Heavy-quark physics with a Twisted Mass QCD valence action

Andrea Bussone

26 July 2018

Collaborators: S. Chaves, G. Herdoíza, C. Pena, J. A. Romero[1] and J. Ugarrio[2]

[1]Poster on the light sector

[2]Next talk on the charm sector

Motivations

The hope is to find New Physics

Heavy masses and decay constants are important for, e.g. at tree level,

CKM matrix unitarity

$$\Gamma(D_s \to \ell\nu) = \frac{G_F^2 f_{D_s}^2 m_\ell^2 M_{D_s}}{8\pi}\left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 |V_{cs}|^2$$

Higgs decay

$$\Gamma(H \to b + \overline{b}) = \frac{3G_F}{4\sqrt{2}\pi} m_H {m_b}^2(\overline{\mathrm{MS}}, m_H)$$

On the lattice different energy scales should fit

small finite volume effects
$m_\pi \gg L^{-1}$

small discretization errors
$m_h \ll a^{-1}$

The ways to reduce the discr. errors are

reduce the lattice spacing $a \lesssim 0.05\ \mathrm{fm}$

(topological freezing problem)

Symanzik improvement programme

(or chiral-preserving regularizations)

Set-up

CLS initiative is generating cnfgs with

[Lüscher and Schaefer, JHEP 1107 036, 2011; Bruno et al. JHEP 1502 (2015) 043]

Tree-level $O(a^2)$-improved gauge action[3]

[Lüscher and Weisz, Commun. Math. Phys. 97 (1985) 59]

$$\mathrm{S}_G[U] = \frac{\beta}{6}\left[c_0 \sum_{\mathrm{plaq.}} \mathrm{Tr}\,(1 - U(p)) + c_1 \sum_{\mathrm{rect.}} \mathrm{Tr}\,(1 - U(r))\right],\ c_0 = \frac{5}{3},\ c_1 = -\frac{1}{2}$$

$N_f = 2 + 1$ non-perturbatively $O(a)$-improved Wilson fermions[3]

[Sheikholeslami and Wohlert, Nucl. Phys. B 259 (1985) 572]

$$\mathrm{S}_F[U, \overline{\psi}, \psi] = a^4 \sum_{f=u,d,s} \sum_x \overline{\psi}_f(x) D_{\mathrm{W}}(m_{0,f}) \psi_f(x)$$

$$D_{\mathrm{W}}(m_0) = \sum_\mu \left[\gamma_\mu \widetilde{\nabla}_\mu - \frac{a}{2} \nabla^-_\mu \nabla^+_\mu\right] + m_0 + a c_{\mathrm{SW}} \sum_{\mu,\nu} \frac{i}{4} \sigma_{\mu\nu} \widehat{F}_{\mu\nu}$$

[3] with tree-level $O(a)$-improved open BCs in time.

O(a) improvement

Wilson-like theories

O(a) effects are tied up to the hard breaking of χ-symmetry in Wilson reg.

[Lüscher et al. Nucl. Phys. B 491 (1997) 323]

Twisted Mass (TM) is automatically O(a)-improved at maximal twist

[Frezzotti and Rossi, JHEP 0408 (2004) 007]

($\mathbf{m} = \mathbf{m}_0 - m_{\rm cr}\mathbf{1} = 0$ and $\boldsymbol{\mu} = \mu^a T^a$, where $T^a \in \mathfrak{su}(N_f)$)

Can we profit from this property by putting TM in the valence sector?

[Frezzotti et al. JHEP 0108 (2001) 058]

$$\mathrm{S}_F[U,\overline{\psi},\psi] = a^4 \sum_x \overline{\psi}(x) \left[\gamma_\mu \widetilde{\nabla}_\mu - \frac{a\,\mathbf{r}}{2} \nabla_\mu^- \nabla_\mu^+ + \mathbf{m}_0 + i\boldsymbol{\mu}\gamma_5\right] \psi(x)$$

(twisted basis is used)

Extension of **[Bhattacharya et al. Phys. Rev. D 73 (2006) 034504]** to the Wilson-like theories

Review automatic O(a)-improvement

Symanzik asymptotic expansion

[Symanzik, Nucl. Phys. B 226 (1983) 187]

$$\langle \Phi \rangle = \langle \Phi_0 \rangle\big|_{\rm cont} - a \int \mathrm{d}^4 z \, \langle \Phi_0 \mathcal{L}_1(z) \rangle\big|_{\rm cont} + a \, \langle \Phi_1 \rangle\big|_{\rm cont} + \mathrm{O}(a^2)$$

(Infinite volume is assumed)

Symmetries of the theory

$R_5 \times [\boldsymbol{\mu} \to -\boldsymbol{\mu}] \times [\mathbf{m} \to -\boldsymbol{m}] \times [\boldsymbol{r} \to -\boldsymbol{r}]$

