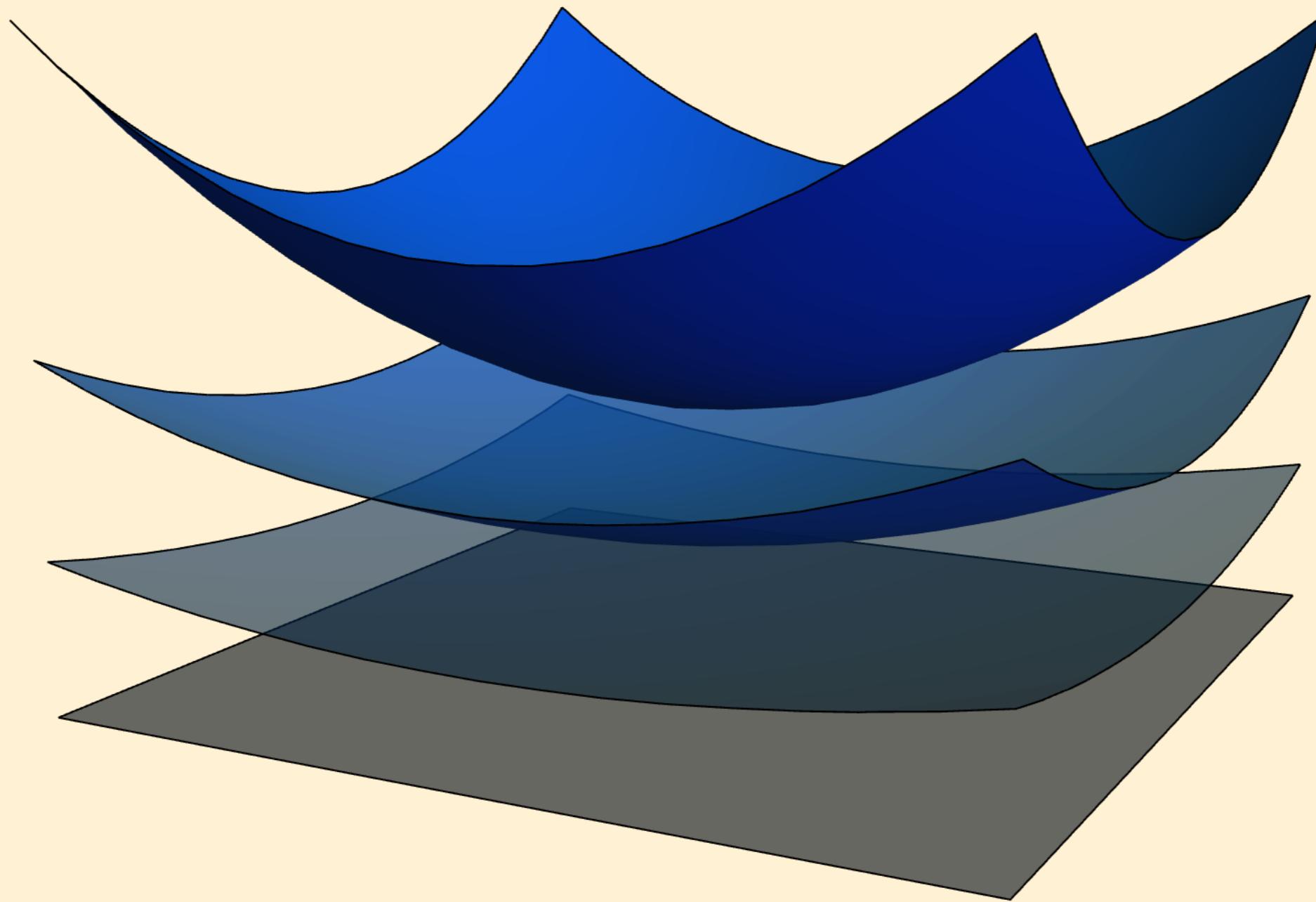


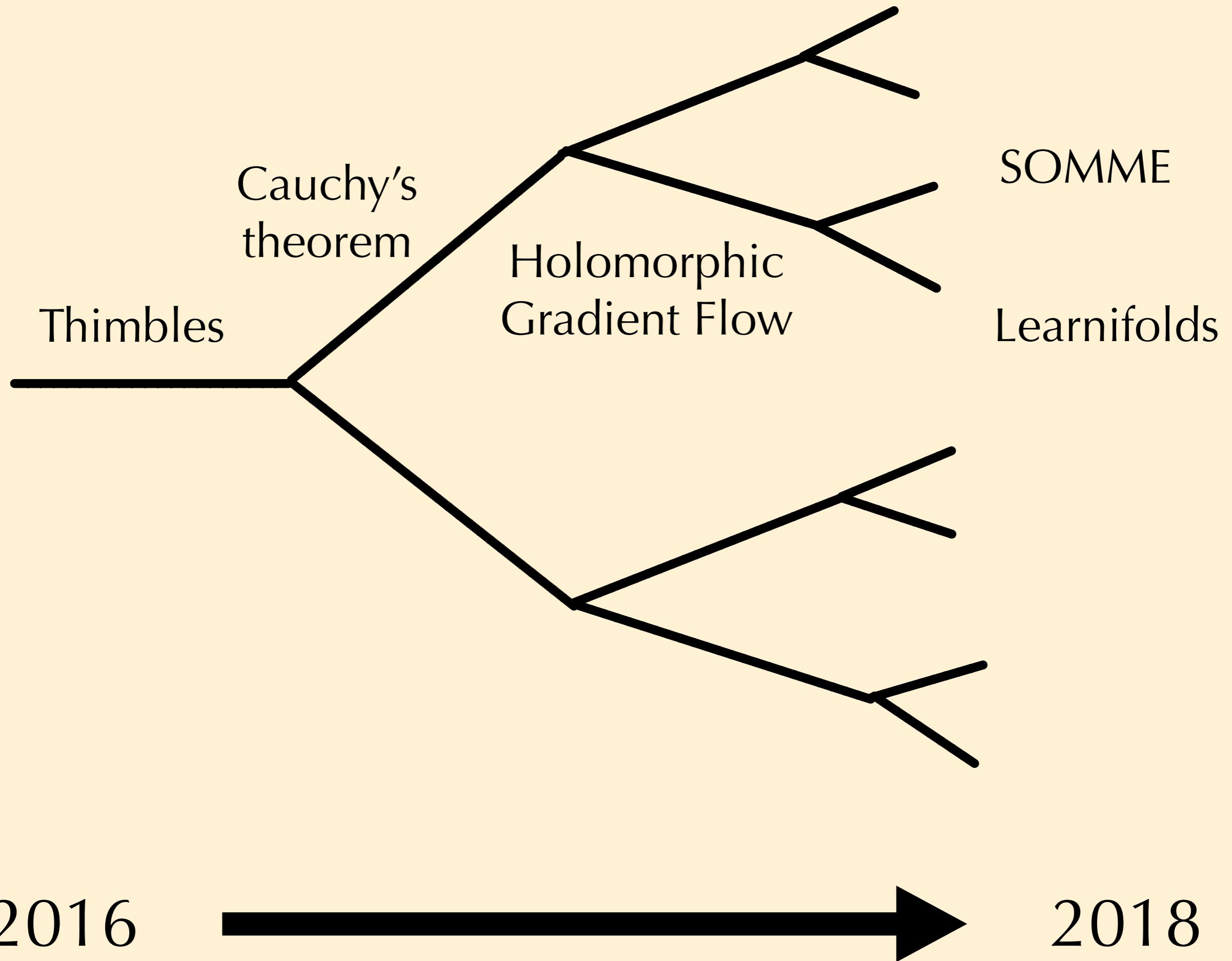
DEFORMING PATH INTEGRALS:
APPLICATION TO THE
(2+1) THIRRING MODEL

NEILL WARRINGTON
UNIVERSITY OF MARYLAND

COLLABORATORS: ANDREI ALEXANDRU, PAULO BEDAQUE,
HENRY LAMM, SCOTT LAWRENCE

$$\int_{\mathbb{R}^N} D\phi \, e^{-S(\phi)} = \int_{\mathcal{M}} D\tilde{\phi} \, e^{-S(\tilde{\phi})}$$





Complex Manifolds so far

(0+1) Fermions

Fujii, Kamata, Kikukawa, Schmidt,
