Radiative corrections to decay amplitudes in lattice QCD

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This talk is based on ongoing work and the following papers:

1. *QED Corrections to Hadronic Processes in Lattice QCD*,
   N.Carrasco, V.Lubicz, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino and M.Testa,
   Phys. Rev. D **91** (2015) no.7, 07450 [arXiv:1502.00257 [hep-lat]].
2. *Finite-Volume QED Corrections to Decay Amplitudes in Lattice QCD*,
   V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula and N.Tantalo,
   Phys. Rev. D **95** (2017) no.3, 034504 [arXiv:1611.08497 [hep-lat]].
3. *First Lattice Calculation of the QED Corrections to Leptonic Decay Rates*,
   D.Giusti, V.Lubicz, G.Martinelli, C.T.Sachrajda, F.Sanfilippo, S.Simula, N.Tantalo and C.Tarantino,
   Phys. Rev. Lett. **120** (2018) 072001 [arXiv:1711:06537]

1. Introduction
2. What is QCD in QCD+QED?
   - Material for discussion by the community
3. Lattice calculations of leptonic decay amplitudes
   - First numerical results
4. Radiative corrections to semileptonic decay amplitudes
5. Summary and conclusions

- Once electromagnetic corrections are included, what is meant by QCD becomes convention dependent.
- The action can be written schematically in the form:

$$S^{\text{full}} = \frac{1}{g_s^2} S^{\text{YM}} + \sum_f \left\{ S_f^{\text{kin}} + m_f S_f^{\text{m}} \right\} + S^A + \sum_\ell \left\{ S_\ell^{\text{kin}} + m_\ell S_\ell^{\text{m}} \right\} .$$

How should we choose the bare quark masses (m_f) and strong coupling (g_s)?

- Without QED, in the 4-flavour theory, for each value of g_s we can e.g., choose the four *physical* bare quark masses $(m_u^0, m_d^0, m_s^0, m_c^0)$ to be those for which the 4 dimensionless ratios:

$$\frac{a_0 m_{\pi^0}}{a_0 m_\Omega}, \quad \frac{a_0 m_{K^0}}{a_0 m_\Omega}, \quad \frac{a_0 m_{K^+}}{a_0 m_\Omega} \quad \text{and} \quad \frac{a_0 m_{D^0}}{a_0 m_\Omega},$$

take their physical values.
  - Dimensional transmutation $\Rightarrow$ define the lattice spacing by imposing, e.g., that

$$a_0 = \frac{a_0 m_\Omega}{m_\Omega^{\text{phys}}} .$$

- QED corrections however, shift the hadronic masses by $O(\alpha) m_H \Rightarrow$ some choice of convention is necessary if we wish to define the QCD and QED contributions separately.

- In the hadronic scheme (which we advocate) we impose the same conditions as in pure QCD and add mass counterterms $m_f = m_f^0 + \delta m_f$ and $a = a_0 + \delta a$.
- For a general observable O, of mass dimension 1 say,

$$O^{\text{phys}} = \frac{\langle aO\rangle^{\text{full}}}{a} = \frac{\langle a_0 O\rangle^{\text{QCD}}}{a_0} + \frac{\delta O}{a_0} - \frac{\delta a}{a_0^2}\langle a_0 O\rangle^{\text{QCD}} + O(\alpha^2)\,.$$

  where δO is the contribution from the electromagnetic corrections and mass counterterms.

- The first term on the right-hand side is one that can be calculated within QCD alone. It has a well defined continuum limit as does the sum.
  - This allows us to answer the question: What is the difference between QCD (defined as above) and the full theory.
  - At no point in the calculation do we have to take a numerical difference between calculations performed in the full theory and in QCD. We calculate the IB terms directly.
  - If ever needed, the scheme can be extended to higher orders in α.

