Moments of pion distribution amplitude using OPE on lattice

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Outline

- Pion distribution amplitude and its moments
- Moments using lattice OPE with a valance heavy quark [1]
- Lattice correlators
- Exploratory numerical results
- Summary

[1] proposed by Detmold and Lin (Phys.Rev. D73 (2006)).

Pion LC wave function/distribution amplitude:

$$\langle 0|\overline{u}(\frac{z}{2})\ \gamma_5\gamma_\mu\ d(-\frac{z}{2})|\pi^+(p)\rangle = -ip_\mu f_\pi \int_0^1 d\xi\ e^{i(\overline{\xi}p\frac{z}{2}-\xi p\frac{z}{2})}\phi_\pi(\xi)$$

$$\overline{\xi} = 1-\xi$$

fraction ξ of pion momentum is carried by u quark.

(Mellin) Moments:

$$a_n = \int_0^1 d\xi\ \xi^n\ \phi_\pi(\xi).$$

OPE:

$$\begin{aligned}\langle 0|O^{\mu_1..\mu_n}|\pi^+(p)\rangle &= f_\pi\ a_{n-1}\ [p^{\mu_1}\dots p^{\mu_n} - \text{Traces}]\\ O^{\mu_1..\mu_n} &= \overline{\psi}\gamma^{\{\mu_1}\gamma^5(iD^{\mu_2})\dots(iD^{\mu_n\}})\psi - \text{Traces}\end{aligned}$$

In the isospin limit $m_u = m_d$:

$$\phi_\pi(\xi) = \phi_\pi(\bar{\xi})$$

$\Longrightarrow$ Odd moments vanish $\rightarrow$ lowest non-trivial moment is a_2.

Lattice calculation:

- second moment is calculated quite precisely (Bali et. al., 2017).
- going beyond is challenging because:
    - operator mixing (reduced symmetry: O(4) $\rightarrow$ H(4)).

Euclidean OPE with a valance heavy quark

Heavy-light axial current:

$$A^{\mu}_{\boldsymbol{\Psi},\psi} = \overline{\boldsymbol{\Psi}}\gamma^{\mu}\gamma^{5}\psi + \overline{\psi}\gamma^{\mu}\gamma^{5}\boldsymbol{\Psi}$$

ψ: light quarks, $\boldsymbol{\Psi}$: fictitious, relativistic, valance quark which is heavy.

- Simplify the lattice calculation:
  - no disconnected contribution.
  - easy to invert the quark matrix.

The relevant (Euclidean) hadronic tensor:

$$U_A^{\mu\nu}(q,p) = \int d^4x \ e^{iqx} \ \langle \pi^+(p) | T[A^{\mu}_{\Psi,\psi}(x) A^{\nu}_{\Psi,\psi}(0)] | 0 \rangle$$

OPE:

$$\mathrm{U}_{\mathrm{A}}^{[\mu\nu]}(\mathrm{p},\mathrm{q}) \quad \sim \quad 2\mathrm{i} \ \mathrm{f}_\pi \ \varepsilon_{\mu\nu\rho\lambda} \ \mathrm{q}^\rho \mathrm{p}^\lambda \sum_{\mathrm{n}=0,2,\cdots}^{\infty} \mathrm{a}_\mathrm{n} \ \frac{\xi^\mathrm{n}}{\mathrm{n}+1} \left[\frac{\mathrm{C}_\mathrm{n}^2(\eta)}{\tilde{\mathrm{Q}}^2} \right],$$

where $\xi = \frac{\sqrt{\mathrm{p}^2\mathrm{q}^2}}{\tilde{\mathrm{Q}}^2}$, $\eta = \frac{\mathrm{p.q}}{\sqrt{\mathrm{p}^2\mathrm{q}^2}}$ and $\tilde{Q}^2 = -q^2 - m_\Psi^2$.