Ziesché, Tanizaki, Hidaka, Hayata
Alexandru, Basar, Bedaque, Lamm, Lawrence,
Ridgway, Warrington + MANY AUTHORS

(1+1) Fermions

Alexandru, Basar, Bedaque, Ridgway, NW [1609.01730]
Alexandru, Bedaque, Lamm, Lawrence [1804.00697]

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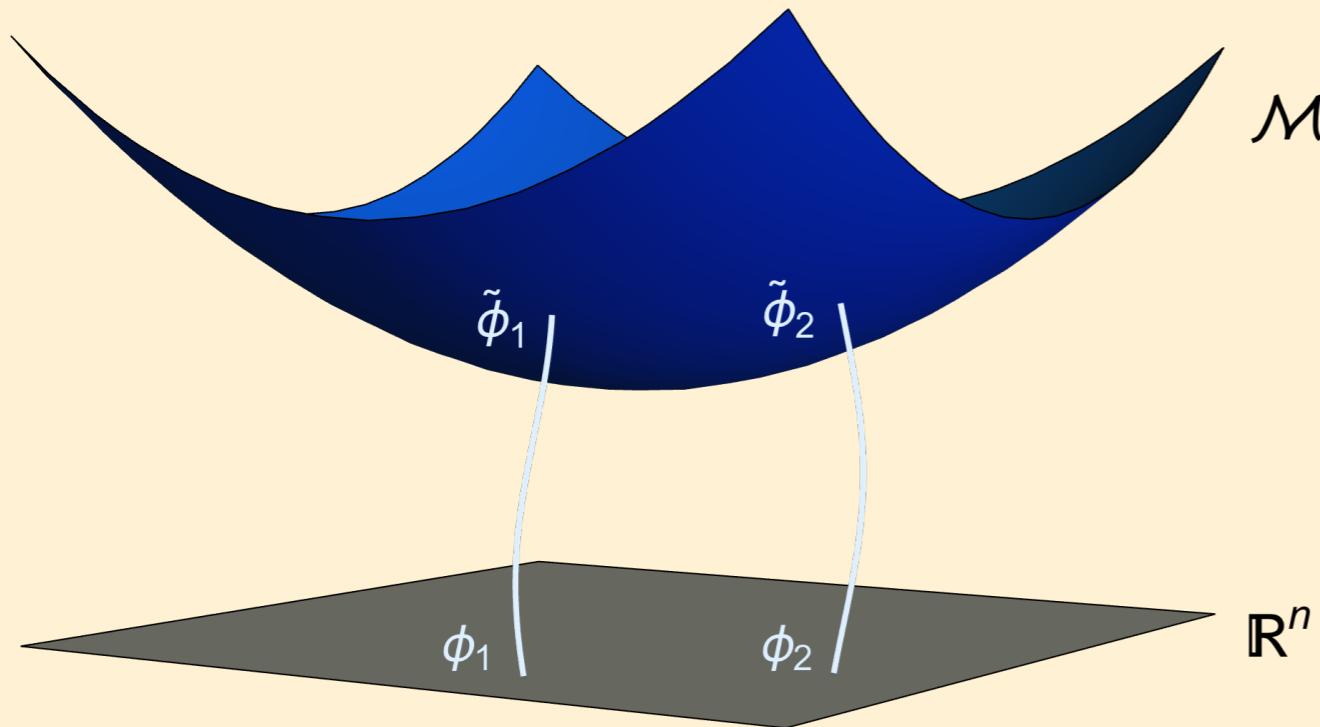
(1+1) Fermions

Alexandru, Basar, Bedaque, Ridgway, NW [1609.01730]
Alexandru, Bedaque, Lamm, Lawrence [1804.00697]

(2+1) & (3+1)?

