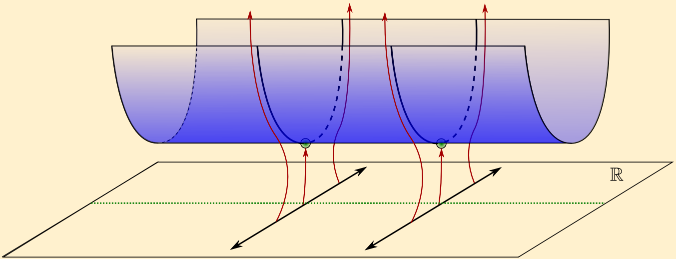
Flowing Gauge Theories: Finite-Density QED_{1+1}

Henry Lamm

w/ Andrei Alexandru, Gökçe Başar, Paulo Bedaque, and Scott Lawrence

1807.02027



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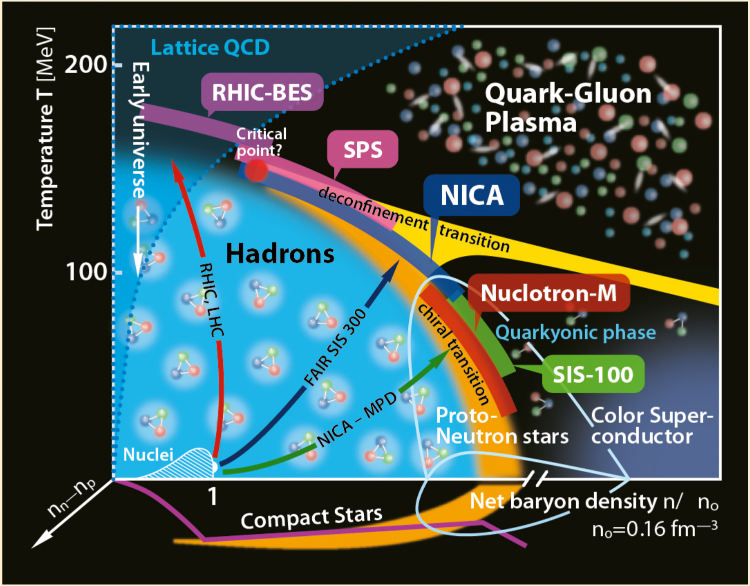
Many interesting problems in QFT exist at $\mu \neq 0$



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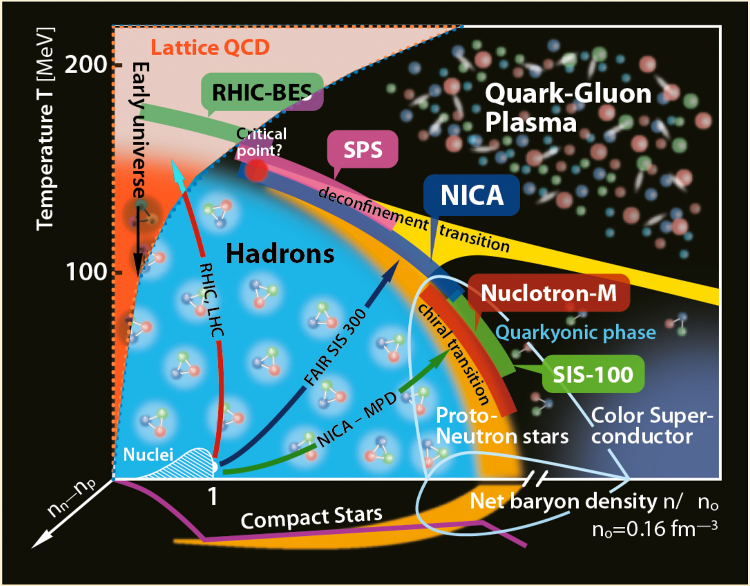

...but analytically, we know ways to deal[1]

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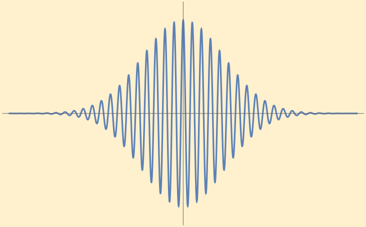

