Exploring the phase diagram of finite density QCD at low temperature by the complex Langevin method

Yuta Ito (KEK)

based on collaboration with

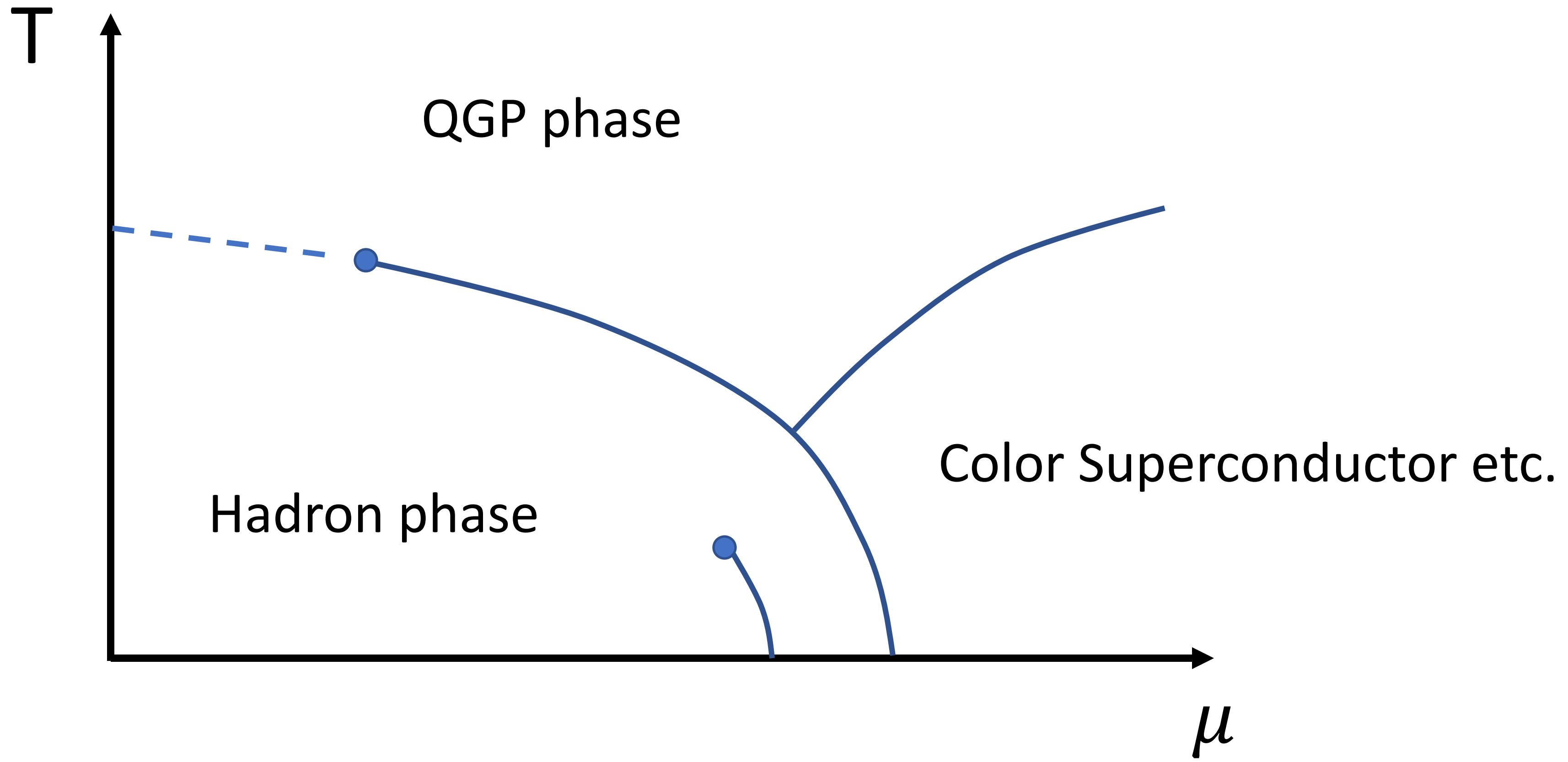
Shoichiro Tsutsui (KEK), Jun Nishimura (KEK, SOKENDAI), Hideo Matsufuru (KEK)

Asato Tsuchiya (Shizuoka University), Shinji Shimasaki (Keio University, KEK)