Holomorphic Gradient Flow

Sign Optimized Manifold Method



Tangent Plane
+
Heavy Dense

$$\frac{d\phi}{ds} = \frac{\partial S^*}{\partial \phi}$$

SOMME

Cauchy's
Theorem

$$\int_{\mathbb{R}^N} D\phi \, e^{-S(\phi)} = \int_{\mathcal{M}} D\tilde{\phi} \, e^{-S(\tilde{\phi})}$$

SOMME

Cauchy's
Theorem

$$\int_{\mathbb{R}^N} D\phi \ e^{-S(\phi)} = \int_{\mathcal{M}} D\tilde{\phi} \ e^{-S(\tilde{\phi})}$$

Manifold

$$\tilde{\phi}_x(\phi) = \phi_x + i \left(\lambda_1 + \lambda_2 \cos(\phi_x) + \lambda_3 \cos(2\phi_x) \right)$$

SOMME

Cauchy's
Theorem

$$\int_{\mathbb{R}^N} D\phi e^{-S(\phi)} = \int_{\mathcal{M}} D\tilde{\phi} e^{-S(\tilde{\phi})}$$

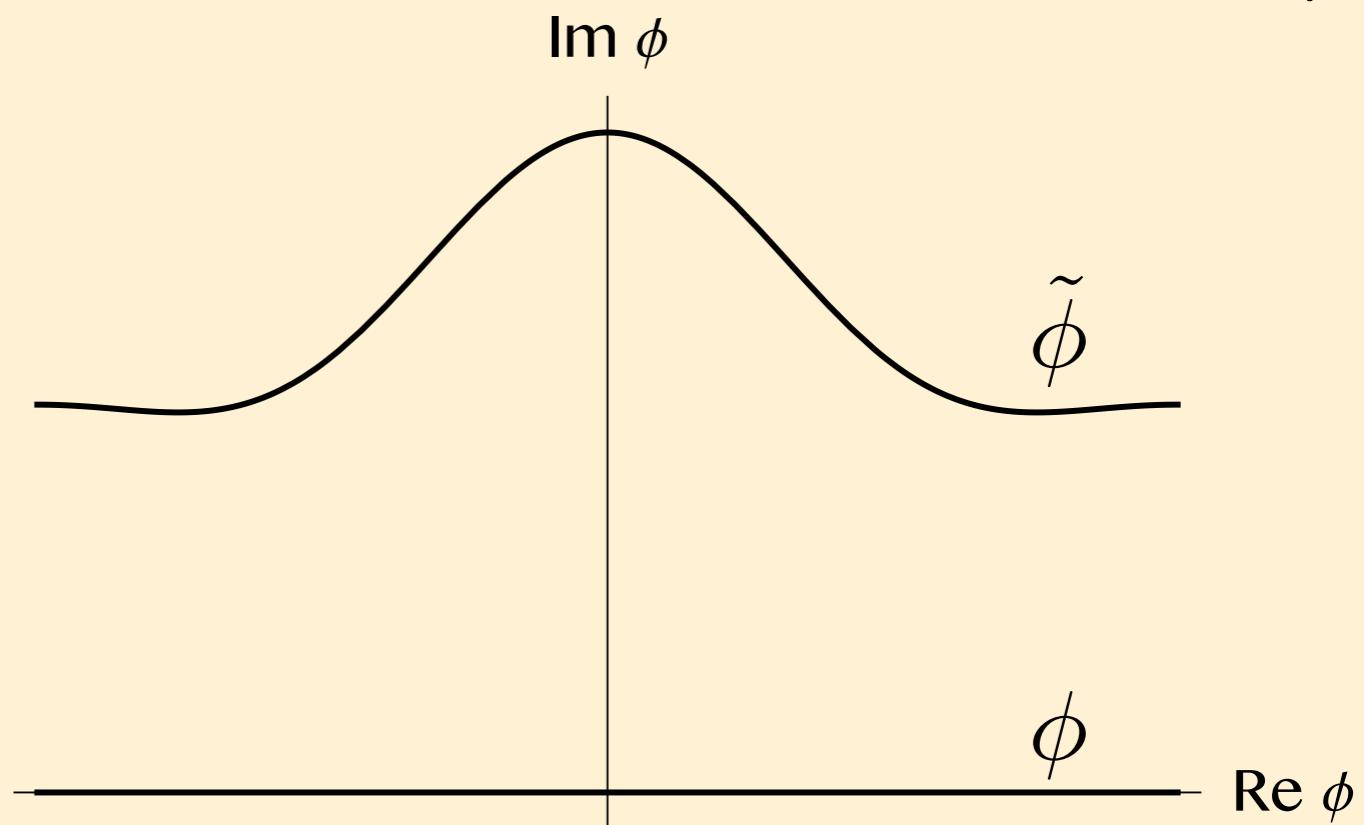
Manifold

$$\tilde{\phi}_x(\phi) = \phi_x + i \left(\lambda_1 + \lambda_2 \cos(\phi_x) + \lambda_3 \cos(2\phi_x) \right)$$