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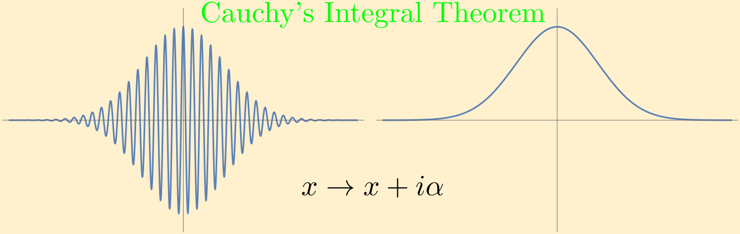

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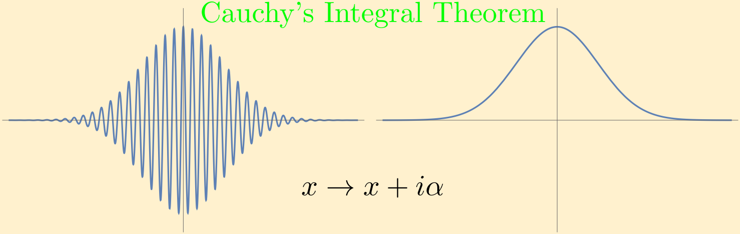

- CIT guarantees holomorphic $f(x)$ (*physics*) unchanged
- **Nonholomorphic** $f(x)$, like the average sign, $\langle\sigma\rangle$, can change!

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Lefschetz thimbles have seemingly optimal properties

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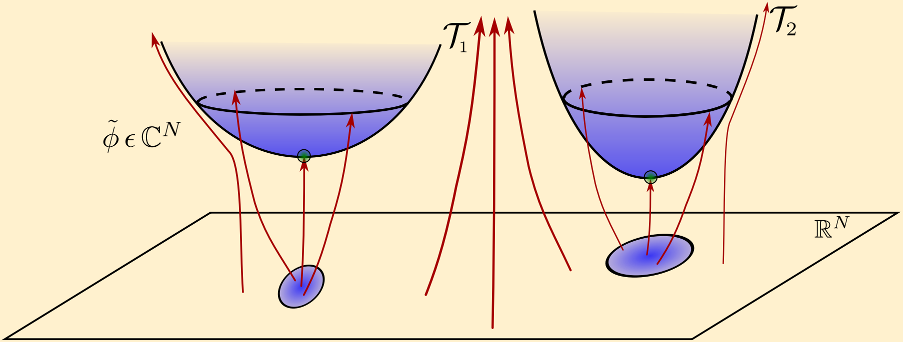

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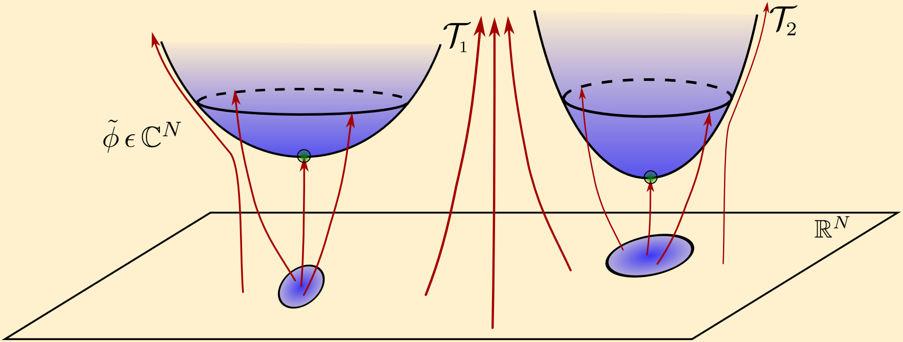

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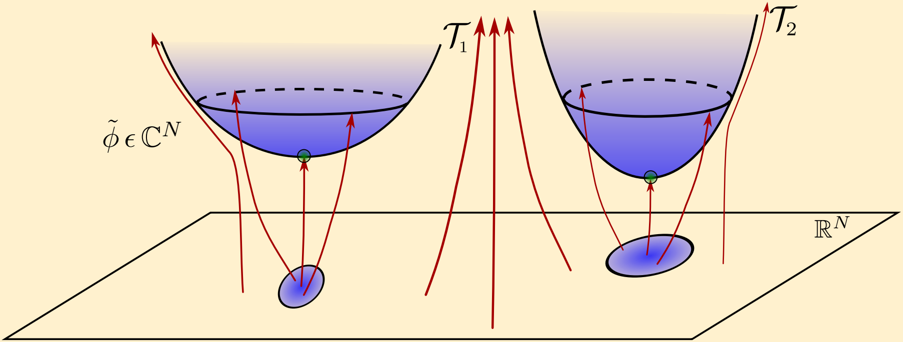

