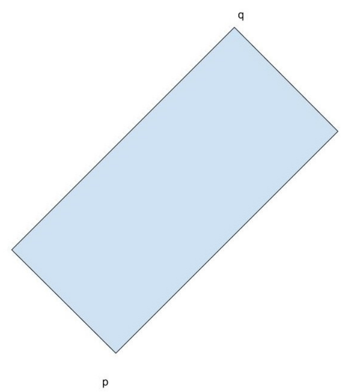
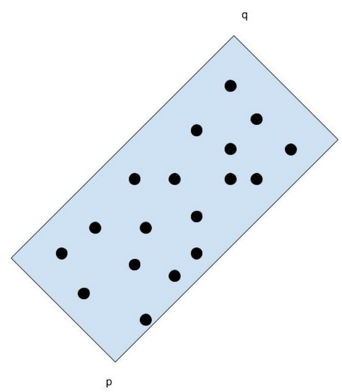
Causal set is a partially ordered set defined as: $a \prec b$ if and only if one can travel from a to b without going faster than the speed of light

Topology is defined by **Alexandrov sets** $\alpha(p,q) = \{r | p \prec r \prec q\}$

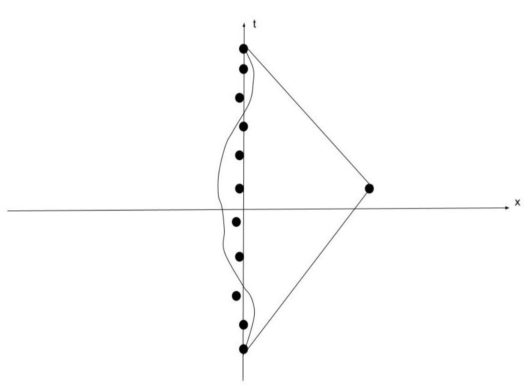


Discreteness is defined through **local finiteness:** $\sharp\alpha(p,q) < \infty$



Metric is defined through $\tau(p,q) = \xi \max\{n | \exists r_1, \cdots, r_{n-1}(p \prec r_1 \prec \cdots \prec r_{n-1} \prec q)\}$

Note: It is max rather than min because of the minus sign in Minkowskian metric. For example, if geodesic is along t-axis, $|dt| \geq \sqrt{(dt)^2 - |d\vec{x}|^2}$ (sign convention is $(+---)$)



Key idea

a) Assume smooth manifold and the presence of coordinates

b) Re-express coordinate-dependent expressions in a way that coordinates aren't explicitly mentioned

c) Copy the result for the non-manifold situation (ex: tree-like causal structure, etc)

Key difference between causal sets and other discrete theories: In manifold situation, the causal set assumption is Poisson scattering $\Longrightarrow$ lack of structure, emphasis on **statistical properties**

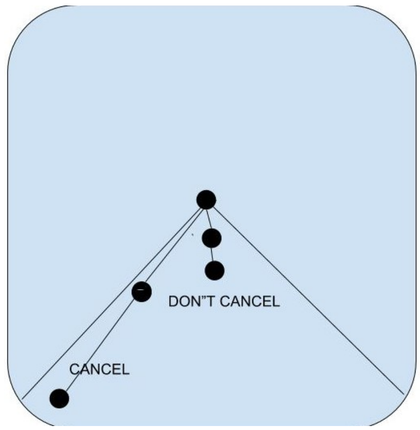
Key difference between my work and other types of causal set theory: I am trying to **re-interpret** causal structure, the definition of fields, etc, while **still** sticking to statistical approach

Conventional version of causal set Lagrangian (Sorkin at el)

Use $-\phi\Delta\phi$ instead of $+\partial^\mu\phi\partial_\mu\phi$

2D case:

$$(\Delta\phi)(p) = \sum_{\{(r,s)|\alpha(r,p)=\{s\}\}} (\phi(p) + \phi(r) - 2\phi(s)) \tag{1}$$



