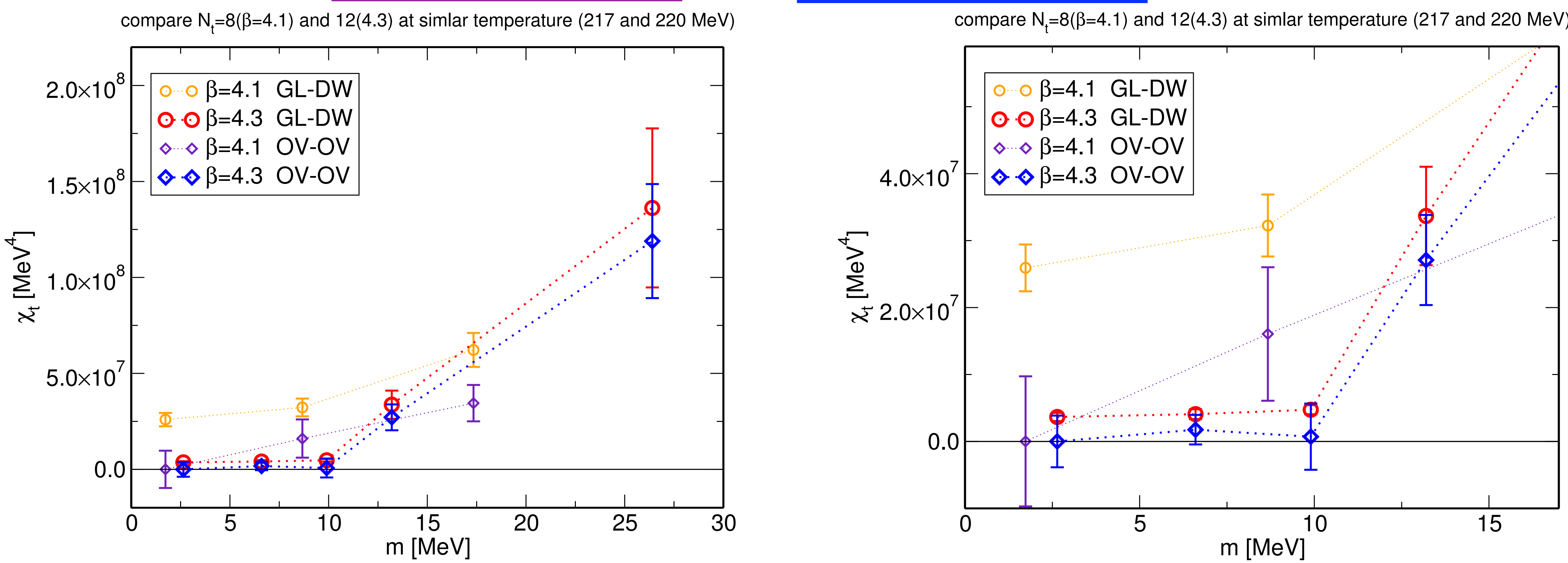
$\chi_t(m)$ T=~220 MeV discretization effect

comparing 1/a=1.7 GeV and 1/a=2.6 GeV ($(3.6\text{fm})^3$ and $(2.4\text{fm})^3$)



- OV-OV: better scaling
- GL-DW: large scaling violation for smaller m
- OV-OV: $\chi_t = 0$ (within error) for $0 \leq m \lesssim 10$ MeV
- GL-DW: $\chi_t > 0$, but, may well decrease as a

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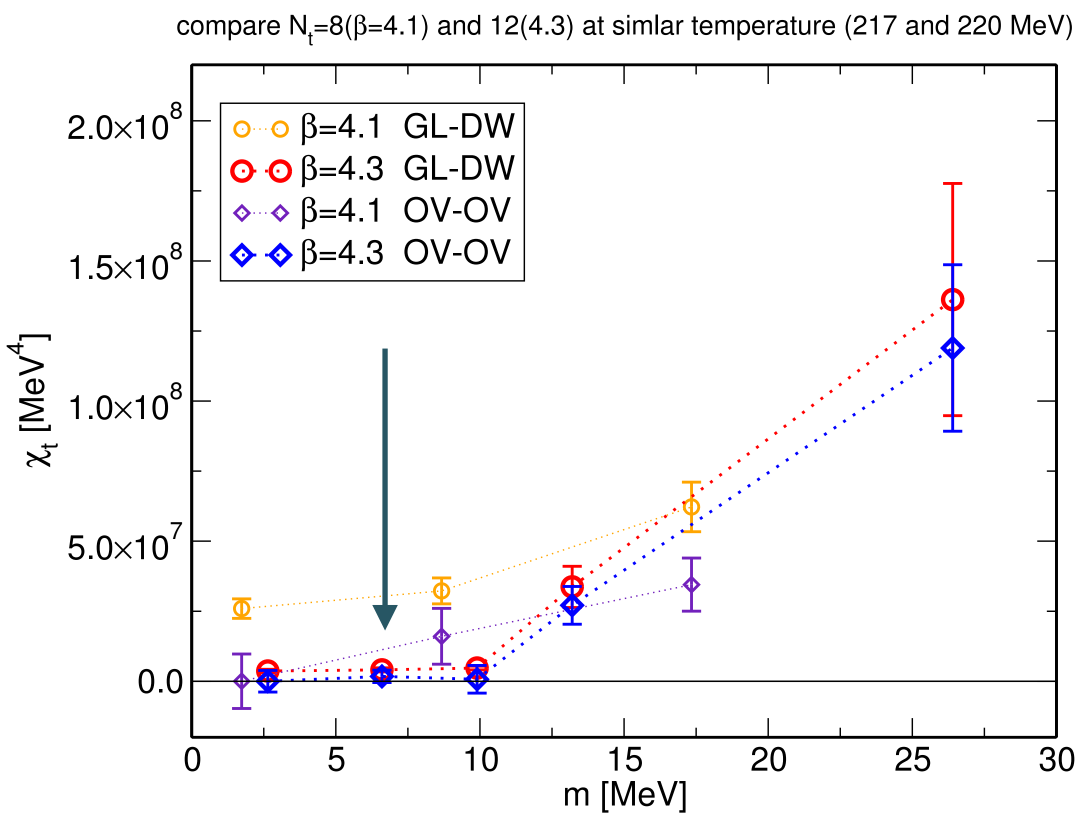

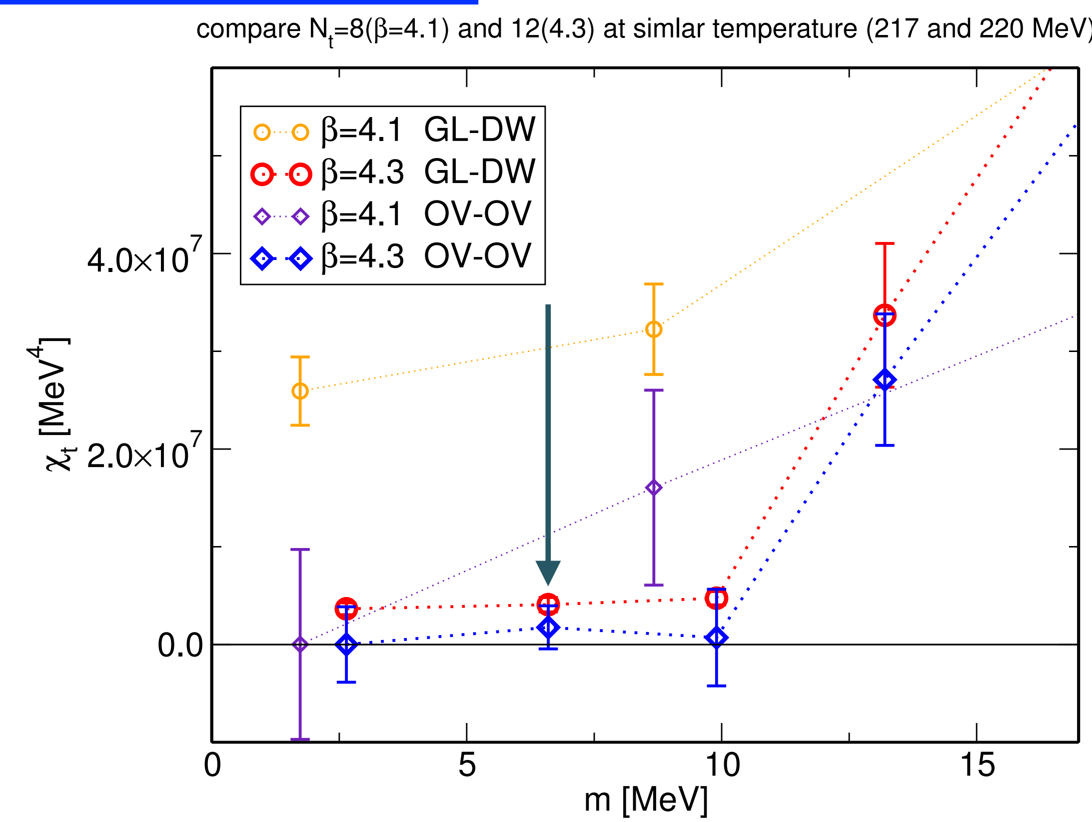

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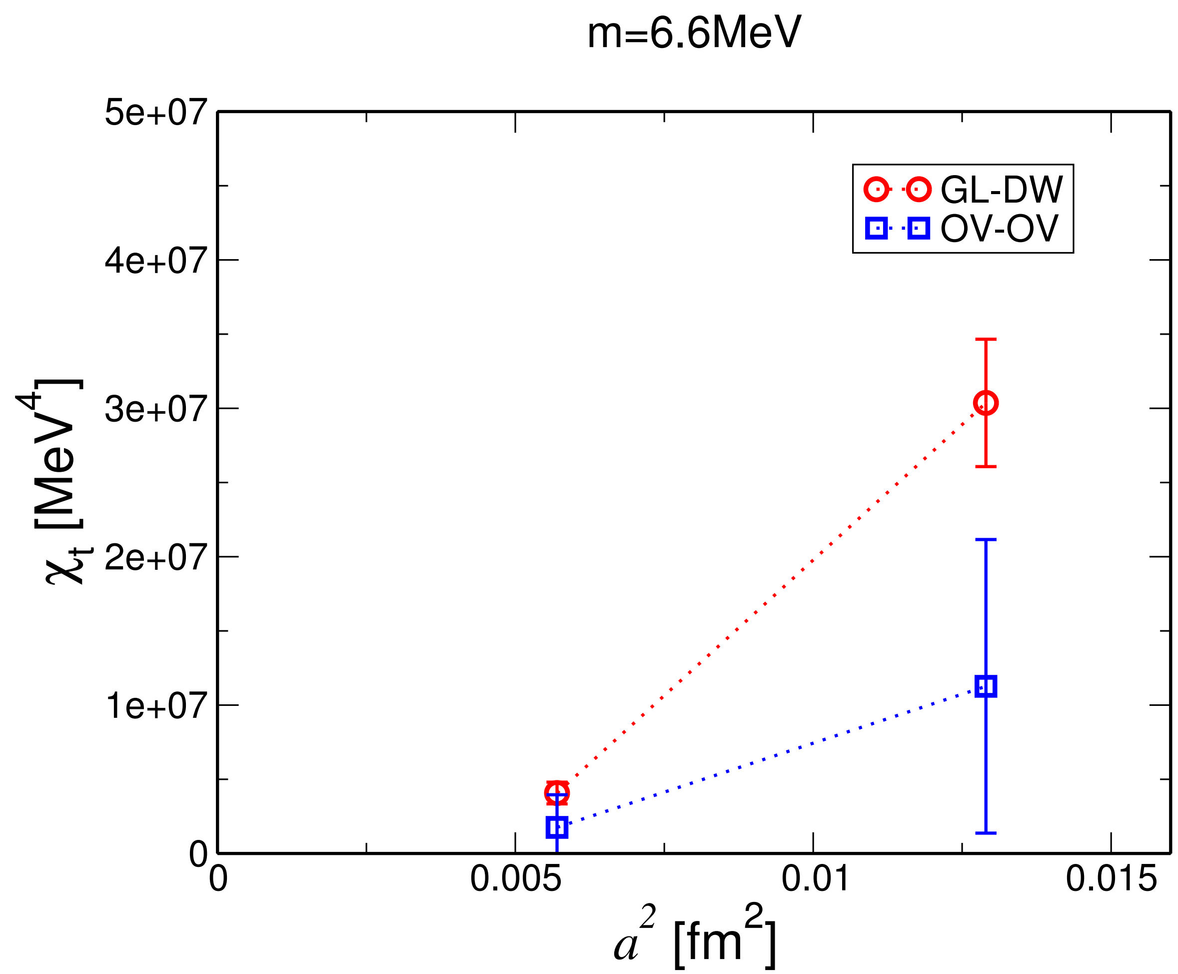
$\chi_t(m)$ T=220 MeV a^2 scaling: m=6.6 MeV



(V=(3.6fm)^3 and (2.4fm)^3)

continuum scaling in 1st region

- m=6.6 MeV
- vanishing towards continuum limit
- caveat: physical volume is different → needs further invest.

$\chi_t(m)$ T=~220 MeV, 32^3x12

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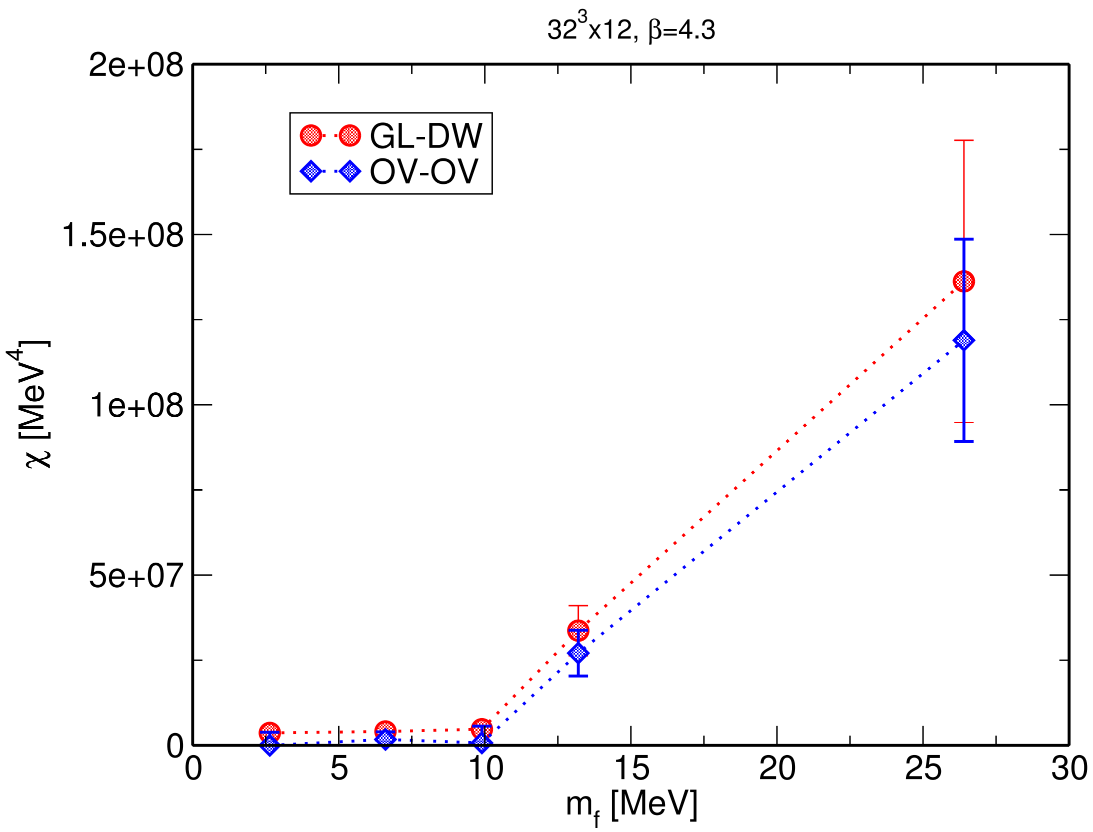

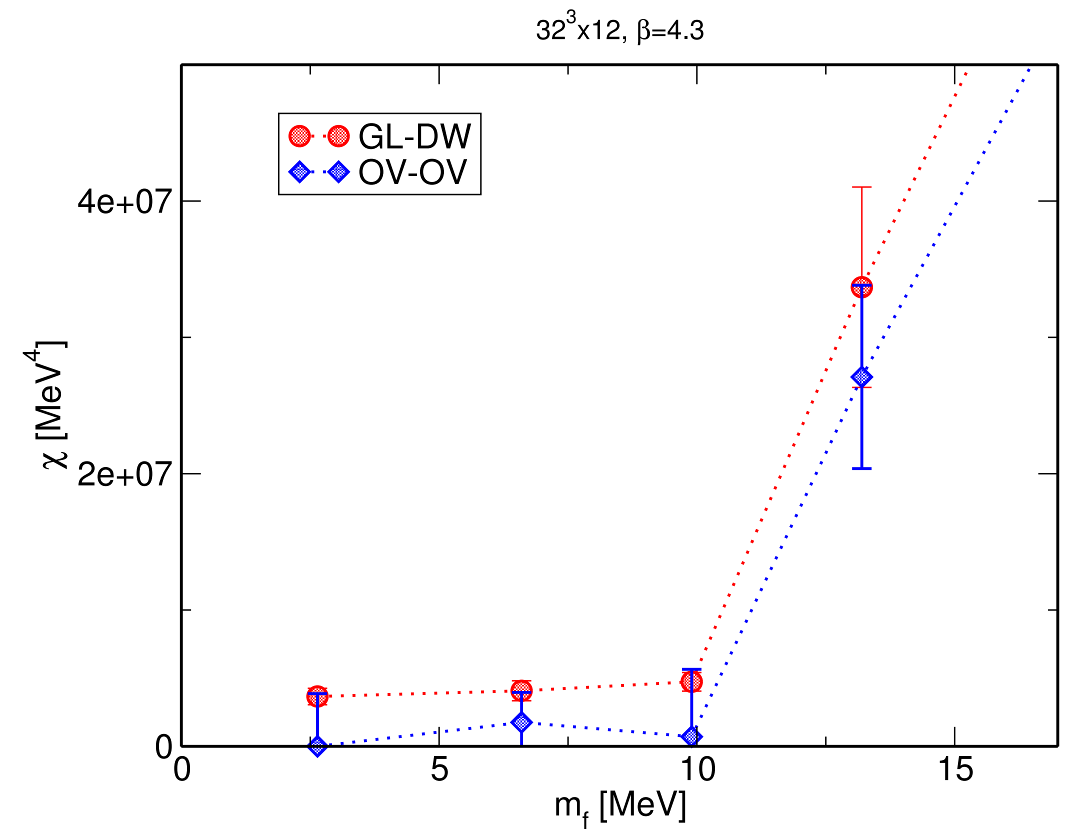

suggesting 2 regions

1: χ_t is very small (may vanish in $a\to 0$): $0 \leq m \lesssim 10$ MeV

(→ consistent w/ Aoki-Fukaya-Tanigchi for $U(1)_A$ symm.)

2: sudden growth of χ_t : 10 MeV $\lesssim$ m

- physical ud mass point: m≃4 MeV

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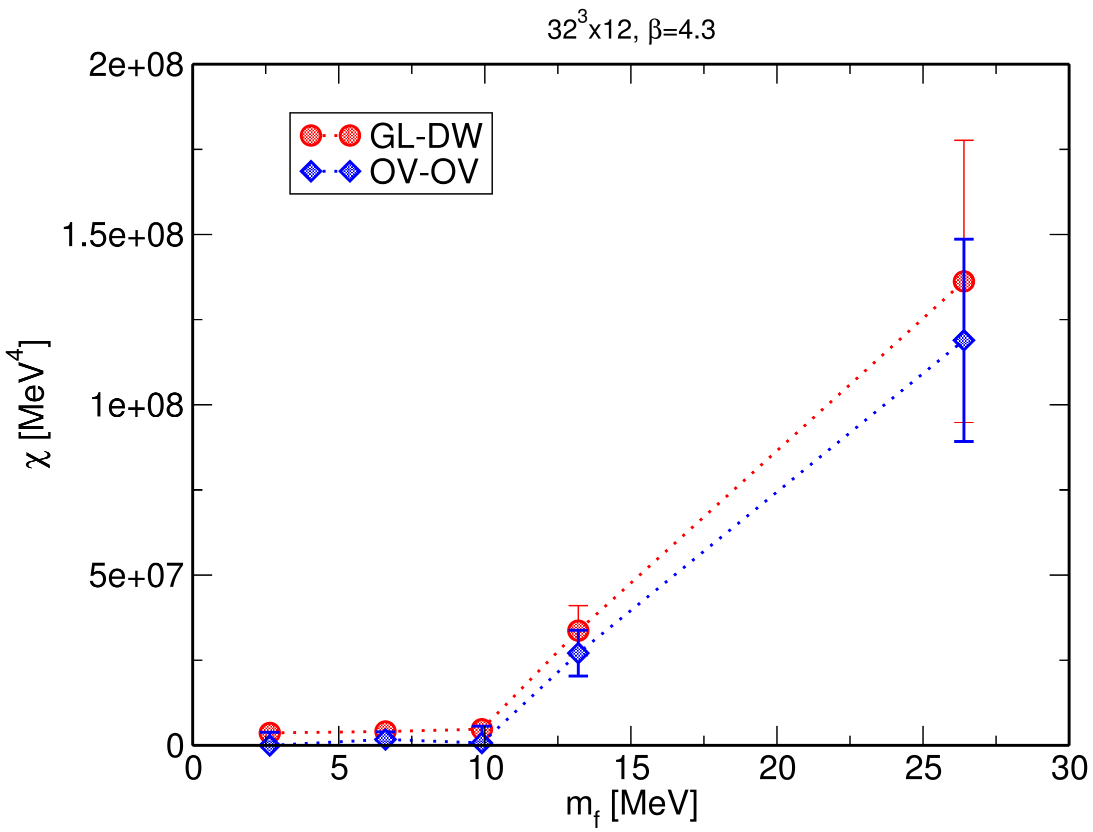

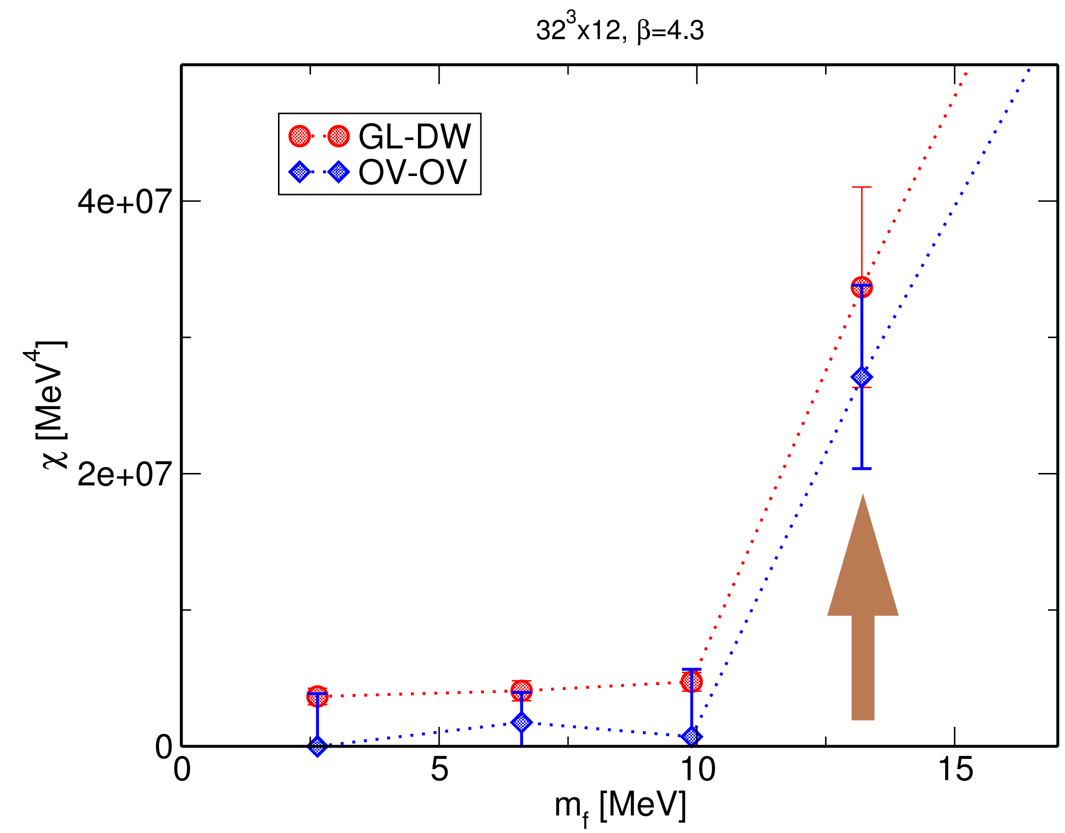

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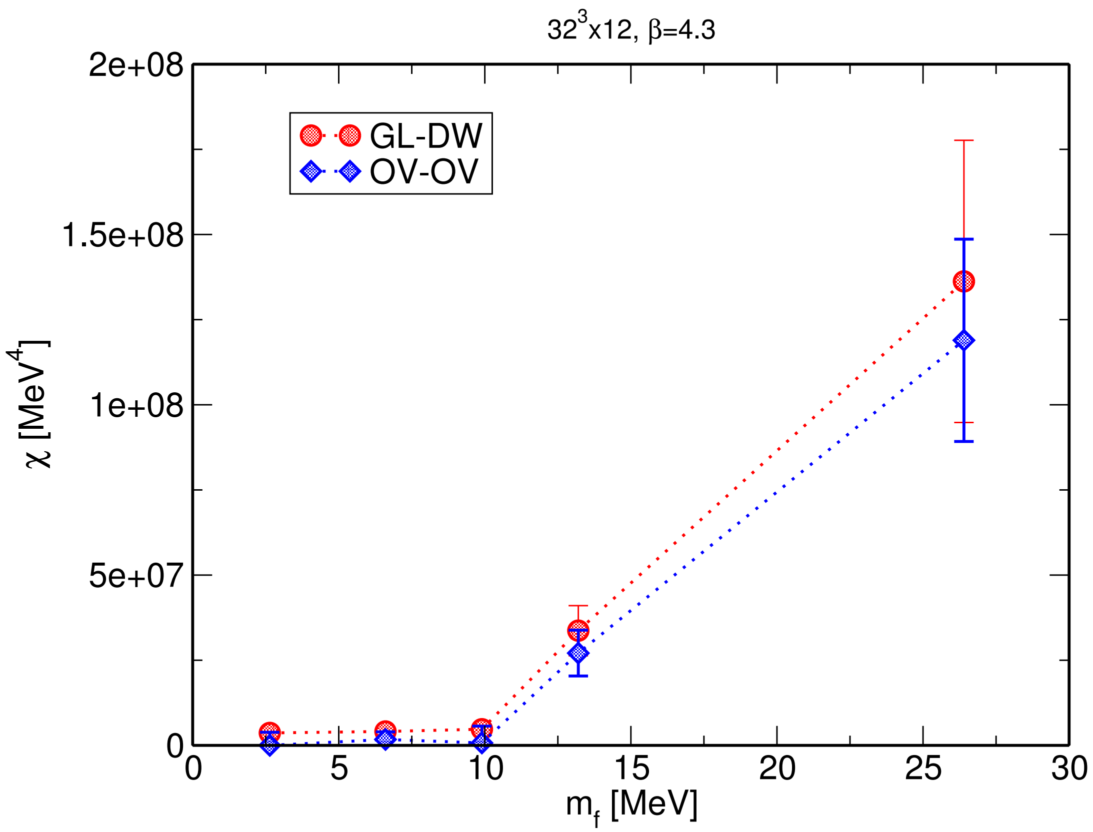