- $\psi(x) \xrightarrow{R_5} \gamma_5 \psi(x)$
- $\overline{\psi}(x) \xrightarrow{R_5} -\overline{\psi}(x)\gamma_5$

$P \times [\boldsymbol{\mu} \to -\boldsymbol{\mu}]$

$\Rightarrow P \times R_5 \times [\mathbf{m} \to -\boldsymbol{m}] \times [\boldsymbol{r} \to -\boldsymbol{r}]$

Wilson Average (WA)

$$\frac{1}{2}\left[\langle \Phi \rangle\Big|_{\substack{r\\ m\\ \mu}} + \langle \Phi \rangle\Big|_{\substack{-r\\ m\\ \mu}} \right] = \langle \Phi_0 \rangle\big|_{\rm cont} + \mathrm{O}(a^2)$$

Mass Average (MA)

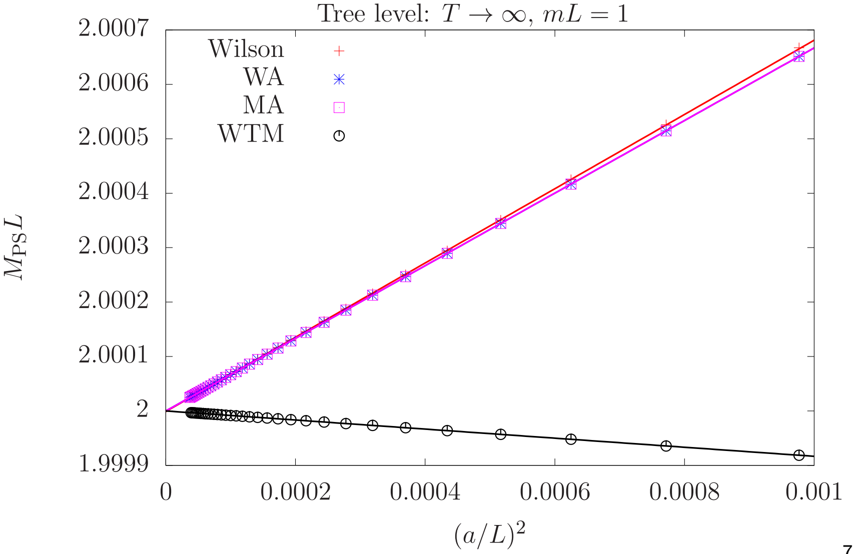
$$\frac{1}{2}\left[\langle \Phi \rangle\Big|_{\substack{r\\ m\\ \mu}} + \eta^P \eta^{R_5} \langle \Phi \rangle\Big|_{\substack{r\\ -m\\ \mu}} \right] = \langle \Phi_0 \rangle\big|_{\rm cont} + \mathrm{O}(a^2)$$

TM at maximal twist

$$\langle \Phi \rangle\Big|_{\substack{r\\ \mu}} = \langle \Phi_0 \rangle\big|_{\rm cont} + \mathrm{O}(a^2)$$

(for physical observables)

Review automatic O(a)-improvement: Tree level



Mixed Action: unitarity & cutoff effects

Wilson reg. transfer matrix
[Lüscher, Commun. Math. Phys. 54 (1977) 283]

Twisted Mass reg. transfer matrix
[Frezzotti et al. JHEP 0107 (2001) 048]

Sea Wilson + Valence TM at maximal twist: non-unitary set-up[4]
Valence determinant is suppressed by ghosts
[Morel, J. Phys. (France) 48 (1987) 1111]

Symmetries of the valence action are easily extended to the ghost sector
[Frezzotti and Rossi, JHEP 0410 (2004) 070]

Valence corr. funct. Φ^V with $m_V = 0$, employing (valence) WA

$$\frac{1}{2}\left[\langle\Phi^V\rangle\Big|_{\substack{r_S,m_S\\ r_V,\mu_V}} + \langle\Phi^V\rangle\Big|_{\substack{r_S,m_S\\ -r_V,\mu_V}}\right] = \frac{1+\eta_V^P\eta_V^{R_5}}{2}\langle\Phi^V\rangle\Big|_{\substack{r_S,m_S\\ r_V,\mu_V}}$$
$$= \langle\Phi_0^V\rangle\Big|_{\mathrm{cont}} + a\langle\Phi_1^S\rangle\Big|_{\mathrm{cont}} + \mathrm{O}(a^2)$$

[4] Matching to a unitary theory can be performed see poster by J. A. Romero.