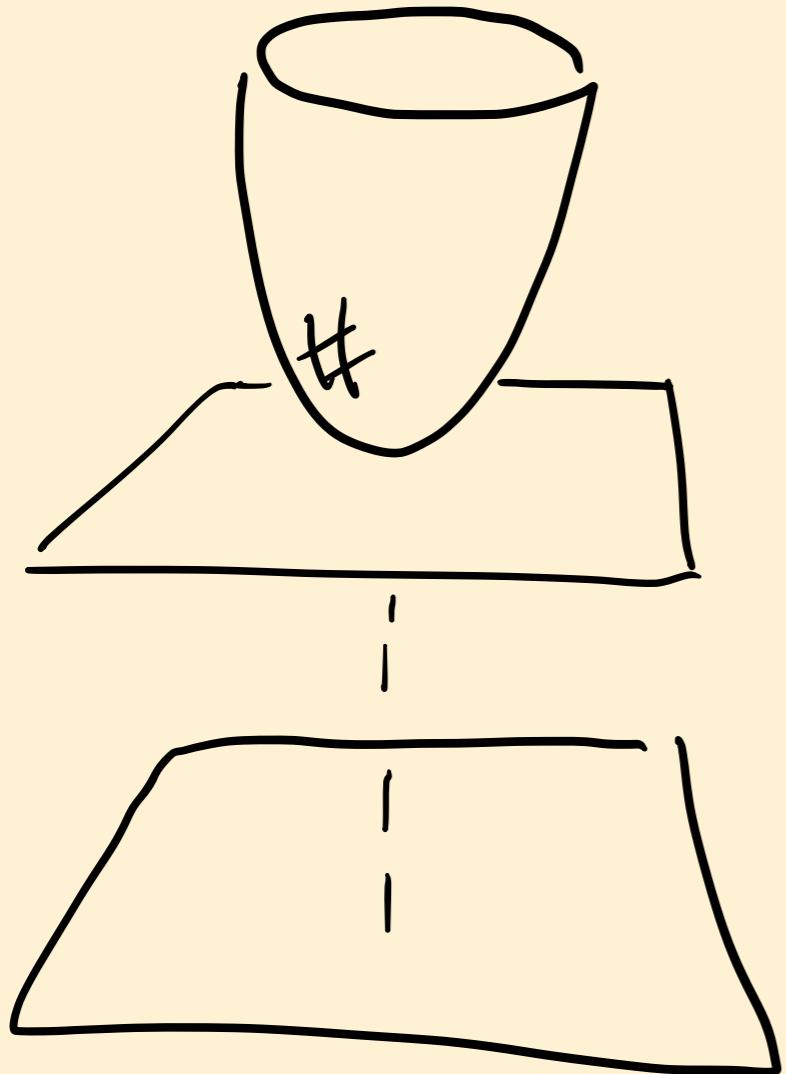
Key fact:

Manifold = factorizable

See Scott Lawrence's
talk for details



Tangent Shift



Heavy Dense

$$Z = \left(\int d\phi \, e^{-S(\phi)} \right)^{\beta V}$$

$$\tilde{\phi}_x(\phi) = \phi_x + i \left(\lambda_1 + \lambda_2 \cos(\phi_x) + \lambda_3 \cos(2\phi_x) \right)$$

(2+1) Thirring Model

$$S = \int dt d^2x \left[\bar{\psi} (\not{\partial} + m + \mu \gamma^0) \psi + \frac{g^2}{4} (\bar{\psi} \gamma^\mu \psi)^2 \right]$$

What is known?

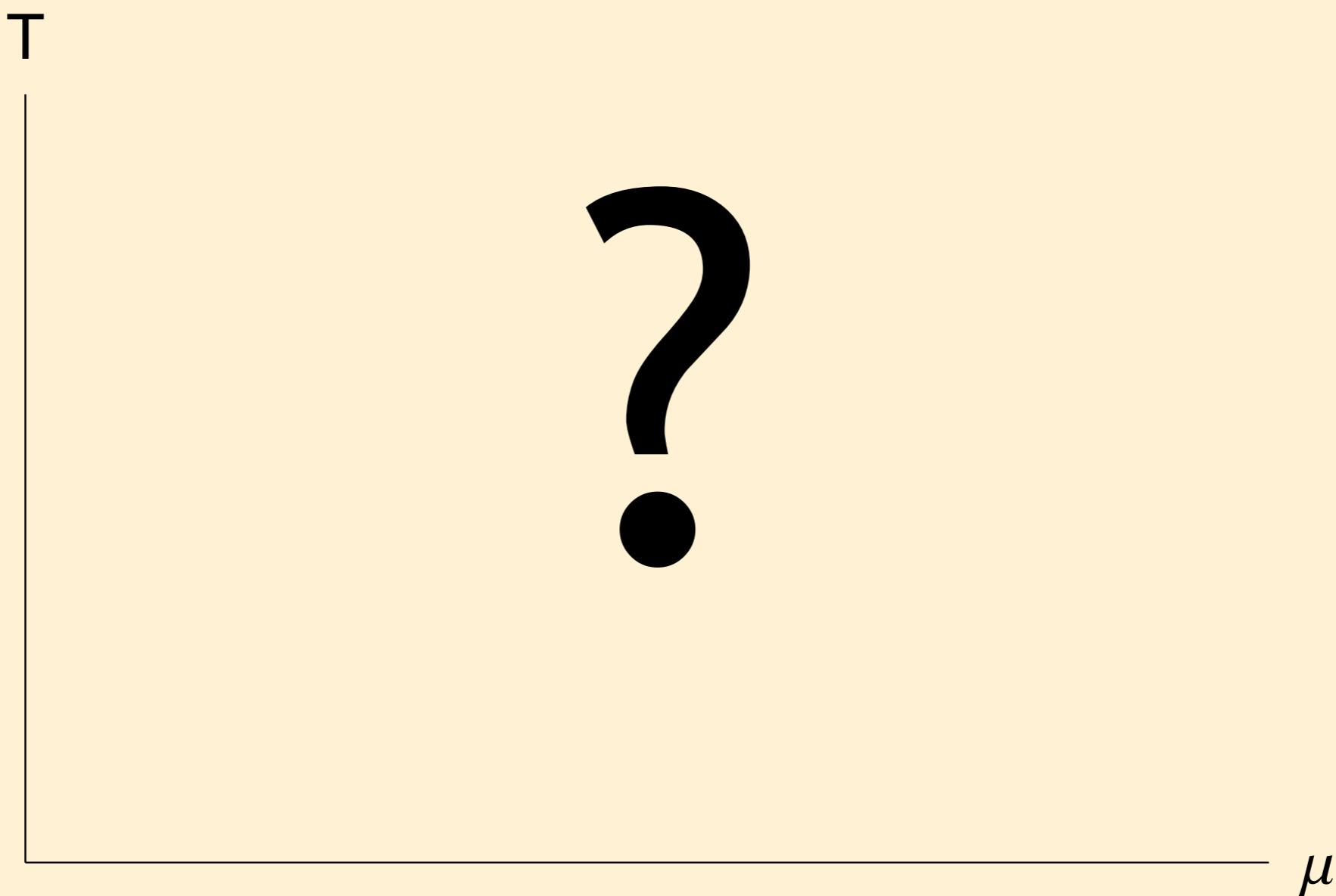
$$\mu = 0$$

- $\langle \bar{\psi} \psi \rangle \neq 0$ spontaneously at $m = 0$ Hands et. al. [9512013]
- Chiral restoration at large N_f Hands et. al. [0701016]

$$\mu \neq 0$$

- Complex Langevin + Heavy Dense Pawłowski et. al. [1302.2249]
- Complex Langevin + Fermion bags Li [1608.03141]