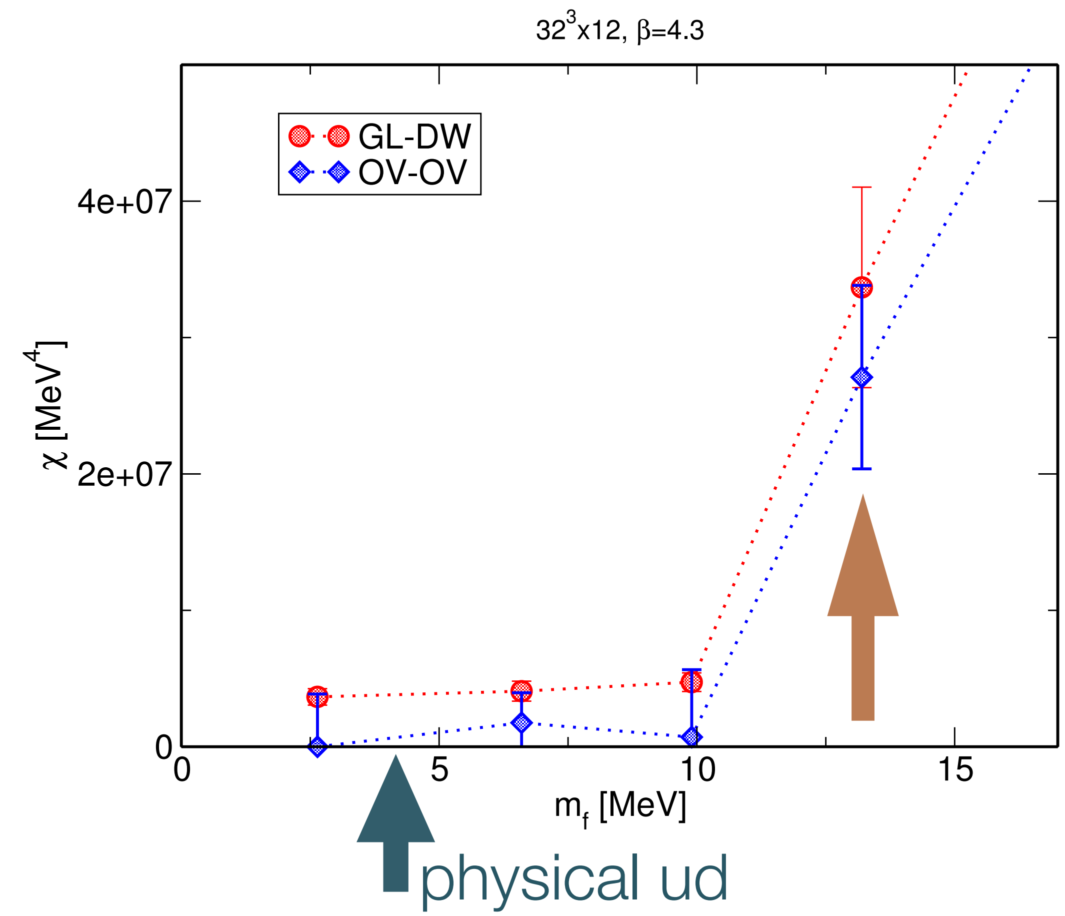

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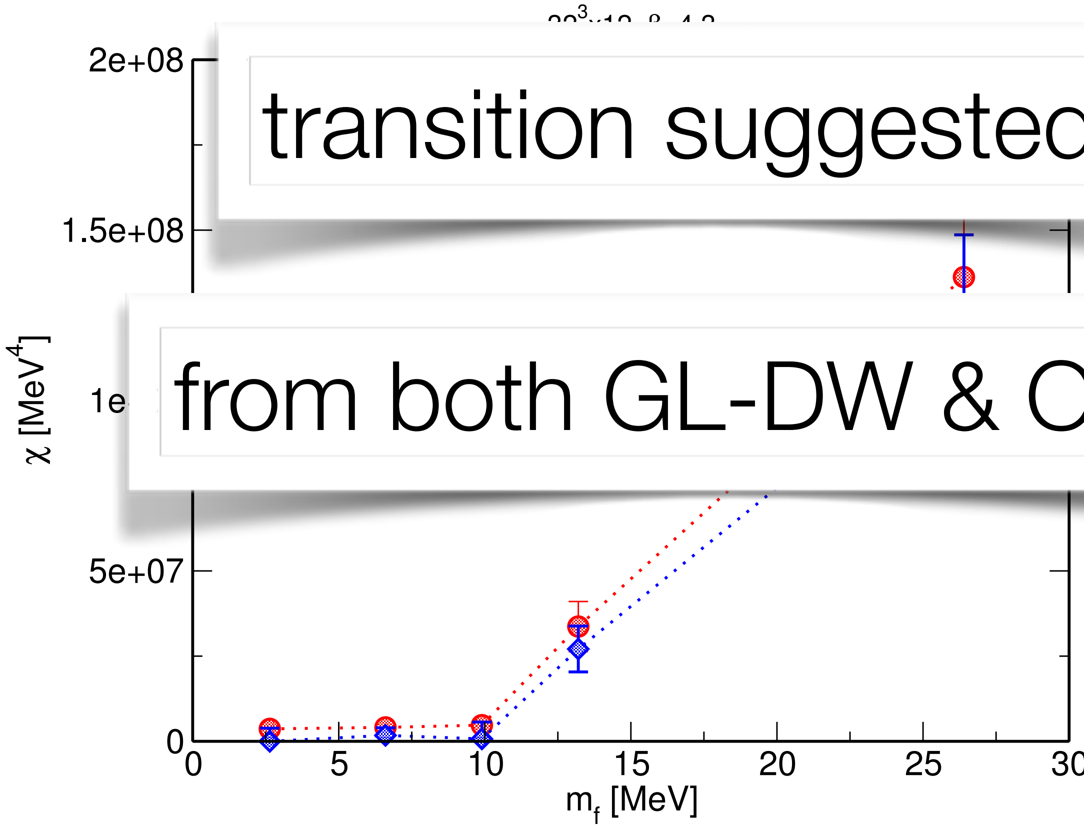
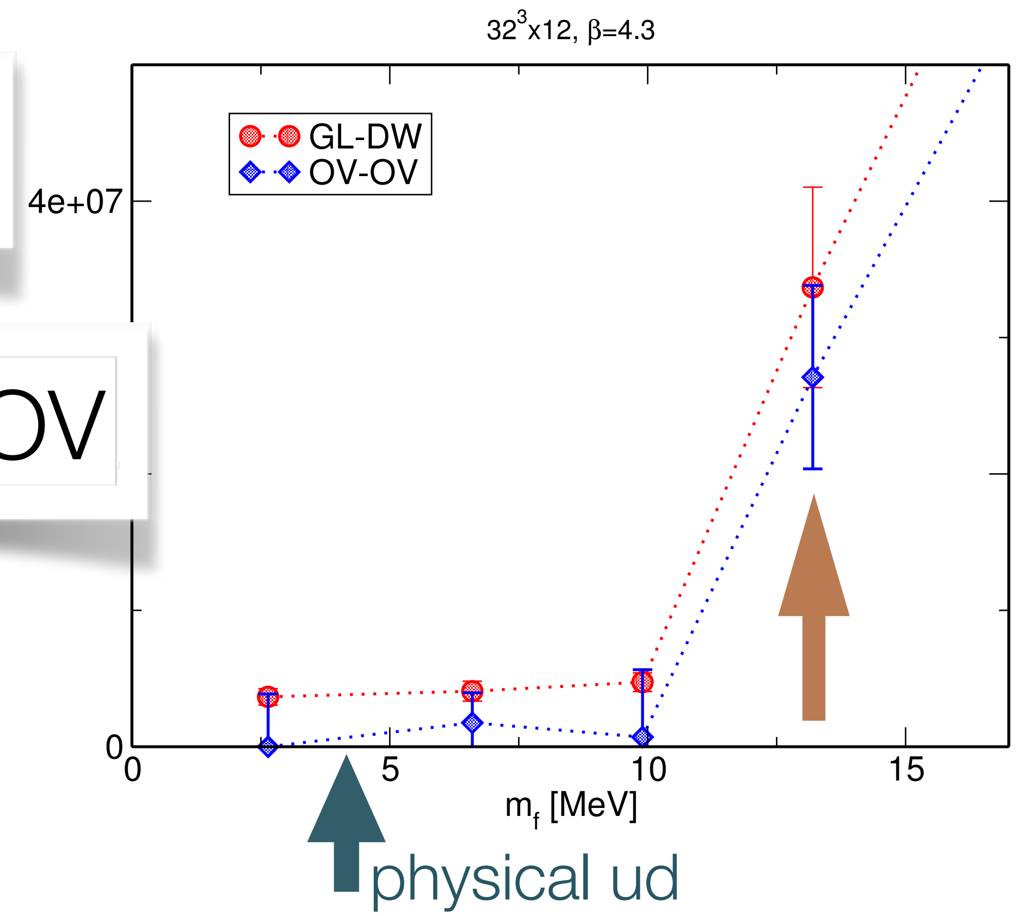
from both GL-DW & OV-OV





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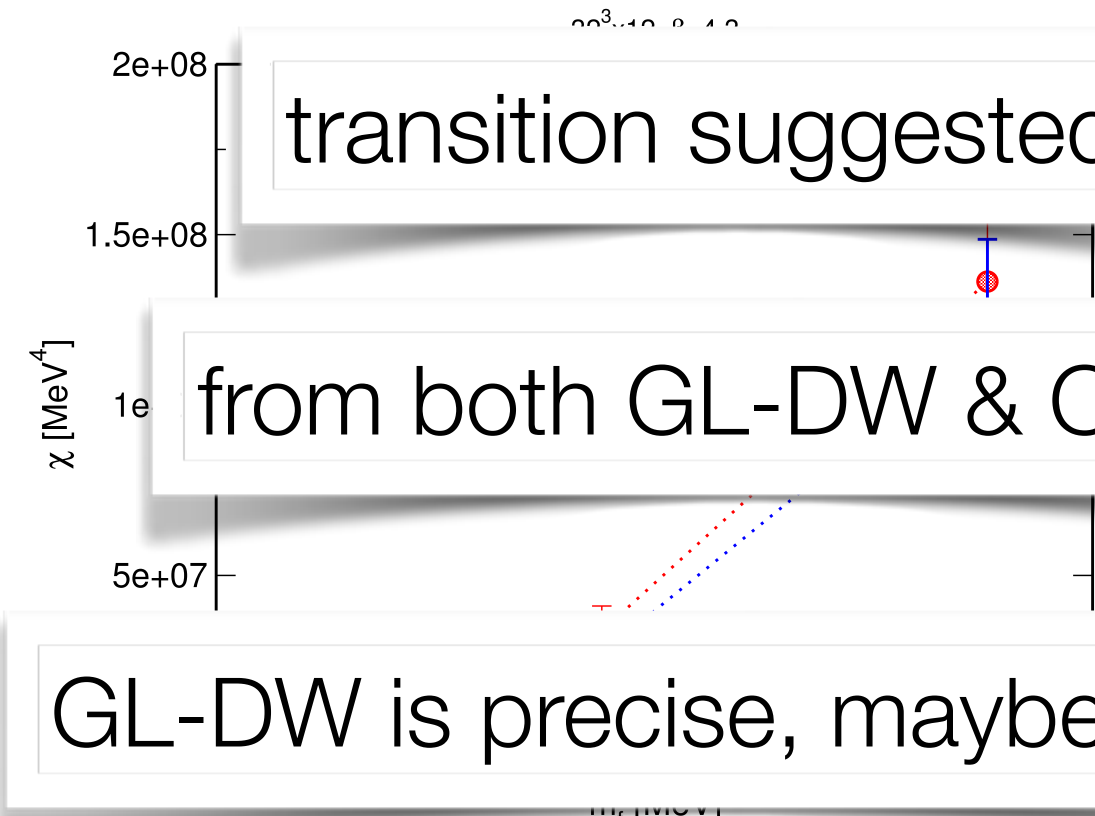

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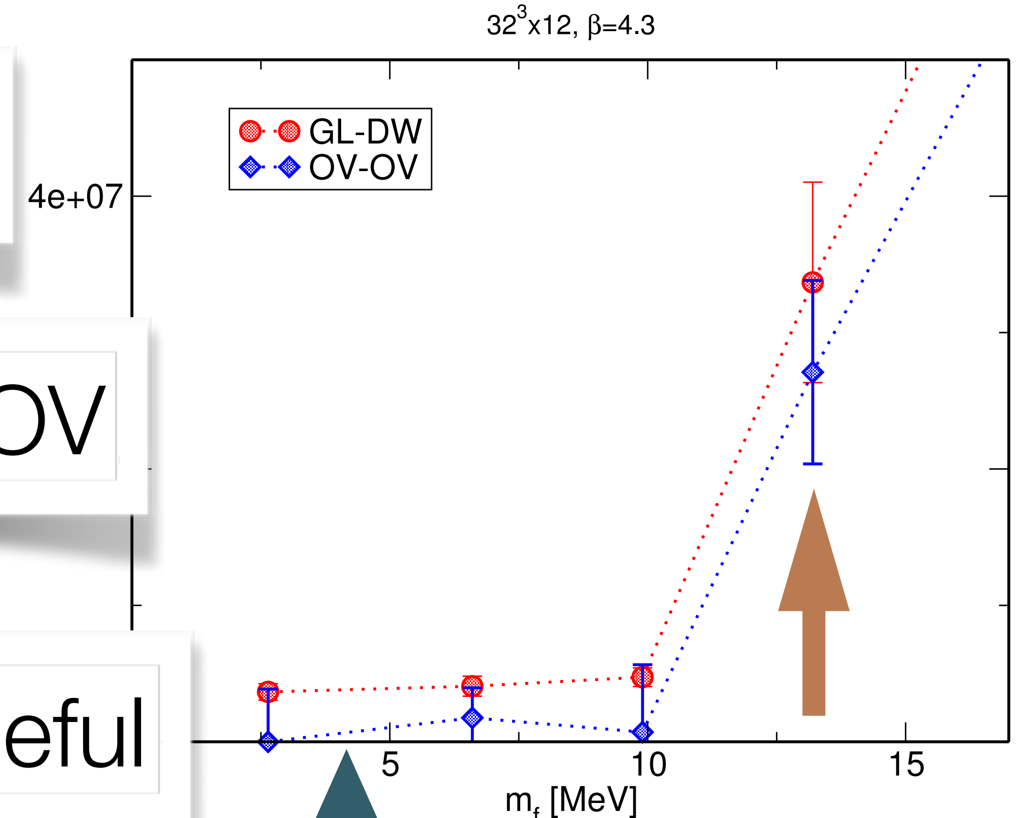
GL-DW is precise, maybe useful



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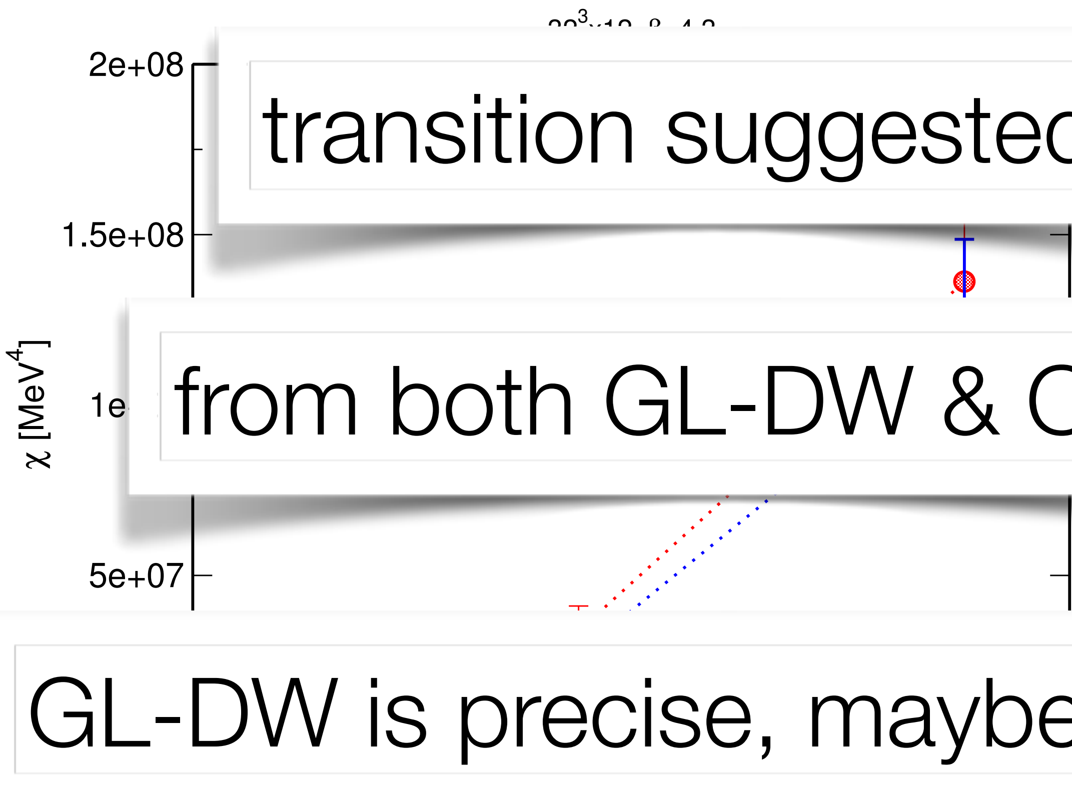

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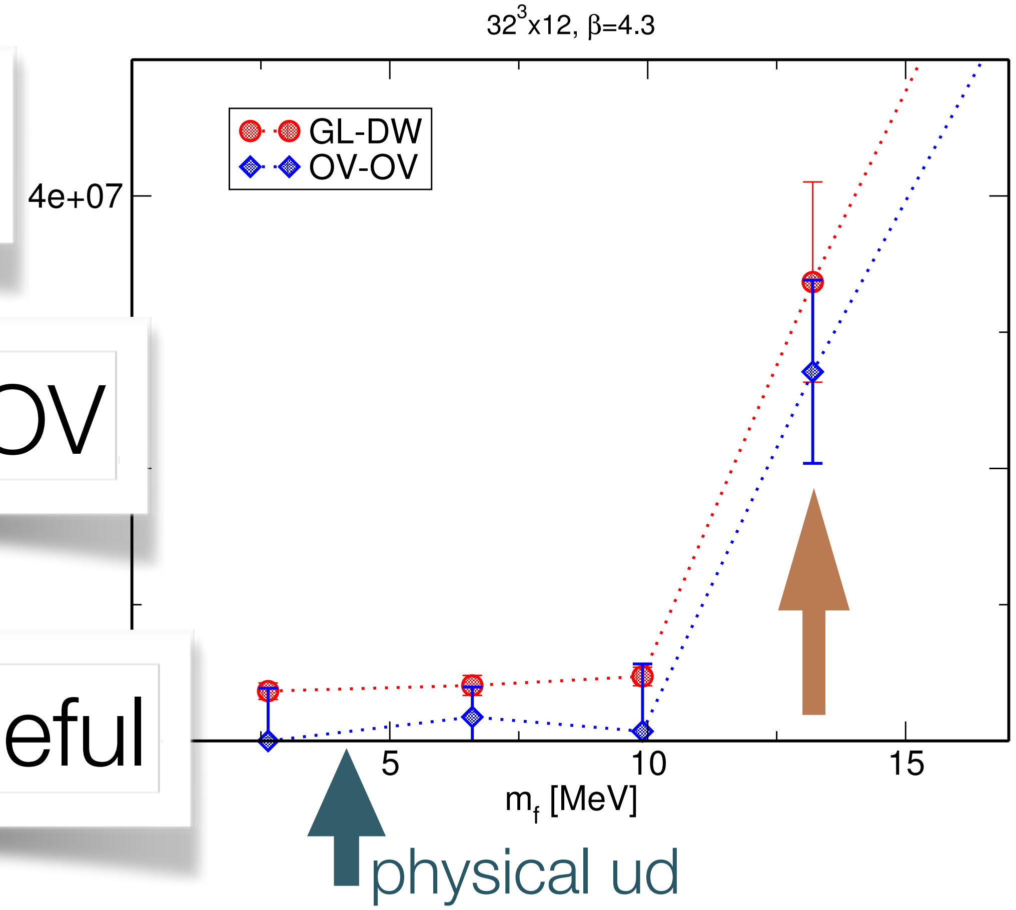

suggesting 2 regions

1: χ_t is v ... m ≲ 10 MeV

Next step: Volume Study

(→ consistent w/ Aoki-Fukaya-Tanigchi for $U(1)_A$ symm.)

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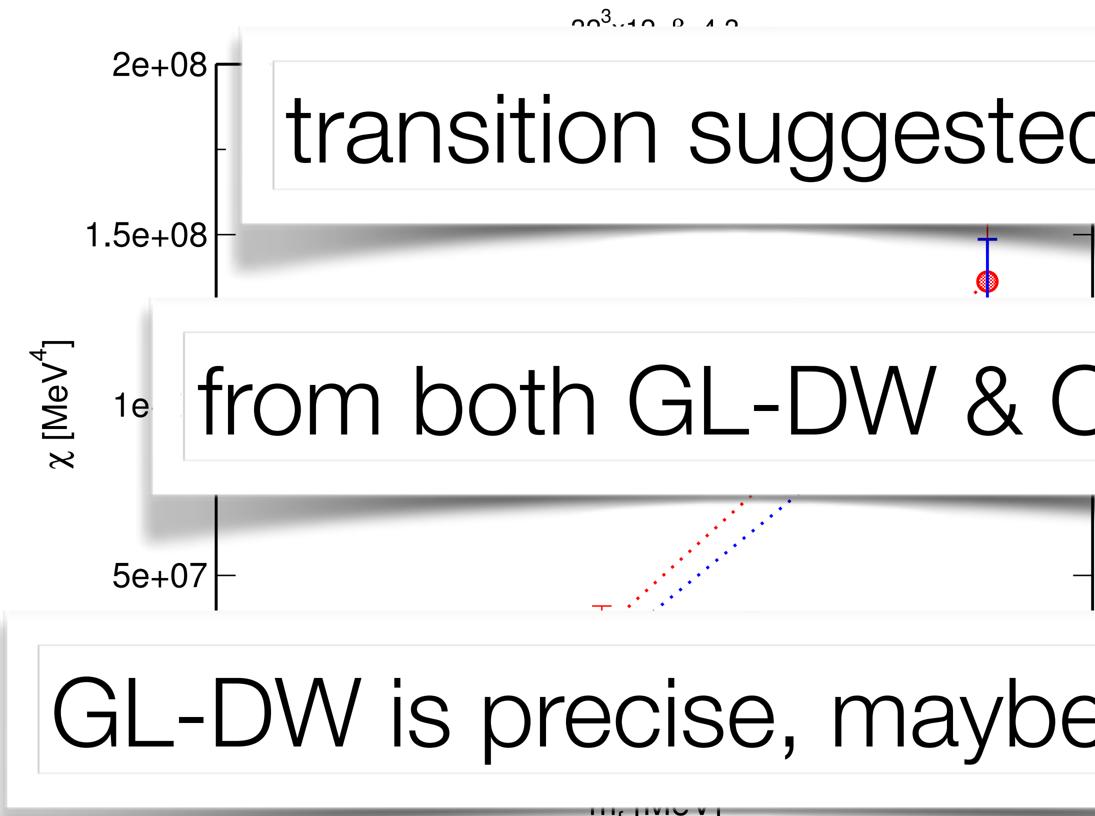

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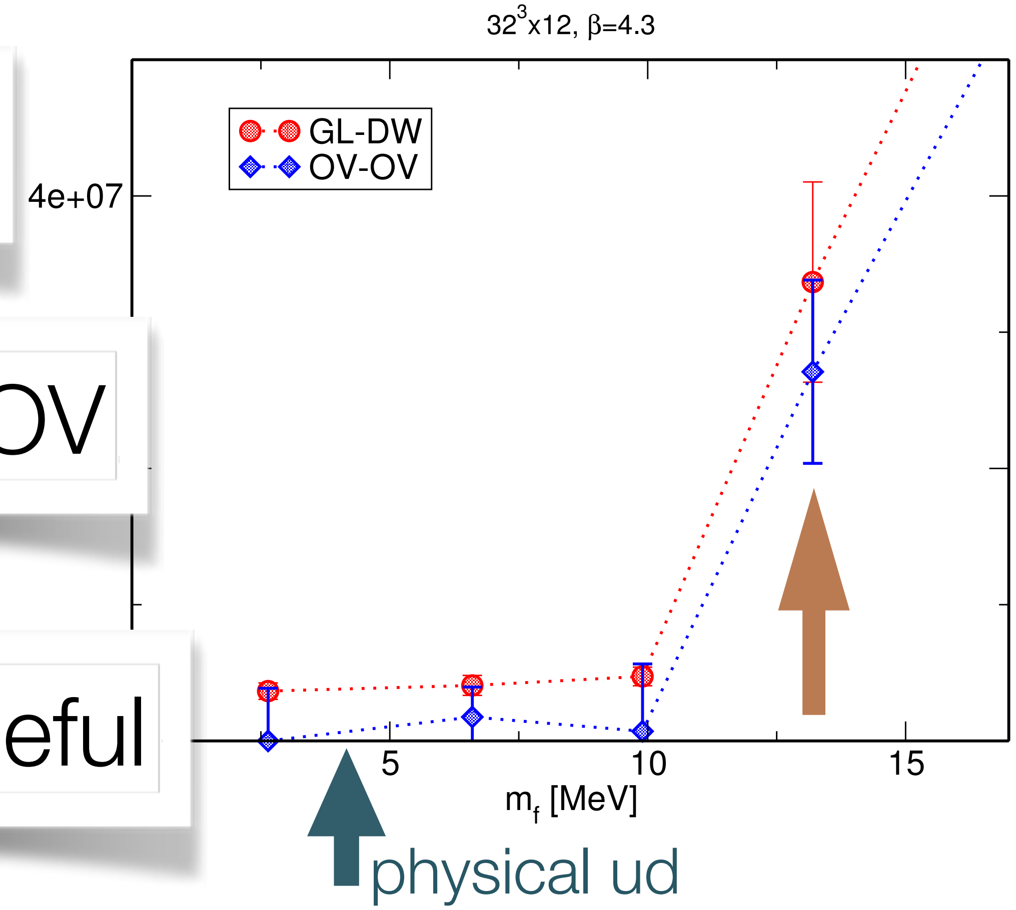

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… symm.)

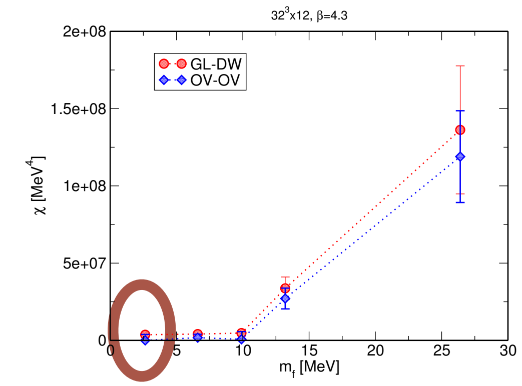
2:

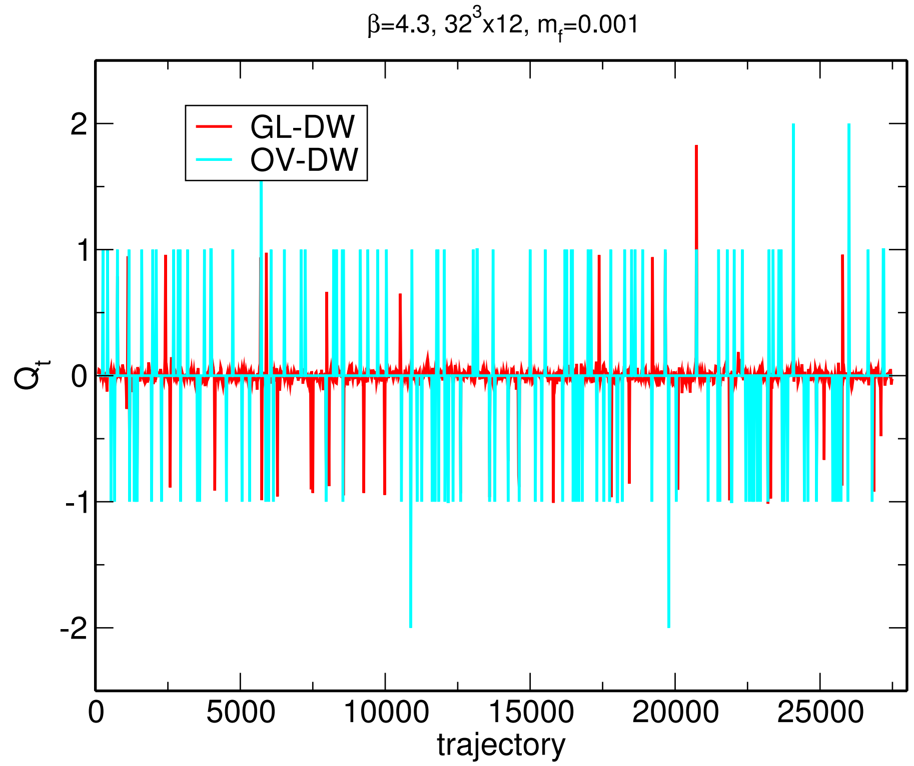
Next step: Volume Study

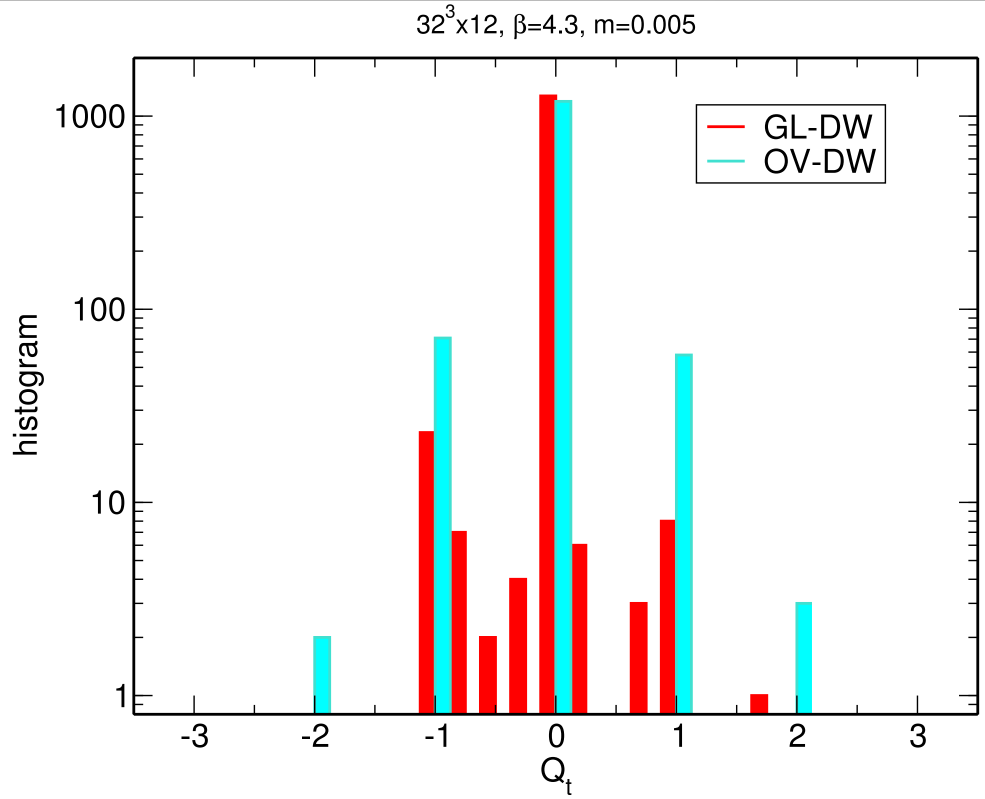
In addition to 32^3, **24^3** & **48^3** are studied