For simplicity, Wilson coefficients are set to one.
a_n is the n^{th} Mellin moment of the pion DA:

$$\langle 0 | \overline{\psi} \gamma^{\{\mu_1} \gamma^5 (\mathrm{i}\mathrm{D}^{\mu_2}) \ldots (\mathrm{i}\mathrm{D}^{\mu_\mathrm{n}\}}) \psi - \mathrm{Traces} | \pi^+(\mathrm{p}) \rangle = \mathrm{f}_\pi \ \mathrm{a}_{\mathrm{n}-1} \ [\mathrm{p}^{\mu_1} \ldots \mathrm{p}^{\mu_\mathrm{n}} - \mathrm{Traces}].$$

$\mathrm{C}_\mathrm{n}^2(\eta)$'s are Gegenbauer polynomials.

- Identical result for the VV type correlator.

We compute the hadronic tensor in a unphysical regime:

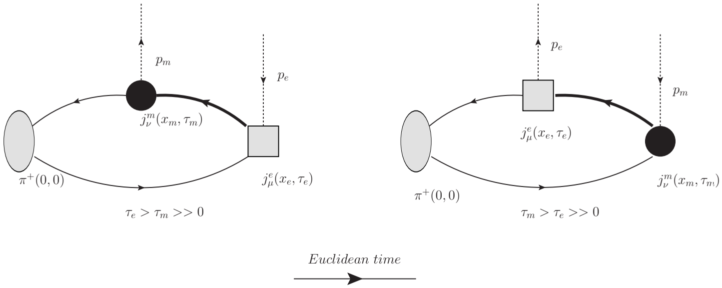
$$(p+q)^2 < (m_\Psi + \Lambda_{QCD})^2$$

The required scale hierarchy:

$$\Lambda_{QCD} << \sqrt{q^2} < m_{\boldsymbol{\Psi}} << \frac{1}{a}$$

$\Longrightarrow$ Fine lattices are required.

The correlators



$$
\begin{aligned}
C_3^{\mu\nu}(\tau_m,\tau_e;\vec{p}_m,\vec{p}_e) &= \sum_{\vec{x}_e}\sum_{\vec{x}_m} e^{i\vec{p}_e\cdot\vec{x}_e}\mathrm{e}^{-i\vec{p}_m\cdot\vec{x}_m}\langle 0|\mathrm{T}[j_e^{\mu}(\vec{x}_e,\tau_e)j_m^{\nu}(\vec{x}_m,\tau_m)\mathscr{O}_\pi^\dagger(\vec{0},0)]|0\rangle \\
C_\pi(\tau_\pi;\vec{p}_\pi) &= \sum_{\vec{x}} e^{i\vec{p}_\pi\cdot\vec{x}}\left\langle 0\left|\mathscr{O}_\pi(\vec{x},\tau)\mathscr{O}_\pi^\dagger(\vec{0},0)\right|0\right\rangle \\
&\xrightarrow{\tau_\pi\to\infty} \frac{\left|\left\langle \pi(\vec{p}_\pi)\left|\mathscr{O}_\pi^\dagger(\vec{0},0)\right|0\right\rangle\right|^2}{2E_\pi}\times\mathrm{e}^{-E_\pi\tau_\pi}.
\end{aligned}
$$

We can form the ratio

$$R_3^{\mu\nu}(\tau;\vec{q},\vec{p}) = \frac{C_3^{\mu\nu}(\tau_m,\tau_e;\vec{p}_e,\vec{p}_m)}{C_\pi(\tau_m;\vec{p})} \times \left\langle \pi(\vec{p}_\pi) \middle| \mathscr{O}_\pi^\dagger(\vec{0},0) \middle| 0 \right\rangle$$

$$\equiv \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}\cdot\vec{x}} \left\langle 0 \middle| T[j_e^\mu(\vec{x},\tau) j_m^\nu(\vec{0},0)] \middle| \pi(\vec{p}_\pi) \right\rangle_{\tau=\tau_e-\tau_m;\vec{q}=\vec{p}_e;\vec{p}=\vec{p}_e-\vec{p}_m}$$

Perform the temporal Fourier transform:

$$\int \mathrm{d}\tau\ \mathrm{e}^{iq_4\tau}\ R_3^{[\mu\nu]}(\tau;\vec{q},\vec{p}) = U_{A/V}^{[\mu\nu]}(q,p)$$

- Analytic continuation: $p_4 \to ip_0$, $q_4 \to iq_0$.

$$U_{A/V}^{[\mu\nu]}(q,p)|_M = iU_{A/V}^{[\mu\nu]}(q,p) = i\int \mathrm{d}\tau\ \mathrm{e}^{-q_0\tau}\ R_3^{[\mu\nu]}(\tau;\vec{q},\vec{p})$$

$\to R_3^{[\mu\nu]}$ is purely imaginary.