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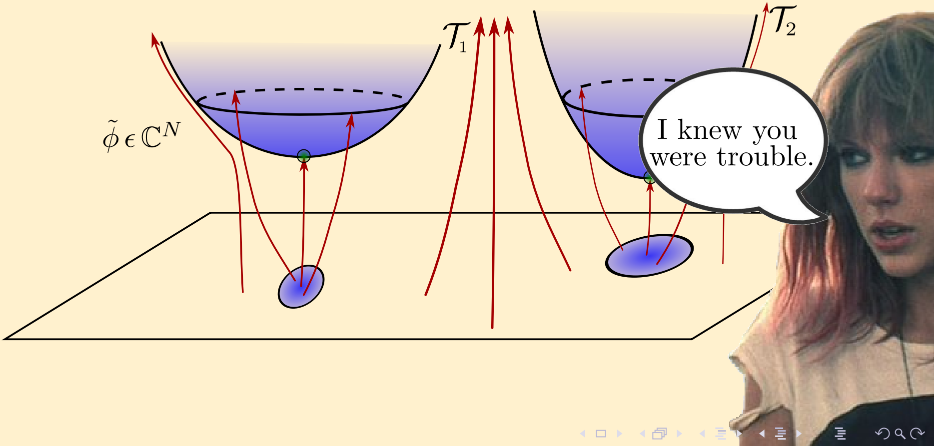

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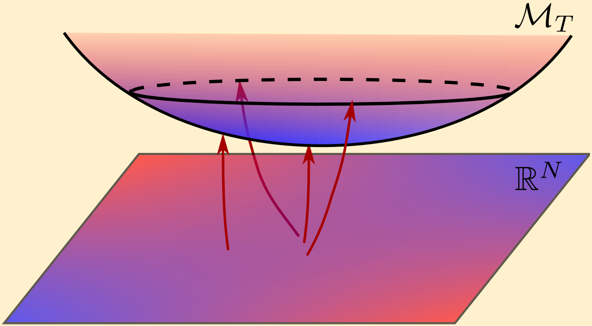

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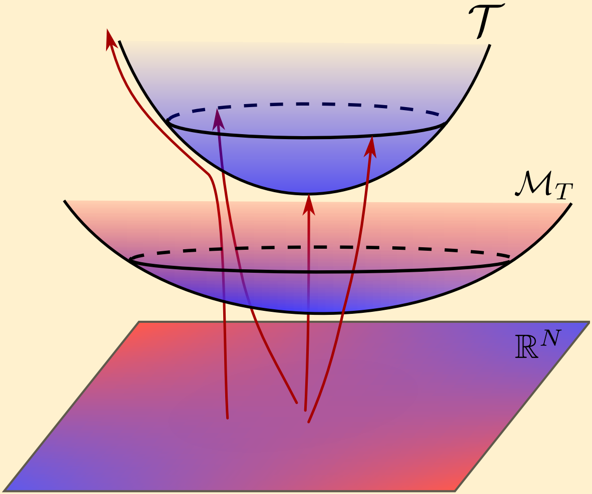

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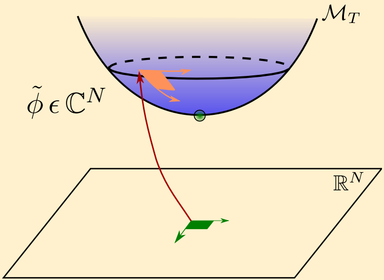

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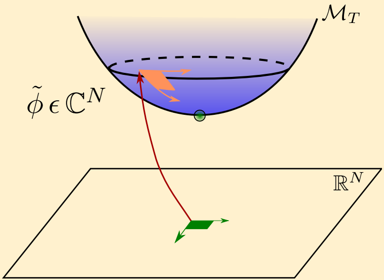