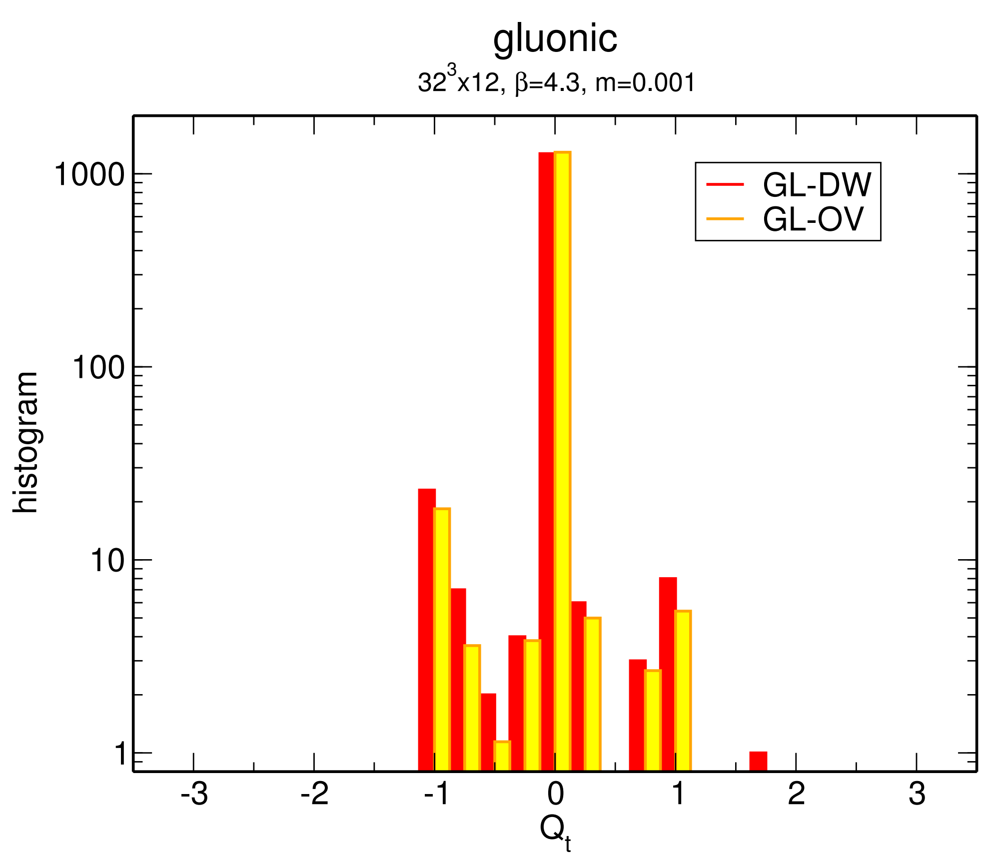
- physical ud mass point: m≃4 MeV

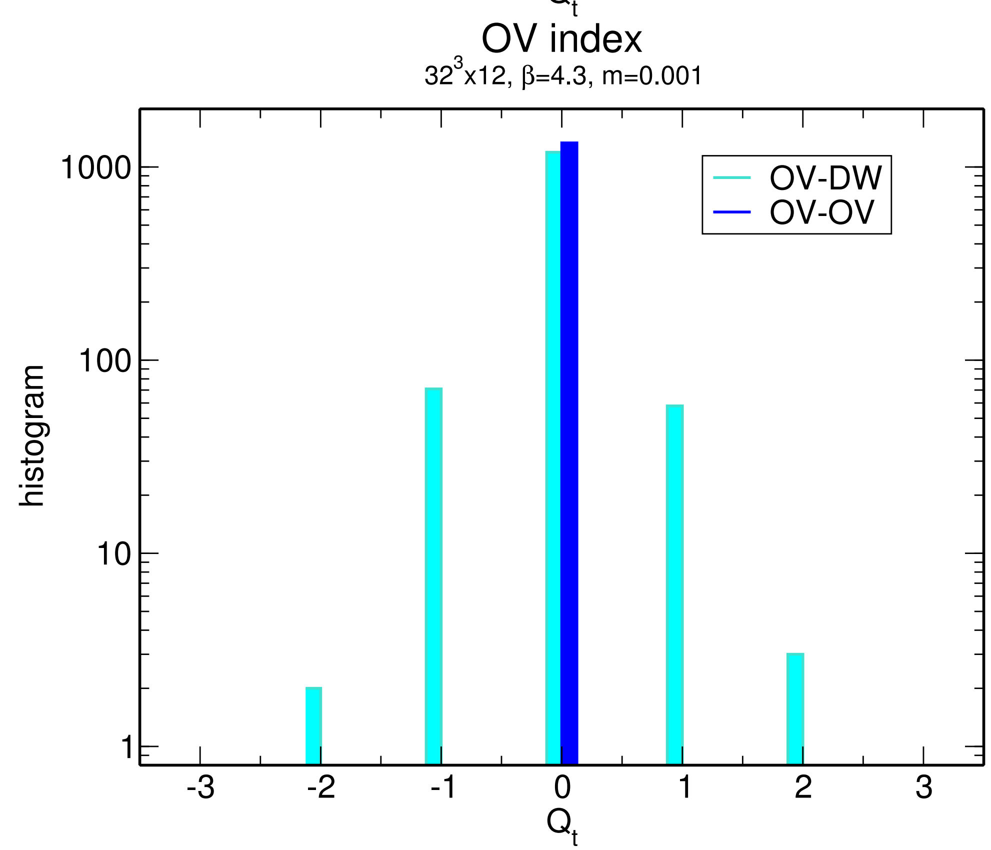
32^3 m=2.6 MeV history and histogram



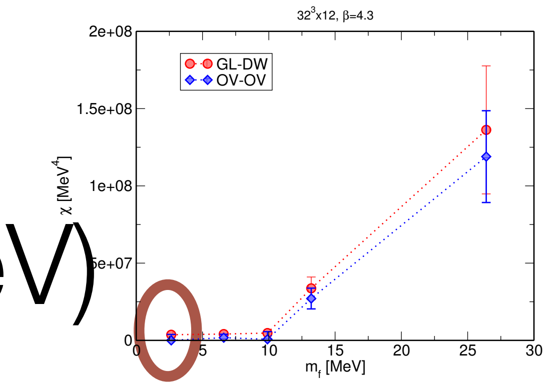


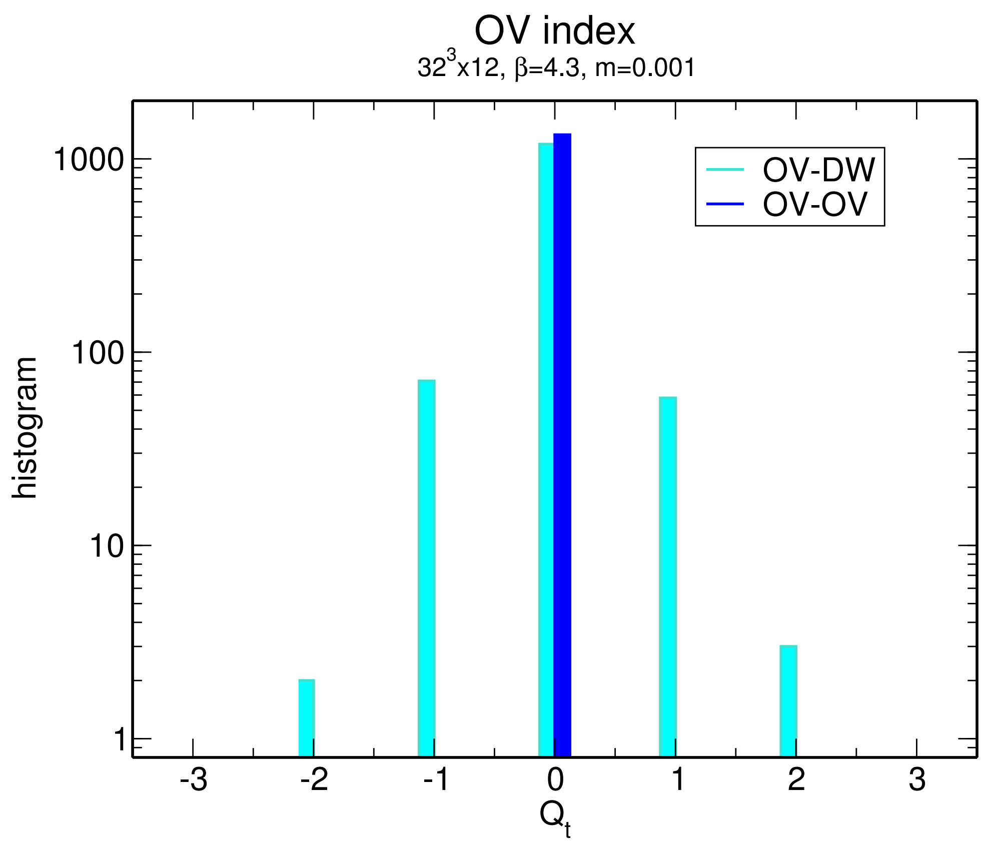






resolution of susceptibility (ex: m=2.6 MeV)





Effective number of statistics

- decreases with reweighting
- $N_{eff}=N_{conf}$ <R>/R_{max}
- N_{conf}=1326 → N_{eff} = 32

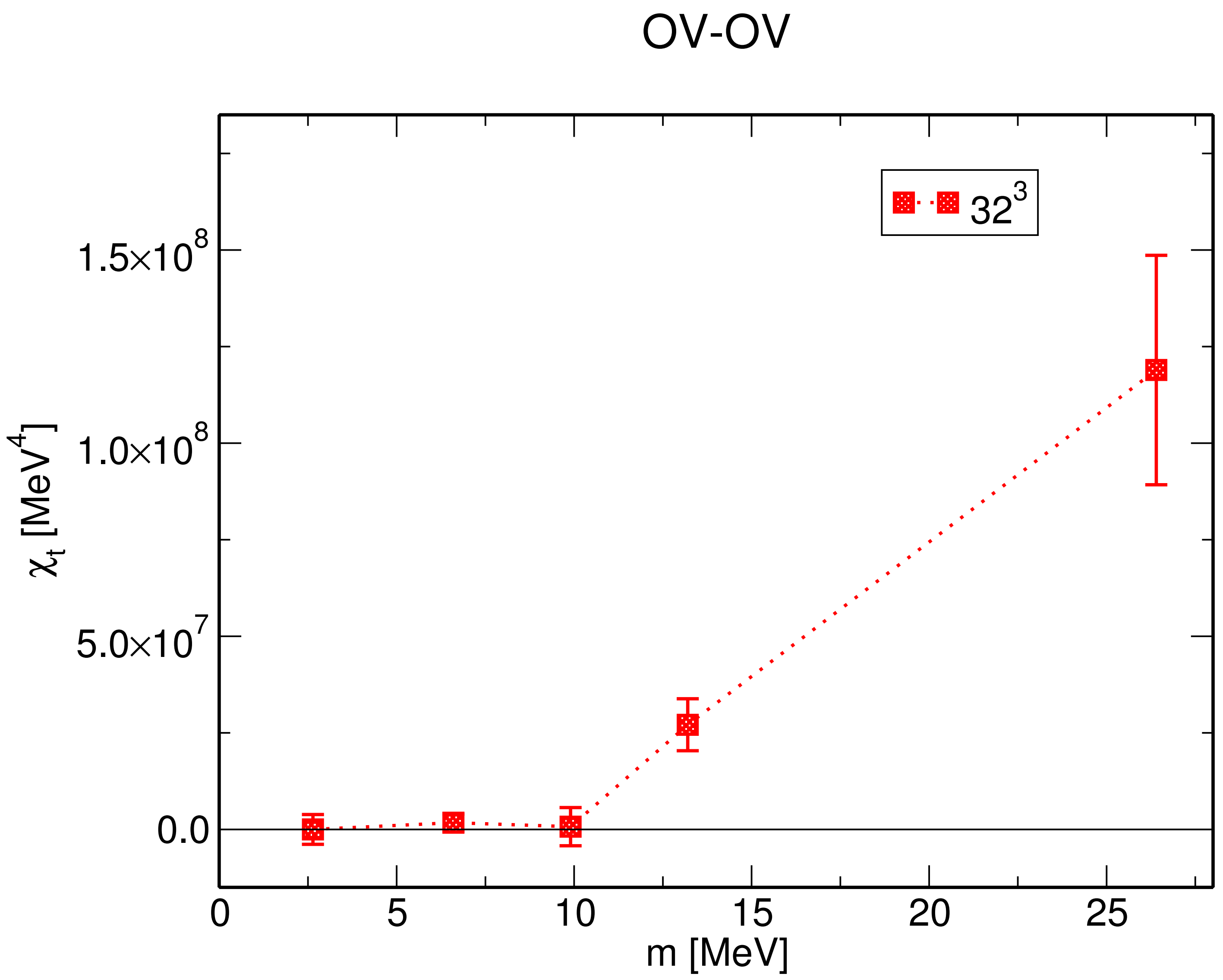
null measurement of topological excitation after reweighting

- does not readily mean χ_t=0:

  (this case <Q^2>=4(4) x10^{-6} ↔ 6(3) x10^{-3} @m=13MeV)

- there must be a resolution of χ_t under given statistics
  - [resolution of <Q^2>] = 1/N_{eff}
- shall take the “statistical error” of <Q^2> = max(Δ<Q^2>, 1/N_{eff})

Results of $\chi_t(m)$ at T=220 MeV; multiple volume



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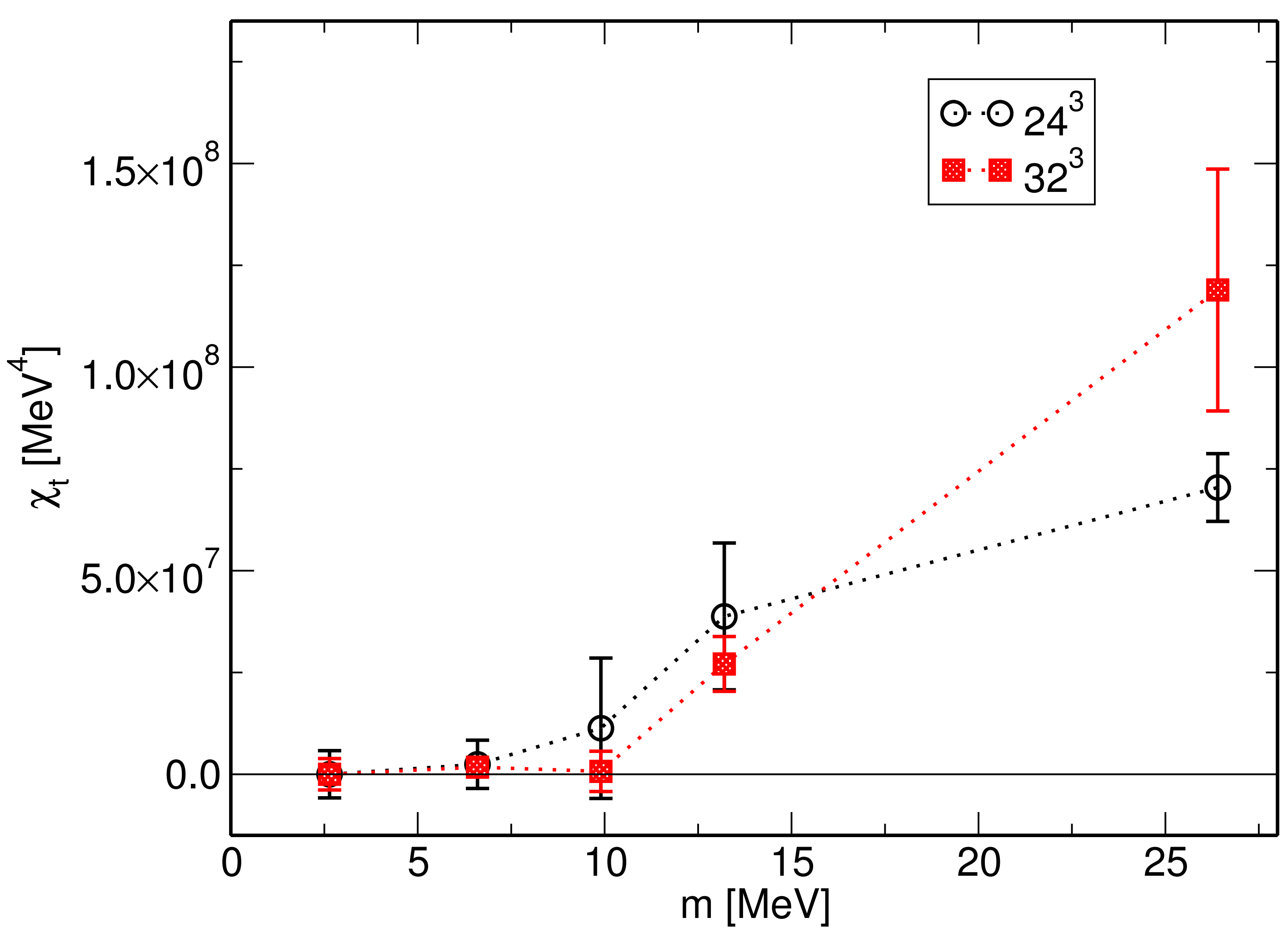
OV-OV



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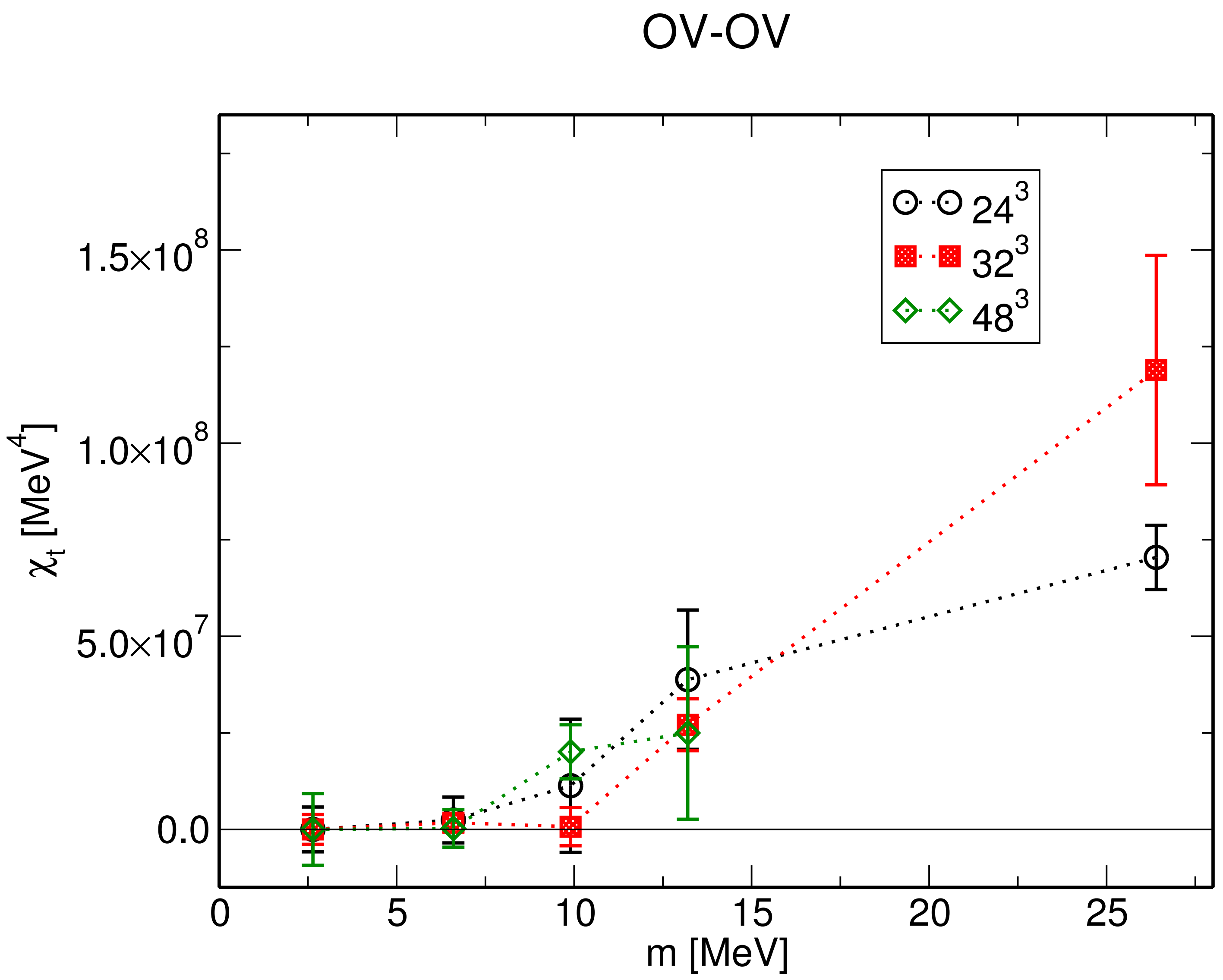

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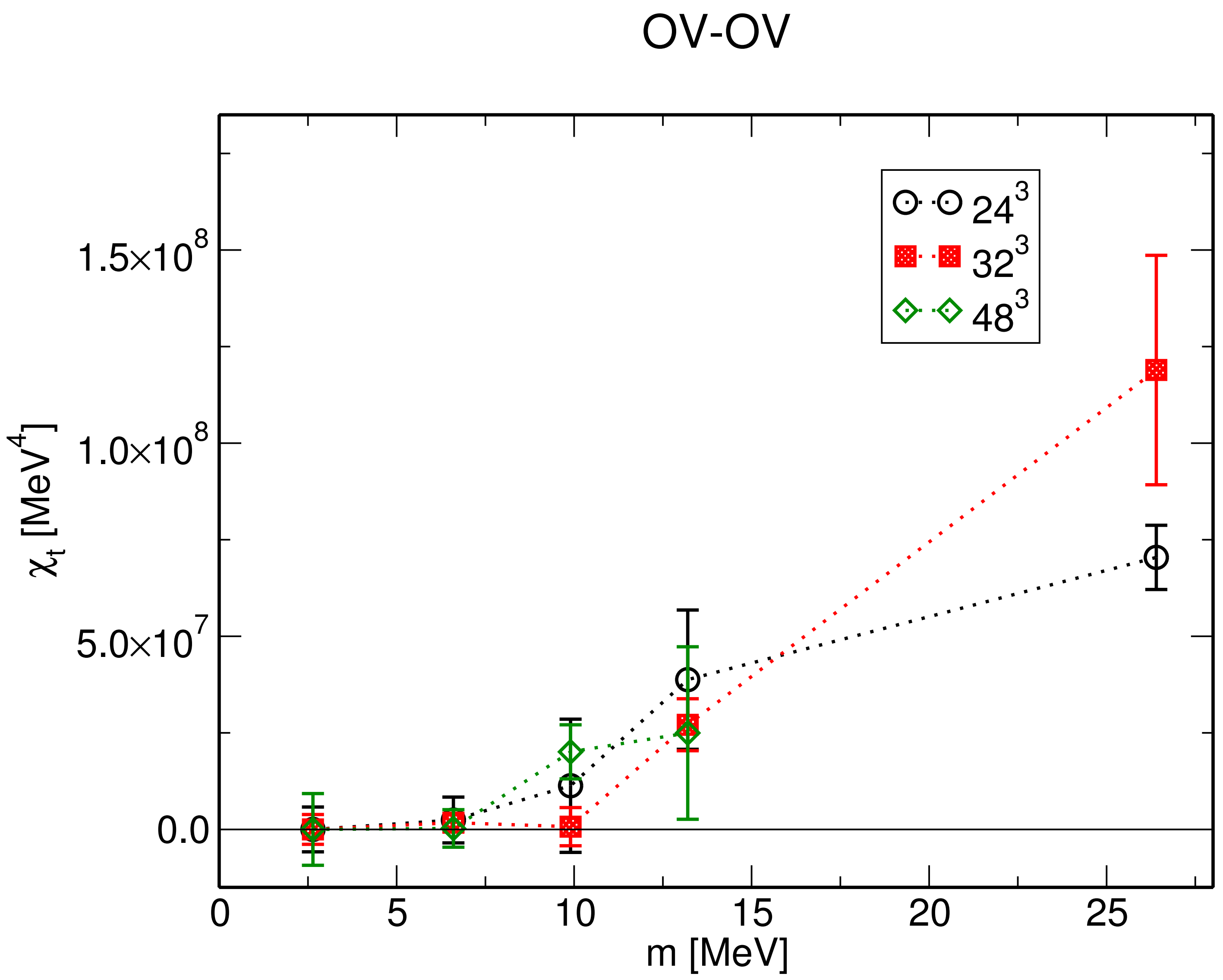

- Statistics in trajectory
  ~30k, 30k, 10k

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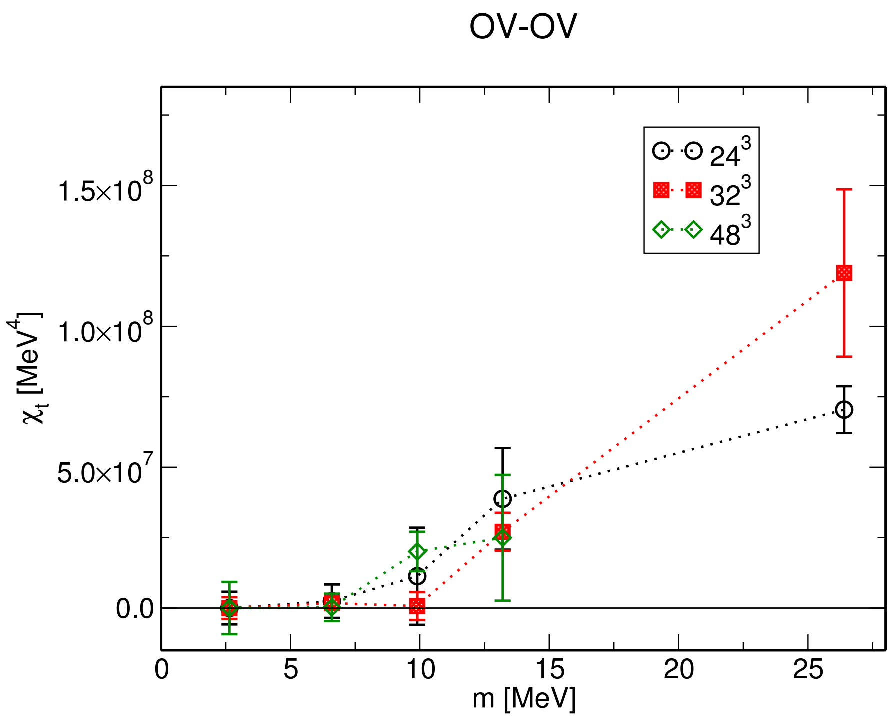

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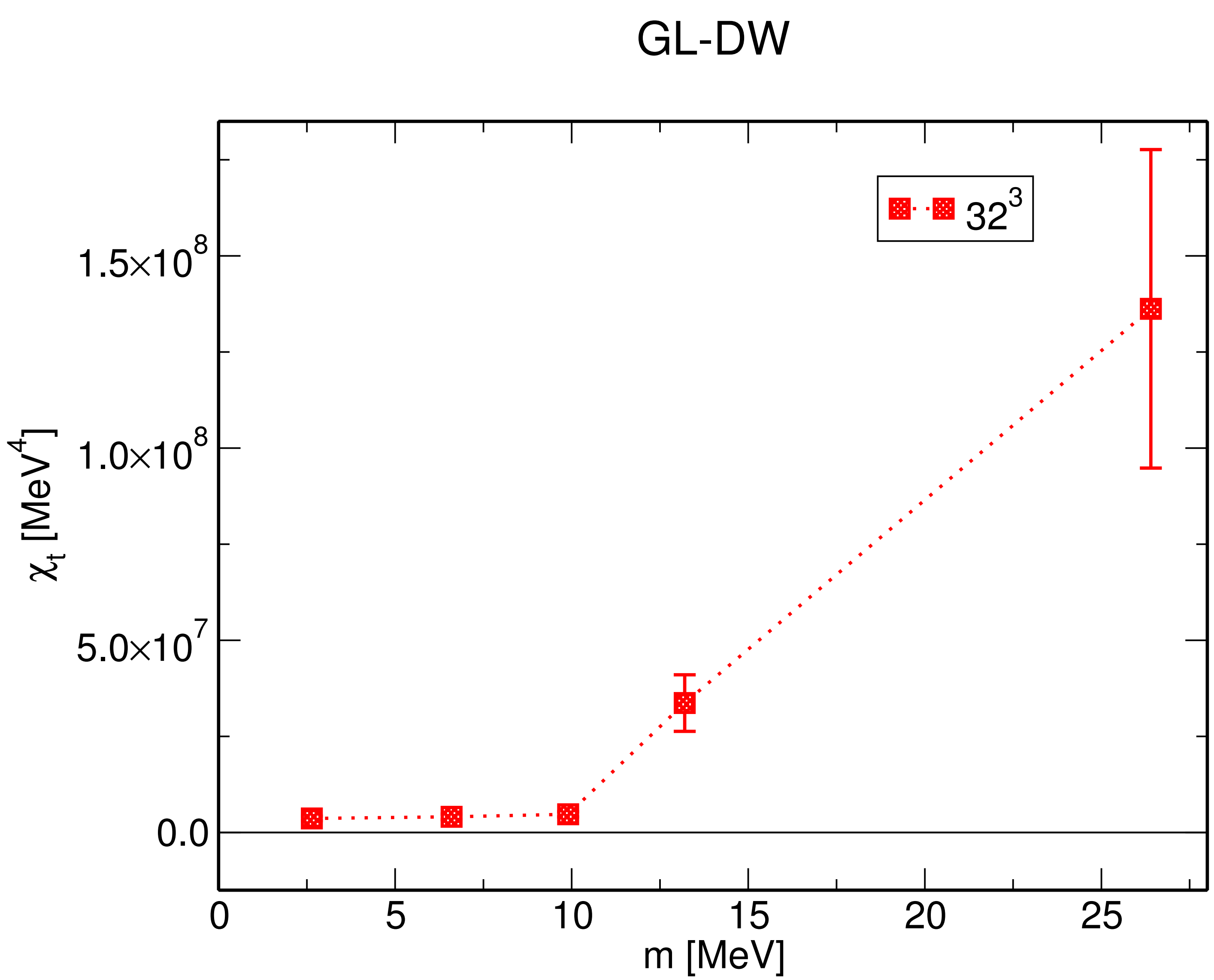