Lattice2018 23/7/2018 at Michigan State University

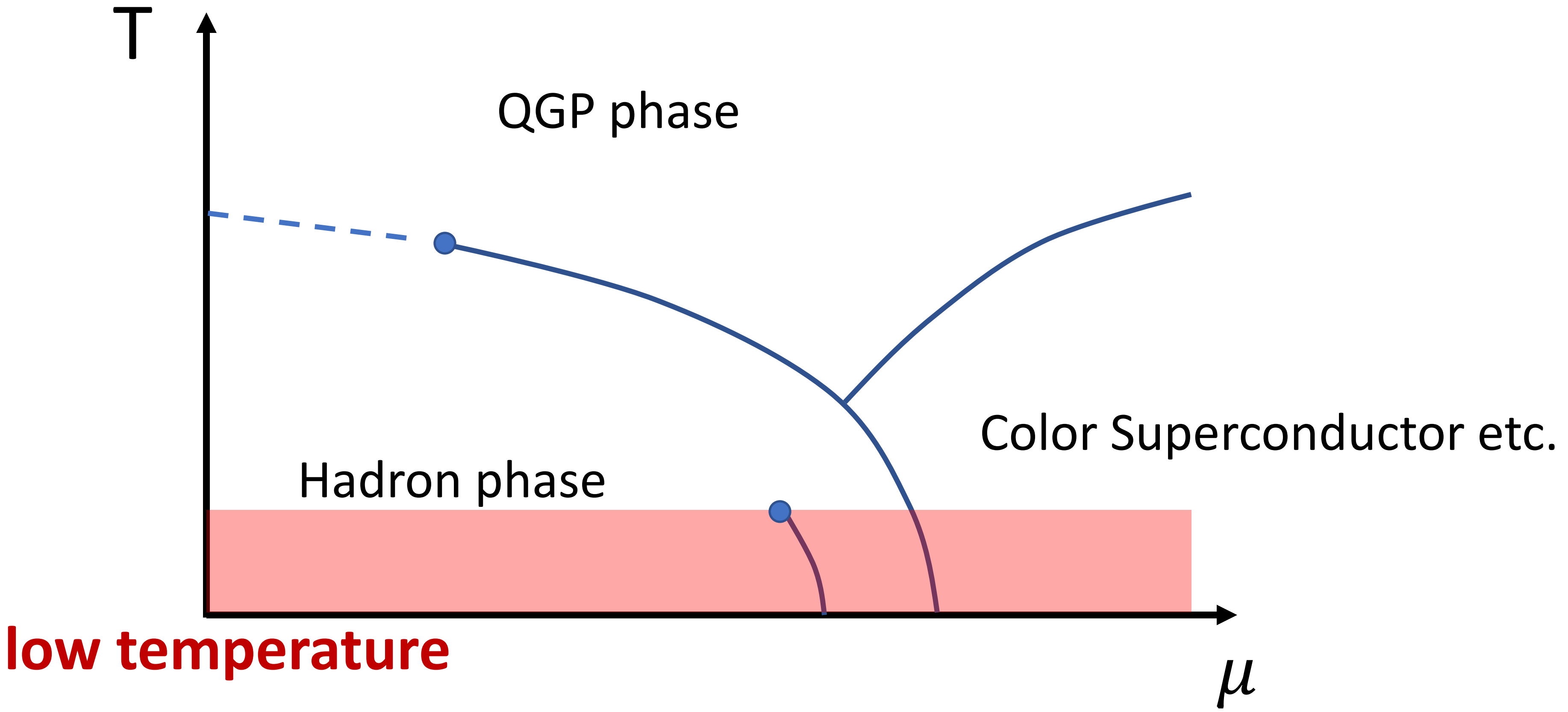
Introduction

- QCD phase diagram (conjectured)



Introduction

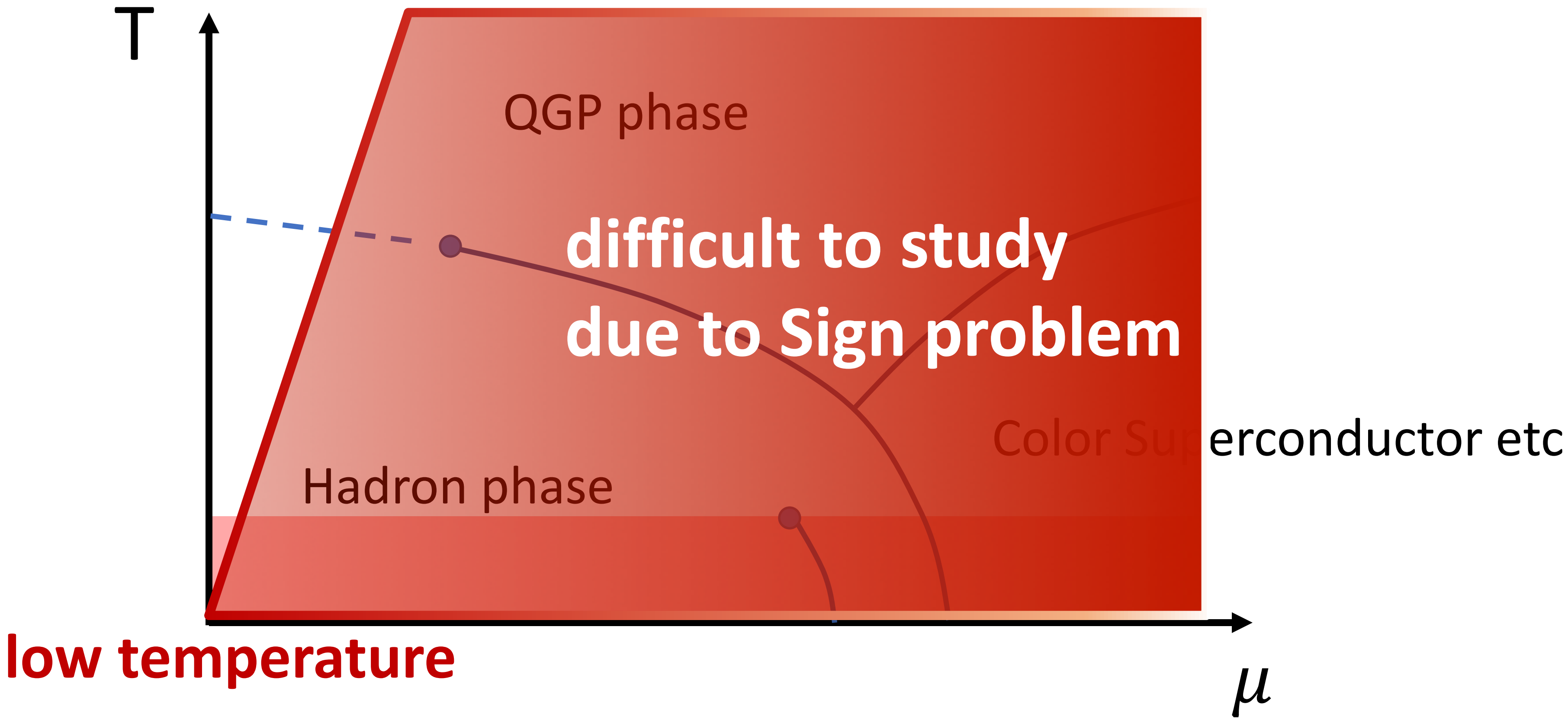
■ QCD phase diagram



- Silver-Blaze phenomenon, neutron star, the quark matter phase, color SC

Introduction

■ QCD phase diagram



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Finite density QCD

Partition function

$$Z = \int dU \; \underline{\det M[U,\mu]} e^{-S_g[U]}$$

$S_g[U]$: gauge action

$\det M[U,\mu]$: fermion determinant

- When $\mu \neq 0$, the the fermion determinant becomes complex, which causes the sign problem.

- Complex Langevin method (CLM) is a promising approach to solve the sign problem.

CLM for lattice QCD

[Parisi '83][Klauder '84]
[Aarts, Seiler, Stamatescu '09]
[Aarts, James, Seiler, Stamatescu '11]
[Sexty '14] [Fodor, Katz, Sexty, Torok '15]
[Nishimura, Shimasaki '15]
[Nagata, Nishimura, Shimasaki '15]

- complexify the link variables

$$U_{x\mu} \in SU(3) \to \mathcal{U}_{x\mu} \in SL(3, C)$$

- consider holomorphic extension of the action

$$S[U] \to S[\mathcal{U}]$$

- update the link variables according to the complex Langevin equation

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i(\epsilon v_{x\mu}(\mathcal{U}) + \sqrt{\epsilon}\eta_{x\mu}(t))\right]\mathcal{U}_{x\mu}(t)$$

t : Langevin time
$\eta_{x\mu}$: Gaussian noise
$v_{x\mu} = -\mathcal{D}_{x\mu}S[\mathcal{U}]$: drift term

Correctness of the results of CLM

The two causes for failure of the CLM.