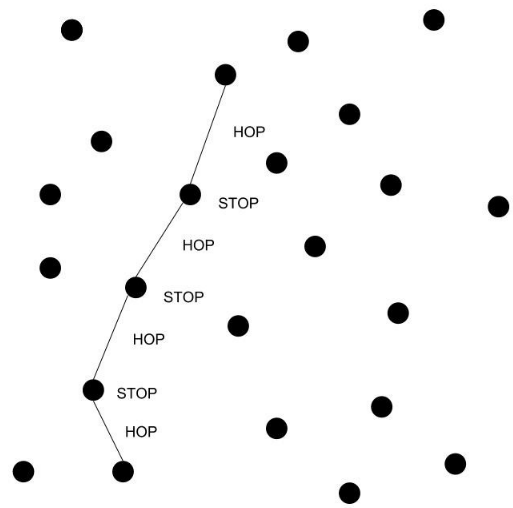
d **dimension**

$$\Delta\phi = \sum_{r_{n(d)} \prec r_{n(d)-1} \prec \cdots \prec r_1 \prec p} (c_0(d)\phi(p) + c_1(d)\phi(r_1) + \cdots + c_{n(d)}(d)\phi(r_{n(d)})) \tag{2}$$

NOTE Cancelation only occurs sufficiently far away from the boundary

What I don't like about it: Existence of the boundary $\Longrightarrow$ Preferred frame $\Longrightarrow$ invalidation of stated claim of causal set theory

Steven Johnston's propagator



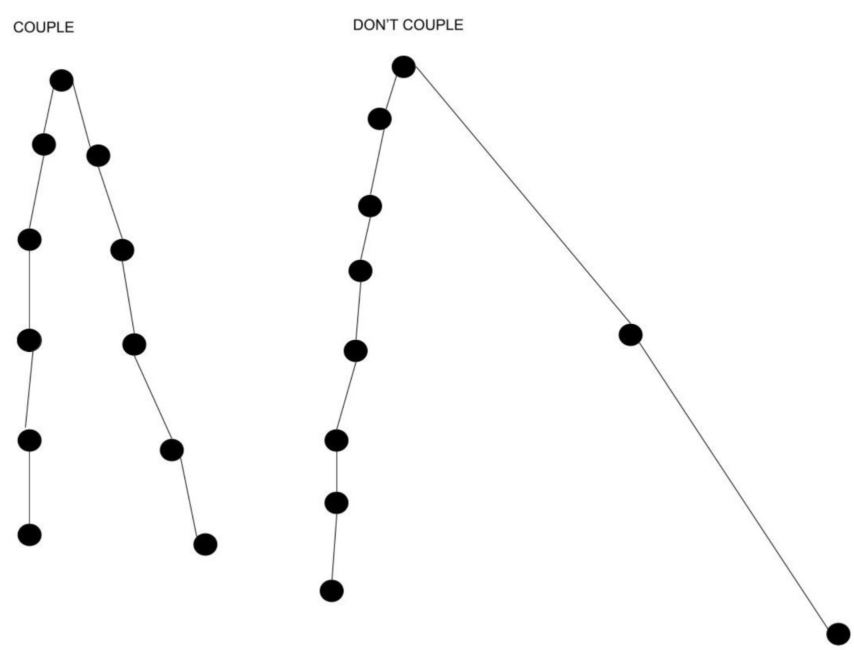
Summing over all possible paths

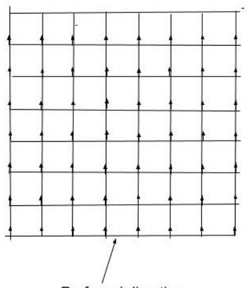
1) Propagators are defined direction, WITHOUT the use of Lagrangians

2) Propagators don't face the problem of non-locality because of the TWO endpoints

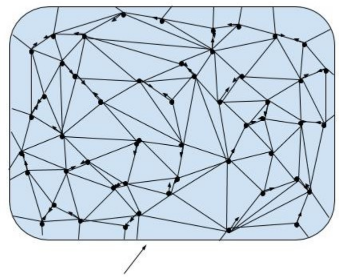
Problem: Coupling different propagators to each other during ϕ^4-coupling

Easy solution: Impose a condition by hand which edges are allowed to be ϕ^4-coupled and which aren't

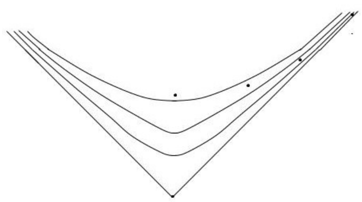




Preferred direction corresponds to edges of the lattice

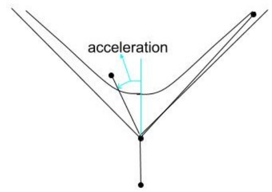


In case of Eucledian Poisson scattering, the preferred direction is a direction to the nearest neighbor

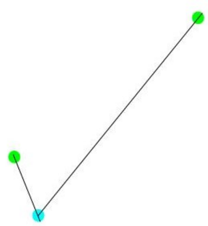


In Minkowski space we have sequence of points that are further and further away coordinate-wise, yet closer and closer in Lorentzian sense and, therefore, there is no nearest neighbor and no UNIQUE preferred frame.