- The range $[\tau_{min},\tau_{max}]$ for the temporal fourier transform is constrained by the regime where we can isolate the one-pion state.

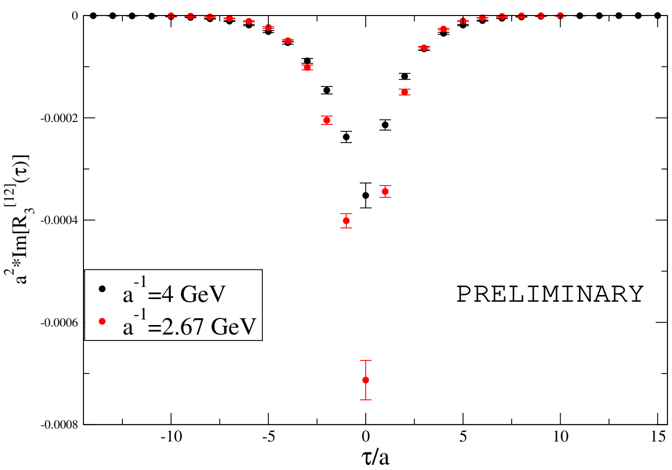
$$U_{A/V}^{[\mu\nu]}(q,p)|_M = i\int_{\tau_{min}}^{\tau_{max}} \mathrm{d}\tau\ \mathrm{e}^{iq_4\tau}\ R_3^{[\mu\nu]}(\tau;\vec{q},\vec{p})$$

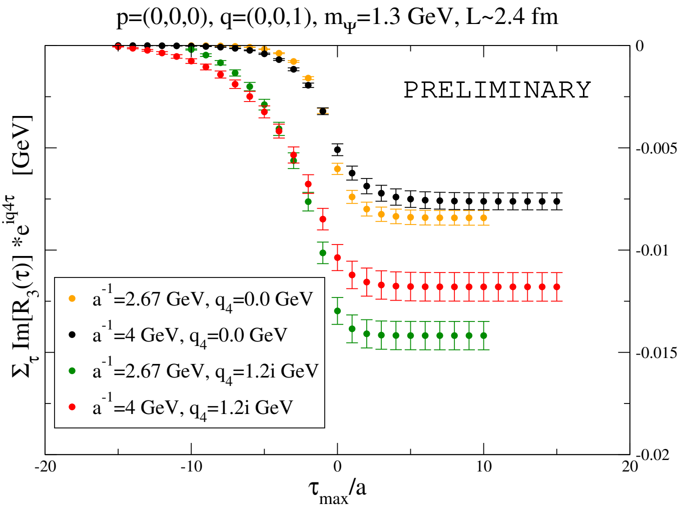
Exploratory numerical results

- Quenched simulation:
  - Pure Wilson gauge action for configuration generation.
  - Non-perturbatively O(a)-improved clover action for valance quarks.
- $m_\pi = 450$ MeV.
- Two values of the lattice spacing: $\mathrm{a}^{-1} = 2.67$ and 4 GeV
- box size: $\mathrm{L} = 2.4$ fm, $\mathrm{T} = 4.8$ fm.
- Two choices of the heavy quark mass, m_{Ψ} =1.3 and 2 GeV.

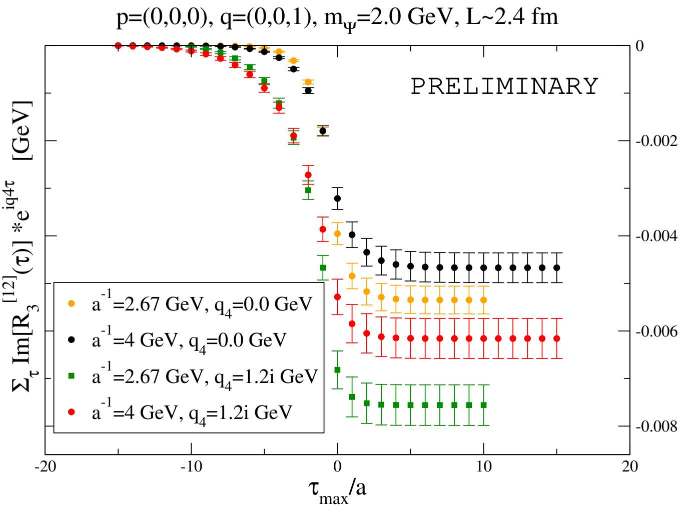
Ensembles available at $\mathrm{L} = 32, 40, 48, 64, 96$ with same physical box size and cut-off scale as high as 8 GeV.