- excursion problem
  large deviation of $\mathcal{U}_{x\mu}$ from SU(3)
- singular drift problem
  appearance of near zero eigenvalues of the Dirac operator

solution ←

Gauge cooling
[Seiler, Sexty, Stamatescu '13]

deformation of the Dirac op.
[YI, Nishimura '16]

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To judge whether the CLM works or not, we consider

- probability distribution of the magnitude of drift term $v_{x\mu} = -\mathcal{D}_{x\mu} S[\mathcal{U}]$

$$p(u) = \frac{1}{4N_V} \left\langle \sum_{x\mu} \delta(u - u_{x\mu}) \right\rangle \qquad u_{x\mu} \equiv \sqrt{\frac{1}{N_\mathrm{c}^2 - 1} \mathrm{tr}(v_{x\mu} v_{x\mu}^\dagger)}$$

- asymptotic behavior of $p(u)$ at large u

[Nagata, Nishimura, Shimasaki '16]
[Nagata, Matsufuru, Nishimura, Shimasaki '16]

  - exponential fall-off → reliable results.
  - power law fall-off → unreliable results.

Setup

YI, Matsufuru, Nishimura, Shimasaki, Tsuchiya, Tsutsui

Lattice setup:

- Lattice size: 8^3 x 16
- Staggered fermion with N_f = 4
- β=5.7
- quark mass: $m_q a$ = 0.01, 0.05
- chemical potential: μa = 0.1,…,0.5

Setup for CLM:

- Langevin step size: ϵ = 10^{-4} , adaptive step size employed
- Total number of Langevin steps: 5 x 10^5 ~ 15 x 10^5
- gauge cooling at every Langevin step

The aim of this work

- We measure the baryon number density

$$\langle n \rangle = \frac{1}{N_V N_c} \frac{\partial}{\partial(\mu a)} \log Z$$

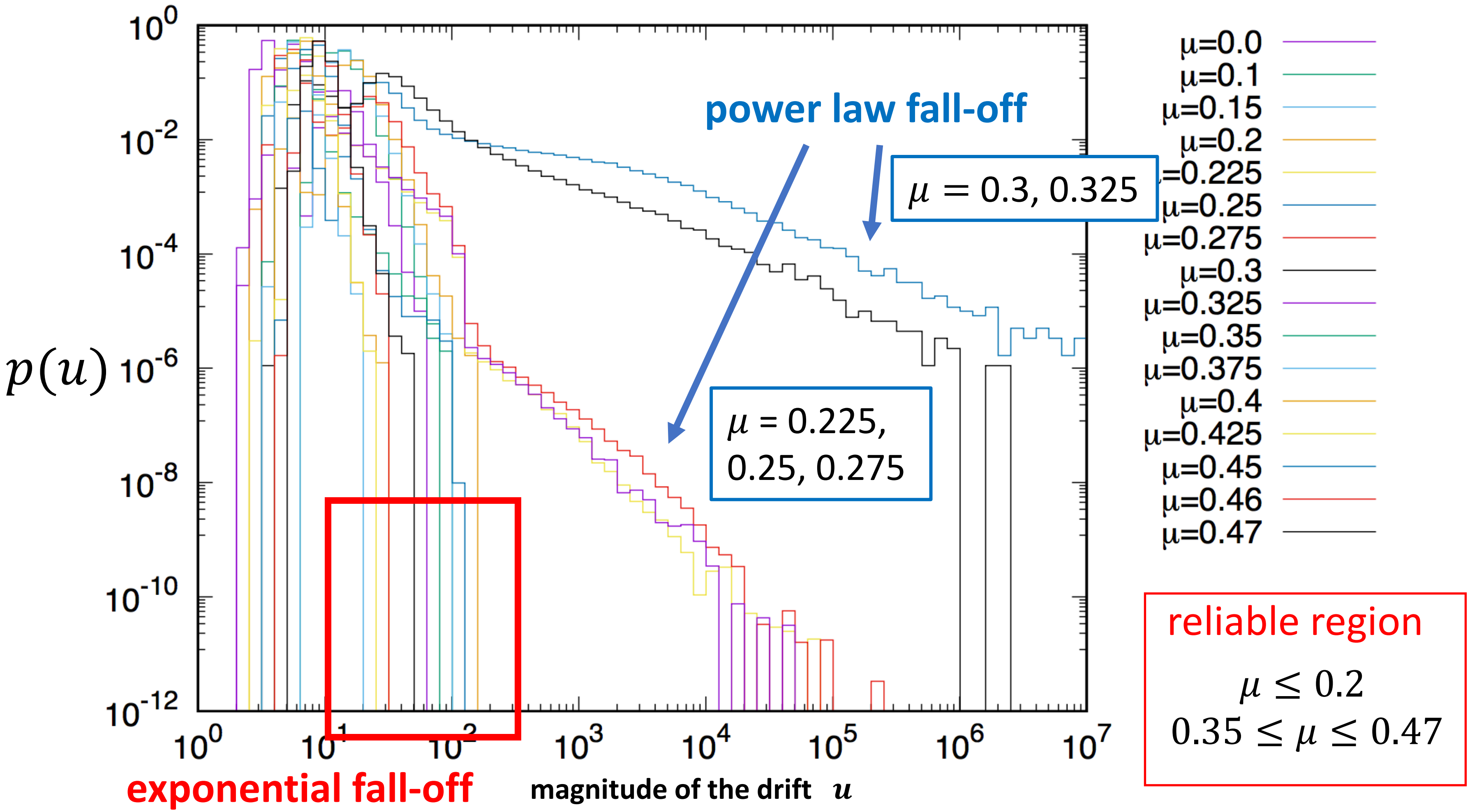
- We focus on the low temperature region.

cf.) Tsutsui's talk: analysis at high temperature region

Silver-Blaze phenomenon?
transition to nuclear matter/quark matter phase?

- We compare the results with RHMC results of the phase quenched model.

Histogram of the drift ($m_q a = 0.05$)