What is not known



Strategy

- Use SOMME manifold
- Combine with HMC
- High statistics

HMC on Manifolds

$$Z = \int_{\mathcal{M}} D\tilde{\phi} e^{-S(\tilde{\phi})} = \int_{\mathbb{R}^N} D\phi |\det J(\phi)| e^{-S(\phi)}$$

↑ ↑
coordinates Jacobian

HMC on Manifolds

$$Z = \int_{\mathcal{M}} D\tilde{\phi} e^{-S(\tilde{\phi})} = \int_{\mathbb{R}^N} D\phi |\det J(\phi)| e^{-S(\phi)}$$

↑ ↑
coordinates Jacobian

Factorizable manifold

$$\tilde{\phi}_x(\phi) = \phi_x + i(\lambda_1 + \lambda_2 \cos(\phi_x) + \lambda_3 \cos(2\phi_x))$$

HMC on Manifolds

$$Z = \int_{\mathcal{M}} D\tilde{\phi} e^{-S(\tilde{\phi})} = \int_{\mathbb{R}^N} D\phi |\det J(\phi)| e^{-S(\phi)}$$

↑ ↑
coordinates Jacobian

Factorizable manifold

$$\tilde{\phi}_x(\phi) = \phi_x + i(\lambda_1 + \lambda_2 \cos(\phi_x) + \lambda_3 \cos(2\phi_x))$$

$$\implies J_{xy}(\phi) = \delta_{xy} [1 - i\lambda_1 \sin(\phi_x) - 2i\lambda_2 \sin(2\phi_x)]$$

HMC on Manifolds

Standard HMC:
$$H(\pi, \phi) = \frac{1}{2} \sum_x \pi_x^2 + S(\phi)$$

$$P(\pi, \phi) \propto e^{-H(\pi, \phi)} \xrightarrow{\text{marginalize}} \sum_{\pi} P(\pi, \phi) \propto e^{-S(\phi)}$$

HMC on Manifolds

Standard HMC:
$$H(\pi, \phi) = \frac{1}{2} \sum_x \pi_x^2 + S(\phi)$$

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Kinetic:
$$\frac{1}{2} \sum_x \pi_x^2 \xrightarrow{\quad} \frac{1}{2} \sum_x \pi_x [J(\phi) J^\dagger(\phi)]_{xy}^{-1} \pi_y$$

HMC on Manifolds

Standard HMC: $H(\pi, \phi) = \frac{1}{2} \sum_x \pi_x^2 + S(\phi)$

$$P(\pi, \phi) \propto e^{-H(\pi, \phi)} \xrightarrow{\text{marginalize}} \sum_{\pi} P(\pi, \phi) \propto e^{-S(\phi)}$$

Kinetic: $\frac{1}{2} \sum_x \pi_x^2 \xrightarrow{} \frac{1}{2} \sum_x \pi_x [J(\phi) J^\dagger(\phi)]_{xy}^{-1} \pi_y$

Manifold HMC: $H(\pi, \phi) = \frac{1}{2} \sum_x \pi_x [J(\phi) J^\dagger(\phi)]_{xy}^{-1} \pi_y + S(\phi)$

$$P(\pi, \phi) \propto e^{-H(\pi, \phi)} \xrightarrow{\text{marginalize}} \sum_{\pi} P(\pi, \phi) \propto |\det J(\phi)| e^{-S(\phi)}$$

$$H(\pi, \phi) = \frac{1}{2} \sum \pi_x [J(\phi) J^\dagger(\phi)]_{xy}^{-1} \pi_y + S(\phi)$$