Mixed Action: quark-mass lattice artifacts

EXAMPLE: charm quark mass

Wilson

$$\widehat{m}_{ij} = \frac{Z_A}{Z_P} m_{ij} \left[1 + a(\widetilde{b}_A - \widetilde{b}_P) m_{ij} + a(\overline{b}_A - \overline{b}_P) \mathrm{Tr}\,\mathbf{m}\right] + \mathrm{O}(a^2)$$

$$\overline{b}_A = \mathrm{O}(g_0^4) = \overline{b}_P$$

$$\widetilde{b}_A - \widetilde{b}_P = -0.0012 g_0^2 + \mathrm{O}(g_0^4)$$

[Taniguchi and Ukawa, Phys. Rev. D 58 (1998) 114503]

TM at maximal twist

$$\widehat{\mu}_j = \frac{1}{Z_P} \mu_j + \mathrm{O}(a^2)$$

Sea Wilson + Valence TM at maximal twist

$$\widehat{\mu}_j = \frac{1}{Z_P} \mu_j \left(1 + a\overline{b}_\mu \mathrm{Tr}\,\mathbf{m}\right) + \mathrm{O}(a^2), \qquad \overline{b}_\mu = \mathrm{O}(g_0^4)$$

(no $a\mu_h$ effects)

Continuum-limit extrap. of light-quark obs. show consistency with $\mathrm{O}(a^2)$ scaling[5]

[5] See poster by J. A. Romero.

Inaccurate inversions

Distance preconditioning[6]

For large time-sep. the solution becomes inaccurate because contributes little to the condition

[Juttner and Della Morte, PoS LAT 2005 (2006) 204]

$\longrightarrow$ Propagator decays exponentially $|S_h(t)| \propto e^{-m_{hh}t/2}$

Global residue

$$\left| \sum_z D(x,z)S(z) - \eta(x) \right| < r$$

Idea given in

[de Divitiis et al. Phys. Lett. B 692 (2010) 157]

Technical implementation

[Collins et al. PoS LATTICE 2016 (2017) 368]

Preconditioned system $\left(PDP^{-1}\right)(PS) = P\eta$,

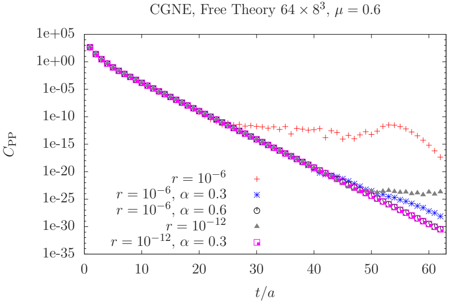
where $P = \mathrm{diag}(e^{+\alpha|y_0 - x_0^{(1)}|}, e^{+\alpha|y_0 - x_0^{(2)}|}, \dots)$

Now the heavy propagator might decay as a pion, number of MVMs will increase

[6] Another option is to increase the working precision.

Distance preconditioning: CGNE example

(residuum normalized by the norm of the source spinor field)



hh PS correlator

Free theory

$V = 64 \times 8^3$

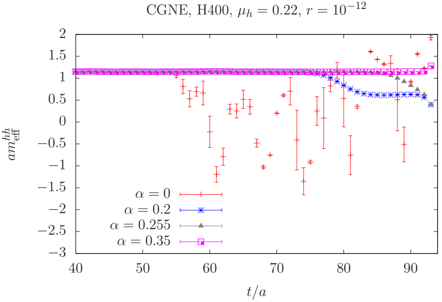
$\mu L = 4.8$

$\eta_c^{(c)}$ mass

H400: 100 cnfgs

$V = 96 \times 32^3$

$\mu_c = 0.22$



Excited states contamination

Smearing of interpolating operators

Signal to noise ratio problem

[Parisi, Phys. Rept. 103 (1984) 203; Lepage, Boulder ASI 1989:97-120 (1989)]

Gradient flow of gauge and quark fields

[Lüscher, JHEP 1304 (2013) 123]

Flow equations

$$\partial_t B_\mu = D_\nu G_{\nu\mu}, \qquad \partial_t \chi = \Delta\chi, \qquad \partial_t \bar{\chi} = \bar{\chi}\overleftarrow{\Delta},$$
$$\Delta = D_\mu D_\mu, \qquad D_\mu = \partial_\mu + B_\mu$$

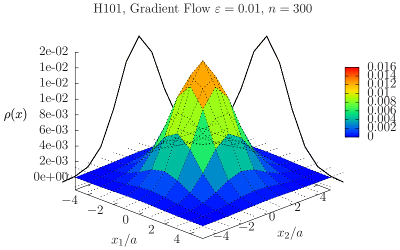
Initial conditions

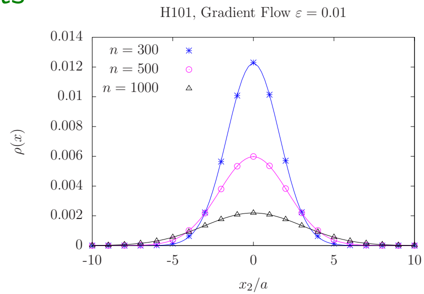
$$B_\mu\big|_{t=0} = A_\mu\,, \qquad \chi\big|_{t=0} = \psi\,, \qquad \bar{\chi}\big|_{t=0} = \bar{\psi}$$