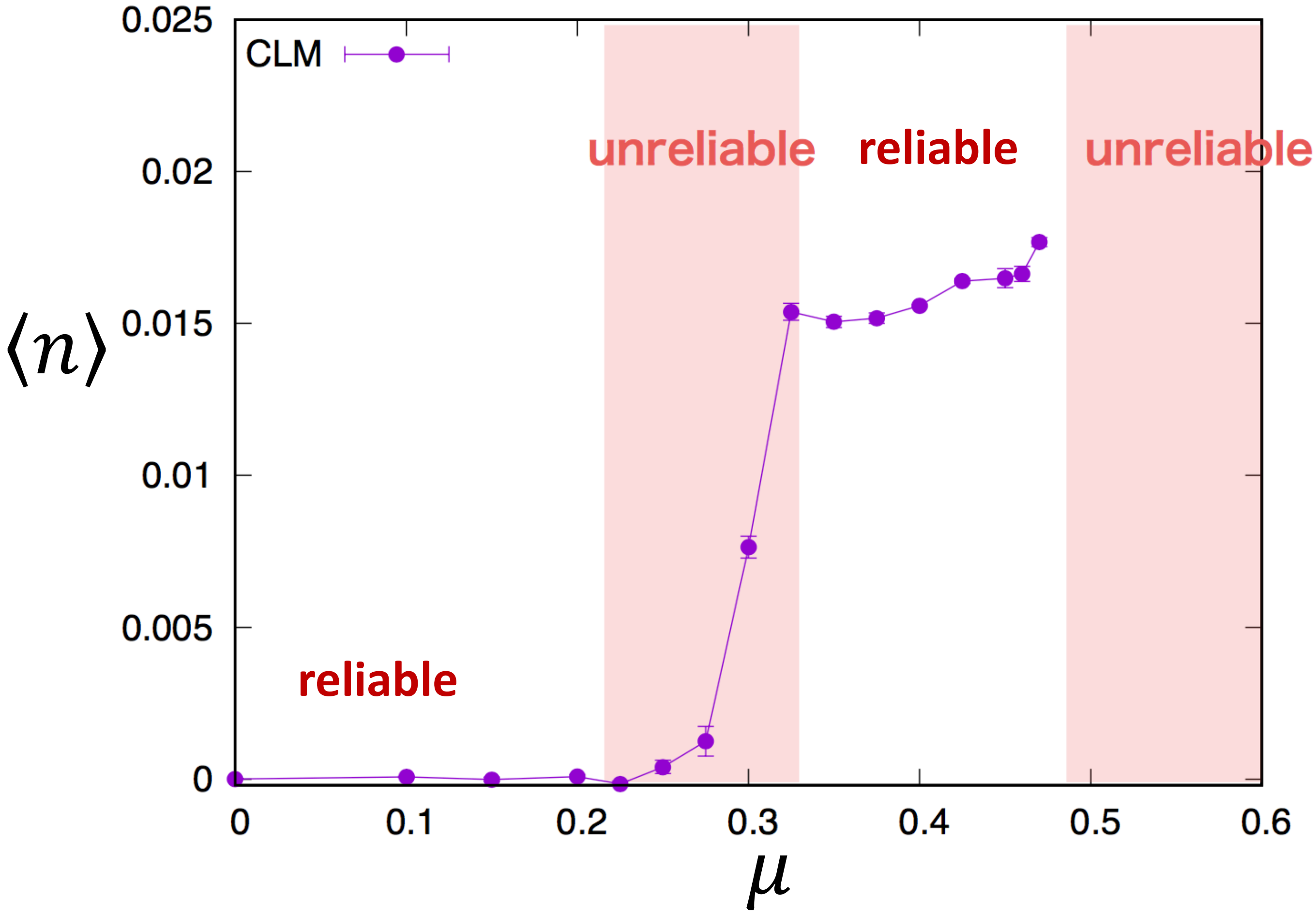
reliable region

$\mu \leq 0.2$

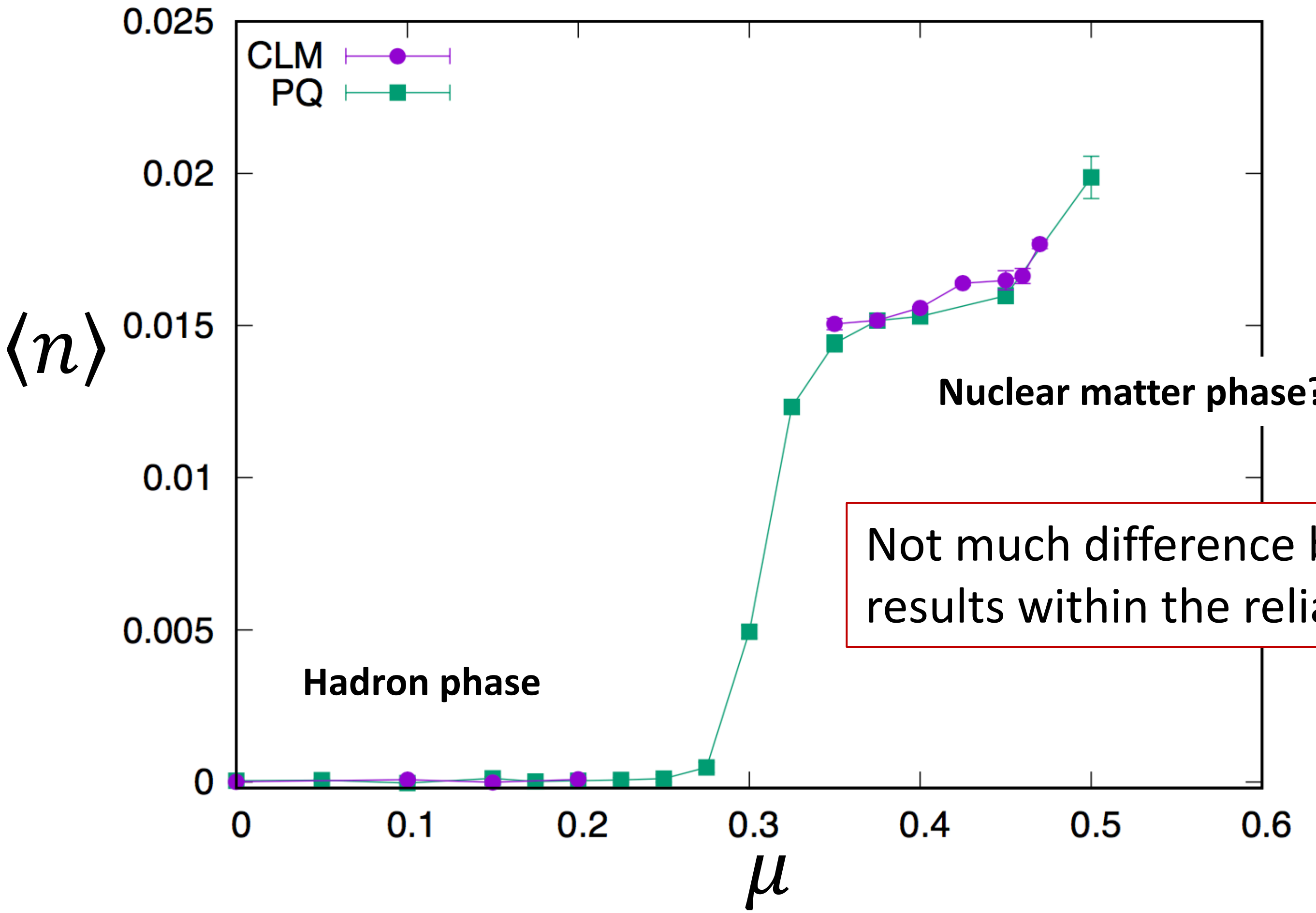
$0.35 \leq \mu \leq 0.47$

The CLM successfully works even at the intermediate region of μ.