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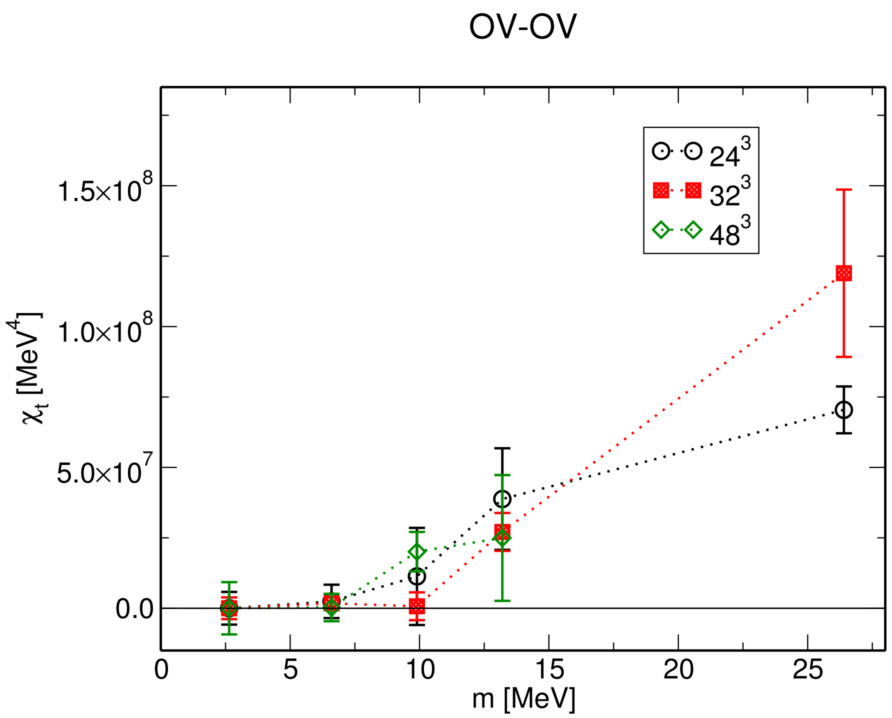

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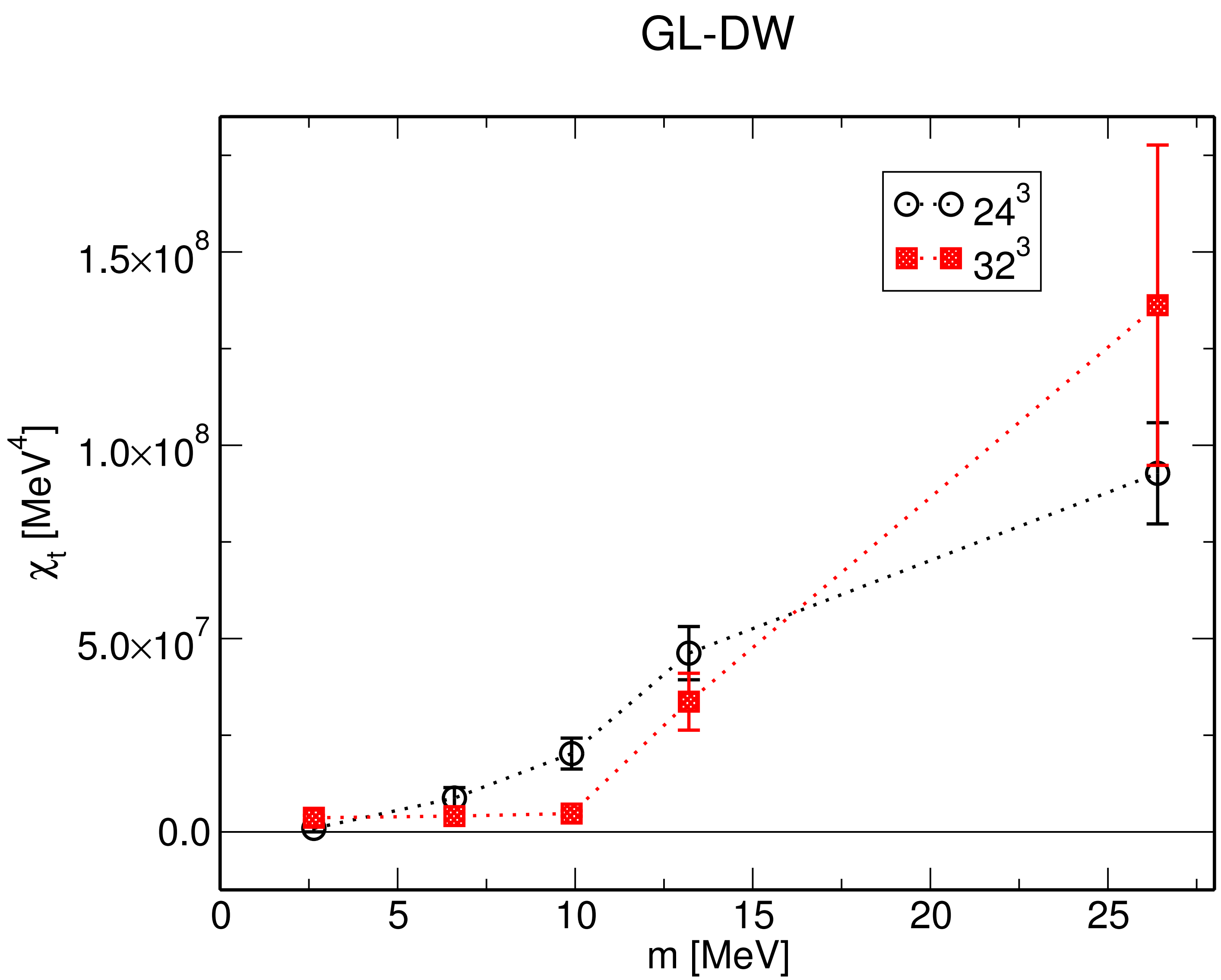

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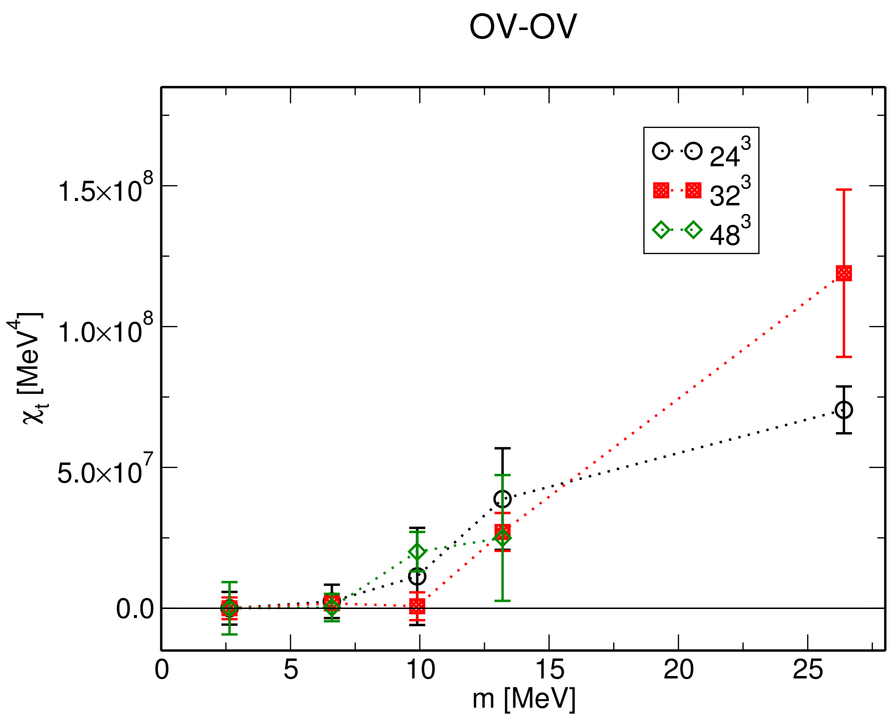

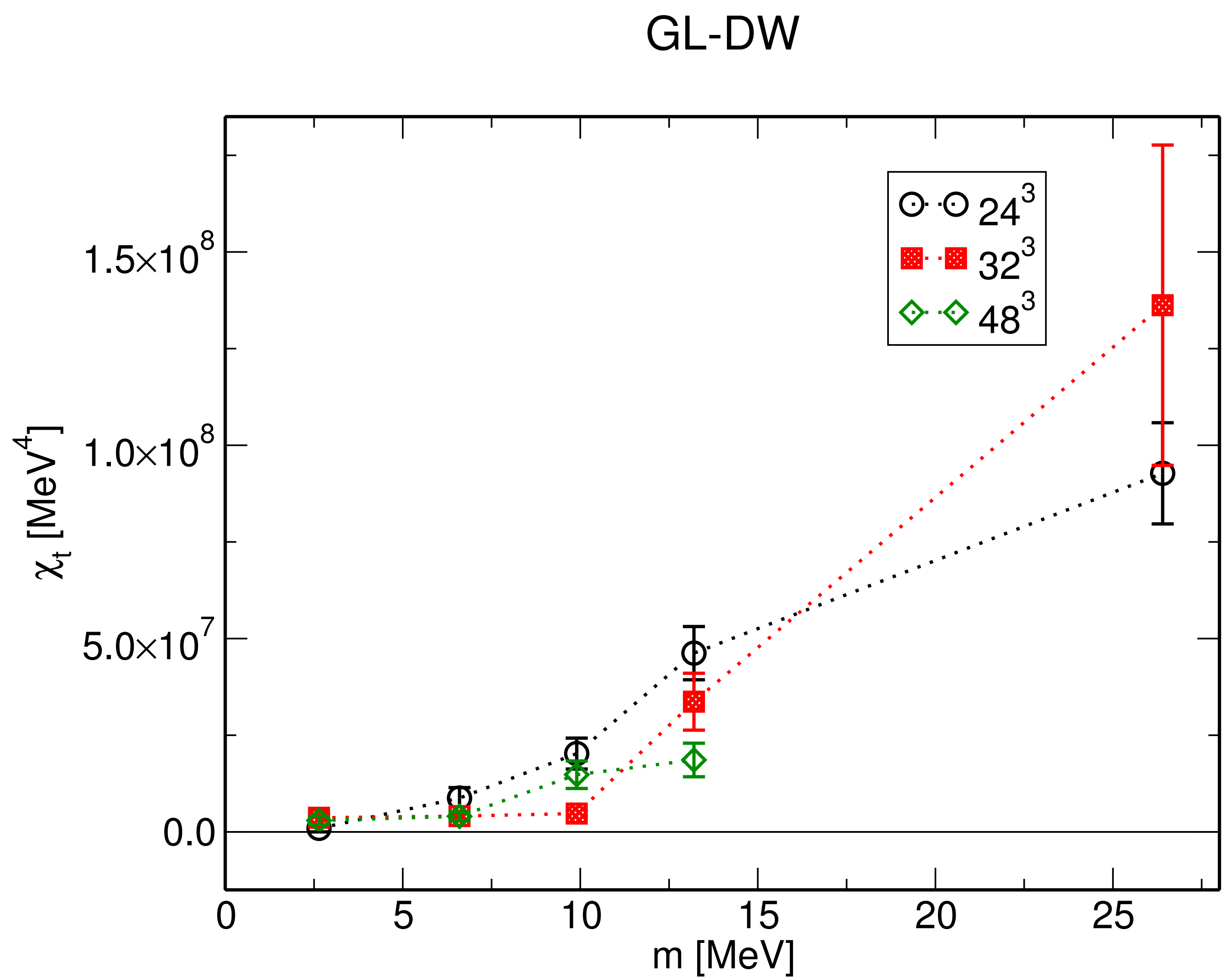

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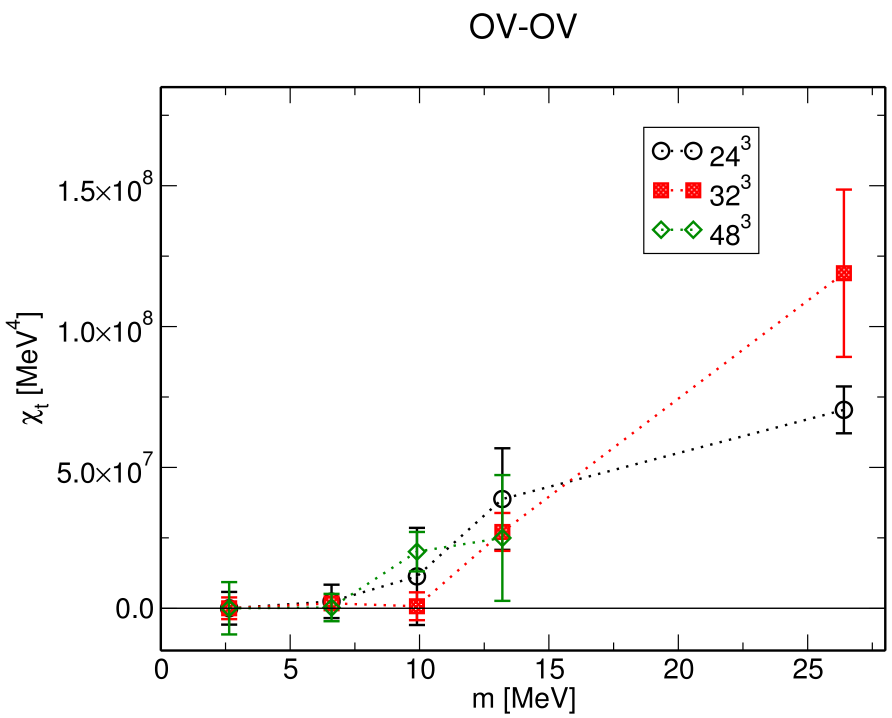

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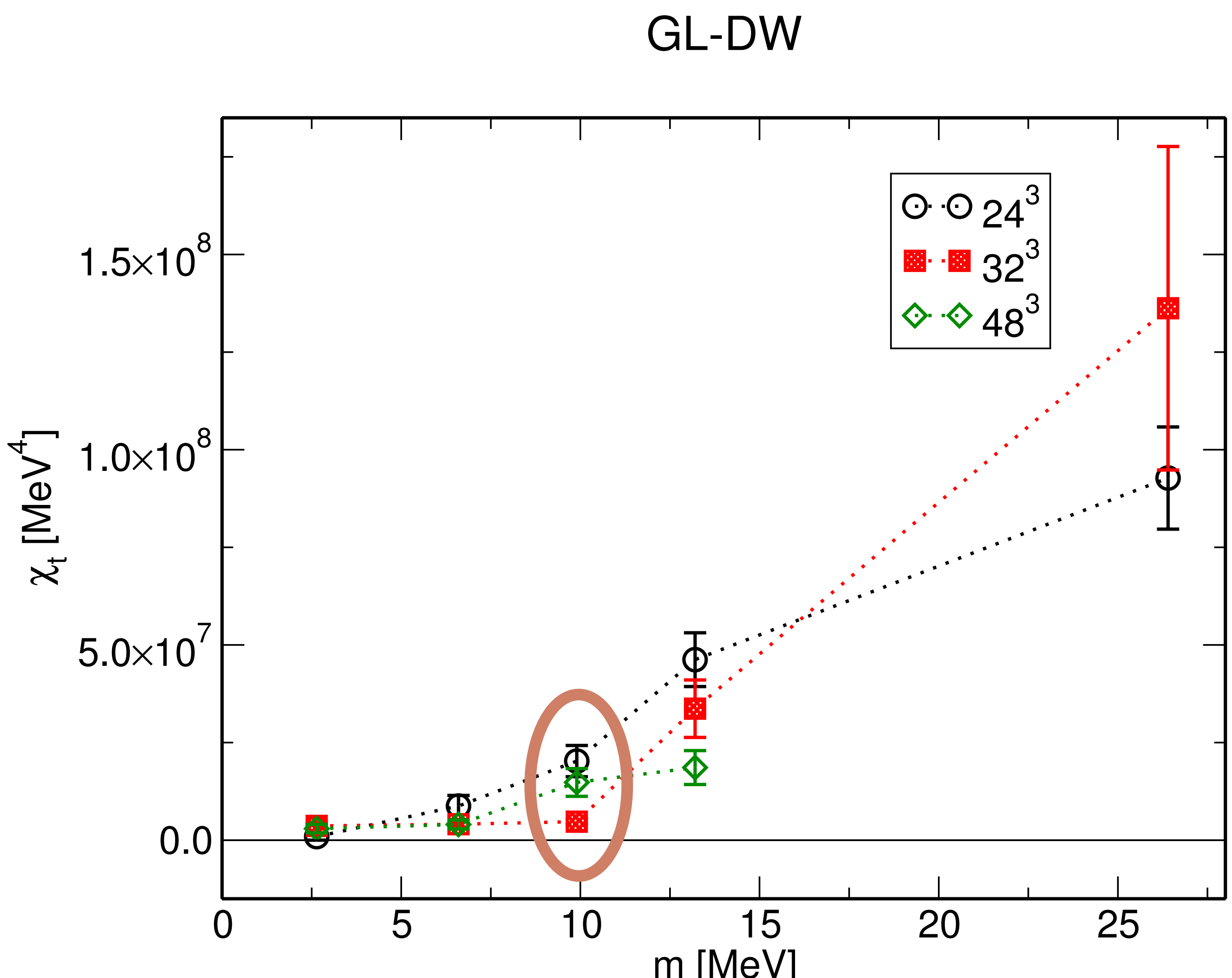

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  - non-monotonic
  - important region, where a phase boundary was suggested w/ 32^3

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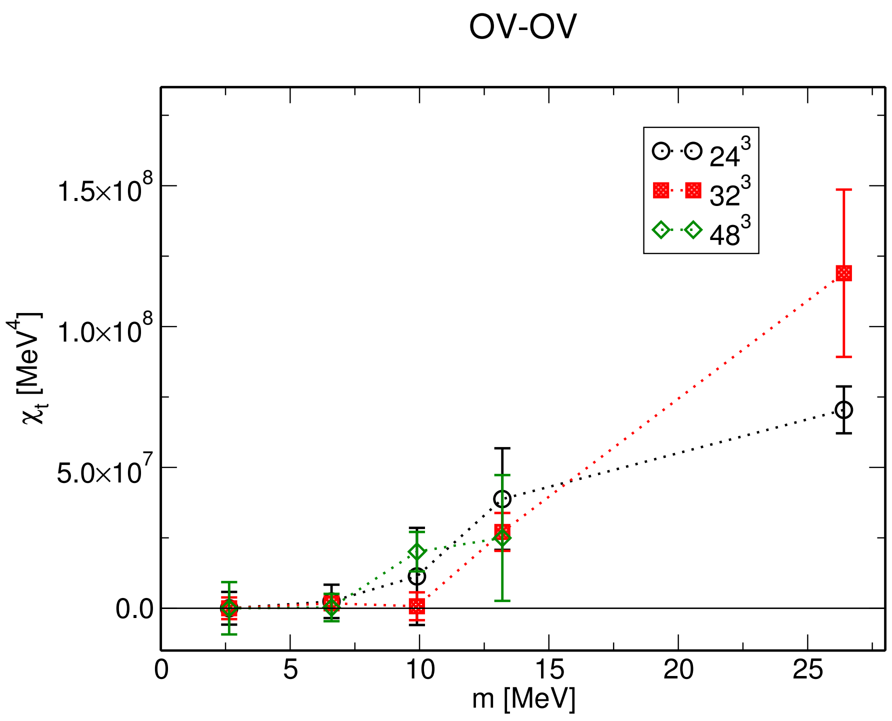

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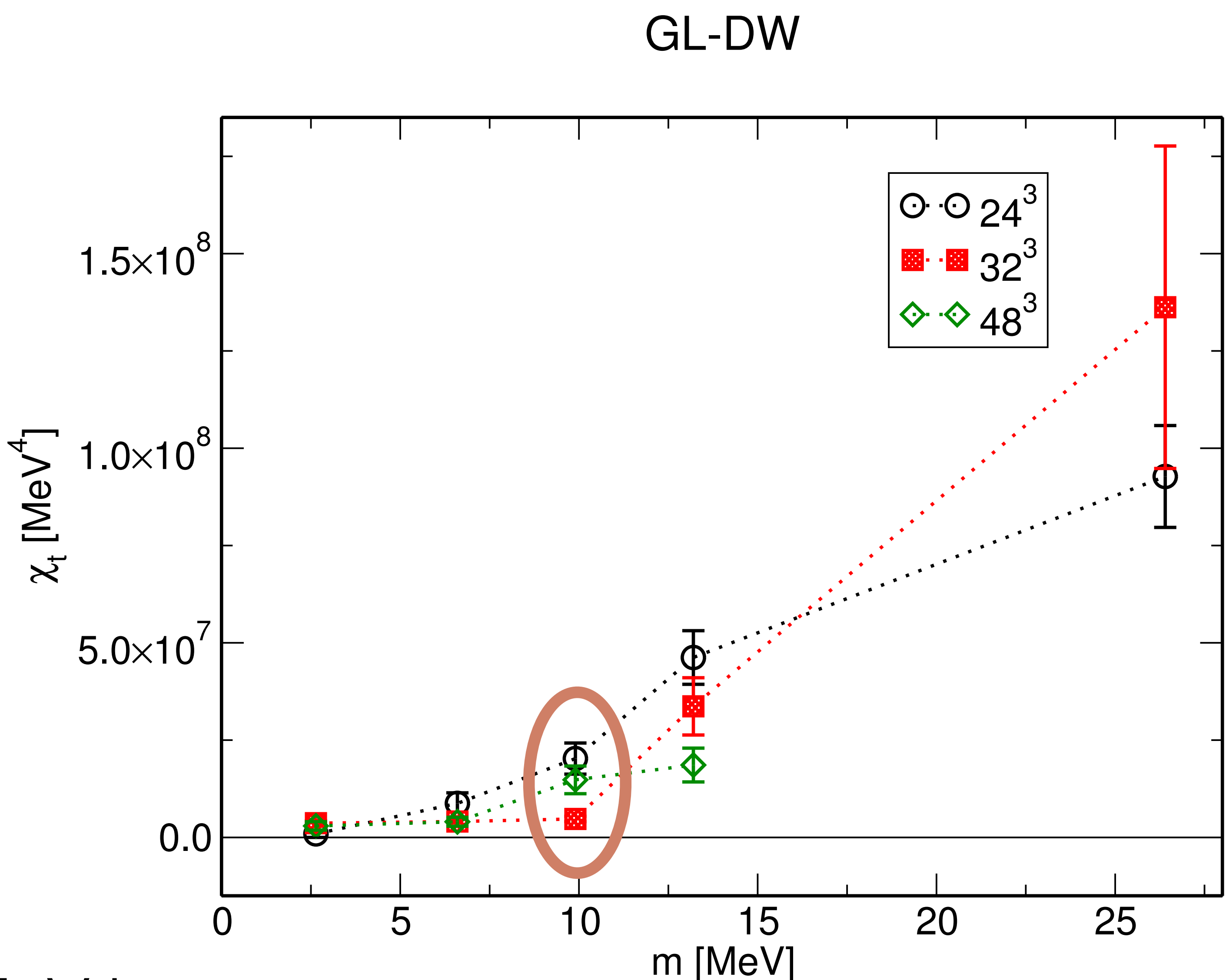

- V dependence at m=10 MeV is strange
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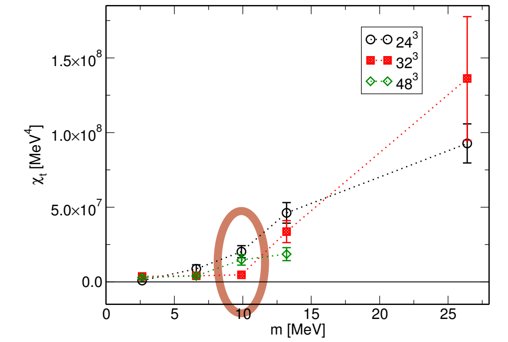
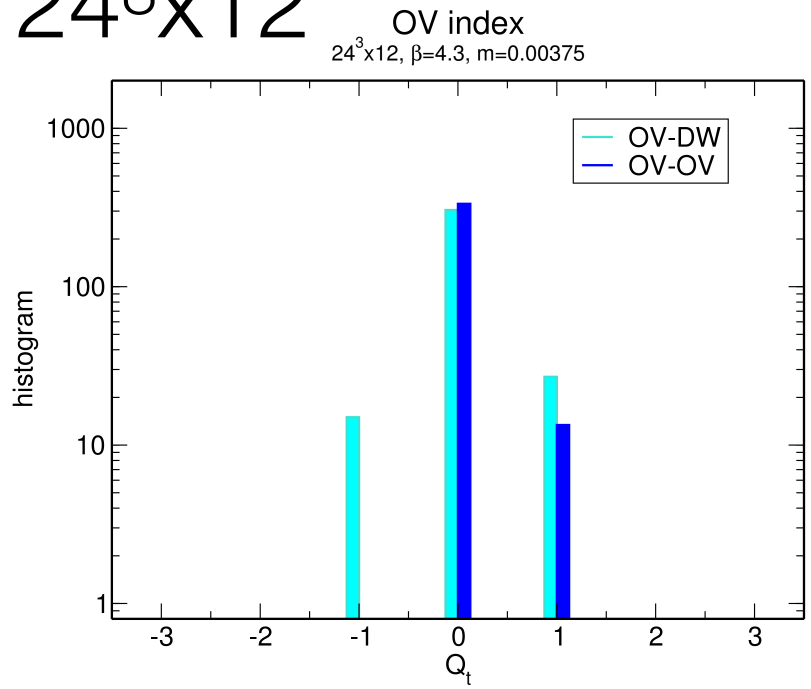
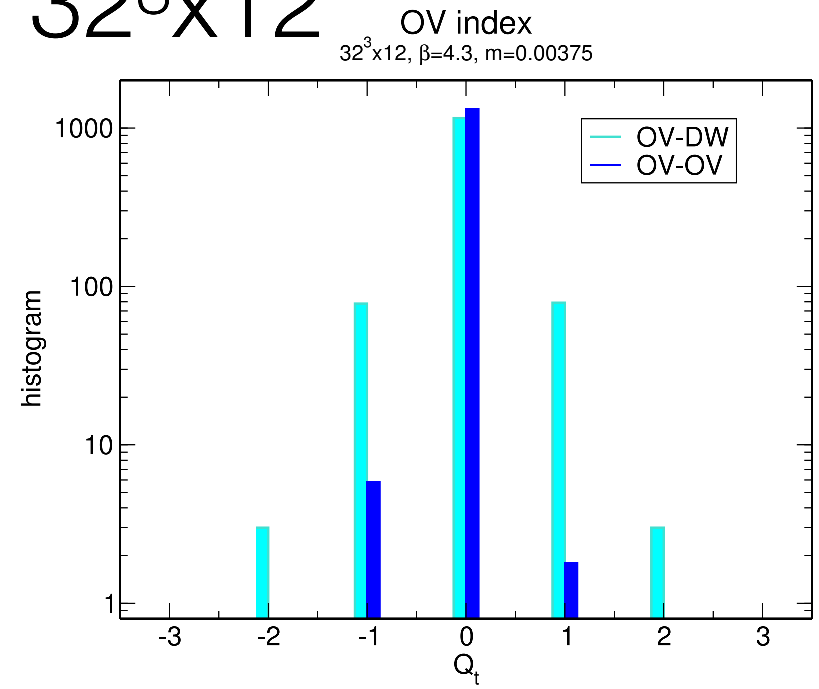
  - important region, where a phase boundary was suggested w/ 32^3
- Let's look at the histogram of Q

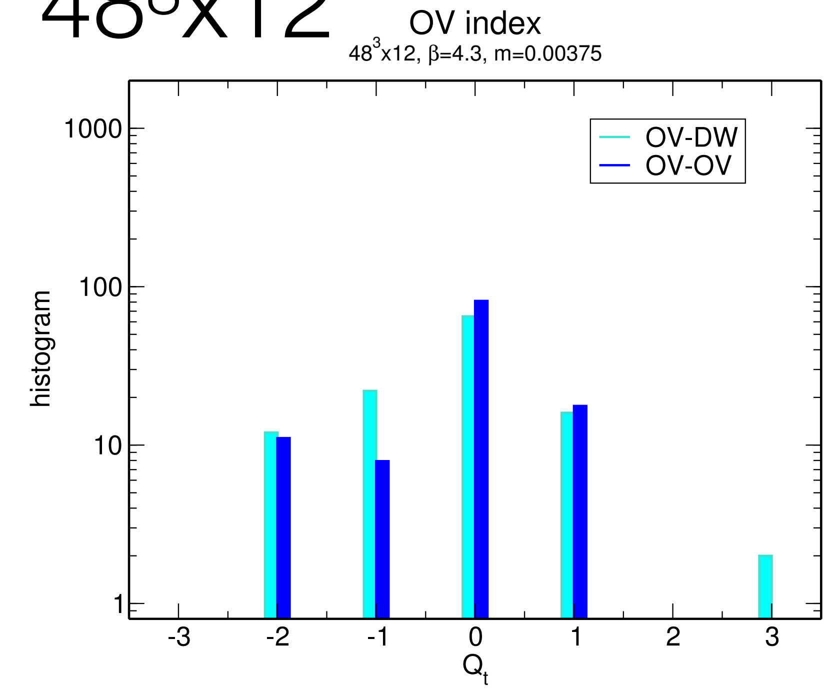
summary of histogram: T=220 MeV, m=10 MeV