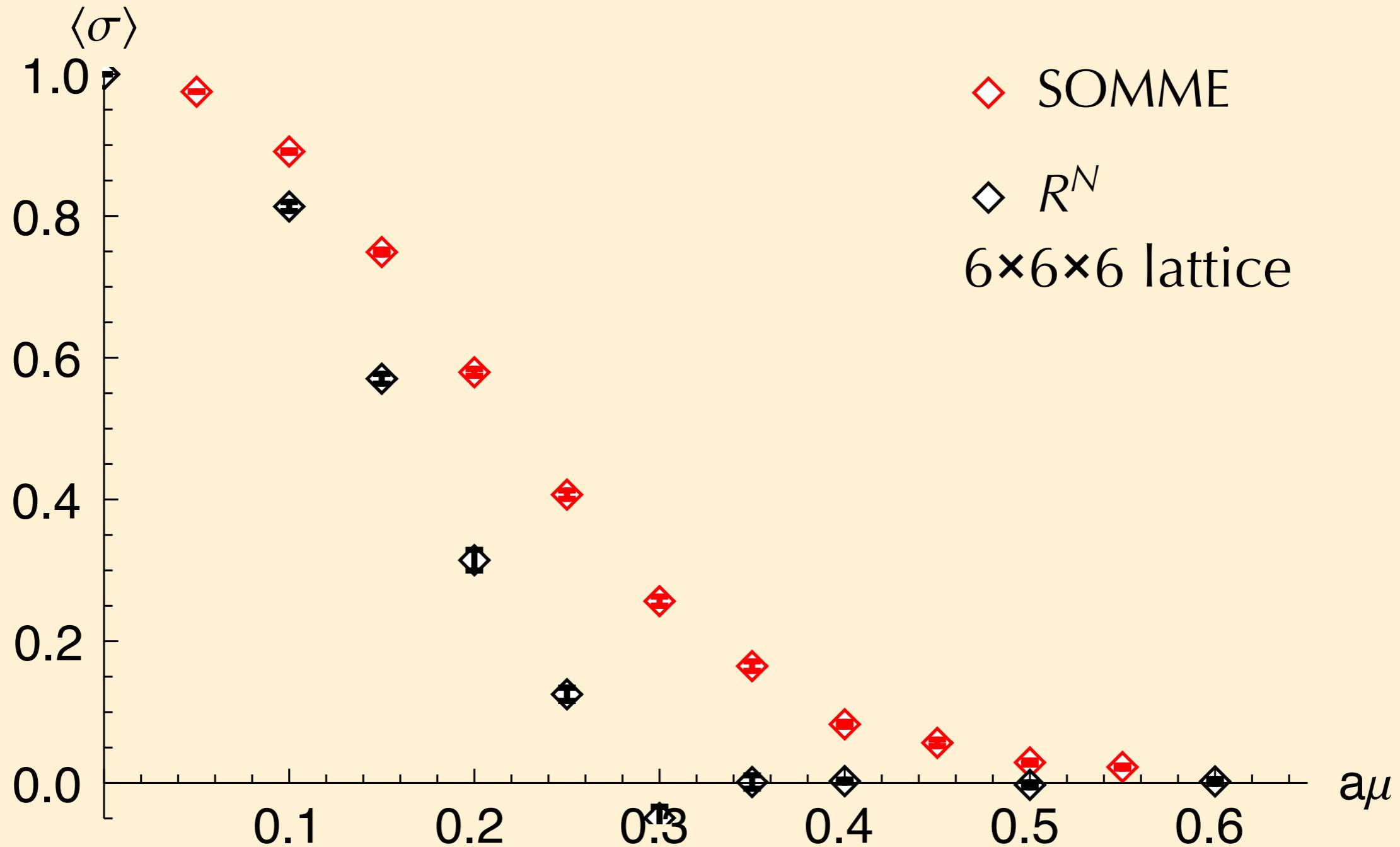


difficult if non-separable

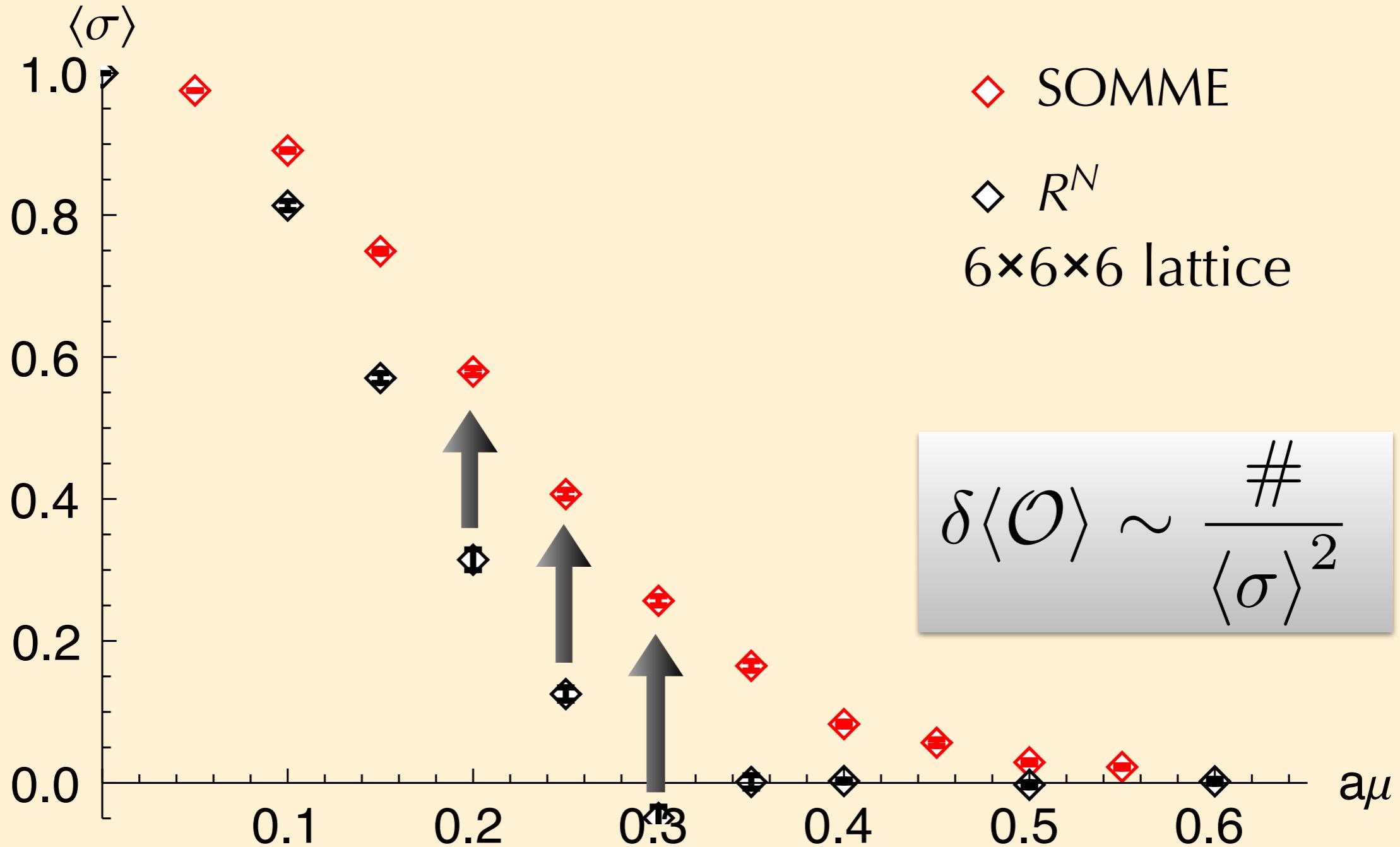
SOMME is key
to HMC on manifolds

Results