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J.Gasser, A.Rusetsky and I.Scimemi, hep-ph/0305260

- Of course, other schemes are possible e.g. defined by requiring that

$$g(\mu) = Z_g(0,g_0,\mu)\,g_0 = Z_g(e,g_s,\mu)\,g_s$$
$$m_f(\mu) = Z_{m_f}(0,g_0,\mu)\,m_{f,0}(g_0) = Z_{m_f}(e,g_s,\mu)\,m_f(e,g_s)\,,$$

  i.e. that the renormalised coupling and masses are equal in some scheme and at some renormalisation scale.
  - FLAG has adopted this with the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV.
  - The four dimensionless ratios R_i ($i = 1$-4)

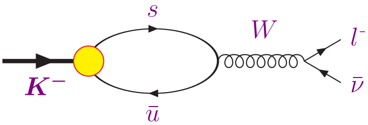
$$\frac{a_0 m_{\pi^0}}{a_0 m_\Omega},\ \frac{a_0 m_{K^0}}{a_0 m_\Omega},\ \frac{a_0 m_{K^+}}{a_0 m_\Omega}\quad \text{and}\quad \frac{a_0 m_{D^0}}{a_0 m_\Omega},$$

    no longer take their physical values and we can write:

$$R_i = R_i^{\mathrm{phys}}(1+\varepsilon_i)\,,$$

    with much discussion as to what the ε_i are.
- Before precise non-perturbative calculations of hadronic masses were possible, schemes such as GRS, based on setting conditions at perturbative scales, were natural.
  - However, we suggest that hadronic schemes are now more natural and should be used instead.

Consider as an example $K_{\ell 2}$ decays in pure QCD:

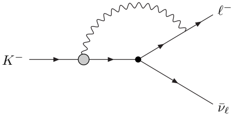


- In pure QCD

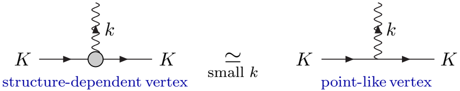
$$\Gamma(K^- \to \ell^- \bar{\nu}_\ell) = \frac{G_F^2\,|V_{us}|^2 f_K^2}{8\pi}\, m_K\, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2 ,$$

where the leptonic decay constant f_K contains all the QCD effects ($\langle 0|\,\bar{s}\gamma^\mu\gamma^5 u\,|K(p)\rangle = if_K\, p^\mu$).

- The experimental value of Γ and the lattice computation of $f_K \Rightarrow |V_{us}|$.
  - It is V_{us} which we primarily wish to determine as precisely as possible.
- Beyond $\sim$ 1% precision, radiative corrections must be included $\Rightarrow$ presence of infrared divergences.
  - f_K no longer contains all the QCD effects.



- The observable we calculate is $\Gamma_0(K \to \ell\bar{\nu}_\ell) + \Gamma_1(K \to \ell\bar{\nu}_\ell\gamma)$ where (in the kaon rest-frame) $E_\gamma < \Delta E$ and ΔE is sufficiently small for the structure dependence of K to be neglected ($\Delta E \lesssim 20$ MeV).

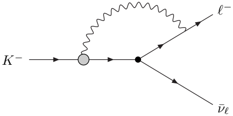


- We now write

$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))\,.$$

where pt stands for *point-like*.

  - The second term on the rhs can be calculated in perturbation theory. It is infrared convergent, but does contain a term proportional to $\log \Delta E$.
  - The first term is also free of infrared divergences.
  - Γ_0 is calculated non-perturbatively and $\Gamma_0^{\rm pt}$ in perturbation theory.



$$\Gamma_0 + \Gamma_1(\Delta E) = \lim_{V\to\infty} (\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty} (\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))\,.$$

- Finite-volume effects take the form:

$$\Gamma_0^{\rm pt}(L) = C_0(r_\ell) + \tilde{C}_0(r_\ell)\log(m_\pi L) + \frac{C_1(r_\ell)}{m_\pi L} + \ldots\,,$$

  where $r_\ell = m_\ell/m_\pi$ and m_ℓ is the mass of the final-state charged lepton.

  The exhibited L-dependent terms are *universal*, i.e. independent of the structure of the meson!

  - We have calculated the coefficients (using the $\mathrm{QED_L}$ regulator of the zero mode).

- The leading structure-dependent FV effects in $\Gamma_0 - \Gamma_0^{\rm pt}$ are of $O(1/L^2)$.