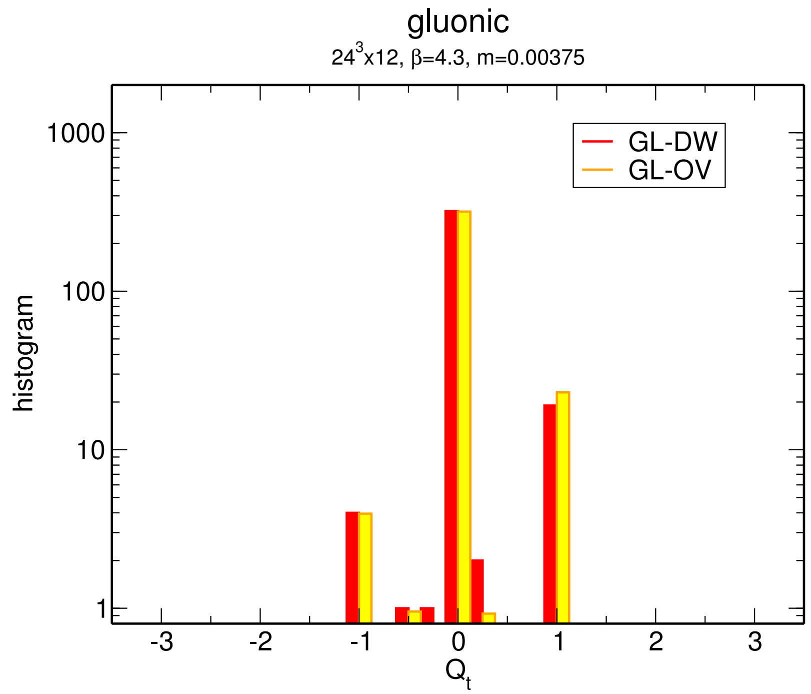


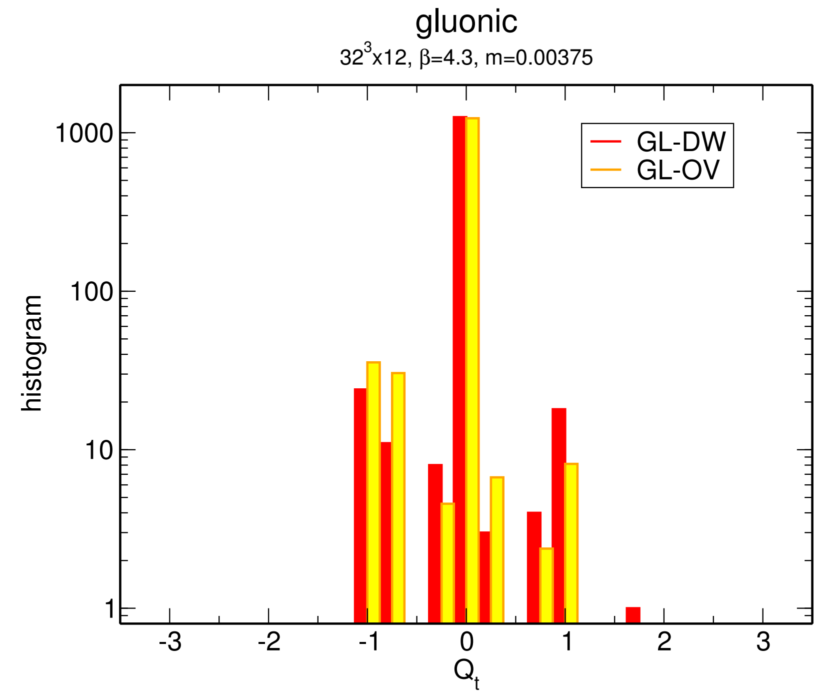
24^3x12	32^3x12	48^3x12

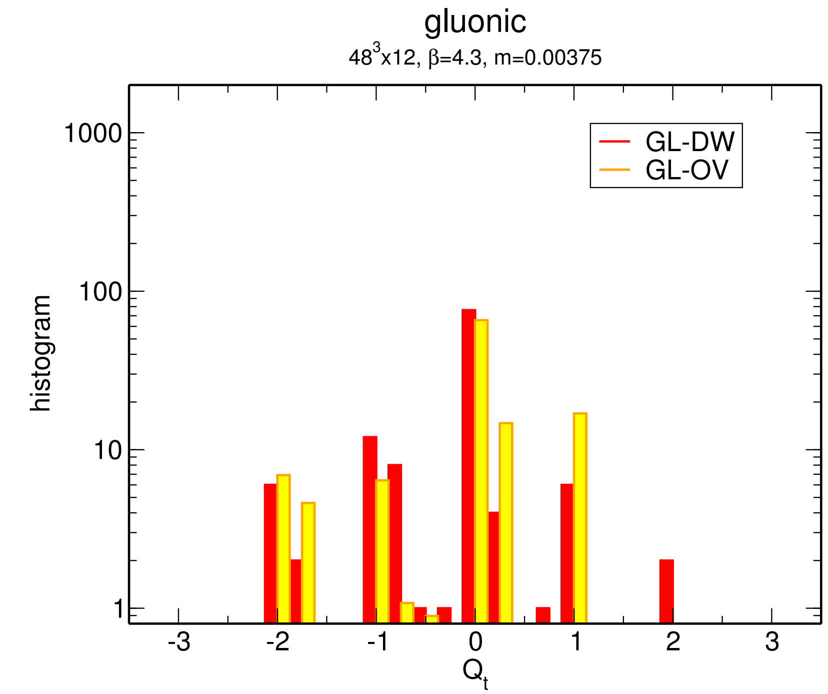






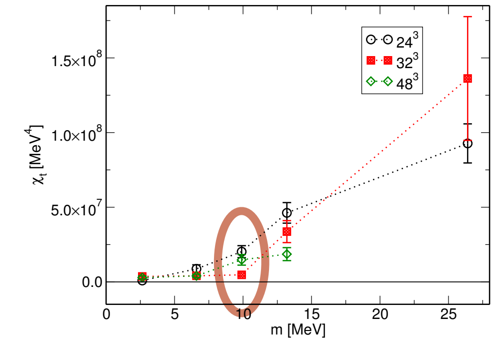




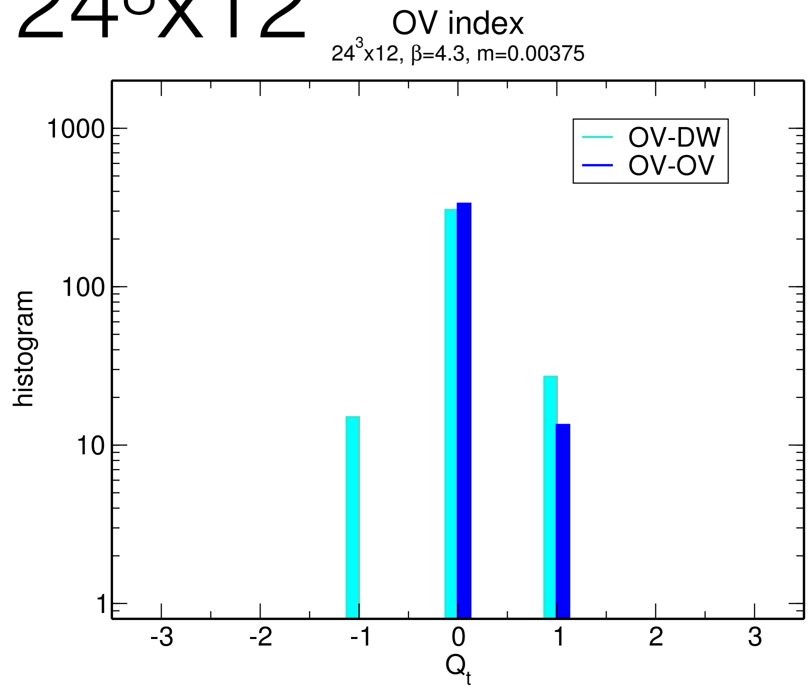


# trajectory:	~30k	~30k	~10k
sample rate:	100	20	100

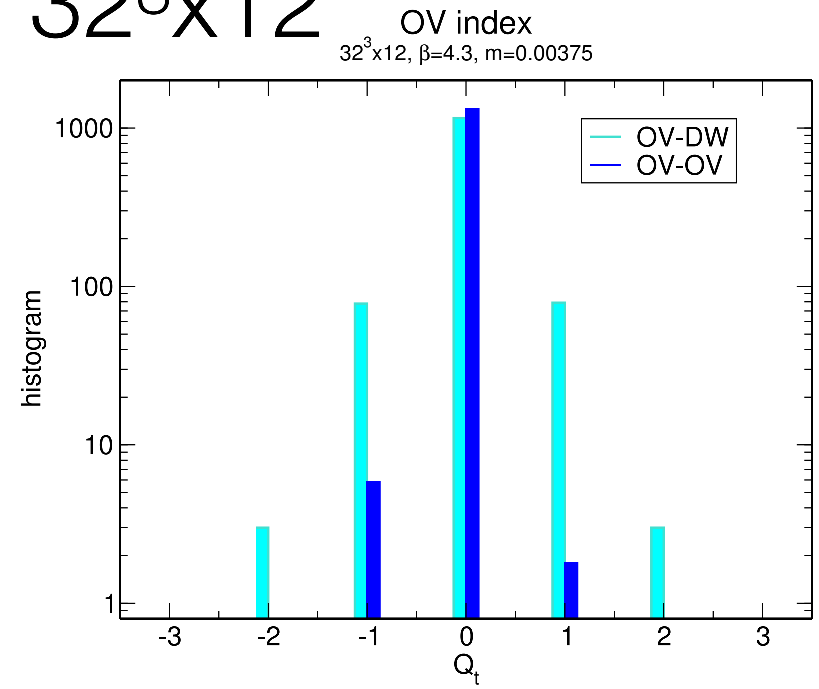
summary of histogram: T=220 MeV, m=10 MeV



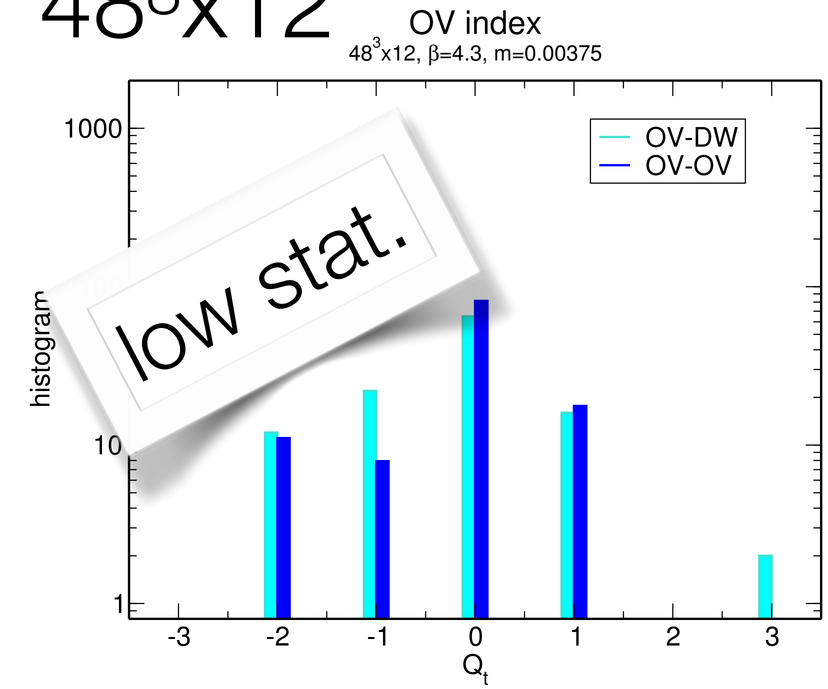
24^3x12

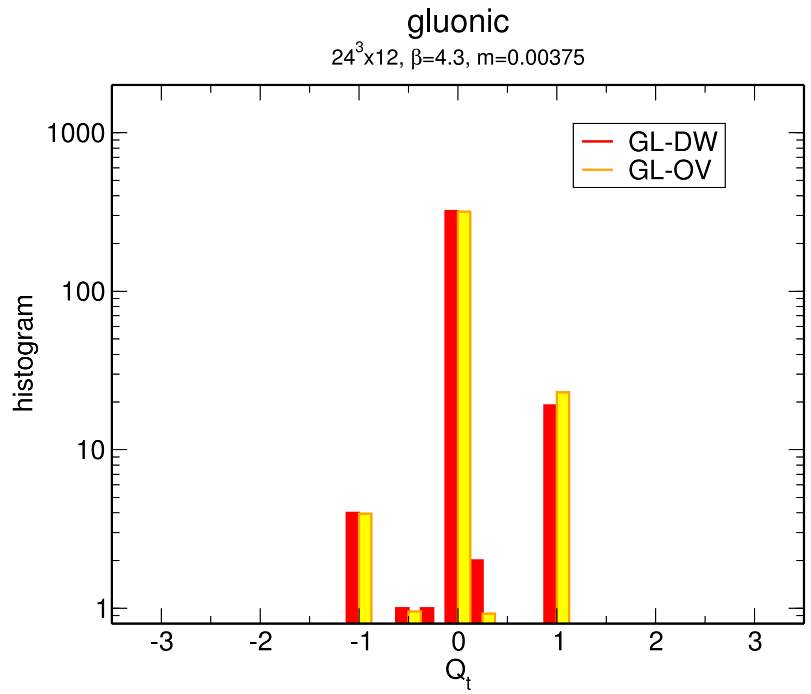


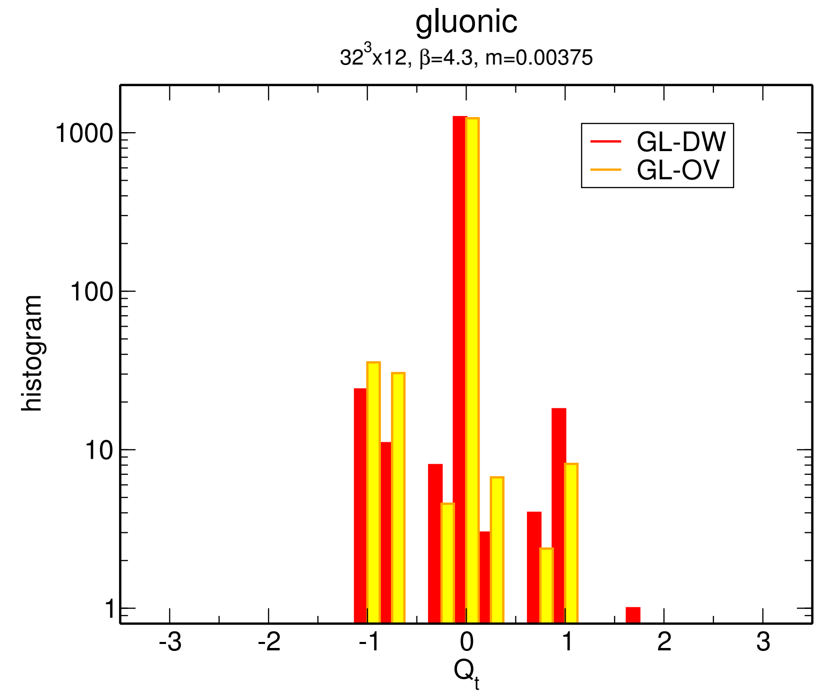
32^3x12

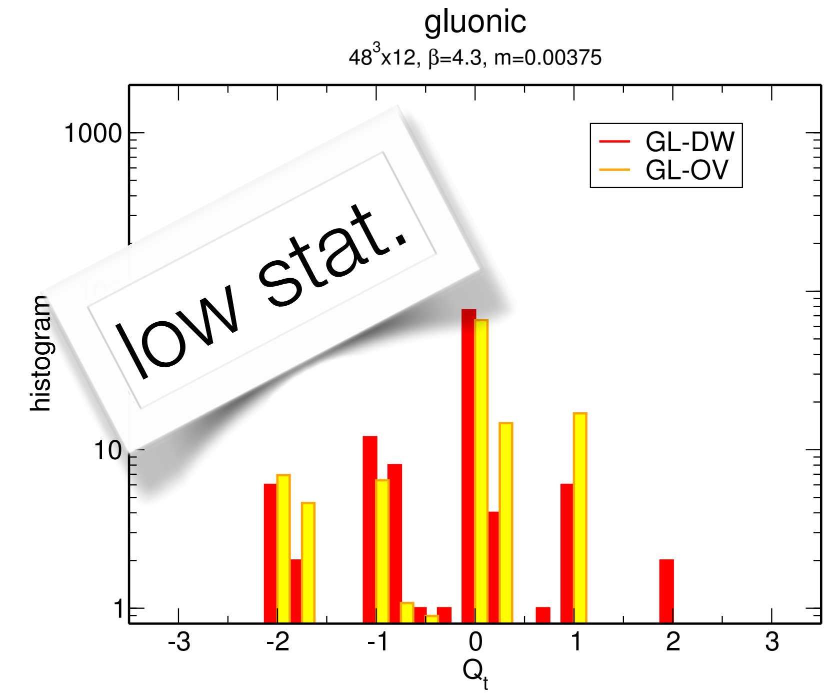


48^3x12



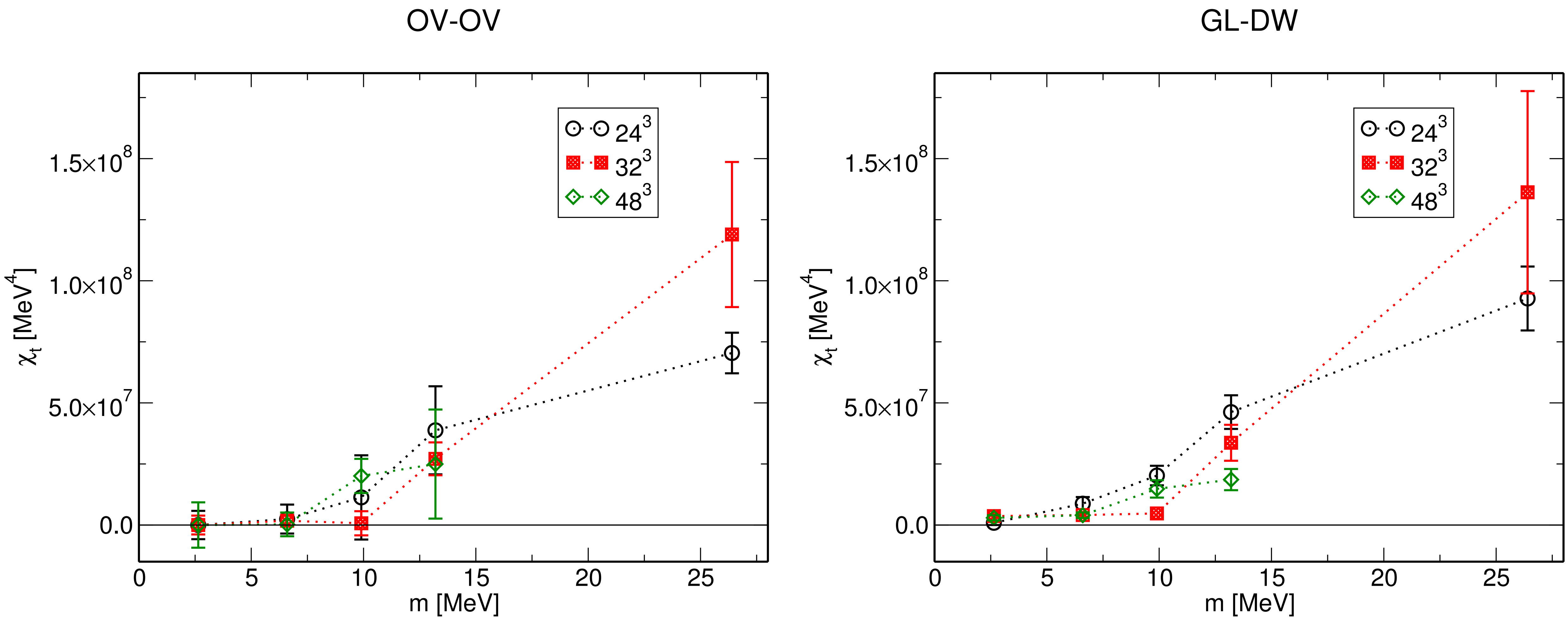






# trajectory:	~30k	~30k	~10k
sample rate:	100	20	100

Results of $\chi_t(m)$ at T=220 MeV; multiple volume



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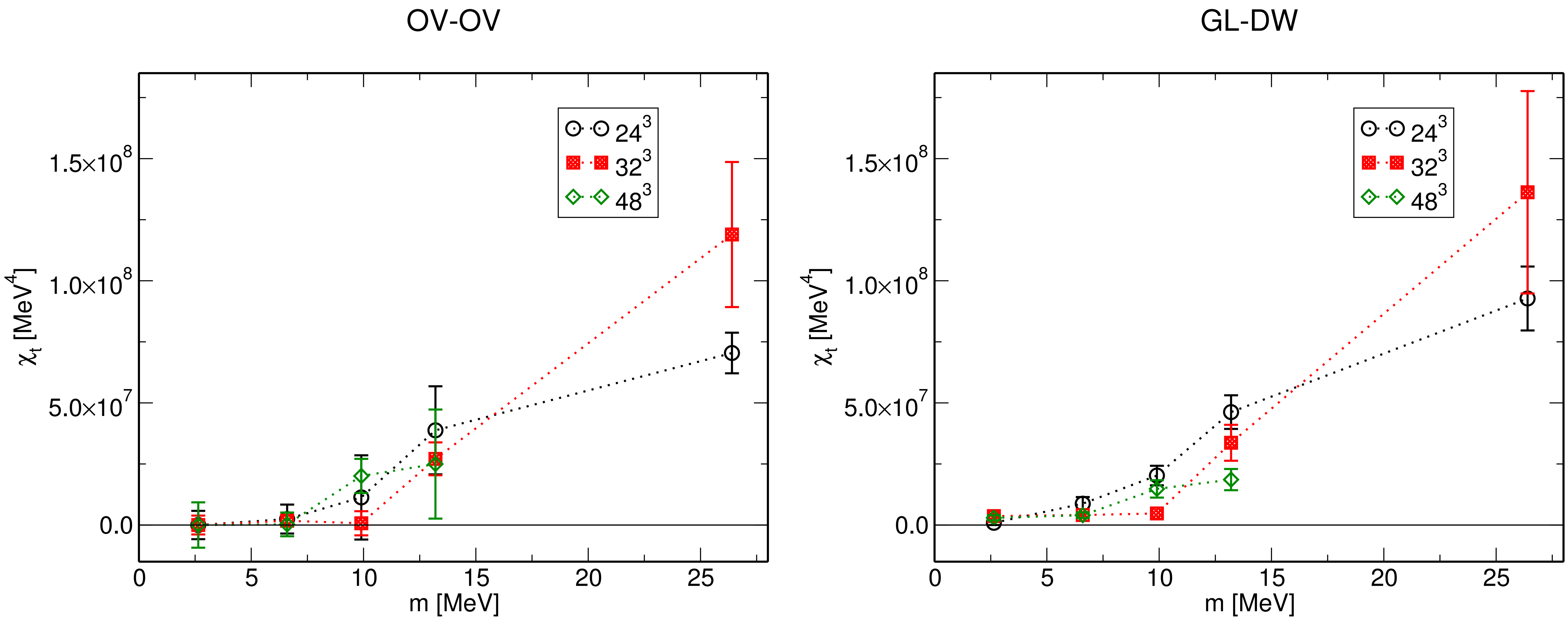
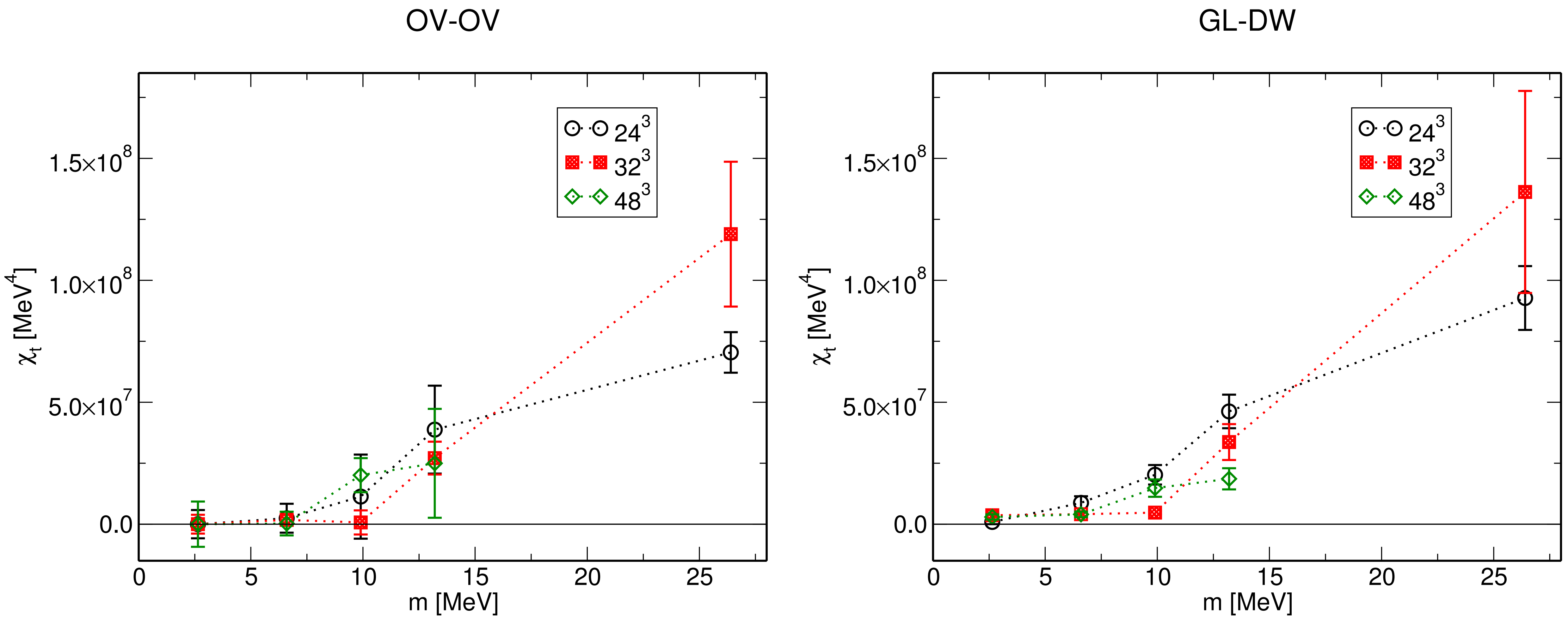


- V dependence at m=10 MeV is strange
  - Low statistics for 48^3 → not really conclusive

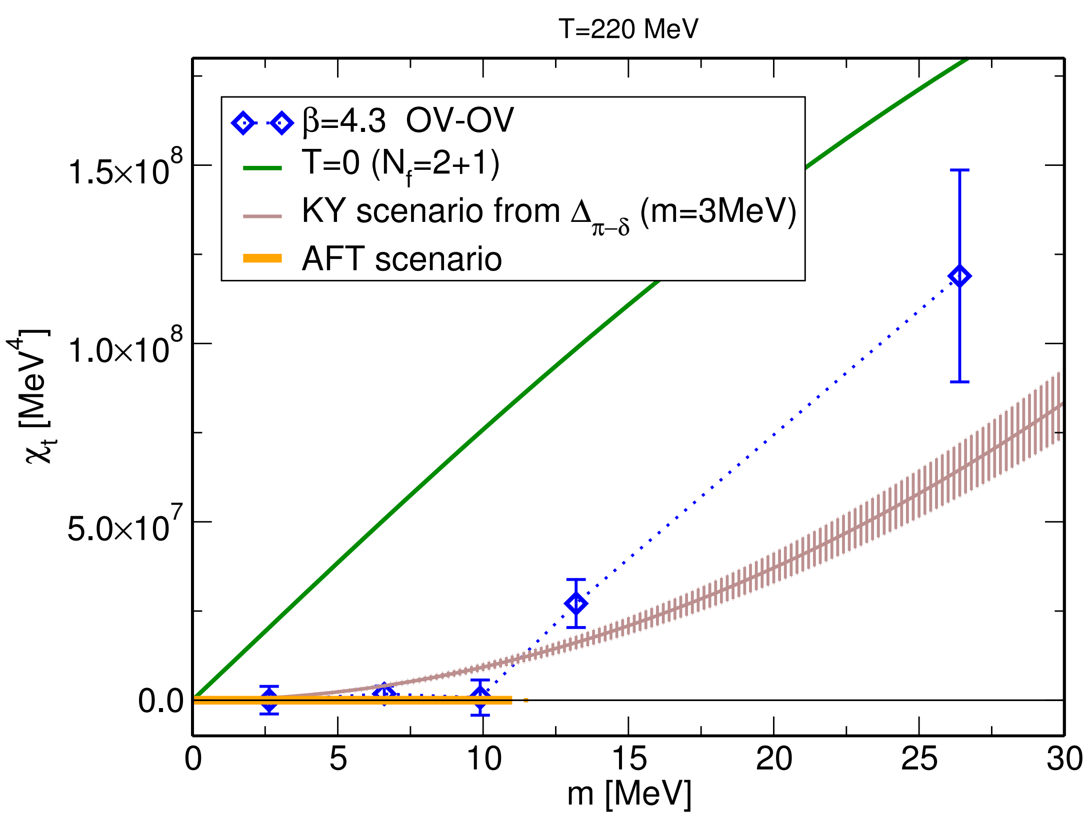
Results of $\chi_t(m)$ at T=220 MeV; multiple volume