Baryon number density ($m_q a = 0.05$)

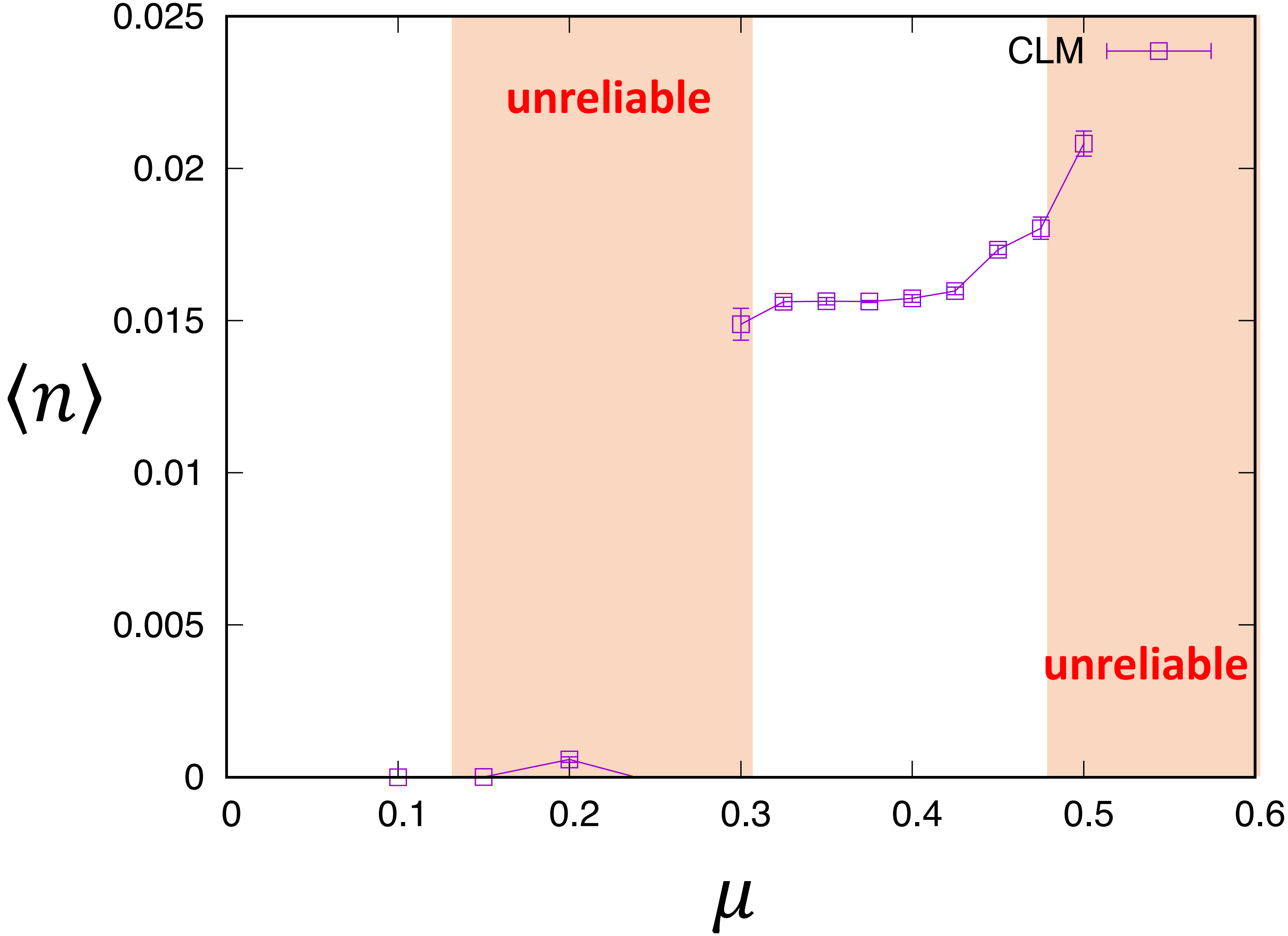


Comparison with phase quenced simulation ($m_q a = 0.05$)

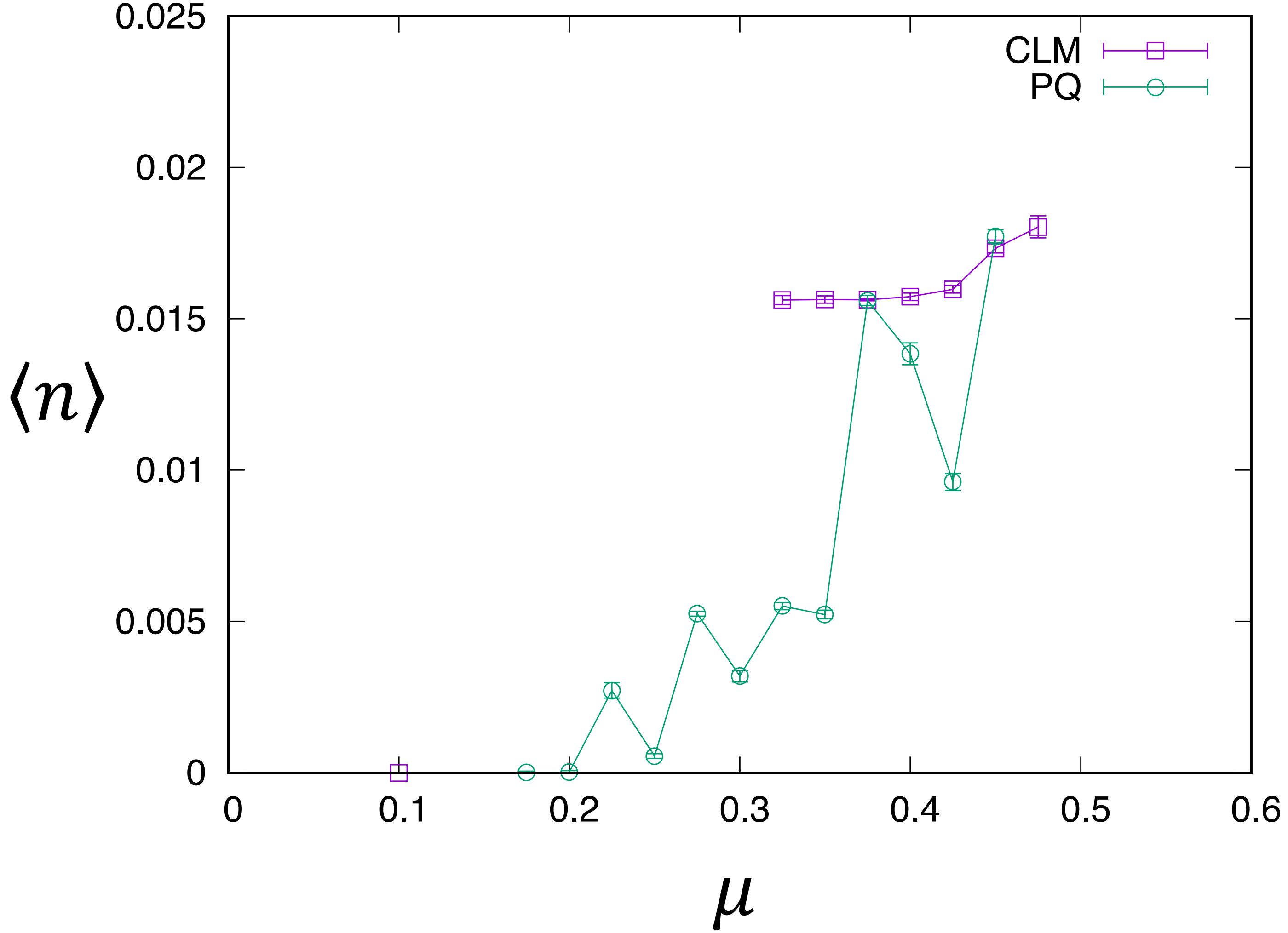


Not much difference between CLM and PQ results within the reliable region.