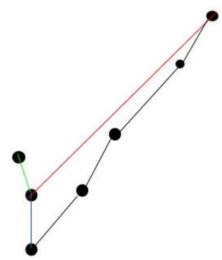
NOTE: this doesn't exclude a HIERARCHY of frames.



While there is no unique preferred velocity for a given position, there *IS* a unique preferred acceleration for any given position+velocity. In other words, equivalence principle is violated



Both points are the direct neighbors



Only one of the two edges is a direct neighbor

DILLEMA: Locality $\Longrightarrow$ Finately many neighbors $\Longrightarrow$ Nearest neighbor $\Longrightarrow$ Preferred frame

MY ANSWER: The price for nearest edge neighbor is violation of Newtons first law INSTEAD OF preferred frame

a) the nearest edge-neighbor relates to the fact that geodesic wiggles

b) wiggling of geodesic is interpretted as gravity

THEREFORE

c) nearest edge-neighbor phenomenon is "explained away" through gravity

Conventional thinking

$$a(p,q) = \int_{\gamma(p,q)} A_\mu(\gamma(\tau))\dot{\gamma}^\mu(\tau) d\tau \tag{3}$$

where γ is a geodesic connecting p and q

$\phi(x)$ is given

My thinking

a) Replace $A_\mu(x)$ and $\phi(x)$ with $A_\mu(x,p)$ and $\phi(x,p)$

b) Assume $A^\mu(x,p_1) \approx A^\mu(x,p_2)$ and $\phi(x,p_1) \approx \phi(x,p_2)$ if the relative velocity of reference frames corresponding to p_1 and p_2 is not too close to c

c) Assume that in the reference frames, with resect to which $p/|p|$ isn't too close to c, $\phi(x,p)$ and $A^\mu(x,p)$ are both locally linear

d) Define

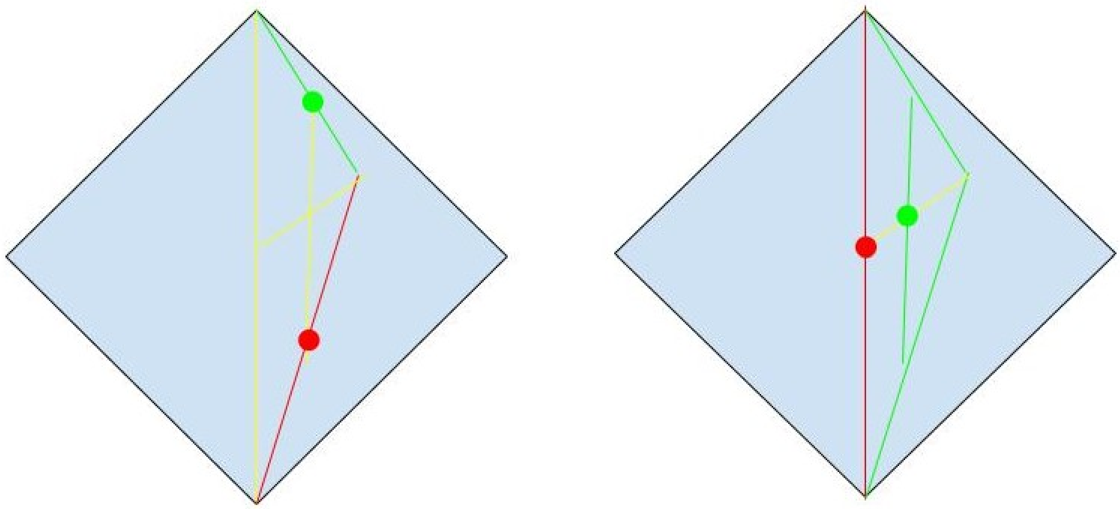
$$a(p,q) = \int_{\gamma(p,q)} A_\mu(\gamma(\tau),\dot{\gamma}(\tau))\dot{x}^\mu(\tau) d\tau d\tau \tag{4}$$

$$\phi(p,q) = \frac{1}{\tau(p,q)} \int_{\gamma(p,q)} \phi(\gamma(\tau),\dot{\gamma}(\tau))\dot{x}^\mu(\tau) d\tau \tag{5}$$

NOTE: Since path integral is dominated by NON-DIFFERENTIABLE paths, the ssumptions b and c are dropped once we are under the path integral; those assumptions ONLY apply to "well behaved" functions we are thinking of in order to "motivate" our definition of the action.