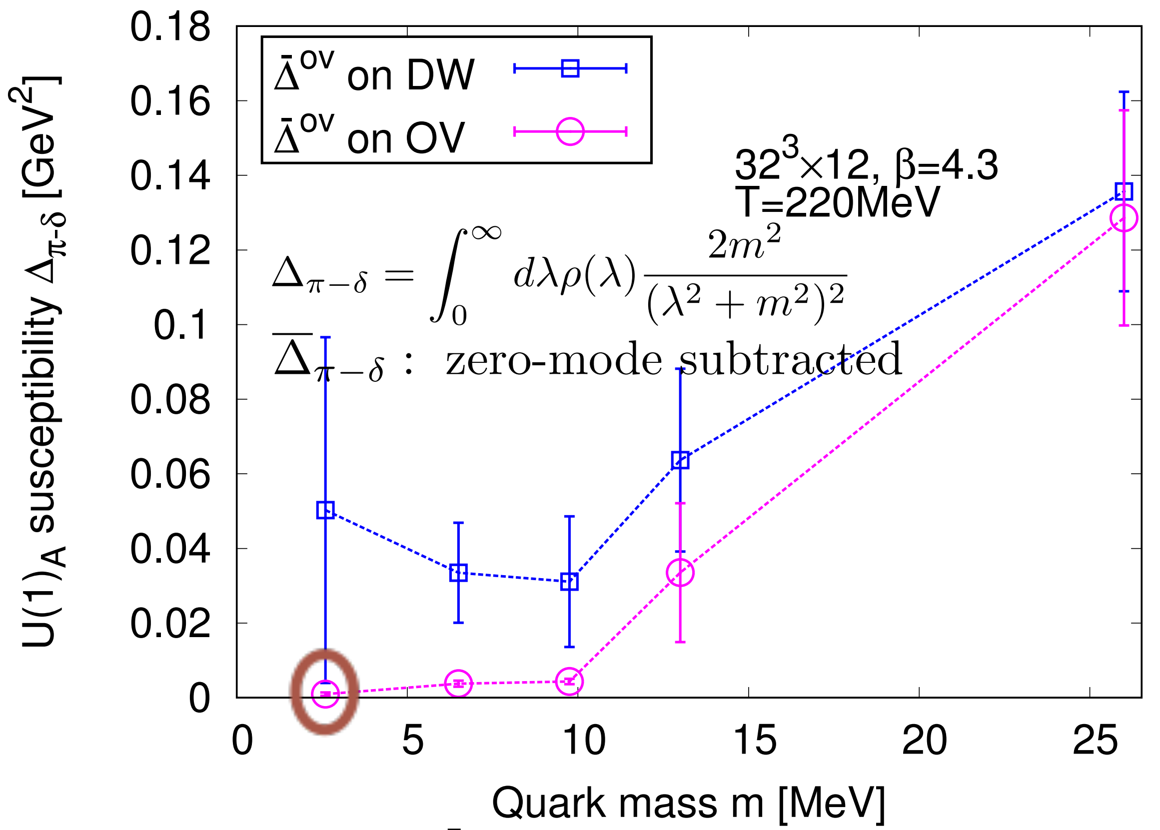


- V dependence at m=10 MeV is strange
  - Low statistics for 48^3 → not really conclusive
- decrease as V at m=13 MeV
  - but, also low statistics for 48^3 → not really conclusive

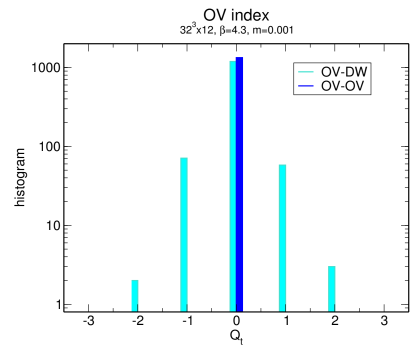
discussion last year

competing scenarios for χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=220 MeV





- KY scenario [Kanazawa, Yamamoto 2016]
  - $\Delta_{\pi\text{-}\delta}$: including zero mode cont. is proper
  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\Delta_{\pi\text{-}\delta} \simeq 8\, V f_A^2\, m^2$ for Q=0 sector (for $2V f_A m^2 < 1$)
- $\Delta_{\pi\text{-}\delta}$ @ lightest point only from Q=0
- $\chi_t = 2\, f_A\, m^2$
- tension at m≥10 MeV χ_t sudden growth



discussion last year

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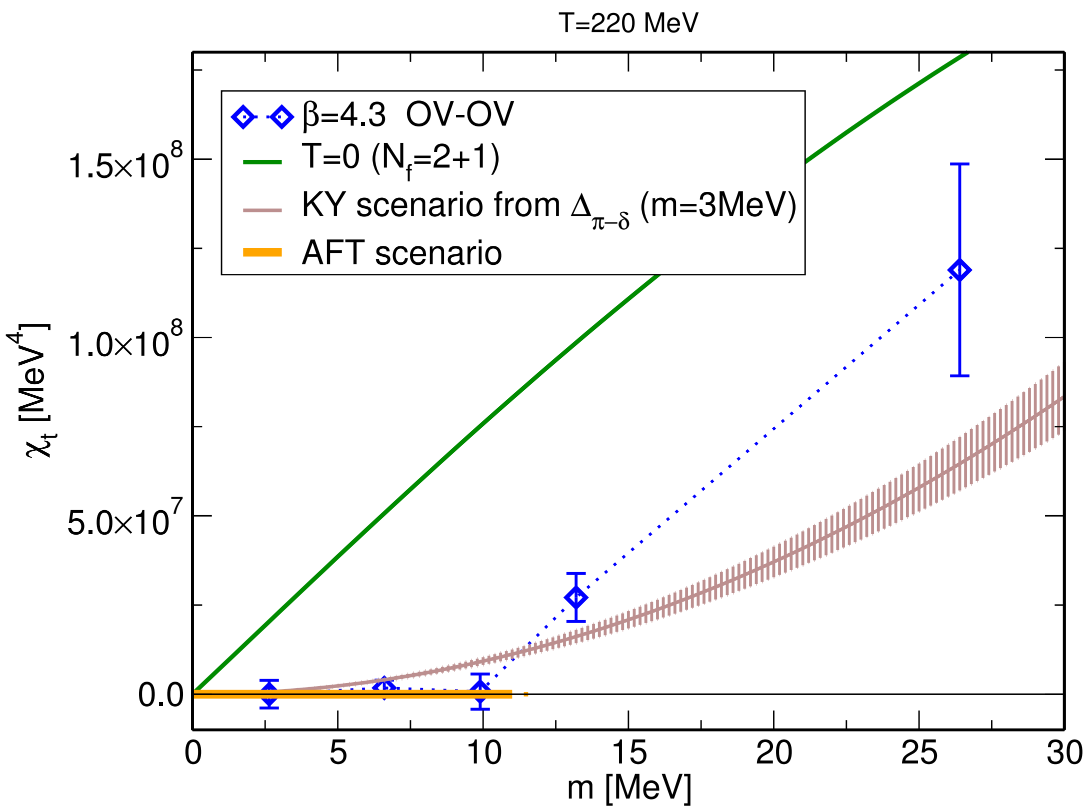

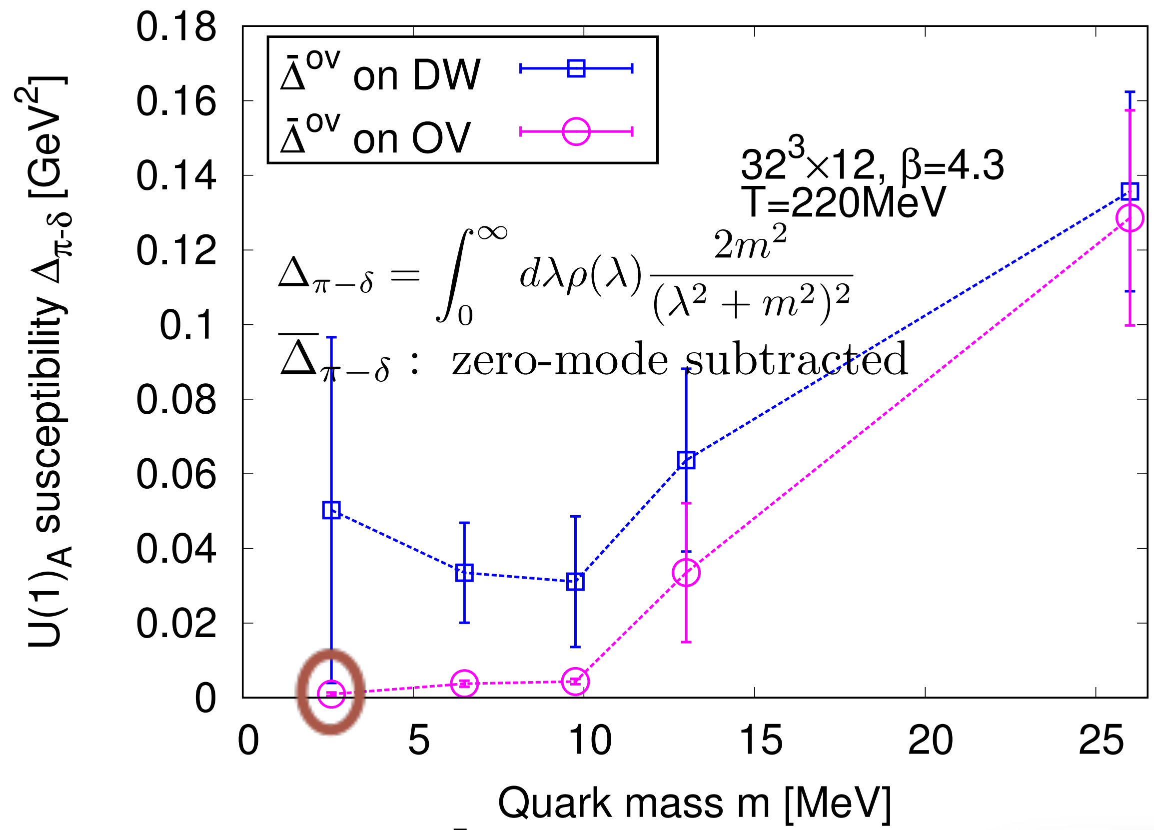

- KY scenario
  - $\Delta_{\pi\text{-}\delta}$: incl

Volume study would be useful to check this

  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\boxed{\Delta_{\pi\text{-}\delta} \simeq 8\, V\, f_A^2\, m^2}$ for Q=0 sector (for $2V f_A m^2 < 1$)
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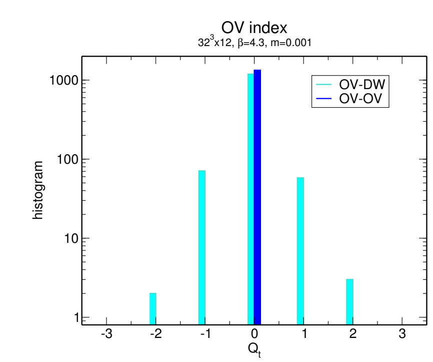
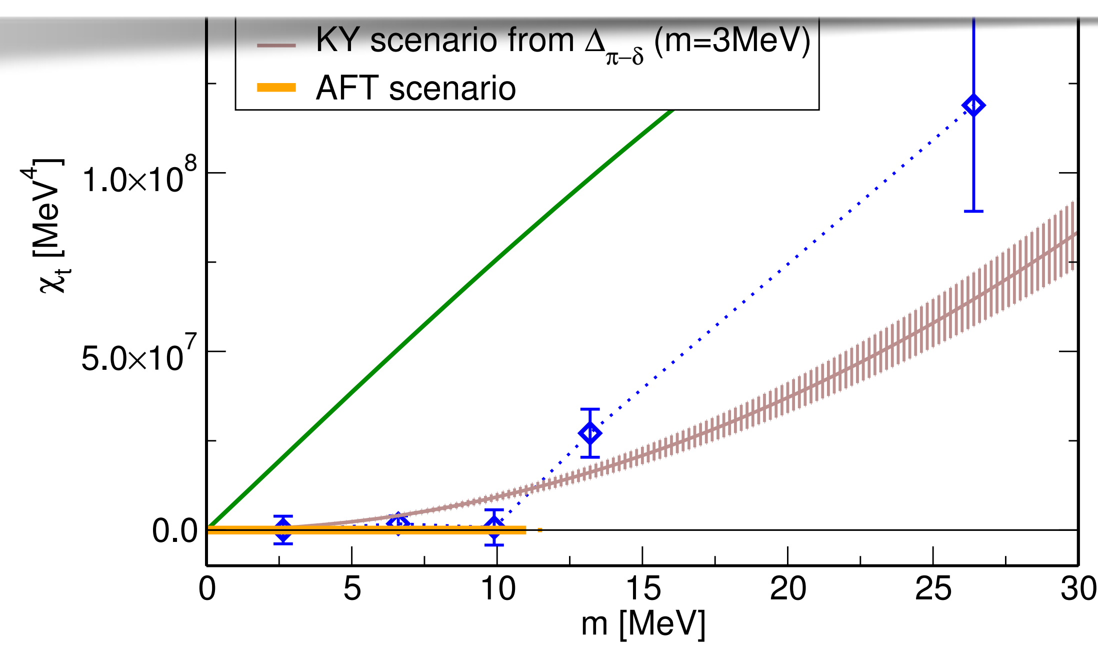
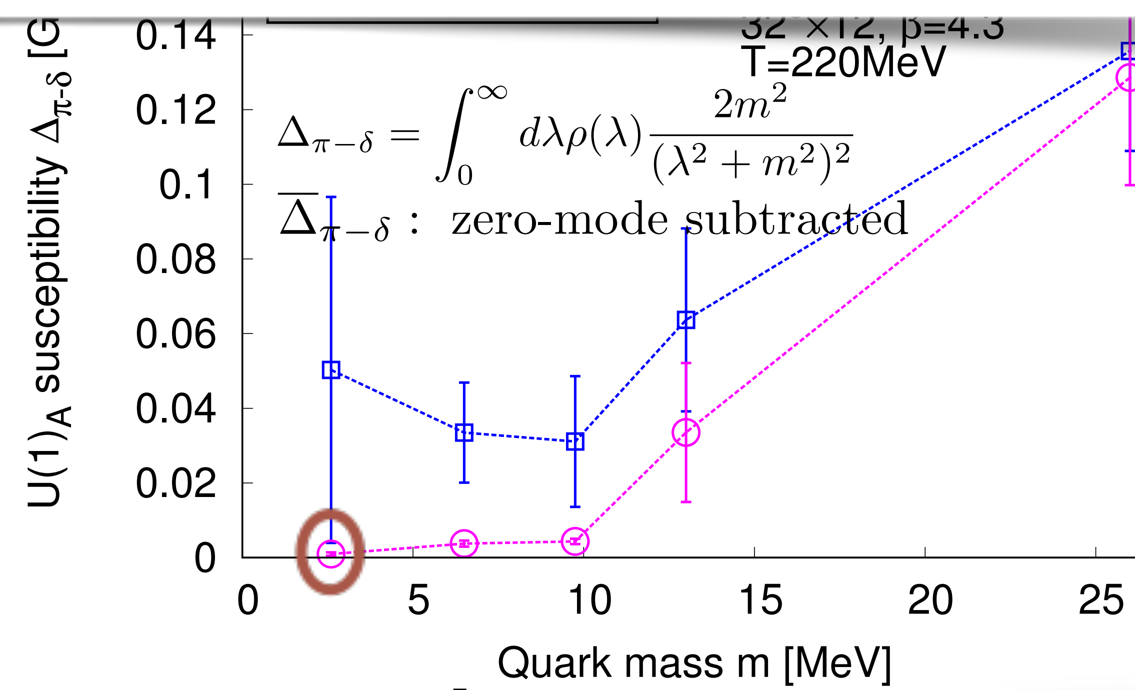


competing scenarios for

discussion last year

How is this changed with multiple volume ?





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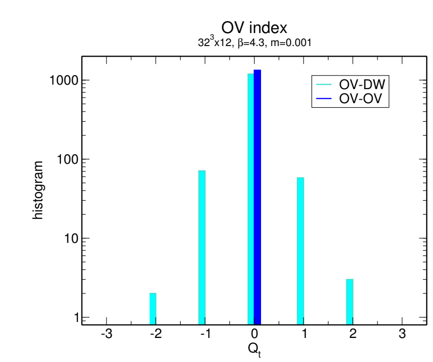

competing scenarios for

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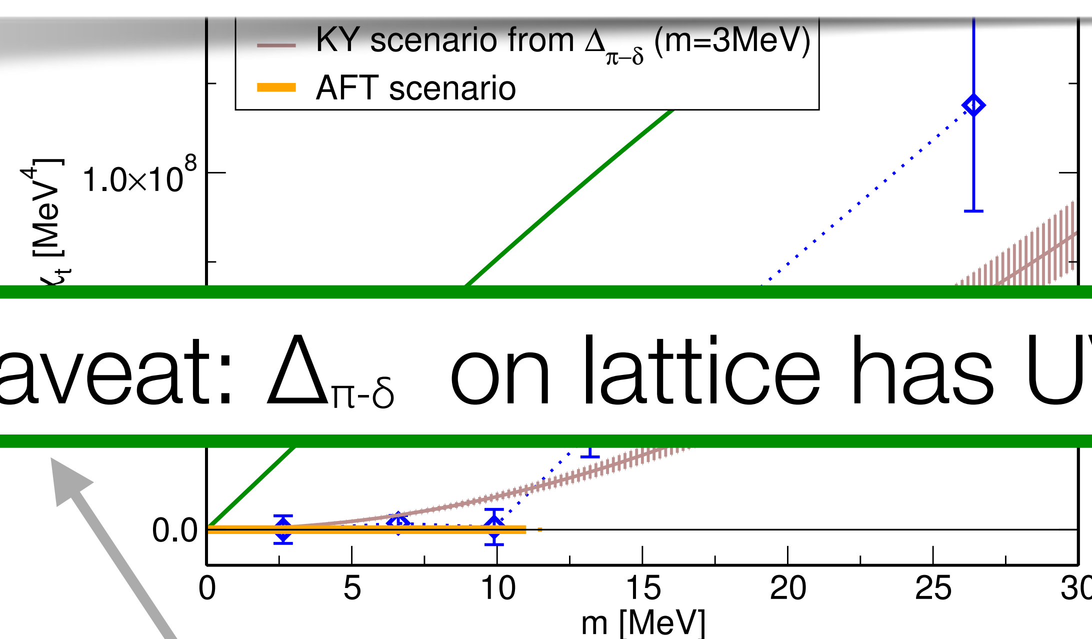

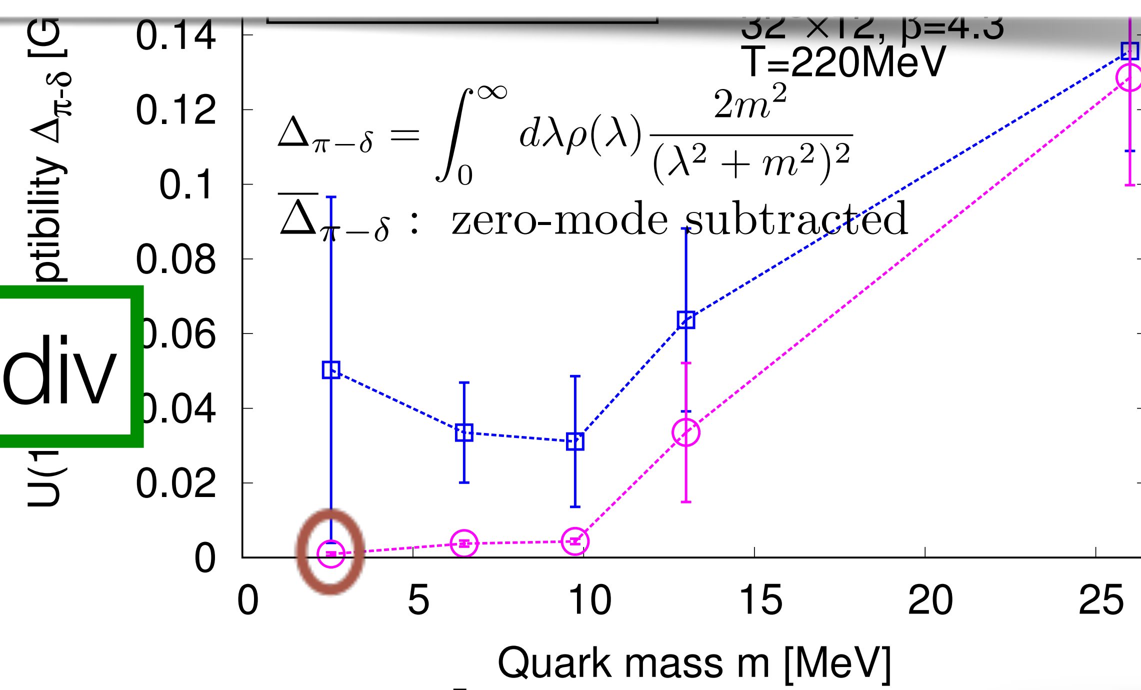

caveat: $\Delta_{\pi\text{-}\delta}$ on lattice has UV div

- KY scenario
  - $\Delta_{\pi\text{-}\delta}$: incl
  - $\Delta_{\pi\text{-}\delta}$ = const >0
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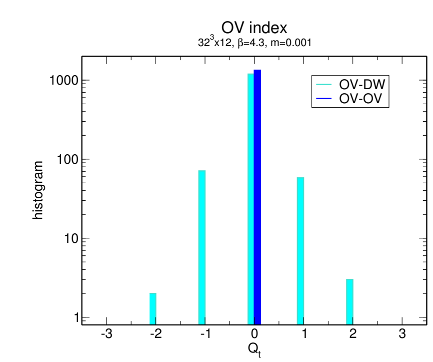

competing scenarios for

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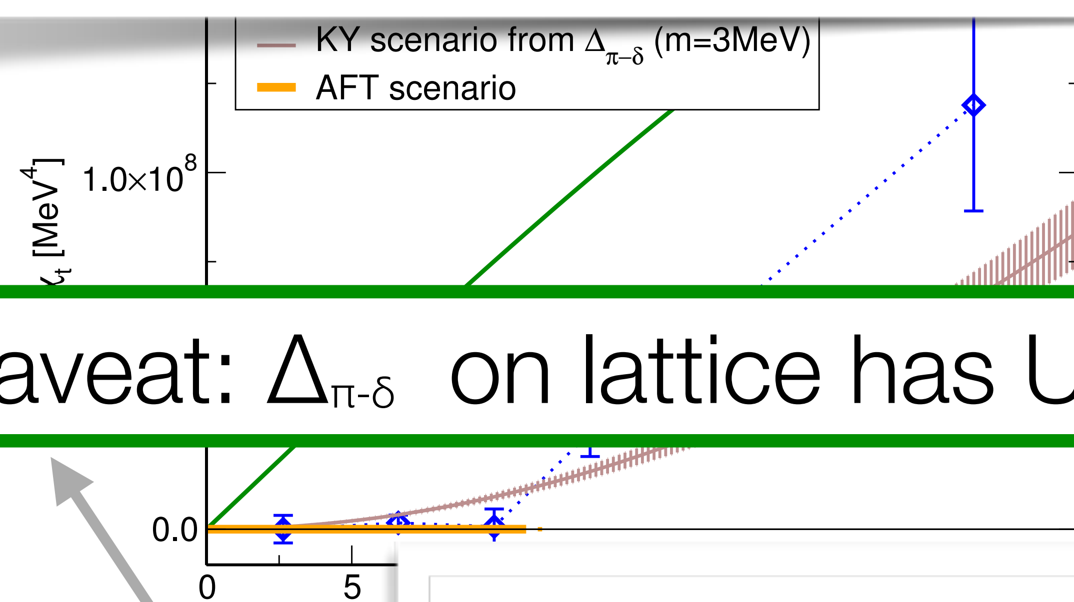

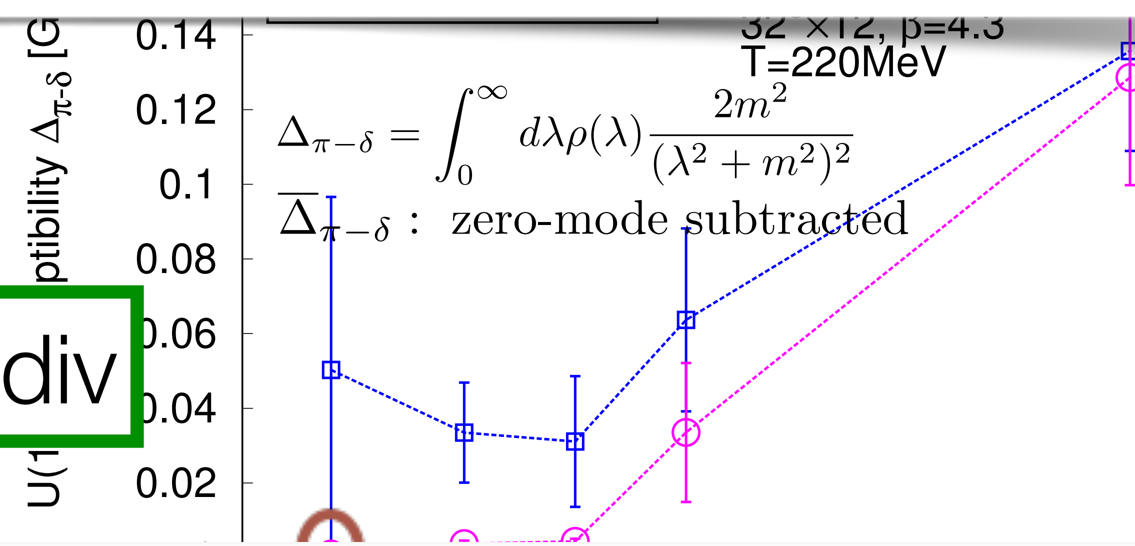

caveat: $\Delta_{\pi\text{-}\delta}$ on lattice has UV div

to remove the UV cut λ_{max} dep.
same λ_{max}, different V data compared

- K
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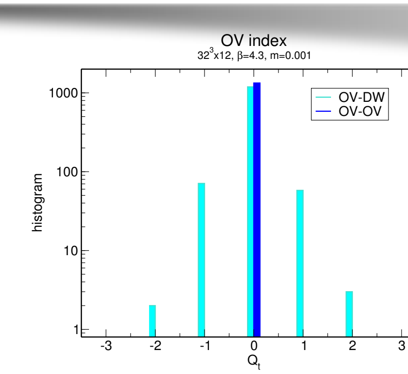

competing scenarios for

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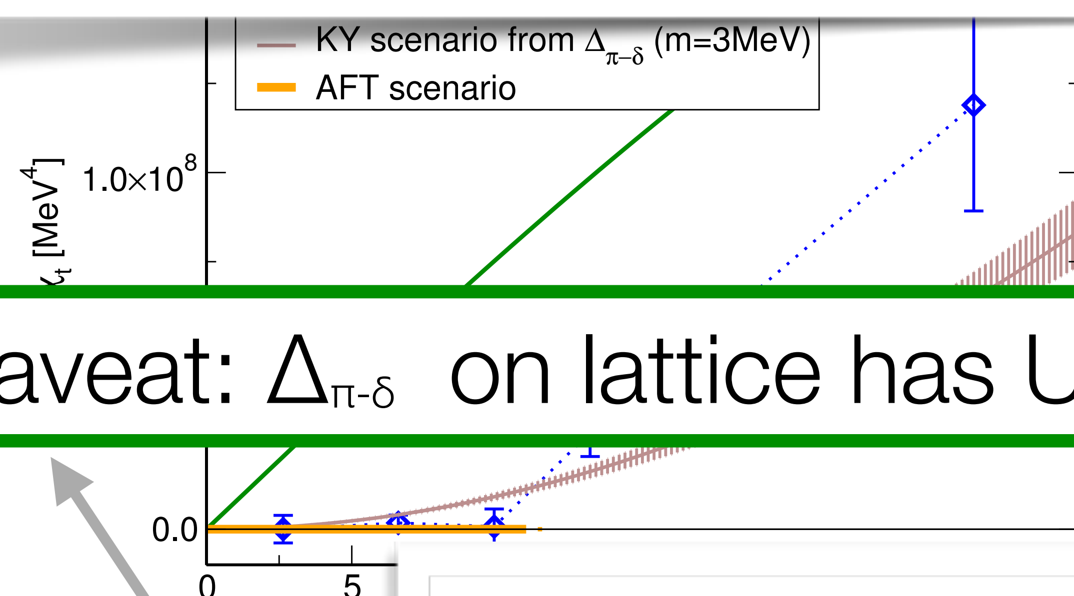