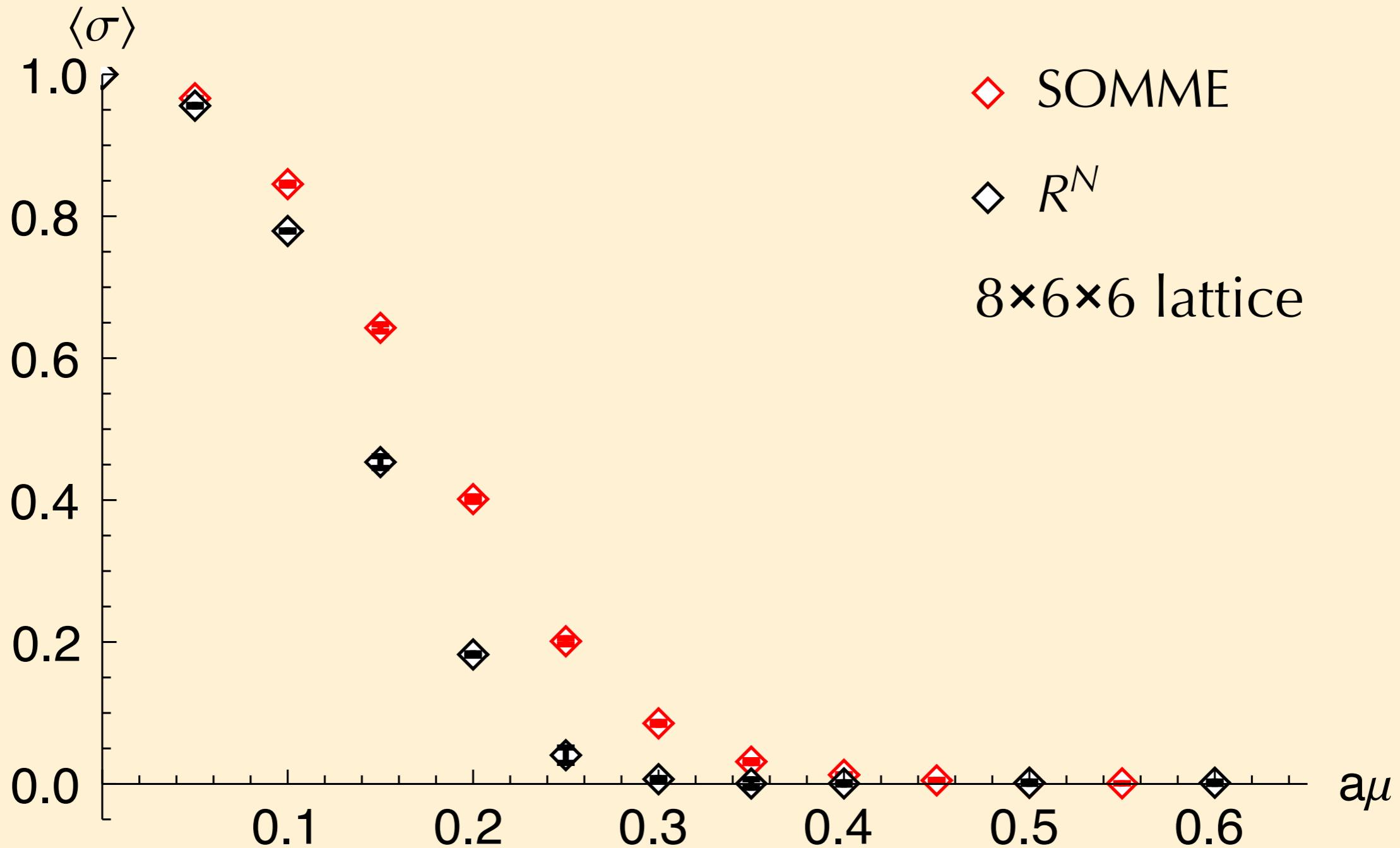
Sign Increases



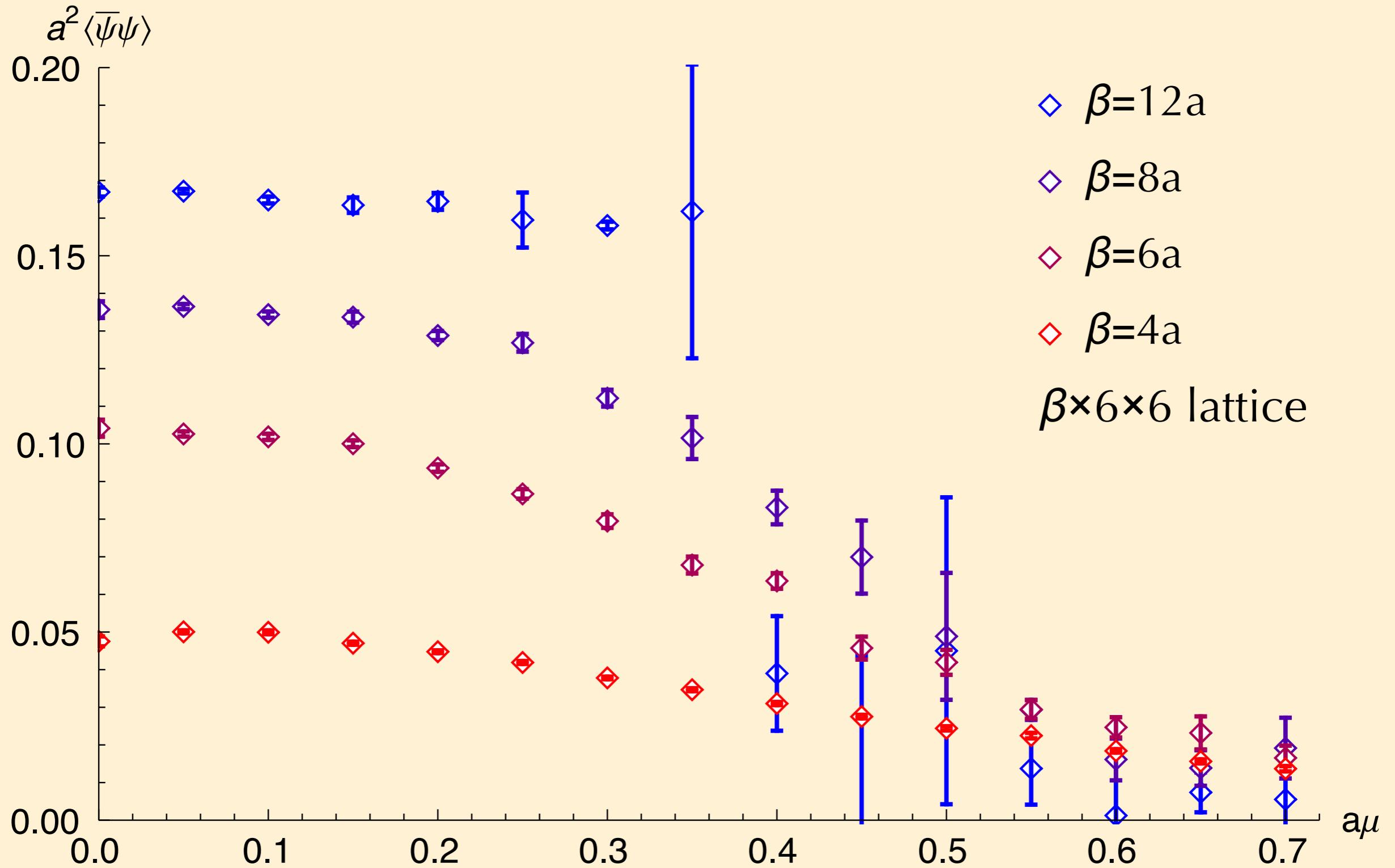
Sign Increases



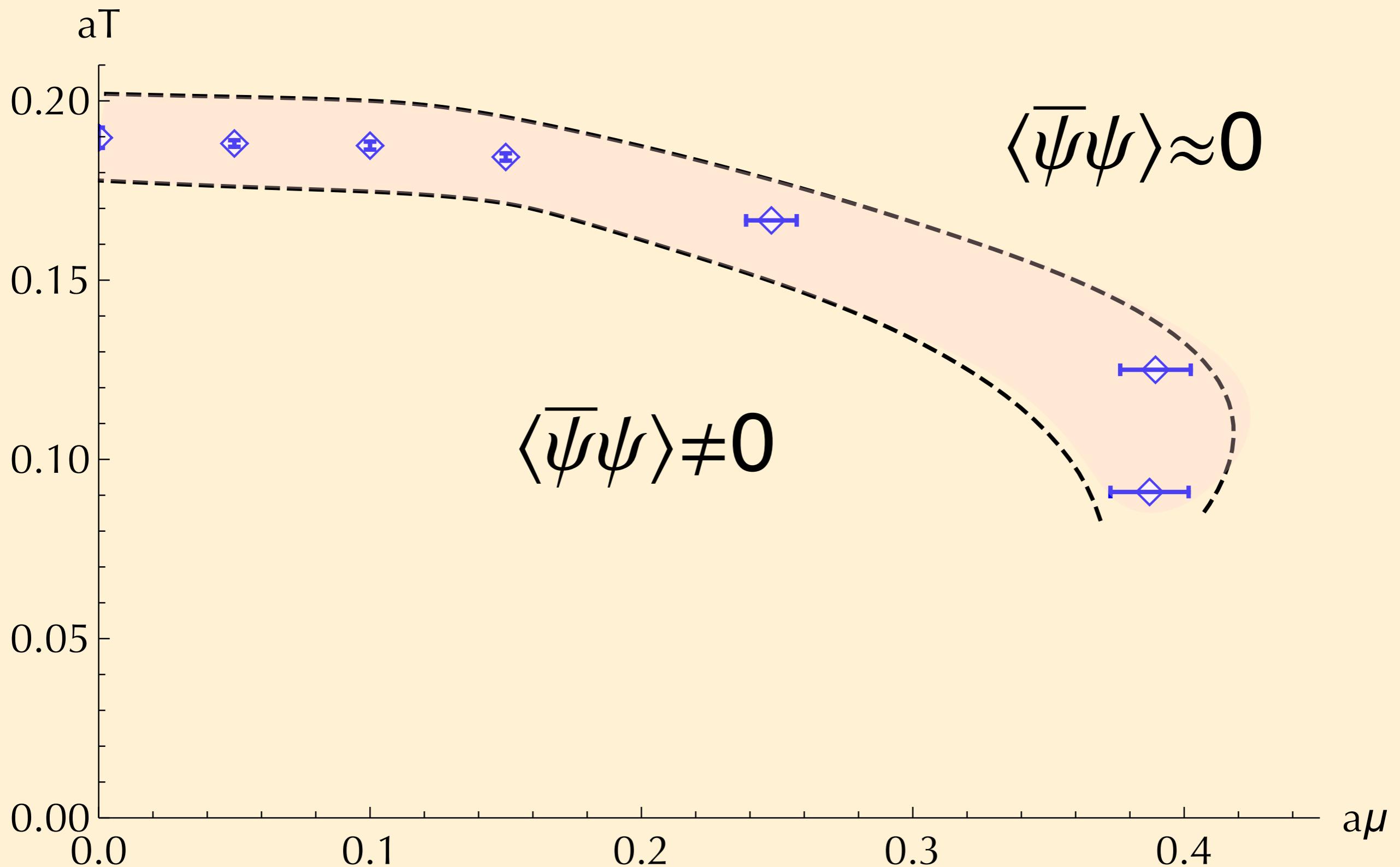
Sign Increases



Chiral Condensate



(T, μ) Phase Diagram



Upshot

- Complex manifolds in (2+1) ✓
- SOMME + HMC = speed
- (T, μ) phase diagram computed

Thanks!