Baryon number density ($m_q a = 0.01$)



Comparison with phase quenched simulation ($m_q a = 0.01$)



$m_q a$=0.01 vs 0.05

For $\mu \lesssim 0.3$

PQ

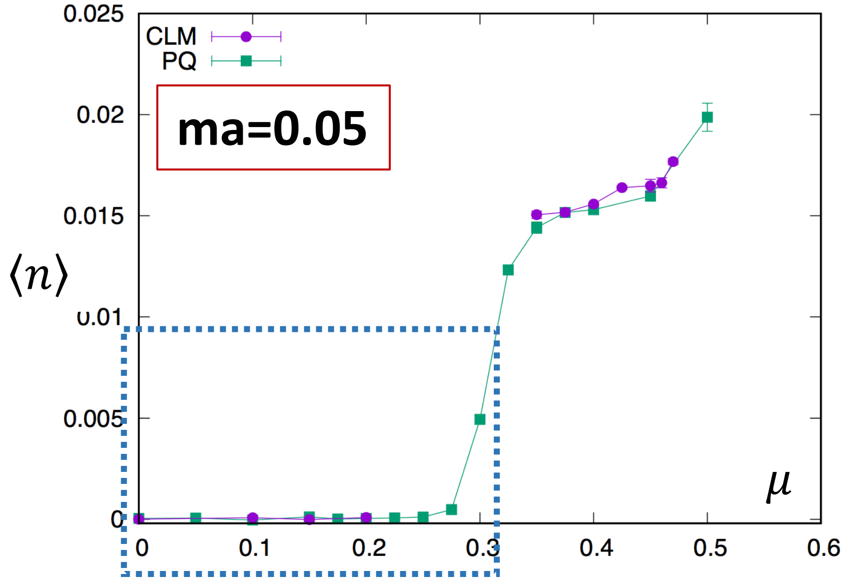
- <n> starts to grow earlier for $m_q a$=0.01 than $m_q a$=0.05.

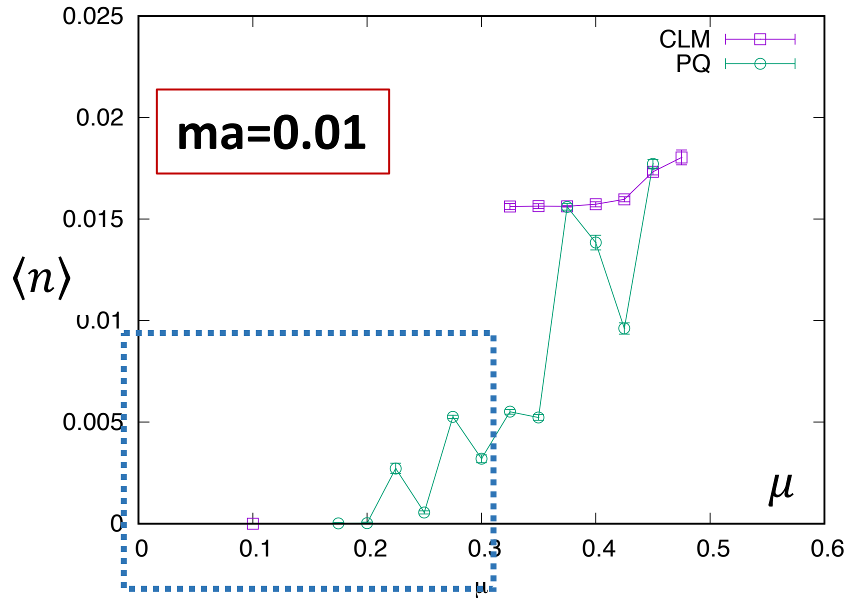
CLM

- The region of μ in which the CLM works depends on the quark mass.

  Singular drift problem becomes severer for small m_q .

➢ In the Silver-Blaze region, the CLM is not reliable.





$m_q a$=0.01 vs 0.05

For $\mathbf{0.3 \lesssim \mu \lesssim 0.45}$

PQ

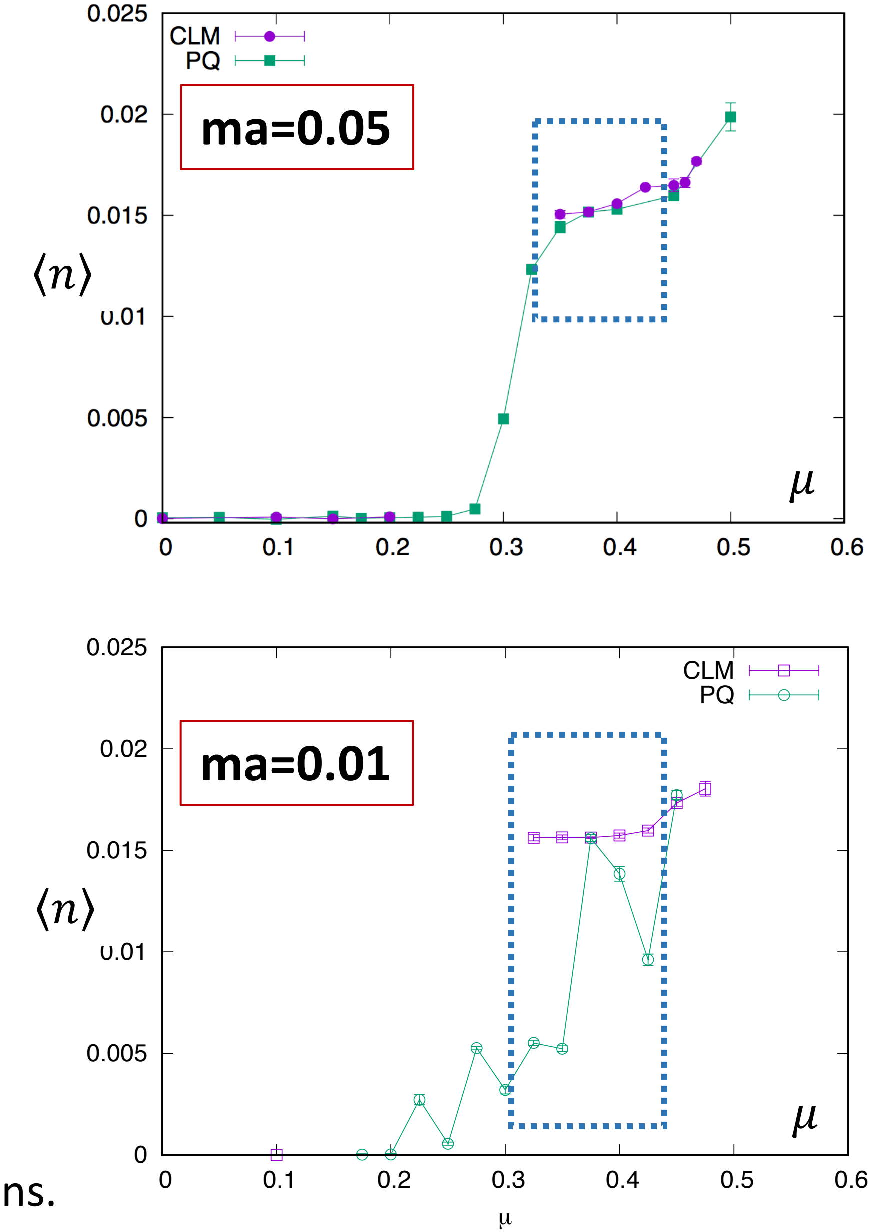
- <n> is sensitive to quark mass.
  → consistnt with the formation of pion condensate.