$p=(0,0,0)$, $q=(0,0,1)$, $m_\Psi=1.3$ GeV, $L\sim 2.4$ fm



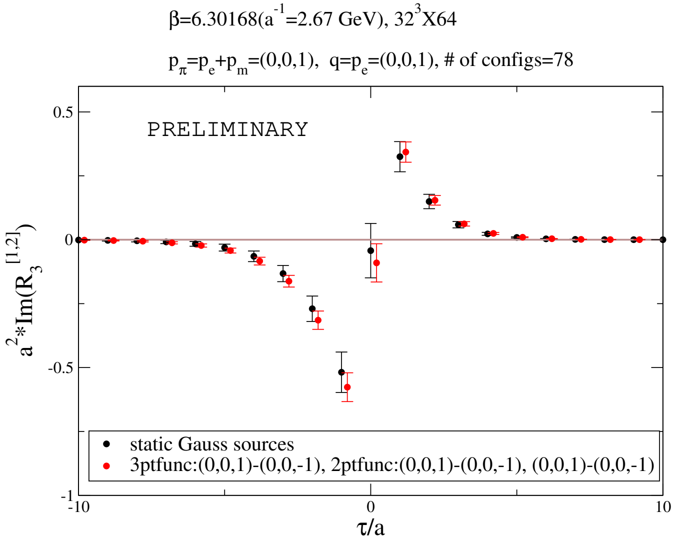


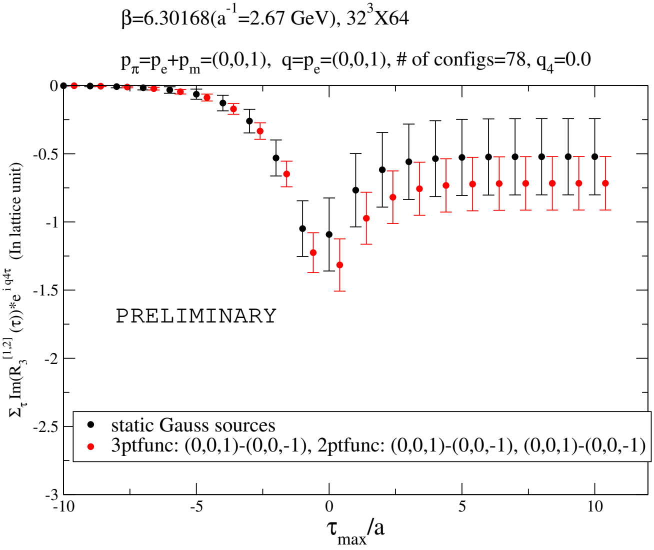
p=(0,0,0), q=(0,0,1), m$_\Psi$=1.3 GeV, L~2.4 fm
PRELIMINARY
Σ_τ Im[R$_3$(τ)] *e$^{iq4\tau}$ [GeV]
τ_{max}/a
a^{-1}=2.67 GeV, q$_4$=0.0 GeV
a^{-1}=4 GeV, q$_4$=0.0 GeV
a^{-1}=2.67 GeV, q$_4$=1.2i GeV
a^{-1}=4 GeV, q$_4$=1.2i GeV
-20
-10
0
10
20
0
-0.005
-0.01
-0.015
-0.02



p=(0,0,0), q=(0,0,1), m_Ψ=2.0 GeV, L~2.4 fm
PRELIMINARY
Σ_τ Im[R_3^[12](τ)] *e^iq4τ [GeV]
τ_max/a
a^-1=2.67 GeV, q_4=0.0 GeV
a^-1=4 GeV, q_4=0.0 GeV
a^-1=2.67 GeV, q_4=1.2i GeV
a^-1=4 GeV, q_4=1.2i GeV
0
-0.002
-0.004
-0.006
-0.008
-20
-10
0
10
20

Momentum smearing (G. Bali et. al., 2016.)