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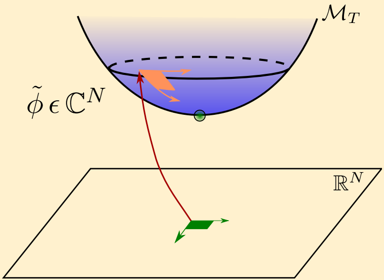

Prohibitive reweighting from $W - J$ from **exceptional** configurations when T is large

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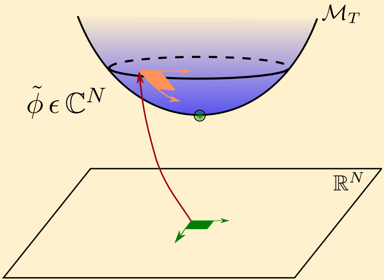

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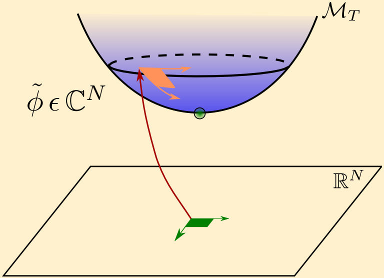

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These are technical complications...

...but gauge theories have conceptual issues

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QED_{1+1} with $N_f = 3$ staggered fermions

In the continuum:

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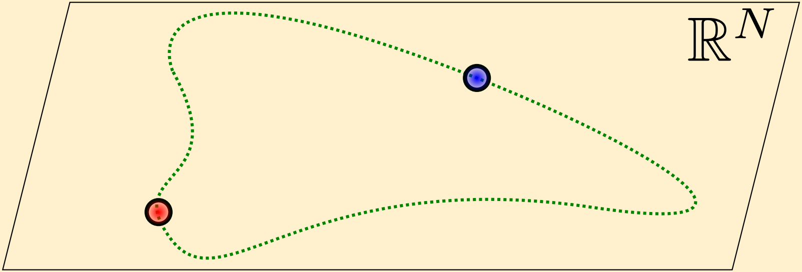

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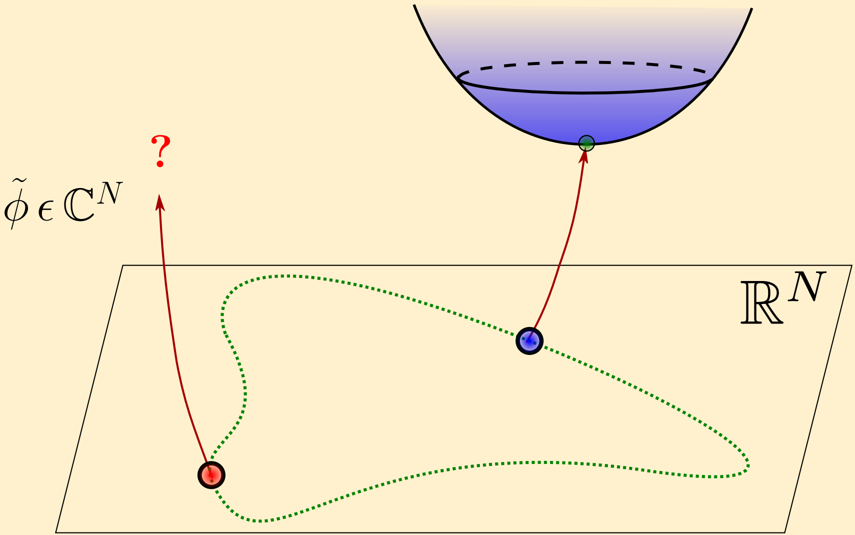