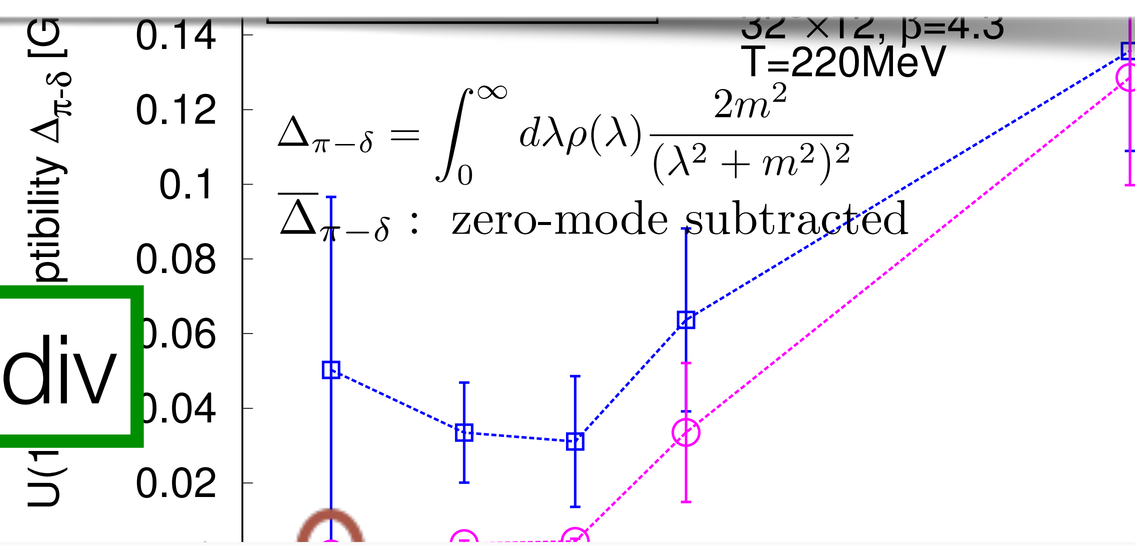

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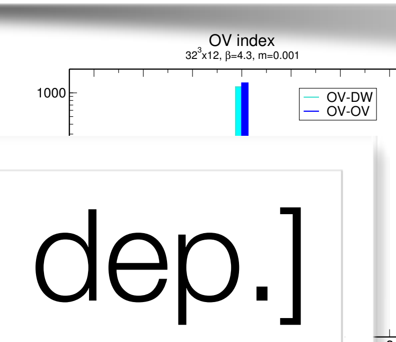
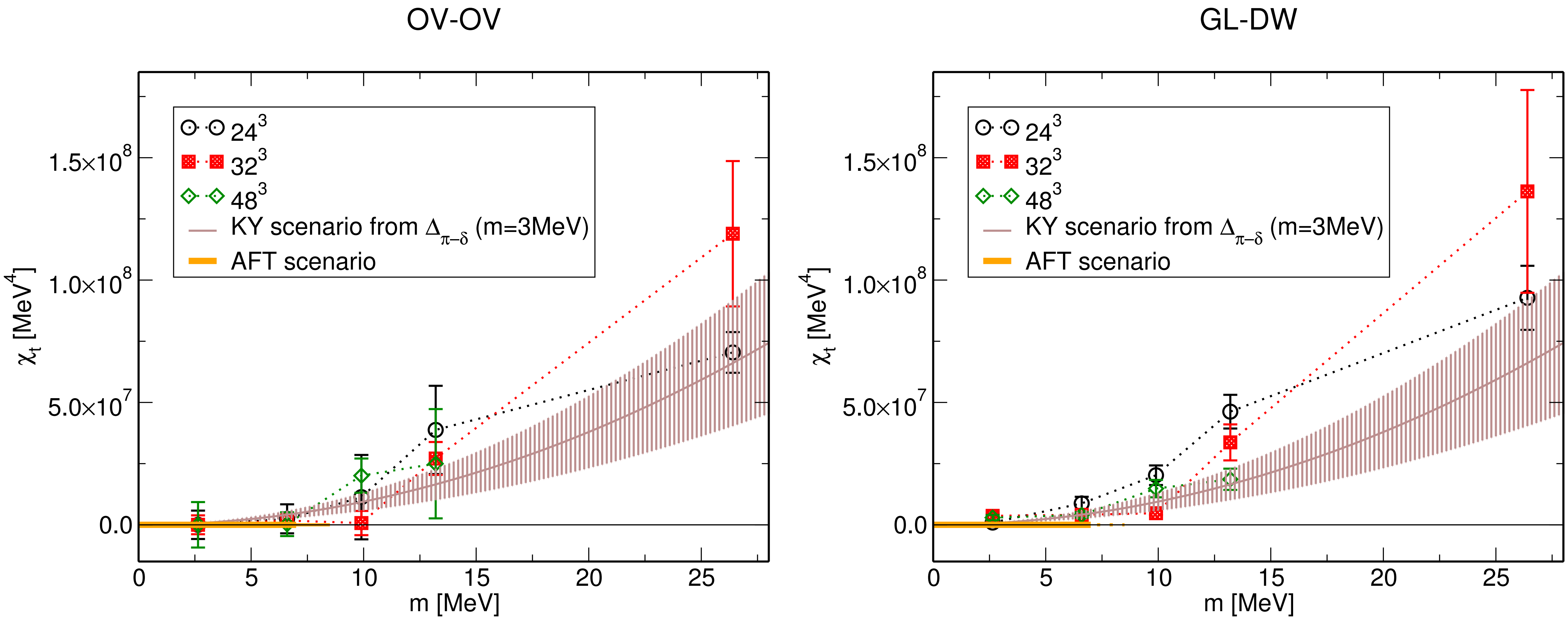


$$\Delta_{\pi\text{-}\delta}(V_1,\lambda_{max}) - \Delta_{\pi\text{-}\delta}(V_0,\lambda_{max}) = [\text{no } \lambda_{max} \text{ dep.}]$$
$$= 8(V_1 - V_0)\, f_A{}^2\, m^2 \;\rightarrow\; \chi_t = 2\, f_A\, m^2$$

competing scenarios with multiple volumes for χ_t given $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=220 MeV



- AFK scenario: $\chi_t = 0$ for $0<m<m_c$
- KY scenario: $\chi_t = 2\, f_A\, m^2$
- There are no strong tensions
- Neither scenario is excluded

Summary and Outlook

Summary and Outlook

- χ_t investigated w/ unitary overlap fermion through reweighting from JLQCD DWF
- focus on T=220 MeV and N_t=12 (1/a = 2.64 GeV): ** still preliminary **
- 32^3 lattice: phase transition like behavior at m≃10 MeV (last year)
- 24^3 & 48^3 are newly studied
- in m≃10 MeV region, volume dependence is not conclusive
    - likely due to poor statistics of 48^3 lattices
- in $V \to \infty$ limit one cannot eliminate either

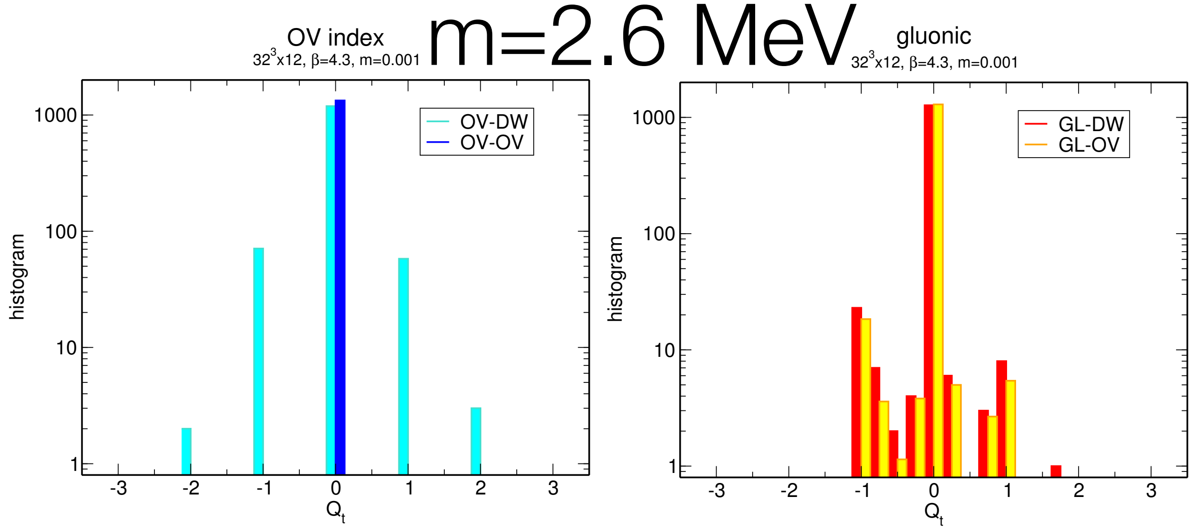
$$\chi_{top} = 0 \quad \text{for } 0<m<m_C \quad || \quad \chi_{top} = 2 f_A m^2$$

- significant improvement of the statistics is required
    - determinant breakup of reweighting factor tested (sometimes works)
- lower temperature may be easier (χ_{top} increases $\to$ easier topology sampling)

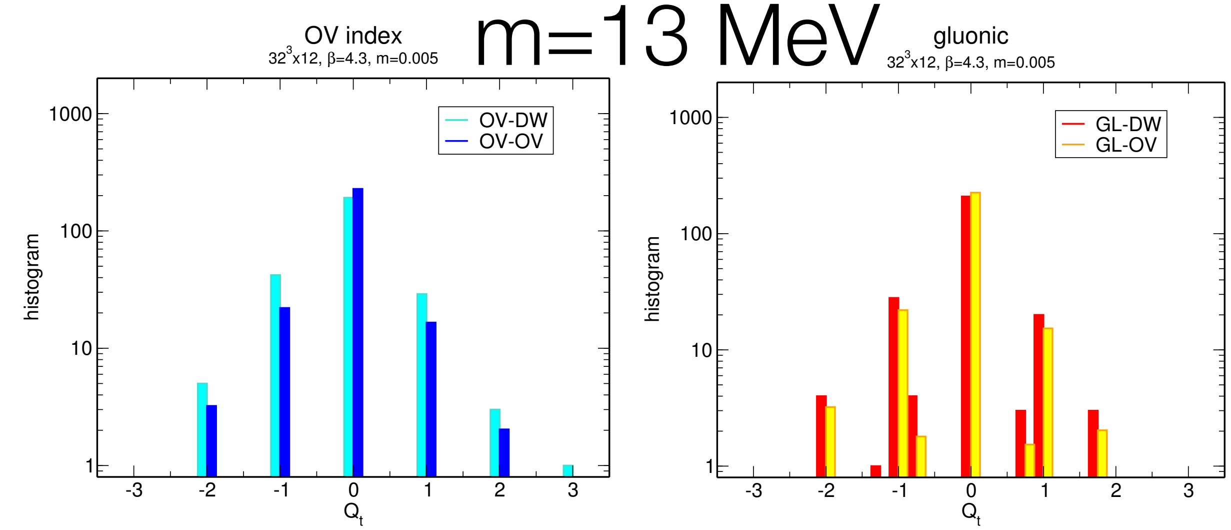
Thank you very much for your attention !

summary of histogram: T=220 MeV, 32^3x12

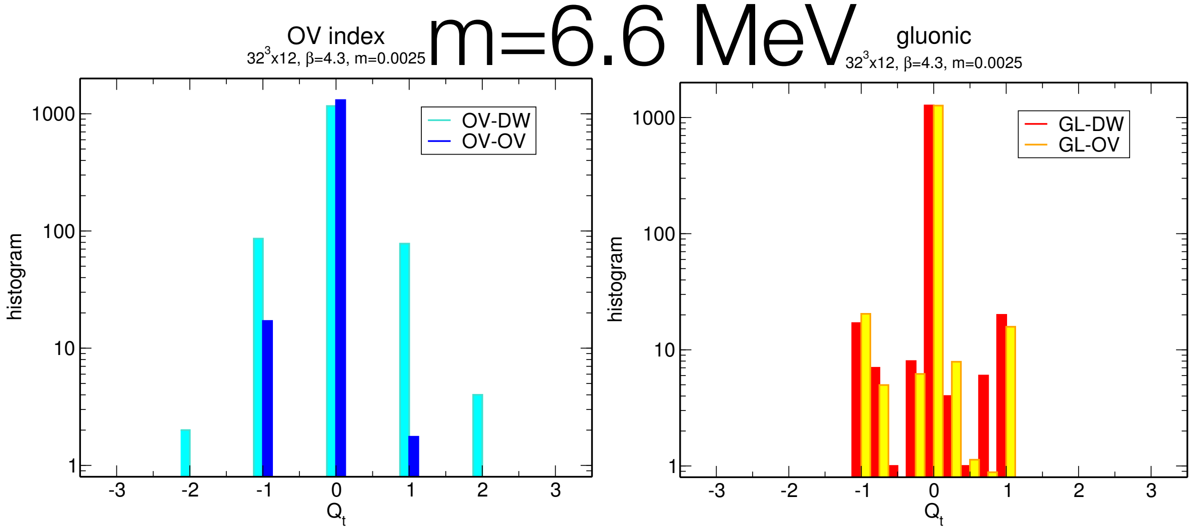
m=2.6 MeV



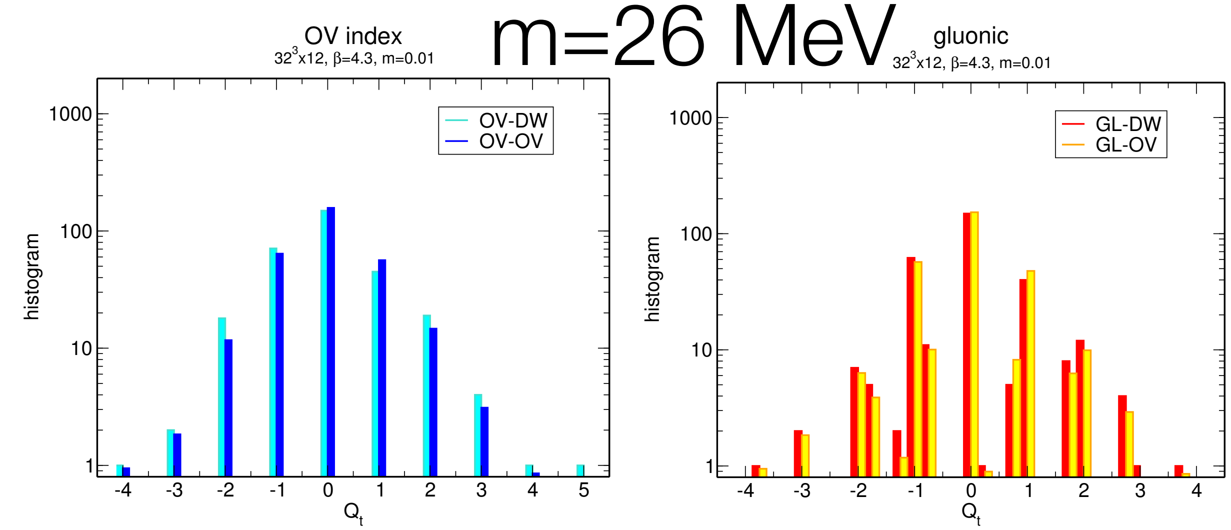
m=13 MeV



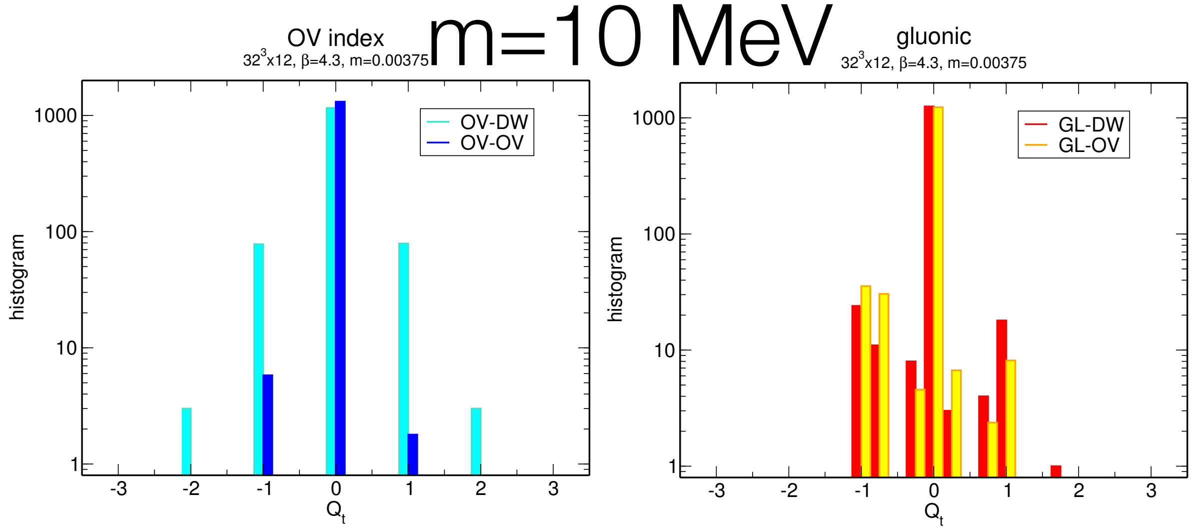
m=6.6 MeV



m=26 MeV



m=10 MeV



sample rate=20 # trajectory≃30k sample rate=100

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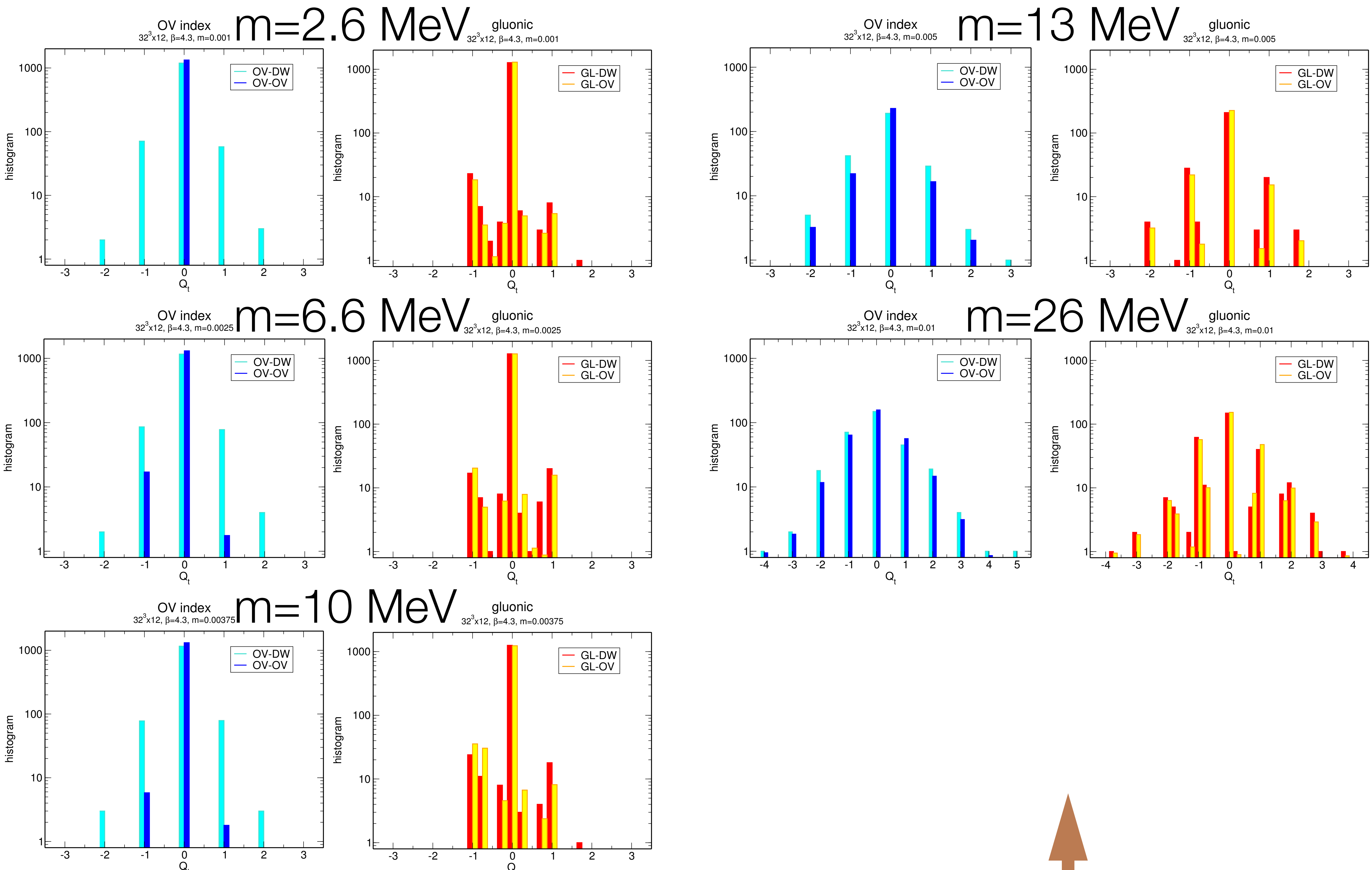

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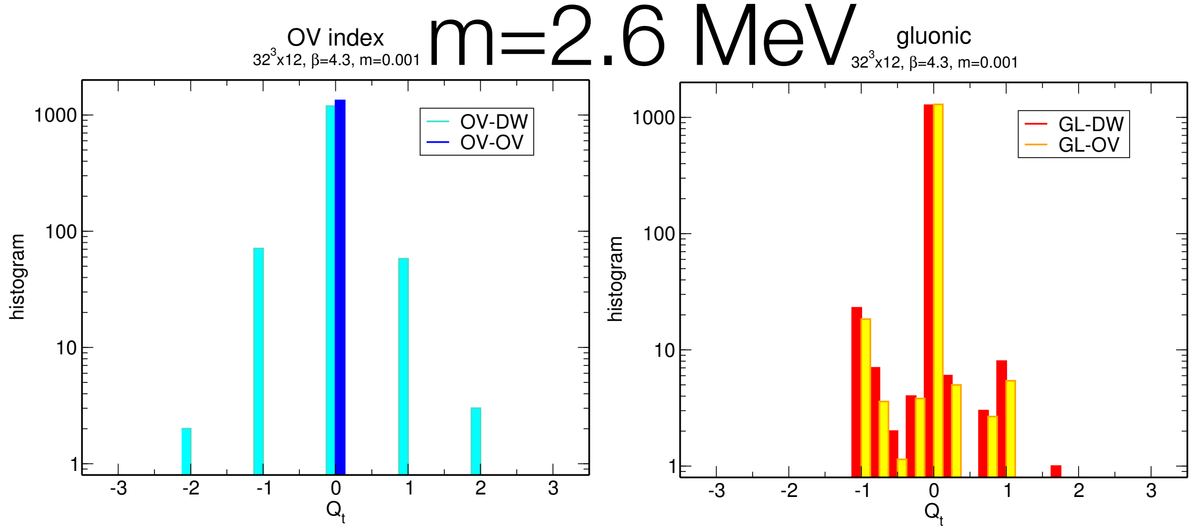

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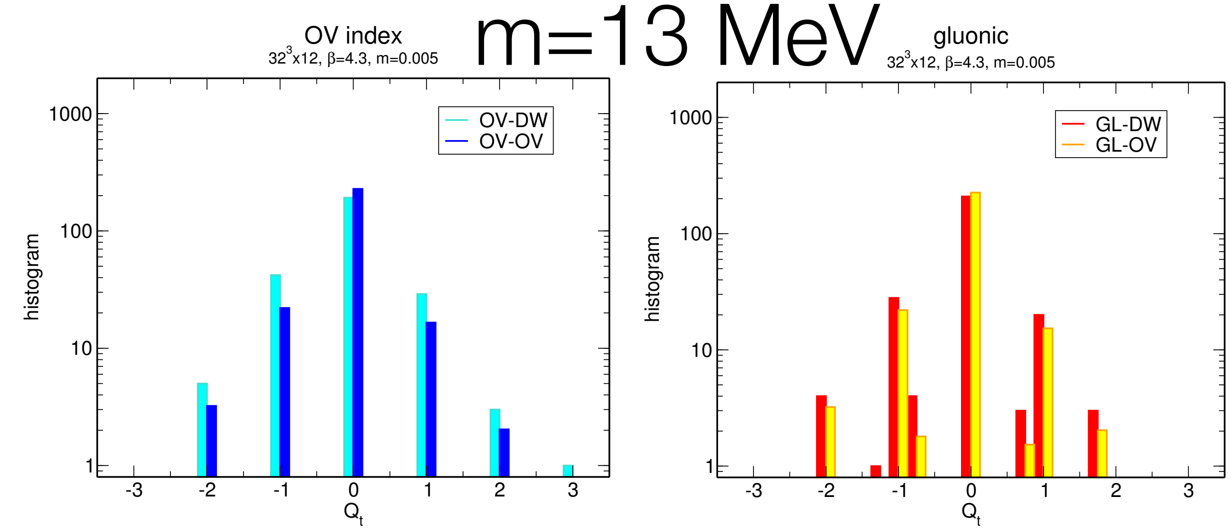

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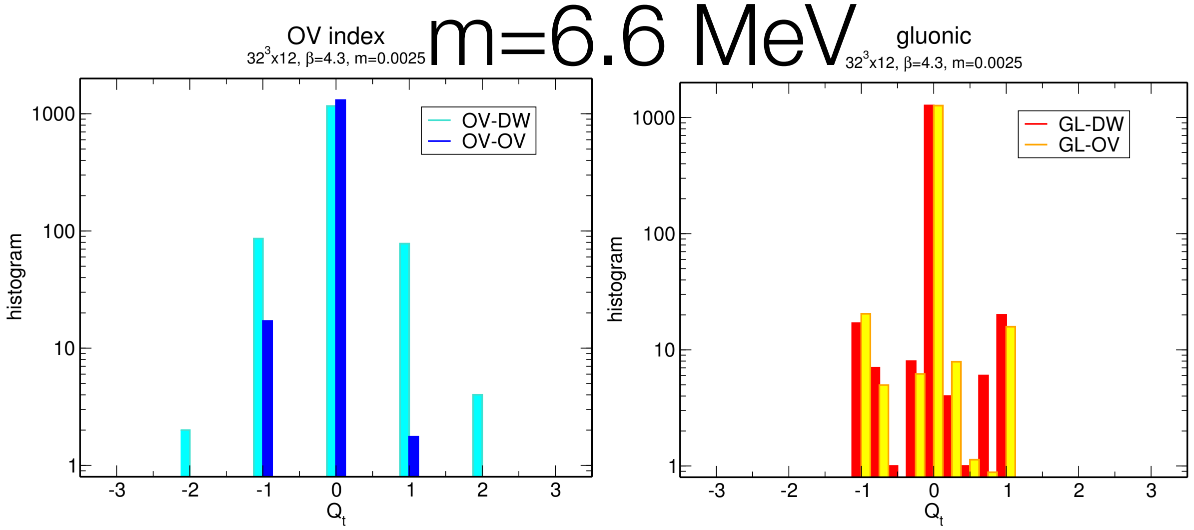
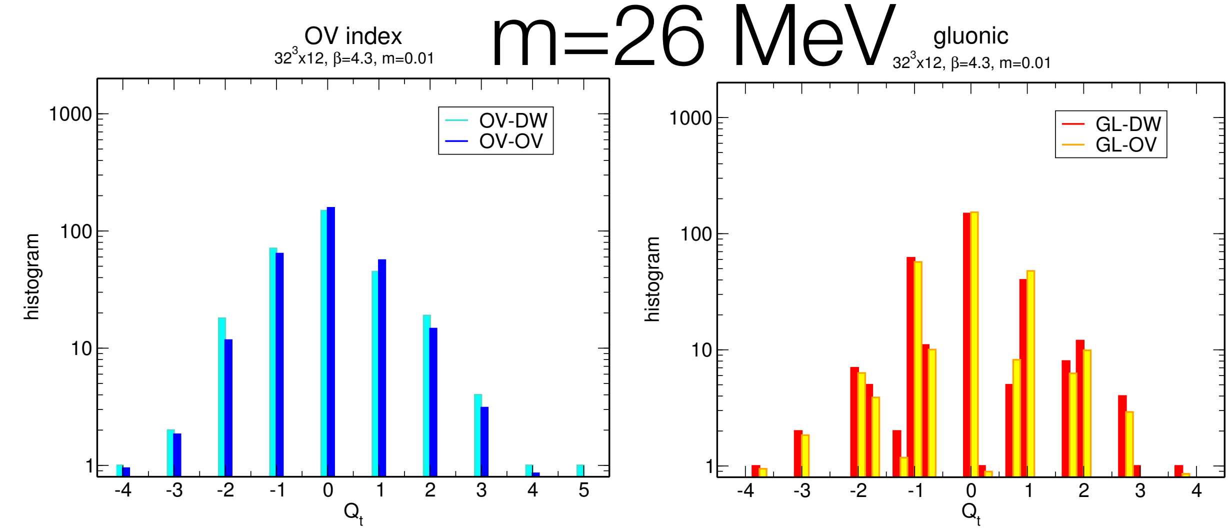
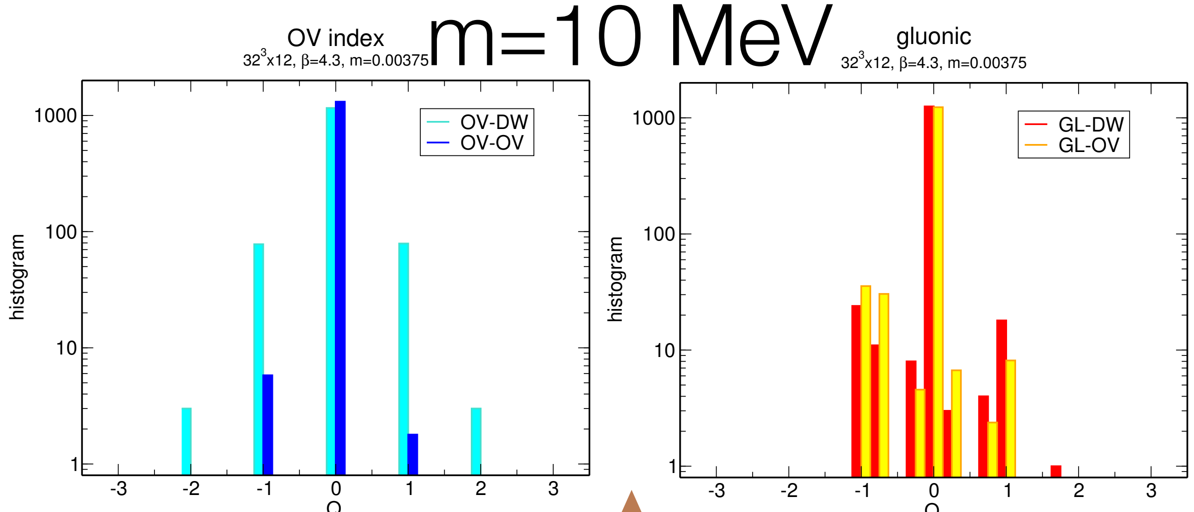


m=26 MeV



m=10 MeV



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Implication of $\chi_t(m_f)=0$ for $0 < m_f < m_c$