β=6.30168(a^-1=2.67 GeV), 32^3X64
p_π=p_e+p_m=(0,0,1), q=p_e=(0,0,1), # of configs=78, q_4=0.0
Σ_τ Im(R_3^[1,2](τ))*e^(i q4τ) (In lattice unit)
τ_max/a
PRELIMINARY
static Gauss sources
3ptfunc: (0,0,1)-(0,0,-1), 2ptfunc: (0,0,1)-(0,0,-1), (0,0,1)-(0,0,-1)
0
-0.5
-1
-1.5
-2
-2.5
-3
-10
-5
0
5
10

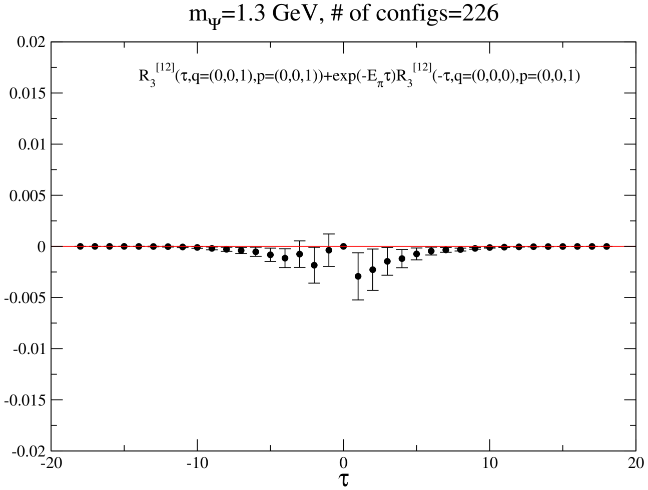
Summary

- The method: study of current-current matrix elements and OPE on the lattice.
- A fictitious, valance heavy quark to make lattice calculation simpler, to give more flexibility, for easy analytic continuantion, ..
- In this talk, the results are shown for two values of the lattice spacing. The lattice artifacts look under control, although they can be non-negligible.
- The method requires the hadronic tensor in the $a \to 0$ limit to be matched with the OPE formula. The data at more values of the lattice spacings will be analyzed in the near future in order to perform continuum extrapolation of the hadronic tensor reliably.
- The approach described here looks promising, and has the potential to make important contributions to the study of hadron structure.

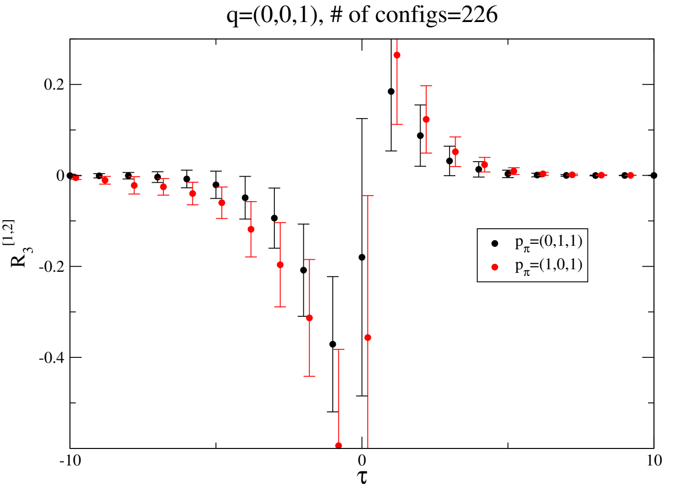
Thanks for your attention!

Translation invariance:

$$R_3^{[\mu\nu]}(\tau, q, p) + e^{-E_\pi(p)\tau} R_3^{[\mu\nu]}(\tau, p-q, p) = 0$$



$m_\Psi \sim 2$ GeV $U_A^{[12]} = f_\pi \, \frac{q^3p^4 - p^3q^4}{2} \, \sum_{n=0,\ even}^{\infty} a_n \, f_n(q^2, p.q, m_\Psi)$



All have same $p^3q^4 - q^3p^4$, p.q and q^2.