NOTE: $a(p,q) = -a(q,p)$, BUT $\phi(p,q) = +\phi(q,p)$

Setup



$$\mathcal{L} = \eta_{scal} \int_{\alpha(p,q)} (\mathcal{K}_1(\phi; p, q, r) - C_{scal}(d)\mathcal{K}_2(\phi; p, q)) \tag{6}$$

$$\mathcal{K}_1 = \int_{\alpha(p,q)} d^d r (\text{top} - \text{bottom})^2 = \int_{\alpha(p,q)} d^d r (\phi(r, q) - \phi(p, r))^2 \tag{7}$$

$$\mathcal{K}_1 = \int_{\alpha(p,q)} d^d r \left(\phi\left(r + \frac{\hat{e}_0}{2} \right) - \phi\left(r \frac{\hat{e}_0}{2} \right) \right)^2 = \int_{\alpha(p,q)} d^d r \left(\frac{\partial \phi}{\partial x^0} \bigg|_0 \right)^2 = \left(\frac{\partial \phi}{\partial x^0} \bigg|_0 \right)^2 \int_{\alpha(p,q)} d^d r \tag{8}$$

$$\mathcal{K}_2 = \int_{\alpha(p,q)} d^d r (\text{left} - \text{right})^2 = \int_{\alpha(p,q)} d^d r \left(\frac{\phi(p, r) + \phi(r, q)}{2} - \phi(p, q) \right)^2 \tag{9}$$

$$\mathcal{K}_2 = \int_{\alpha(p,q)} d^d r \left(\phi\left(\frac{r}{2} \right) - \phi(0) \right)^2 = \int_{\alpha(p,q)} d^d r \left(\frac{\partial \phi}{\partial r^0} \bigg|_0 \frac{r^0}{2} + \frac{\partial \phi}{\partial r^0} \bigg|_0 \frac{r^0}{2} \right)^2 =$$

$$= \frac{1}{4} \left[\left(\frac{\partial \phi}{\partial r^0} \bigg|_0 \right)^2 \int_{\alpha(p,q)} d^d r (r^0)^2 + \left(\frac{\partial \phi}{\partial r^1} \bigg|_0 \right)^2 \int_{\alpha(p,q)} d^d r (r^1)^2 + 2 \frac{\partial \phi}{\partial r^0} \frac{\partial \phi}{\partial r^1} \int_{\alpha(p,q)} d^d r r^0 r^1 \right] \tag{10}$$

$$\text{Odd Function} \Longrightarrow \int_{\alpha(p,q)} d^d r \; r^0 r^1 = 0 \tag{11}$$

$$\mathcal{K}_2 = \frac{1}{4} \left(\left(\frac{\partial \phi}{\partial r^0} \bigg|_0 \right)^2 \int_{\alpha(p,q)} d^d r (r^0)^2 + \left(\frac{\partial \phi}{\partial r^1} \bigg|_0 \right)^2 \int_{\alpha(p,q)} d^d r (r^1)^2 \right) \tag{12}$$

Finding $C_{scal}(d)$

$$\left(\frac{\partial\phi}{\partial x^0}\bigg|_0\right)^2 - \frac{C_{scal}(d)}{4}\left(\langle t^2\rangle\left(\frac{\partial\phi}{\partial x^0}\bigg|_0\right)^2 + \langle (r^1)^2\rangle\left(\frac{\partial\phi}{\partial x^1}\bigg|_0\right)^2\right) =$$
$$= \left((1 - \frac{C_{scal}(d)}{4}\langle t^2\rangle\right)\left(\frac{\partial\phi}{\partial x^0}\bigg|_0\right)^2 - \frac{C_{scal}(d)}{4}\langle (r^1)^2\rangle\left(\frac{\partial\phi}{\partial x^1}\bigg|_0\right)^2 \tag{13}$$

$$1-\frac{C_{scal}(d)}{4}\langle t^2\rangle = \frac{C_{scal}(d)}{4}\langle (r^1)^2\rangle \Longrightarrow 1 = \frac{C_{scal}(d)}{4}(\langle t^2\rangle+\langle (r^1)^2\rangle \Longrightarrow C_{scal}(d) = \frac{4}{\langle t^2\rangle + \langle (r^1)^2\rangle}$$