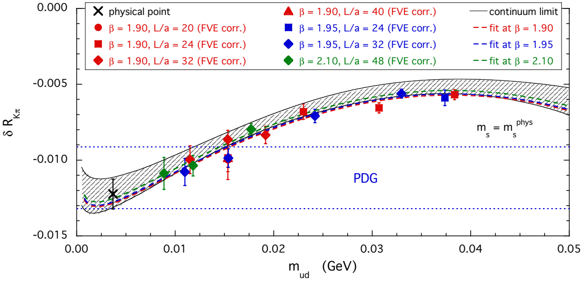
V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula, N.Tantalo, arXiv:1611.08497

- Writing

$$\frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} = \left| \frac{V_{us}}{V_{ud}} \frac{f_K^{(0)}}{f_\pi^{(0)}} \right|^2 \frac{m_\pi^3}{m_K^3} \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \right)^2 (1 + \delta R_{K\pi})$$

where $m_{K,\pi}$ are the physical masses, using numerous twisted mass ensembles we find

$$\delta R_{K\pi} = -0.0122(16)\,.$$ D.Giusti et al., arXiv:1711.06537



- After subtracting the universal FV effects, the ansatz for the combined chiral, continuum and infinite-volume extrapolations is given in (13) of arXiv:1711.06537.

- Writing

$$\frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} = \left| \frac{V_{us}}{V_{ud}} \frac{f_K^{(0)}}{f_\pi^{(0)}} \right|^2 \frac{m_\pi^3}{m_K^3} \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \right)^2 (1 + \delta R_{K\pi})$$

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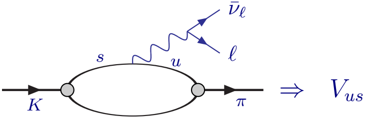
D.Giusti et al., arXiv:1711.06537

- $f_P^{(0)}$ are the decay constants obtained in iso-symmetric QCD with the renormalized $\overline{\mathrm{MS}}$ masses and coupling equal to those in the full QCD+QED theory extrapolated to infinite volume and to the continuum limit.
- This first calculation can certainly be improved.
  - In particular the renormalization into W-regularization has been performed only at $O(\alpha)$. In addition to NPR (in progress), determining the $O(\alpha\alpha_s)$ corrections requires a two-loop perturbative calculation.
- This result can be compared to the PDG value, based on ChPT, is $\delta R_{K\pi} = -0.0112(21)$. V.Cirigliano and H.Neufeld, arXiv:1102.0563
- Our result, together with $V_{ud} = 0.97417(21)$ from super-allowed nuclear β-decays gives $V_{us} = 0.22544(58)$ and

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99985(49)\,.$$

- We are now expanding this framework to semileptonic decays, such as $\bar{K}^0 \to \pi^+ \ell \bar{\nu}_\ell$, where several new features arise.
- Without QED the amplitude depends on two form factors $f_\pm(q^2)$, where q is the momentum transfer between the $\bar{K}^0$ and the π^+; $q = p_K - p_\pi = p_\ell + p_\nu$.



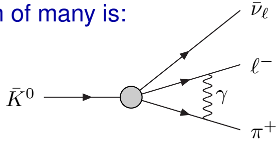
$$\begin{aligned} \langle \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\,\rangle &= f_0(q^2)\,\frac{M_K^2-M_\pi^2}{q^2}q_\mu + f_+(q^2)\left[(p_\pi+p_K)_\mu - \frac{M_K^2-M_\pi^2}{q^2}q_\mu\right] \\ &= f_+(q^2)\,(p_K+p_\pi)_\mu + f_-(q^2)\,(p_K-p_\pi)_\mu \end{aligned}$$

- The natural observable is $d^2\Gamma/dq^2 ds_{\pi\ell}$, where $s_{\pi\ell} = (p_\pi + p_\ell)^2$. Without QED:

$$\frac{d^2\Gamma}{dq^2 ds_{\pi_\ell}} \propto a_+(q^2,s_{\pi\ell})\left|f_+(q^2)\right|^2 + a_0(q^2,s_{\pi\ell})\left|f_0(q^2)\right|^2 + a_{0+}(q^2,s_{\pi\ell})f_0(q^2)f_+(q^2)\,,$$

where the coefficients $a_{+,0,0+}$ are readily determined.