Remarks:

- Restriction to equal-time hyperplane
  [Alexandrou et al. Phys. Lett. B 256 (1991) 60]
- Fermionic evolution depends on the evolved gauge
- Gaussian smearing with $R^2 = 2Dt$
- Good renormalization properties
  [Lüscher and Weisz, JHEP 1102 (2011) 051]
- No extra O(a) effects in TM
  [Shindler, Nucl. Phys. B 881 (2014) 71]

Smearing: numerical experiments



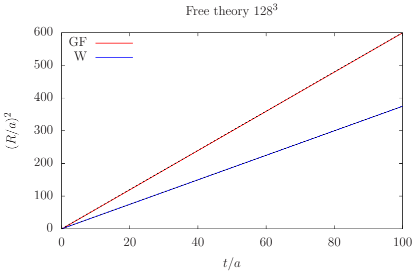


$$\rho(x) = \frac{|\psi(x)|^2}{\sum_x |\psi(x)|^2}$$

In free theory, starting from δ in position and color space one gets a Gaussian with radius

$$R^2_{\mathrm{GF}} = 6t$$

$$R^2_{\mathrm{W}} = \frac{6n\varepsilon}{1+6\varepsilon} = \frac{6t}{1+6\varepsilon}$$



Conclusions

- The mixed action set-up presented is O(a)-improved modulo cutoff effects coming from the sea
- We have generalized a formalism to identify sea O(a)-effects in Wilson-like theories
- We have used the Gradient Flow as smearing tool to fight the signal to noise ratio problem

Outlook

- Check $O(a^2)$ scaling of observable in the charm sector
- Use smearing in the computation
- See if one can profit from the good renormalization properties of the Gradient Flow

Next talk by J. Ugarrio about preliminary numerical results in the charm sector

Back-up slides

Boundary conditions

Generation of cnfgs by CLS

for small lattice spacings the topology freezing is overcame by the use of openBC

$$
\begin{aligned}
F_{0k}(x)|_{x_0=0} = 0 = F_{0k}(x)|_{x_0=T} \qquad \forall\ k = 1,2,3 \\
P_{+}\psi(x)|_{x_0=0} = 0 = P_{-}\psi(x)|_{x_0=T} \\
\overline{\psi}(x)P_{-}|_{x_0=0} = 0 = \overline{\psi}(x)P_{+}|_{x_0=T} \\
P_{\pm} = \frac{1}{2}(1 \pm \gamma_0)
\end{aligned}
$$

Diagonal masses O(a)-improvement in Wilson-like theories

Spurionic analysis: masses transform in the adjoint repr. of $\mathbf{SU}(N_f)$

$$\mathbf{m} \to U^\dagger \mathbf{m} U \qquad i\gamma_5 \boldsymbol{\mu} \to i\gamma_5 U^\dagger \boldsymbol{\mu} U$$

$$\begin{aligned}
\widehat{m}_j = Z_m \Bigg\{ &\left[m_j + (r_m - 1)\frac{\mathrm{Tr}\,\mathbf{m}}{N_f} \right] + a\Bigg[b_m m_j^2 + \widetilde{b}_m \mu_j^2 + \overline{b}_m m_j \mathrm{Tr}\,\mathbf{m} \\
&+ (r_m d_m - b_m)\frac{\mathrm{Tr}\,\mathbf{m}^2}{N_f} + (r_m \overline{d}_m - \overline{b}_m)\frac{(\mathrm{Tr}\,\mathbf{m})^2}{N_f} + (r_m \widetilde{d}_m - \widetilde{b}_m)\frac{\mathrm{Tr}\,\boldsymbol{\mu}^2}{N_f} \Bigg] \Bigg\}
\end{aligned}$$

$$\begin{aligned}
\widehat{\mu}_j = Z_\mu \Bigg\{ &\left[\mu_j + (r_\mu - 1)\frac{\mathrm{Tr}\,\boldsymbol{\mu}}{N_f} \right] + a\Bigg[b_\mu \mu_j m_j + \overline{b}_\mu \mu_j \mathrm{Tr}\,\mathbf{m} \\
&+ (r_\mu d_\mu - b_\mu)\frac{\mathrm{Tr}(\mathbf{m}\,\boldsymbol{\mu})}{N_f} + (r_\mu \overline{d}_\mu - \overline{b}_\mu)\frac{(\mathrm{Tr}\,\mathbf{m}\mathrm{Tr}\,\boldsymbol{\mu})}{N_f} \Bigg] \Bigg\}
\end{aligned}$$

Review automatic O(a)-improvement: Tree level