$$\xi = 1-t \Longrightarrow \langle (1-t)^k\rangle = \frac{\int_0^1 \xi^k \xi^{d-1} d\xi}{\int_0^1 \xi^{d-1} d\xi} = \frac{\int_0^1 \xi^{d+k-1} d\xi}{\int_0^1 \xi^{d-1} d\xi} = \frac{\frac{1}{d+k}}{\frac{1}{d}} = \frac{d}{d+k} \tag{14}$$

$$\langle t\rangle = 1 - \langle 1-t\rangle = 1 - \frac{d}{d+1} = \frac{1}{d+1} \tag{15}$$

$$\langle t^2\rangle = \langle (1-(1-t))^2\rangle = 1 - 2\langle 1-t\rangle + \langle (1-t)^2\rangle = 1 - \frac{2d}{d+1} + \frac{d}{d+2} =$$
$$= \frac{(d+1)(d+2) - 2d(d+2) + d(d+1)}{(d+1)(d+2)} = \frac{d^2+3d+2-2d^2-4d+d^2+d}{(d+1)(d+2)} =$$
$$= \frac{(1-2+1)d^2 + (3-4+1)d + 2}{(d+1)(d+2)} = \frac{2}{(d+1)(d+2)} \tag{16}$$

$$\langle r^2\rangle = \frac{\int_0^1 r^2 r^{d-2}(1-r)dr}{\int_0^1 r^{d-2}(1-r)dr} = \frac{\int_0^1 (r^d - r^{d+1})dr}{\int_0^1 (r^{d-2} - r^{d-1})dr} = \frac{\frac{1}{d+1} - \frac{1}{d+2}}{\frac{1}{d-1} - \frac{1}{d}} =$$
$$= \frac{\frac{d+2-d-1}{(d+1)(d+2)}}{\frac{d-d+1}{(d-1)d}} = \frac{\frac{1}{(d+1)(d+2)}}{\frac{1}{(d-1)d}} = \frac{(d-1)d}{(d+1)(d+2)} \tag{17}$$

$$\langle r^2\rangle = \sum_{k=1}^{d-1}\langle (x^k)^2\rangle = (d-1)\langle (x^1)^2\rangle \Longrightarrow \langle (x^1)^2\rangle = \frac{1}{d-1}\langle r^2\rangle = \frac{d}{(d+1)(d+2)} \tag{18}$$

$$C_{scal}(d) = \frac{4}{\langle t^2\rangle + \langle (x^1)^2\rangle} = \frac{4}{\frac{2}{(d+1)(d+2)} + \frac{d}{(d+1)(d+2)}} = \frac{4}{\frac{d+2}{(d+1)(d+2)}} = \frac{4}{\frac{1}{d+1}} = 4(d+1) \tag{19}$$

Avoiding $C(d)$

$$\mathcal{K}_1(f;p_1,q_1) - C(d)\mathcal{K}_2(f;p_1,q_1) = \mathcal{K}_1(f;p_2,q_2) - C(d)\mathcal{K}_2(f;p_2,q_2) \tag{20}$$

$$\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2) = C(d)(\mathcal{K}_2(f;p_1,q_1) - \mathcal{K}_2(f;p_2,q_2)) \tag{21}$$

$$C(d) = \frac{\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2)}{\mathcal{K}_2(f;p_1,q_1) - \mathcal{K}_2(f;p_2,q_2)} \tag{22}$$

$$\mathcal{L} = \eta(\mathcal{K}_1(\phi;p_0,q_0) - C(d)\mathcal{K}_2(\phi;p_0,q_0)) \tag{23}$$

$$\mathcal{L} = \eta\bigg(\mathcal{K}_1(\phi;p_0,q_0) - \frac{\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2)}{\mathcal{K}_2(f;p_1,q_1) - \mathcal{K}_2(f;p_2,q_2)}\mathcal{K}_2(\phi;p_0,q_0)\bigg) \tag{24}$$

$$\mathcal{L} = \eta\sum\left[W(p_1,q_1,p_2,q_2)\bigg(\mathcal{K}_1(\phi;p_0,q_0) - \frac{\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2)}{\mathcal{K}_2(f;p_1,q_1) - \mathcal{K}_2(f;p_2,q_2)}\mathcal{K}_2(\phi;p_0,q_0)\bigg)\right] \tag{25}$$

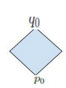
$$w(p_1,p_2,q_1,q_2) = \eta\frac{W(p_1,p_2,q_1,q_2)}{\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2)} \tag{26}$$

$$\begin{aligned}\mathcal{L} = \sum\Big(&w(p_1,p_2,q_1,q_2)(\mathcal{K}_1(\phi;p_0,q_0)(\mathcal{K}_2(f;p_1,q_1) - \mathcal{K}_2(f;p_2,q_2)) - \\ &-\mathcal{K}_2(\phi;p_0,q_0))(\mathcal{K}_1(f;p_1,q_1) - \mathcal{K}_1(f;p_2,q_2))\Big)\end{aligned} \tag{27}$$