- For illustration, one diagram of many is:



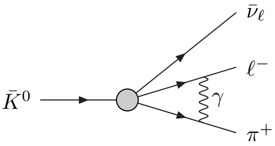
- Following the same procedure as for leptonic decays we write:

$$\frac{d^2\Gamma}{dq^2 ds_{\pi\ell}} = \lim_{V\to\infty}\left(\frac{d^2\Gamma_0}{dq^2 ds_{\pi\ell}} - \frac{d^2\Gamma_0^{\rm pt}}{dq^2 ds_{\pi\ell}}\right) + \lim_{V\to\infty}\left(\frac{d^2\Gamma_0^{\rm pt}}{dq^2 ds_{\pi\ell}} + \frac{d^2\Gamma_1(\Delta E)}{dq^2 ds_{\pi\ell}}\right)$$

- Infrared divergences cancel separately in each of the two terms.
- The second term has been calculated in infinite volume in the eikonal approximation: $(p-k)^2 - m^2 \to -2p\cdot k$.

G.Isidori, arXiv:0709.2439; S.de Boer, T.Kitahara & I.Nišandžic, arXiv:1803.05881

- The $1/L$ corrections depend on $df_\pm/dq^2$ (which however are physical quantities) as well as on the form factors.
  - However, these corrections do not depend on the derivative w.r.t. the masses, which are not physical.
  - The calculation of the $1/L$ corrections is still to be performed.
  - This is likely to require going beyond the eikonal approximation.



- Depending on the volume and $s_{\pi\ell}$, in general there are unphysical contributions from lighter intermediate $\pi\ell(\gamma)$ states, which grow exponentially with the temporal integration region, which must be subtracted.
  - This is a general feature in the calculation of long distance effects.
- For semileptonic decays of heavy mesons however, for much of phase space there are too many lighter intermediate states to handle.
  - This is analogous to the fact that e.g. $B \to \pi\pi$ and $B \to \pi K$ decays amplitudes cannot be calculated whereas $K \to \pi\pi$ amplitudes can.
- We also need to study the FV corrections due to the electromagnetic rescattering.
  - Are the FV corrections $<$ the universal ones?

- We propose that when calculating electromagnetic corrections a hadronic scheme should be used to define what is meant by QCD.
  - The ε_i are naturally obtained in any case.
- For leptonic decays of light mesons the framework is complete and has been shown to be practicable. Corrections are of $O(1\%)$ as expected.
- The priority for improvement is the renormalization. We have:

A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ij}^{\text{CKM}} \left(1 + \frac{\alpha}{\pi} \log \frac{M_Z}{M_W}\right) O_1^{\text{W-reg}}\,.$$

For Wilson & tm fermions:

$$O_1^{\text{W-reg}} = \sum_{i=1}^{5} Z_{1i} O_i^{\text{latt}}(a)$$

and the Z_{1i} are only known to $O(\alpha)$.

- The precision of the calculations would be improved significantly if we knew the $O(\alpha_s\alpha)$ corrections.

- For heavy mesons, heavy-quark (spin) symmetry $\Rightarrow$ vector and pseudoscalar mesons are almost degenerate:

$$m_{D^*} - m_D = 140.603 \pm 0.015\,\text{MeV} \qquad m_{B^*} - m_B = 45.34 \pm 0.23\,\text{MeV}\,.$$

  - Can a suitable ΔE be imposed on the energy of the real photon, or will the structure dependent terms need to be computed?
  - This is both a theoretical and experimental question.
  - Lattice calculation of the real emission diagrams?

- The disconnected diagrams need to be evaluated.

- For semileptonic decays the development of the corresponding framework is well underway. The cancellation of infrared divergences is under control and the structure of the finite-volume corrections is understood.
- There remain a number of significant technical challenges including:
  - i) The explicit evaluation of the $O(1/L)$ finite-volume corrections. We have determined the summands/integrands, but need to evaluate the difference between the sums and integrals.
  - ii) Renormalization of the four-fermion operator is also needed here.
  - iii) An investigation of the subtraction of the unphysical (exponentially growing in time) contributions in an actual computation.
  - iv) A better phenomenological understanding of how much of phase-space is needed to obtain precise determinations of the CKM matrix elements. Will we be able to impose useful cuts on q^2 and $s_{\pi\ell}$ (and corresponding variables for decays of heavy mesons)?
- Summary: We are successfully developing and implementing a framework for the *ab initio* calculation of radiative corrections to leptonic and semileptonic decays.
  - Such a framework is necessary if we are to determine the CKM matrix elements to a precision of better that 1% or so.
- In the following talk, James Richings will describe RBC-UKQCD preparations for including QED in decay amplitudes.