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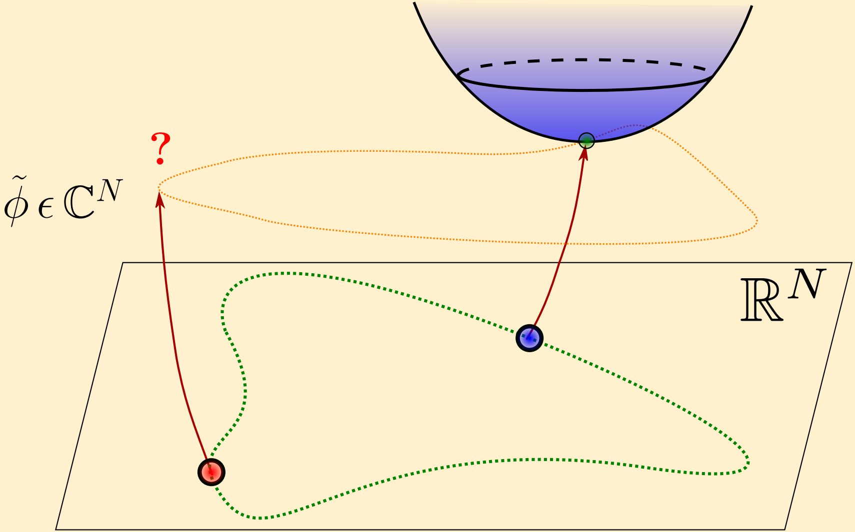

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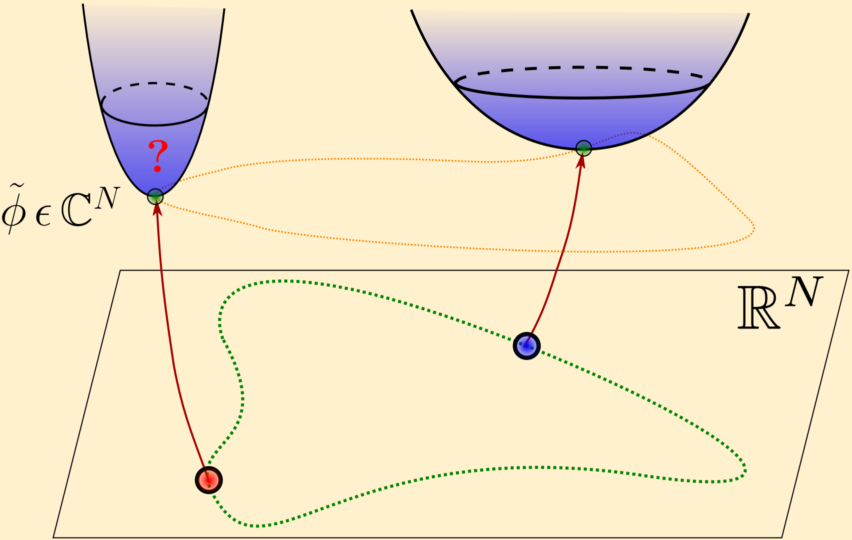

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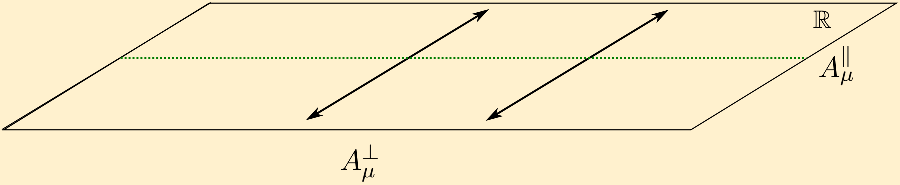

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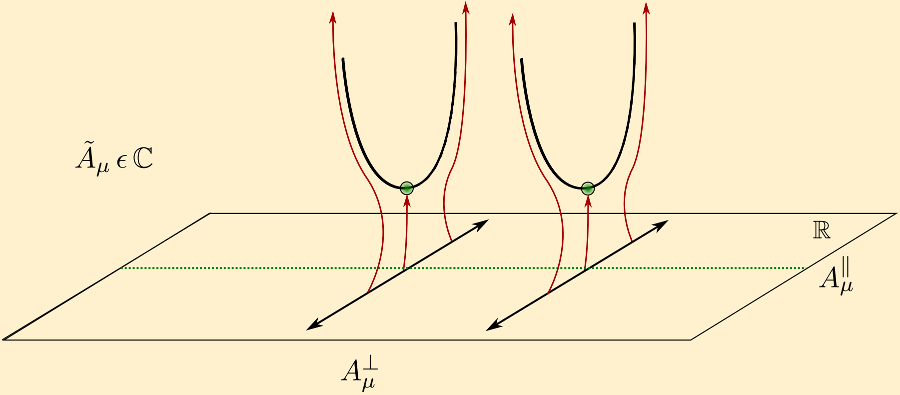