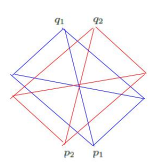
To define f introduce p_3 and write

$$f_{p_3}(s) = \tau(p_3,s) \tag{28}$$

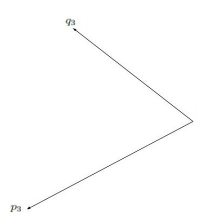
Need both p_3 and q_3 for the **electromagnetic** field



The Alexandrov set where we want to find Lagrangian density: $\alpha(p_0,q_0)$

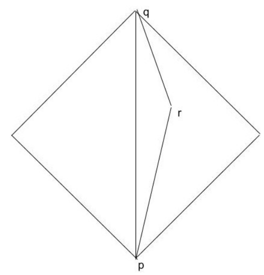


Two test Alexandrov sets. $\alpha(p_2,q_2)$ is a Lorentz boost of $\alpha(p_1,q_1)$ around midpoint.



Two sources of test functions.

Charged scalar field based on short edges



Gauge field on the edge:

$$s_1 \prec s_2 \Longrightarrow \phi(s_1, s_2) = \frac{1}{\tau(s_1, s_2)} \int_{\gamma(s_1,s_2)} \phi(s)|ds| \tag{29}$$

Scalar field at the left: $\phi(p, q)$

Scalar field at the right: $(\phi(p, r) + \phi(r, q))/2$ (Note: $\phi(r, q) = +\phi(q, r)$)

Gauge field from left to right: $(a(p, r) + a(q, r))/2$ (Note: $a(r, q) = -a(q, r)$)

Left-right contribution to the Lagrangian:

$$\int_{\alpha(p,q)} d^d r \left| \left(1 + \frac{i}{2}(a(p, r) + a(q, r))\right)\phi(p, q) - \frac{1}{2}(\phi(p, r) + \phi(r, q))\right|^2 \tag{30}$$

Scalar field at the top: $(\phi(p, q) + \phi(r, q))/2$

Scalar field at the bottom: $\phi(p, r)$

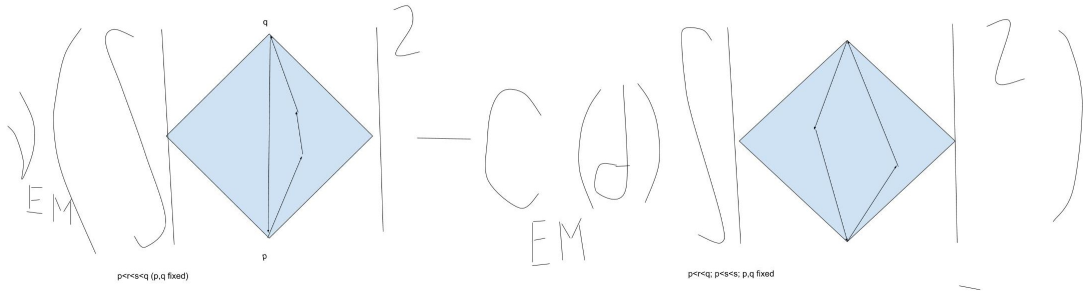
Gauge field from bottom to top: $(a(p, r) + a(p, q))/2$

Bottom-top contribution to the Lagrangian:

$$\left| \left(1 - \frac{i}{2}(a(p, r) + a(p, q))\right)\phi(r, q) - \frac{1}{2}(\phi(p, q) + \phi(r, q))\right|^2 \tag{31}$$

Total charged scalar field contribution to the Lagrangian:

$$\mathcal{L}_{scal} = \nu_{scal}\Bigg(\int_{\alpha(p,q)} d^d r \left| \left(1 - \frac{i}{2}(a(p, r) + a(p, q))\right)\phi(r, q) - \frac{1}{2}(\phi(p, q) + \phi(r, q))\right|^2$$
$$-C(d) \int_{\alpha(p,q)} d^d r \left| \left(1 + \frac{i}{2}(a(p, r) + a(q, r))\right)\phi(p, q) - \frac{1}{2}(\phi(p, r) + \phi(r, q))\right|^2 \Bigg) \tag{32}$$