CLM

- Not sensitive to quark mass.
  → nucleon condensate (not pion)

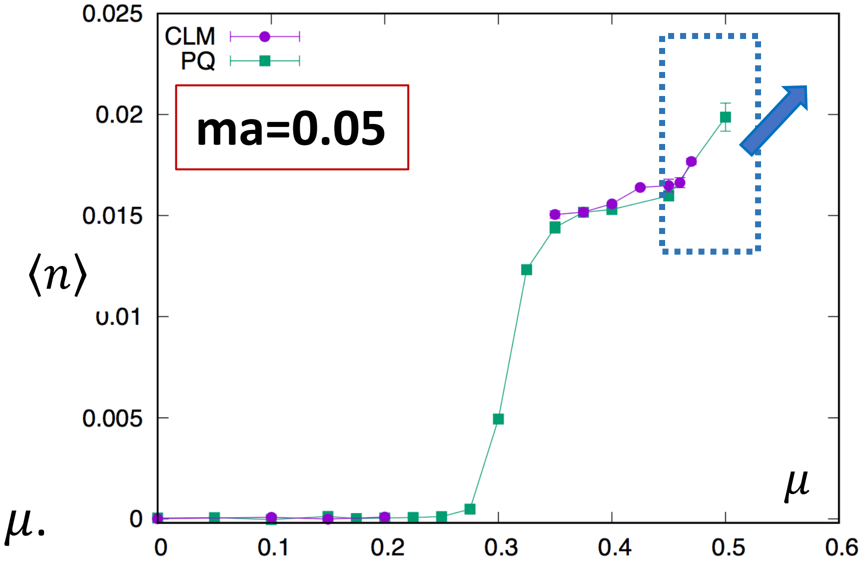
- The plateau region w.r.t. μ is observed.

→ The finite box is already filled with nucleons.



$m_q a$=0.01 vs 0.05

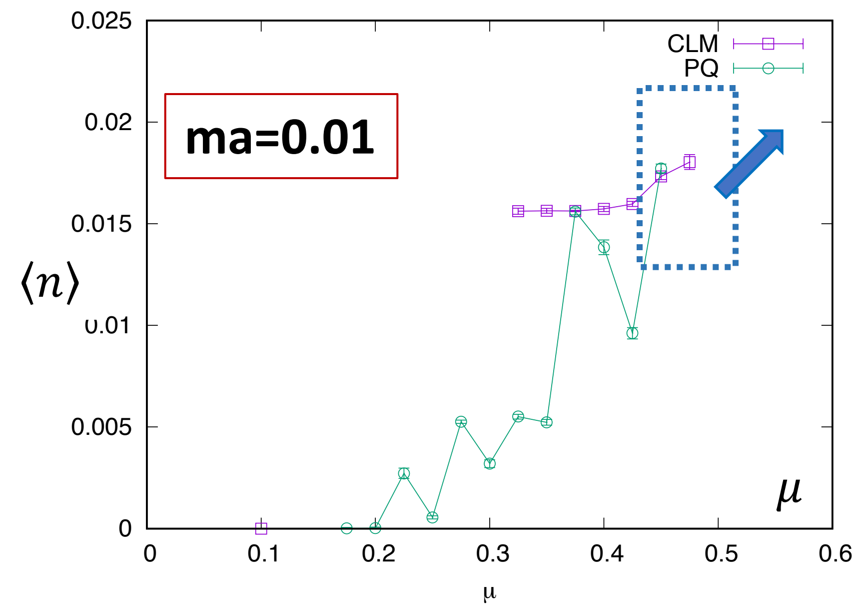
For $\mu \gtrsim 0.45$



CLM

- Singular drift problem occurs at large μ.
- <n> starts to grow again within the reliable region.

trends toward some transition?



To confirm this, we need to simulate with larger volume.

Summary

- Using the CLM, we have explored finite density QCD at low temperature with N_f = 4 staggered fermions.

- Even at the intermediate region of μ, the CLM works without the singular drift problem, where <n> has a plateau region suggesting nuclear matter phase.

- <n> starts to grow rapidly at the end of the plateau region, which may be a trend toward some transition.