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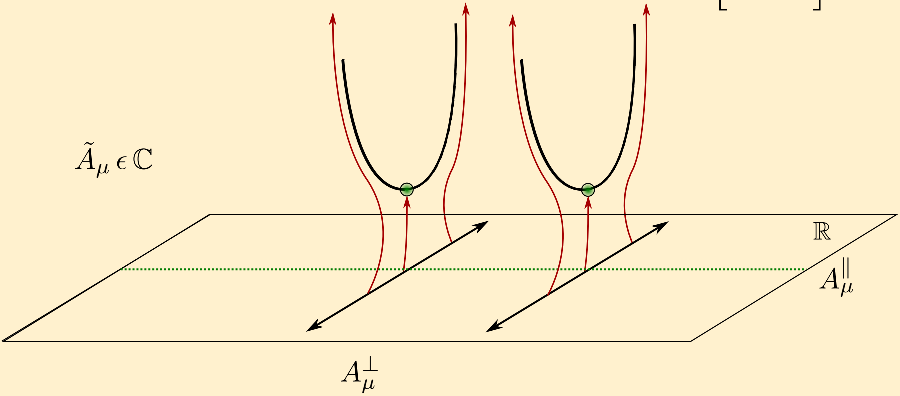
$$\left[\frac{\partial S}{\partial A^{\perp}}, \mathcal{G}\right] = 0$$



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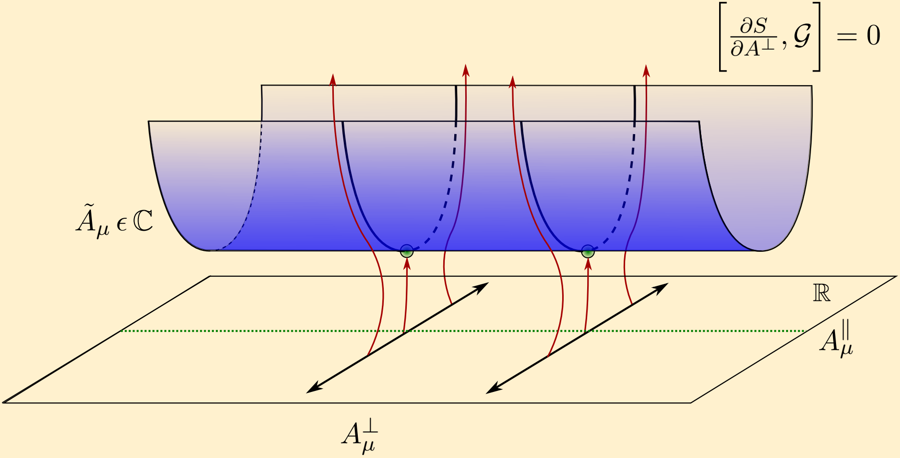
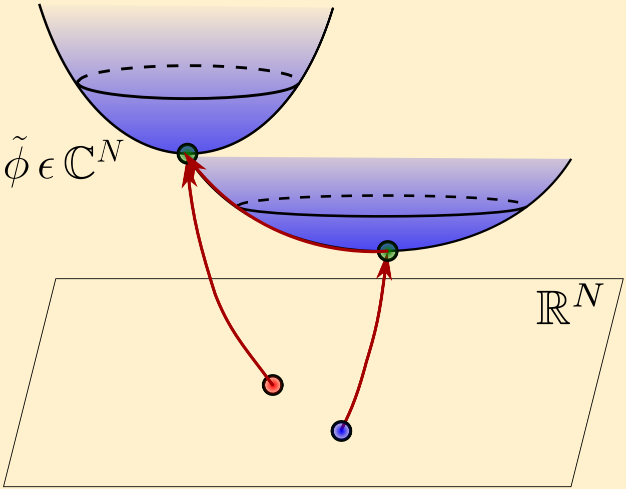


Manifold under flow is just $\mathcal{M}_g \oplus \mathcal{G}$!