- **axion cosmology scenario may fail for $U(1)_A$ restoration**

  due to vanishing / suppressed topological susceptibility

  - $\chi_t|_{m=0} = 0$ & $d^n\chi_t / dm^n|_{m=0} = 0$ Aoki-Fukaya-Tanigchi

    ➡ $\chi_t = 0$ for small non-zero m OR

    ➡ exponential decay for $T>T_c$

$$\chi_t(T) \sim \begin{cases} m_q \Lambda^3_{\rm QCD}, & T < T_c, \\ m_q^2 \Lambda^2_{\rm QCD} e^{-2c(m_q)T^2/T_c^2}, & T > T_c, \end{cases}$$

$$c(m_q) \to \infty \text{ as } m_q \to 0,$$

  - axion mass and decay constant: $\chi_t = m_a^2 f_a^2$

  ➡ axion window can possibly be closed

Kitano-Yamada JHEP [1506.00370]

- see also for θ=π QCD non-standard case with rich implications

Di Vecchia et al. JHEP [1709.00731]

a $U(1)_A$ order parameter

- symmetry in switching flavor non-singlet pseudoscalar and scalar
- order parameter:

$$\Delta_{\pi-\delta} = \int d^4x[\langle \boldsymbol{\pi}^a(x)\boldsymbol{\pi}^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle],$$

➡ 0 for $U(1)_A$ restoration

- as a result, screening masses for these channel will degenerate
  - not a sufficient condition for $U(1)_A$ restoration

relation with Dirac eigenmode spectrum ρ(λ)

$$-\langle \overline{q} q \rangle = \lim_{m \to 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

$$\Delta_{\pi - \delta} = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \to \sim \rho'(0)$$

relation with Dirac eigenmode spectrum ρ(λ)

- chiral condensate : order parameter of $SU(2)_A$

$$-\langle \overline{q}q \rangle = \lim_{m \to 0} \int_0^\infty d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

- $U(1)_A$:

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \to\sim \rho'(0)$$

very roughly speaking

  - very sensitive to the spectrum near λ=0
  - overlap fermion, able to distinguish zero/nonzero modes, is ideal

Analytic works

- Aoki-Fukaya-Taniguchi
  - QCD with OV regulator
  - assuming analyticity of $\rho(0)$
- $f_A \to 0$: $U(1)_A$ br. parameter
- $\chi_{top} = 0$ for $0 < m < m_c$

- Kanazawa-Yamamoto
  - assuming $f_A \neq 0$
  - expansing free energy in m
- discussing
  - contributions of topological sectors
  - finite m and V effect
- $\chi_{top} = 2 f_A m^2$
- yields AFK results

  ← same assumption on ρ

Kanazawa - Yamamoto

- assuming $f_A \neq 0$
- expansing free energy in m

$$Z(T, V_3, M) = \exp\left[-\frac{V_3}{T} f(T, V_3, M)\right],$$

$$f(T, V_3, M) = f_0 - f_2 \operatorname{tr} M^\dagger M - \underline{f_A(\det M + \det M^\dagger)} + \mathcal{O}(M^4),$$

$$M \to e^{-2i\theta_A} V_L M V_R^\dagger \qquad \det M \to \mathrm{e}^{4i\theta_A} \det M$$ breaks $U(1)_A$

other terms are invariant under $U(1)_A$
all invariant under $SU(2)_{L\times R}$

- to study topological sectors

$$M \to M\,\mathrm{e}^{i\theta/N_f} \qquad Z_Q(T, V_3, M) \equiv \oint \frac{\mathrm{d}\theta}{2\pi}\, \mathrm{e}^{-iQ\theta}\, Z(T, V_3, M\mathrm{e}^{i\theta/2}).$$

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Kanazawa - Yamamoto: U(1)$_A$ br. scenario

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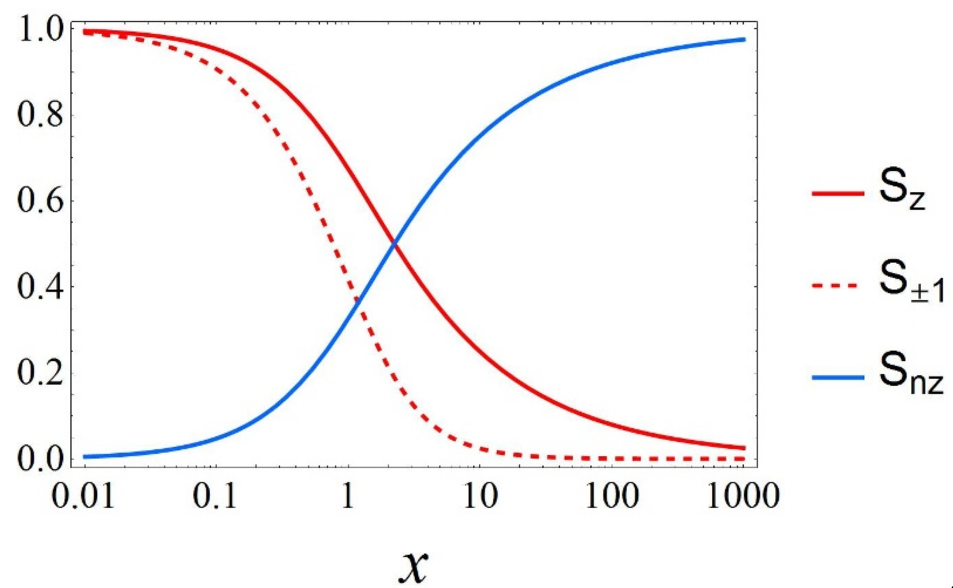
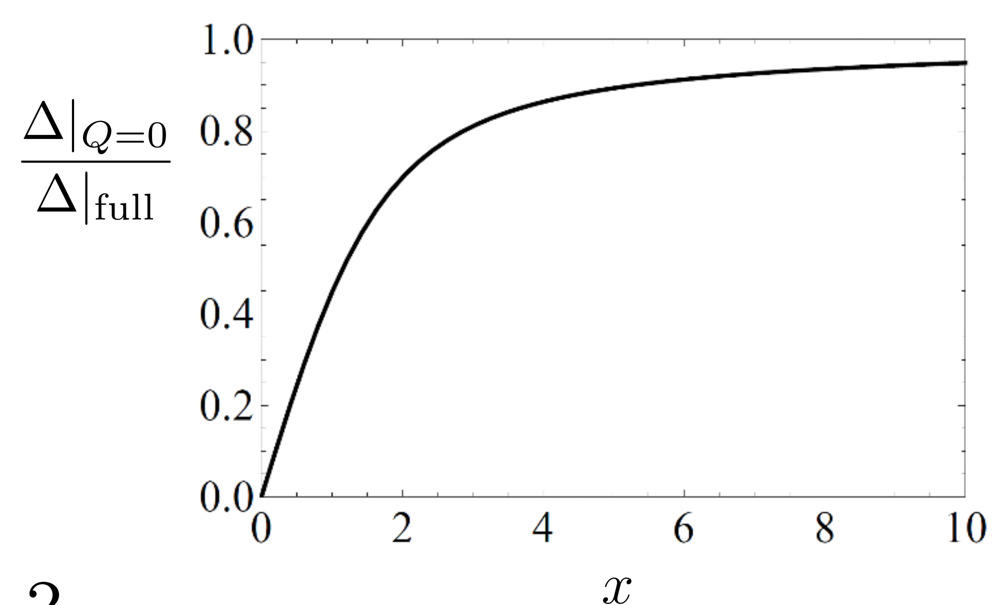
relative contribution of modes





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- adding all Q sector or large enough volume necessary

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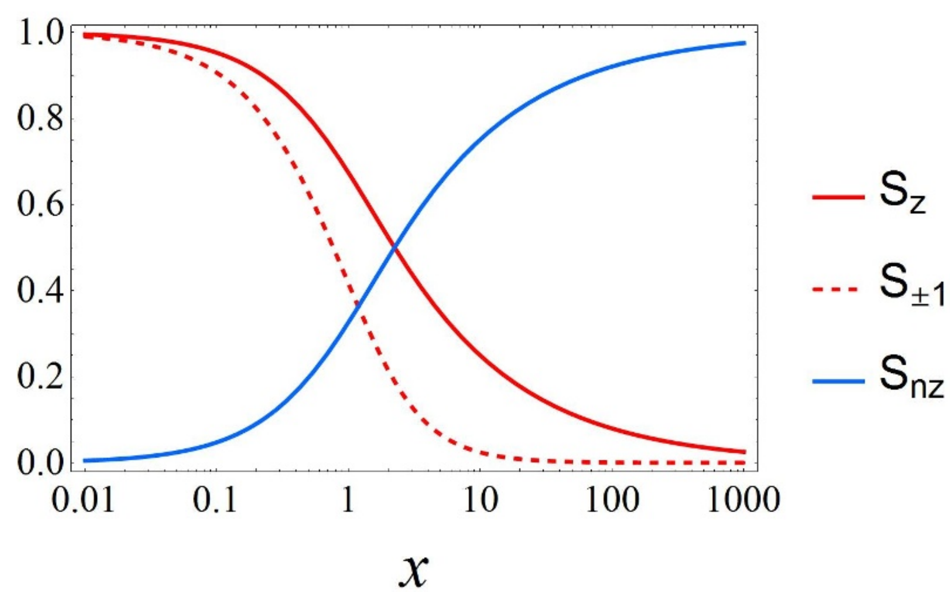

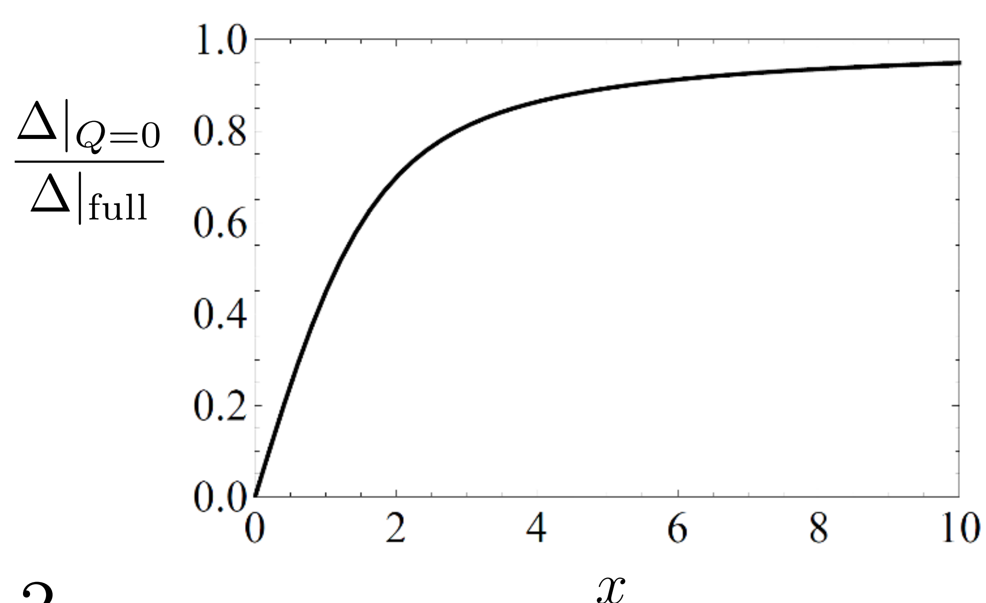

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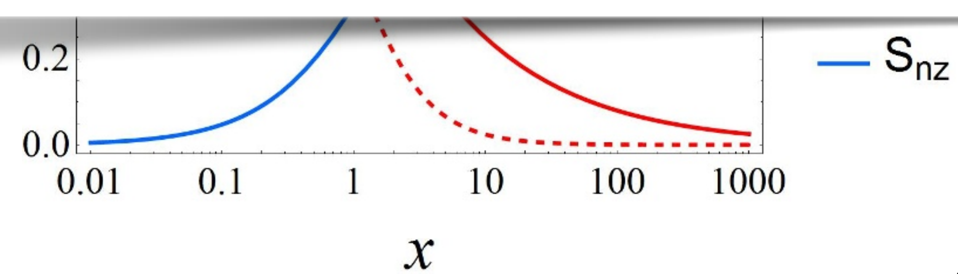
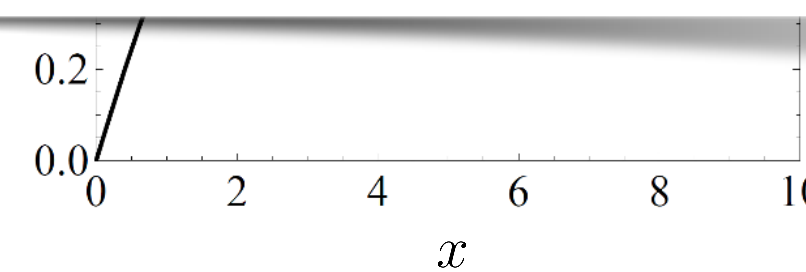
JLQCD

- does not fix topology (DW)
- zero-mode subtraction may have similar effect to fix Q=0
  - for smallest m: actually effectively fixed to Q=0





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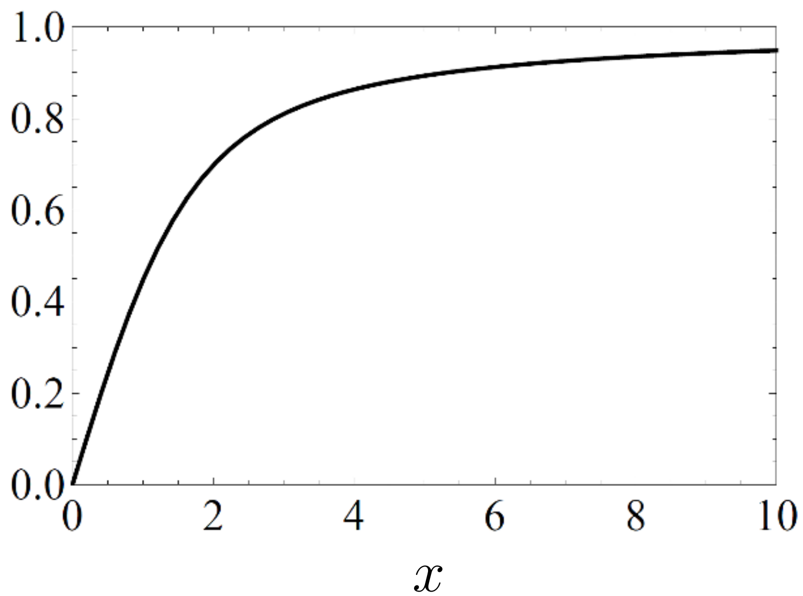
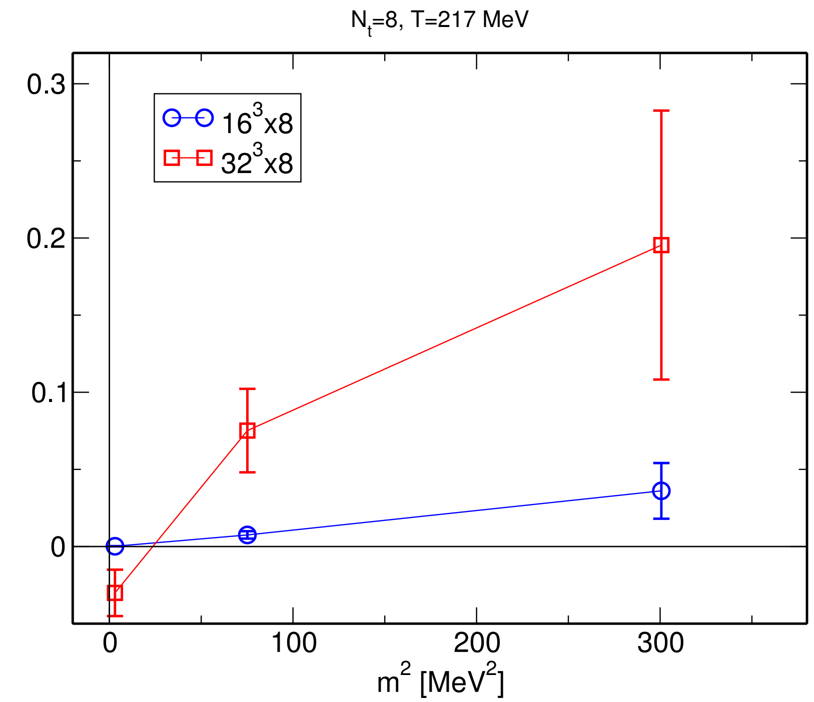
compare with JLQCD Δ with non-zero modes

[JLQCD 2016 Tomiya et al]





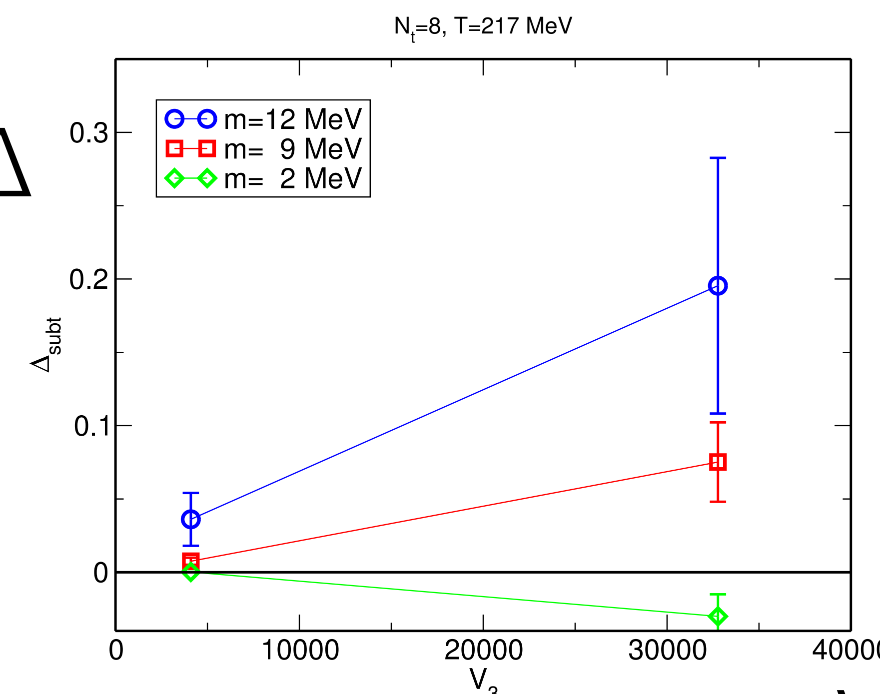
$$x = 2V_4 f_A m^2$$

fix V: Δ→0 as m² for m→0
even for $U(1)_A$ br. case

fix m : Δ∝V

→ NOT inconsistent with JLQCD results

Δ

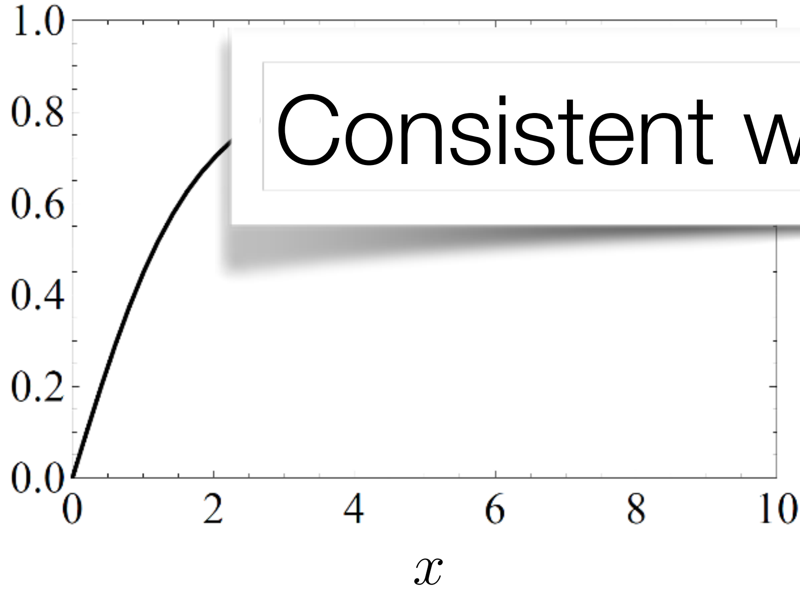


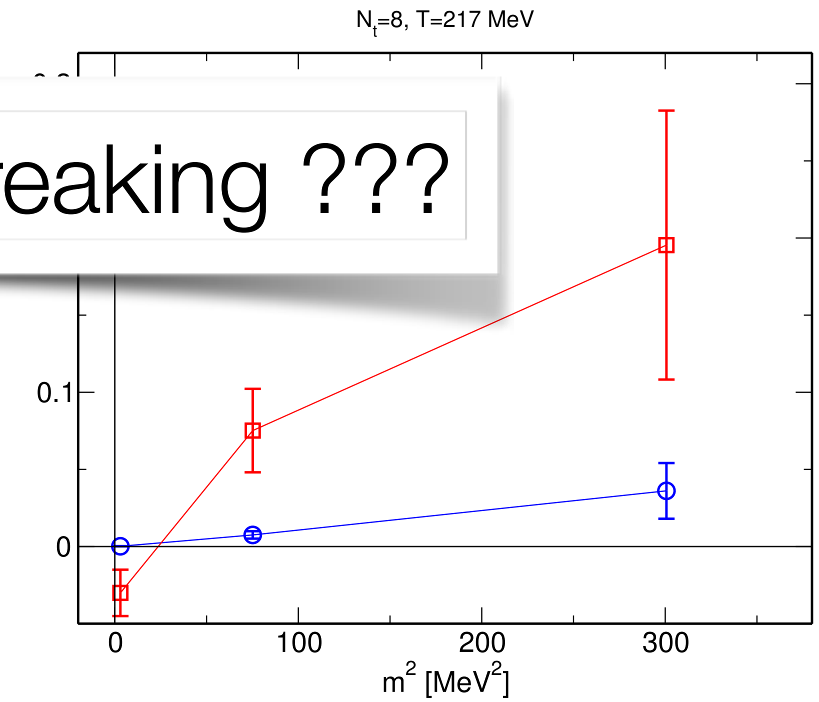
V

compare with JLQCD Δ with non-zero modes

[JLQCD 2016 Tomiya et al]

Consistent with U(1)$_A$ breaking ???





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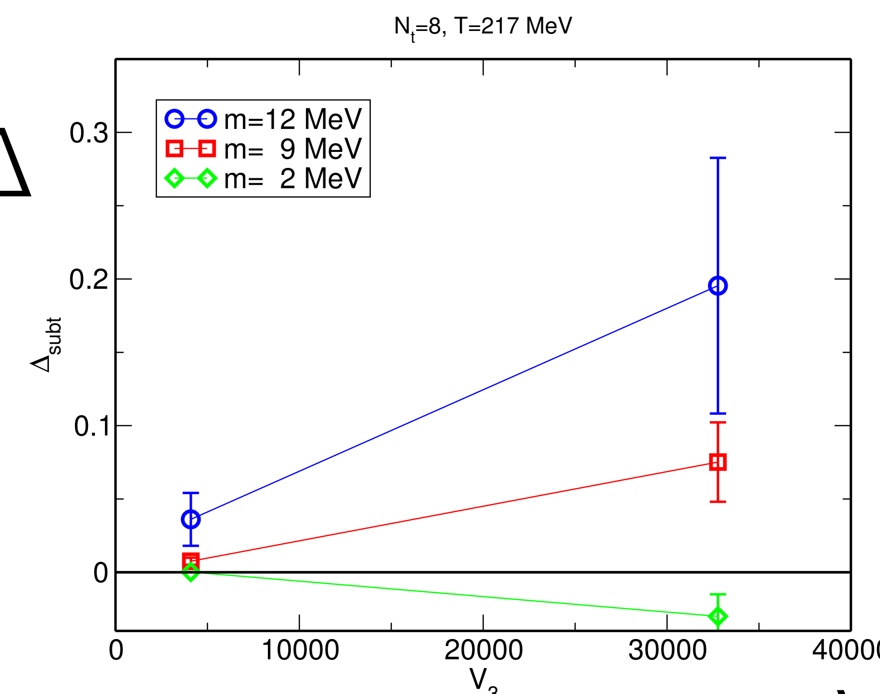

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Lattice framework

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
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λ for $H_M = \gamma_5 \frac{\alpha D_W}{2 + D_W}$

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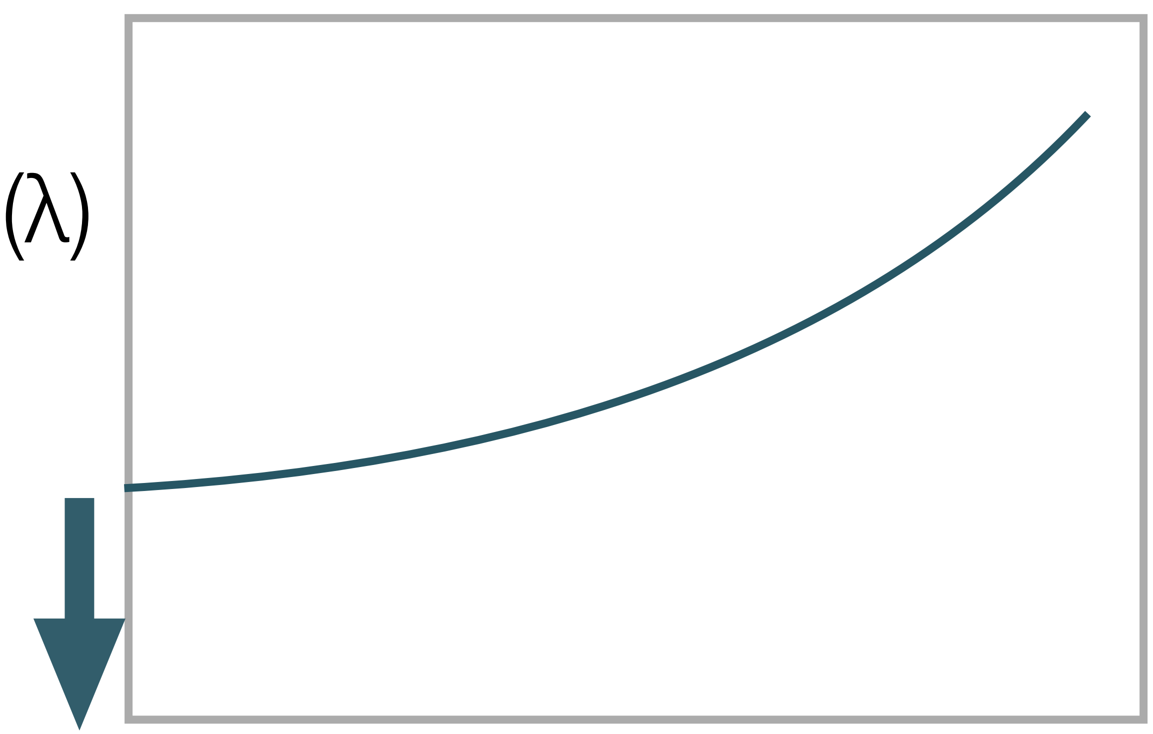
simply speaking,   in the m→0 limit

- U(1)$_A$ restores if

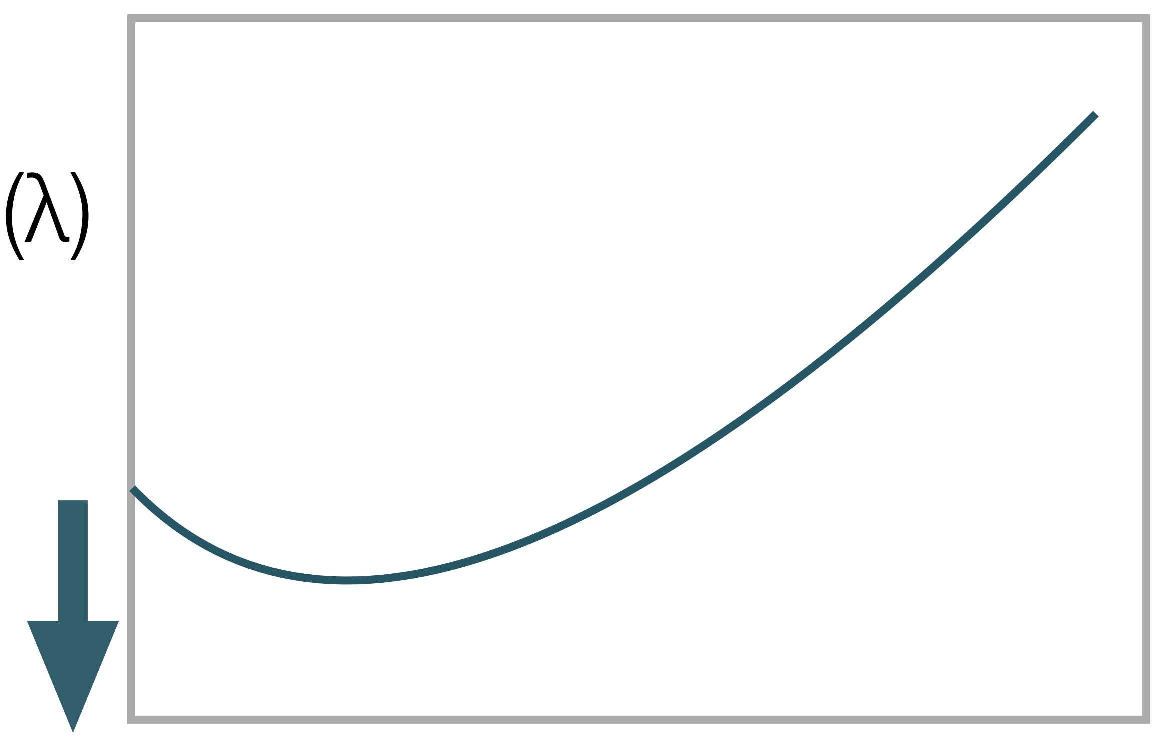
ρ(λ)



with ρ(0)→0 and ρ'(0)→0

- and not if

ρ(λ)



with ρ(0)→0 and ρ'(0)≠ 0

- non-analyticity at λ→0 required