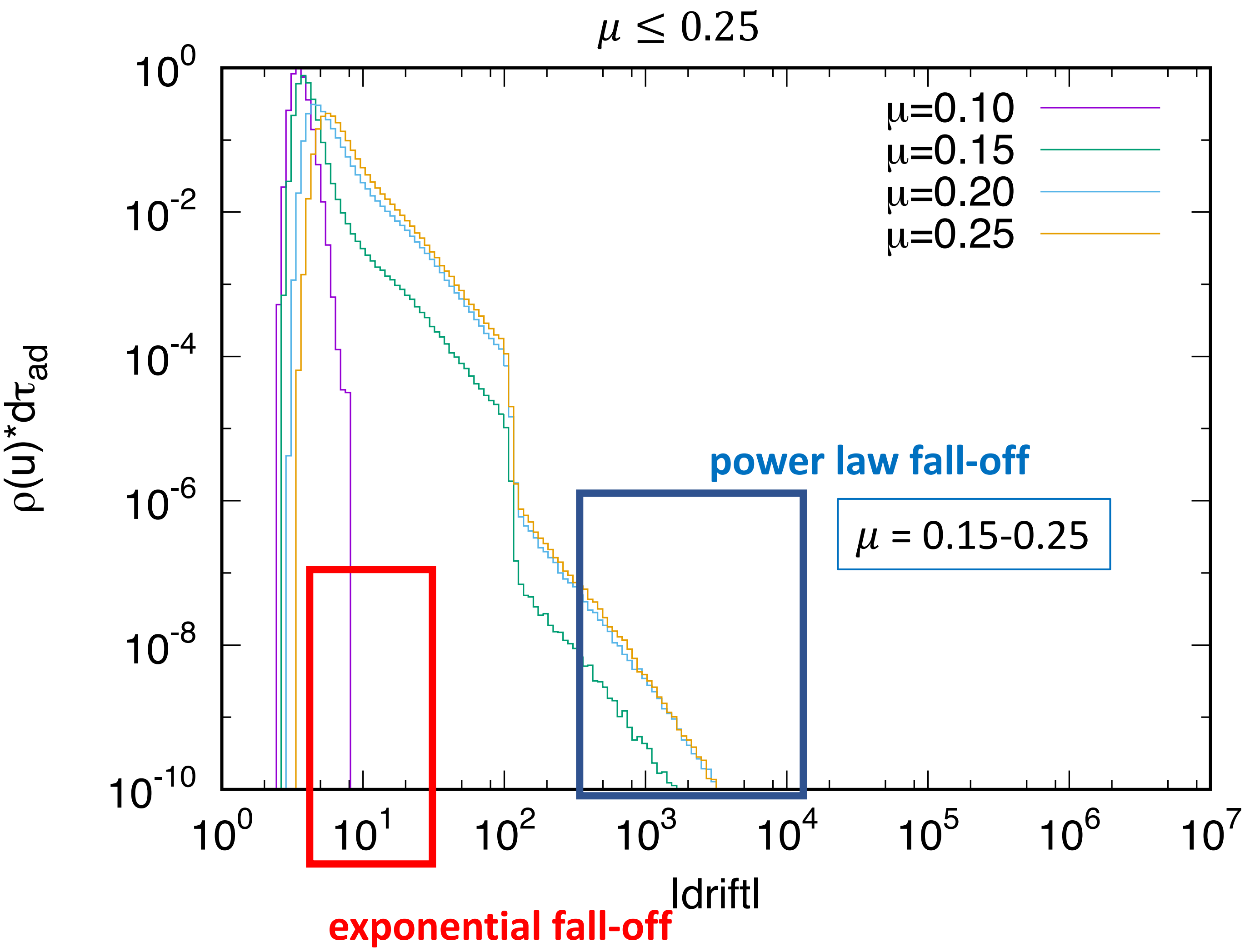
Future work

- measure the pion mass and the nucleon mass
  at the simulated parameter points,
  to understand better the critical behavior w.r.t. μ.

- Understand how the singular drift problem is avoided in the nuclear matter phase by measuring the eigenvalue distribution of the Dirac operator.

- Perform simulations with N_f = 2 Wilson fermions.
  (The CL simulatuon with N_f = 2 staggered fermions is problematic
  even at small μ due to the large unitarity norm. [Kogut, Sinclair '17]
  We have also confirmed this.)

Histogram of the drift ($m_q a = 0.01$)



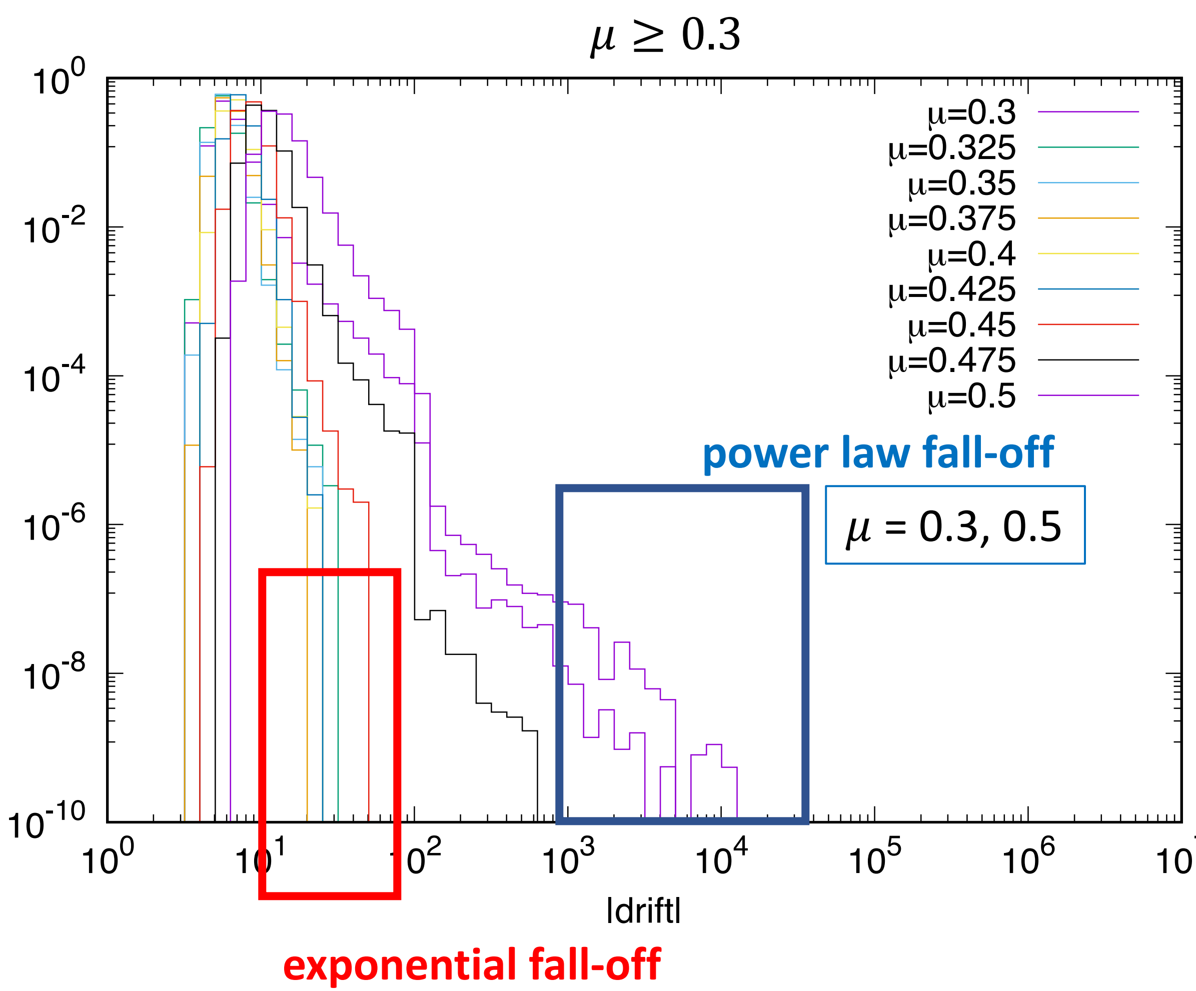
Reliable

$\mu = 0.1$

Unreliable

$\mu = 0.15\text{-}0.25$

Histogram of the drift ($m_q a = 0.01$)



reliable

μ = 0.325 - 0.45

unreliable

μ = 0.3, 0.5