Stokes' phenomenon prevent thimble decomposition



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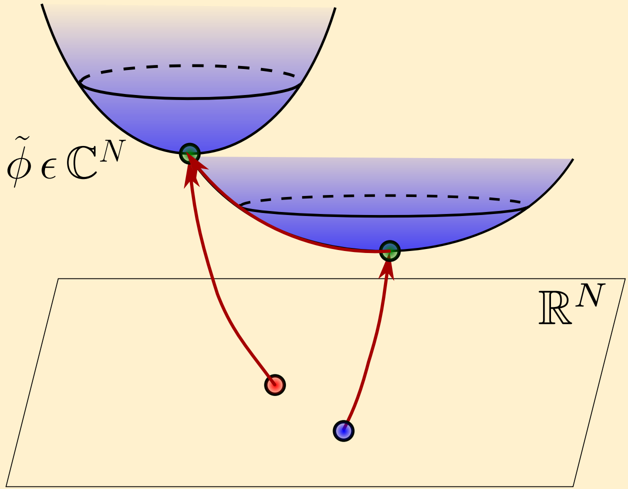
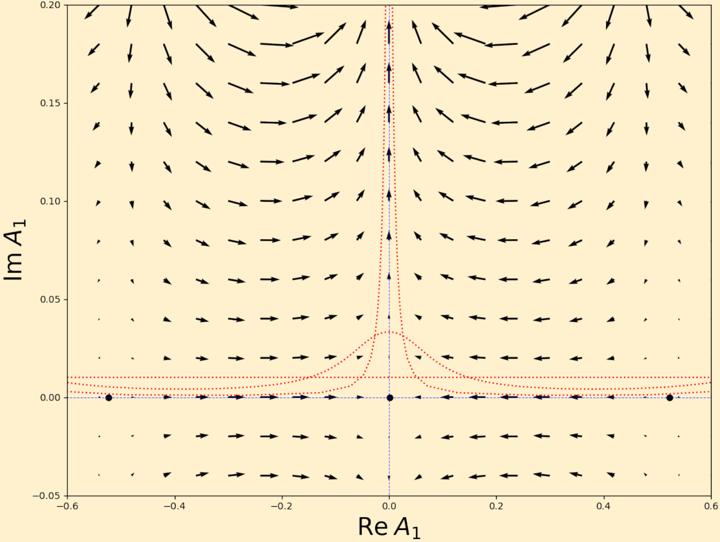


Effect of Stokes' phenomenon on flow is $\langle \sigma \rangle < 1$ due to "bumps"

Ain't no thang but a flow thang.



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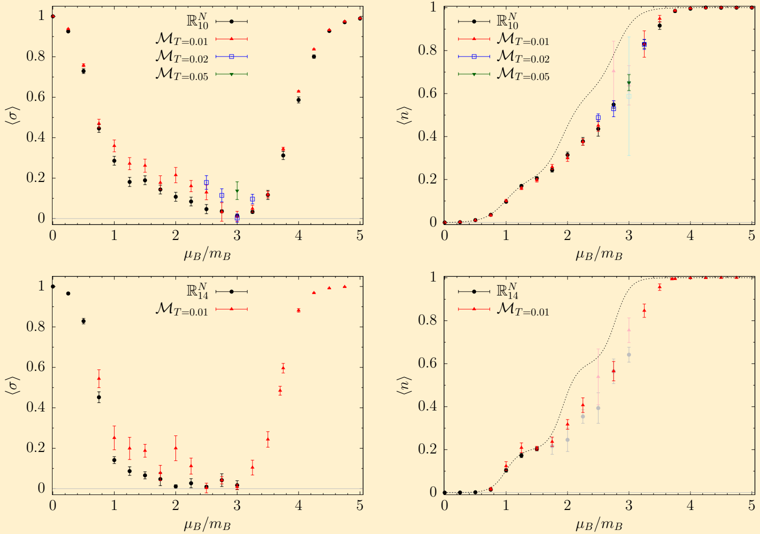

$\langle\sigma\rangle$ and $\langle n\rangle$ as a function of μ for $10, 14\times 10$ for QED_{1+1} with $N_f = 3$

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Questions?