Adjusting coefficients (arXiv:1805.08064)

$$\mathcal{L} = \eta_{EM}\Bigg[\int_{\alpha(p,q)}\left(d^d r\int_{\alpha(r,q)} d^d s(a(p,r)+a(r,s)+a(s,q)+a(q,p))^2\right)$$
$$-C_{EM}(d)\int \alpha(p,q)d^d r d^d s(a(p,r)+a(r,q)+a(q,s)+a(s,p))^2\Bigg] \tag{33}$$

$C_{EM}(d)$ is **very complicated**

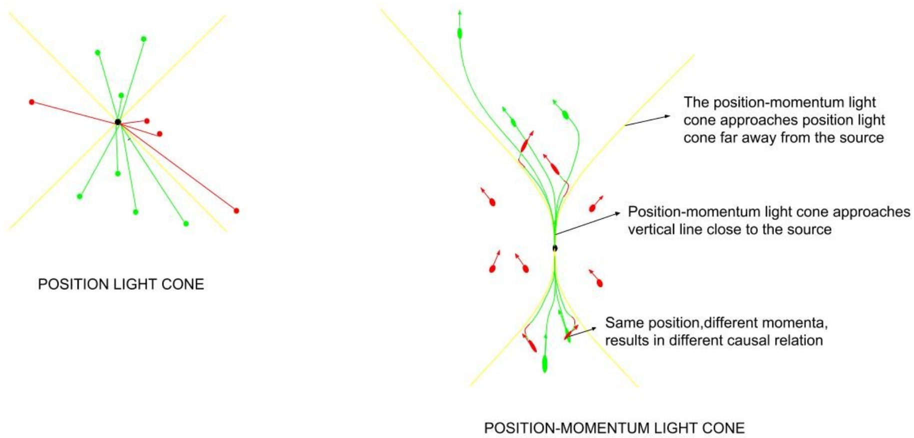
Use of test functions (arXiv:1807.07403)

$$\mathcal{L} = \sum\Bigg\{w(p_1,p_2,q_1,q_2)\Bigg[\left(\int_{\alpha(p_0,q_0)} d^d r_0\int_{\alpha(p_0,q_0)} d^d s_0(a(p_0,r_0)+a(r_0,s_0)+a(s_0,q_0)+a(q_0,p_0))^2\right)\times$$
$$\times\left(\int_{\alpha(p_1,q_1)} d^d r_1 d^d s_1(b(p_1,r_1)+b(r_1,q_1)+b(q_1,s_1)+b(s_1,p_1))^2-\right.$$
$$\left.-\int_{\alpha(p_2,q_2)} d^d r_2 d^d s_2(b(p_2,r_2)+b(r_2,q_2)+b(q_2,s_2)+b(s_2,p_2))^2\right)-$$
$$-\left(\int_{\alpha(p_0,q_0)} d^d r_0 d^d s_0(a(p_0,r_0)+a(r_0,q_0)+a(q_0,s_0)+a(s_0,p_0))^2\right)\times$$
$$\times\left(\int_{\alpha(p_1,q_1)} d^d r_1\int_{\alpha(r_1,q_1)} d^d s_1(b(p_1,r_1)+b(r_1,s_1)+b(s_1,q_1)+b(q_1,p_1))^2-\right.$$
$$\left.-\int_{\alpha(p_2,q_2)} d^d r_2\int_{\alpha(r_2,q_2)} d^d s_2(b(p_2,r_2)+b(r_2,s_2)+b(s_2,q_2)+b(q_2,p_2))^2\right)\Bigg]\Bigg\} \tag{34}$$

test function

$$b_{pq}(r,s) = \frac{1}{2}(\tau(p,r)+\tau(p,s))(\tau(q,s)-\tau(q,r)) \tag{35}$$

Momentum coordinate (arXiv: arXiv:0910.2498)



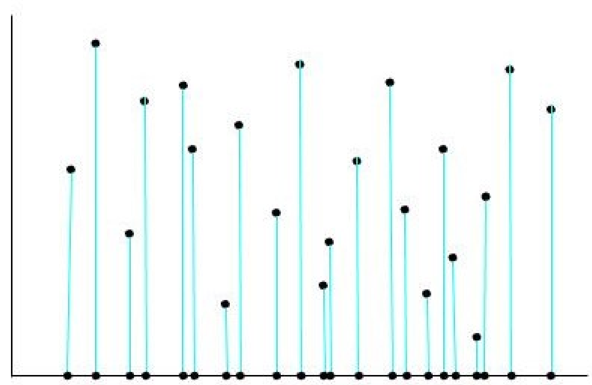
— Sprinkling in the manifold is replaced with sprinkling on a tangent bundle

— An EDGE on a spacetime-based causal set is replaced by a POINT in a phase-spacetime-based causal set

— FINITE density on phase-spacetime becomes INFINITE after projection onto the spacetime (see illustration below)

— Finite denisty on phase spacetime $\Longrightarrow$ nearest neighbor on phase spacetime $\Longrightarrow$ preferred acceleration for any given position and velocity

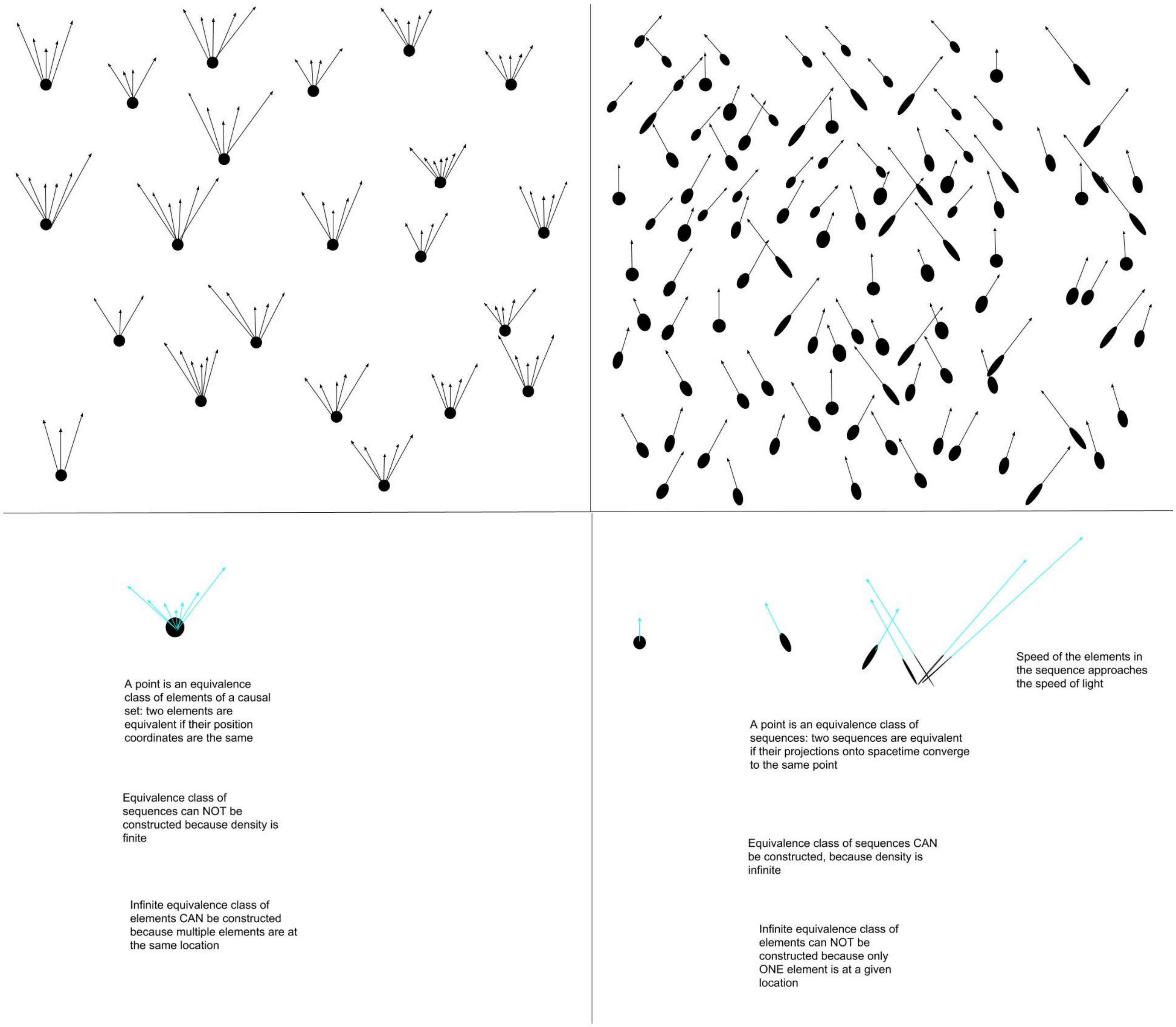
— Infinite density in spacetime $\Longrightarrow$ no nearest neighbor $\Longrightarrow$ absence of THE preferred direction corresponding to a given x



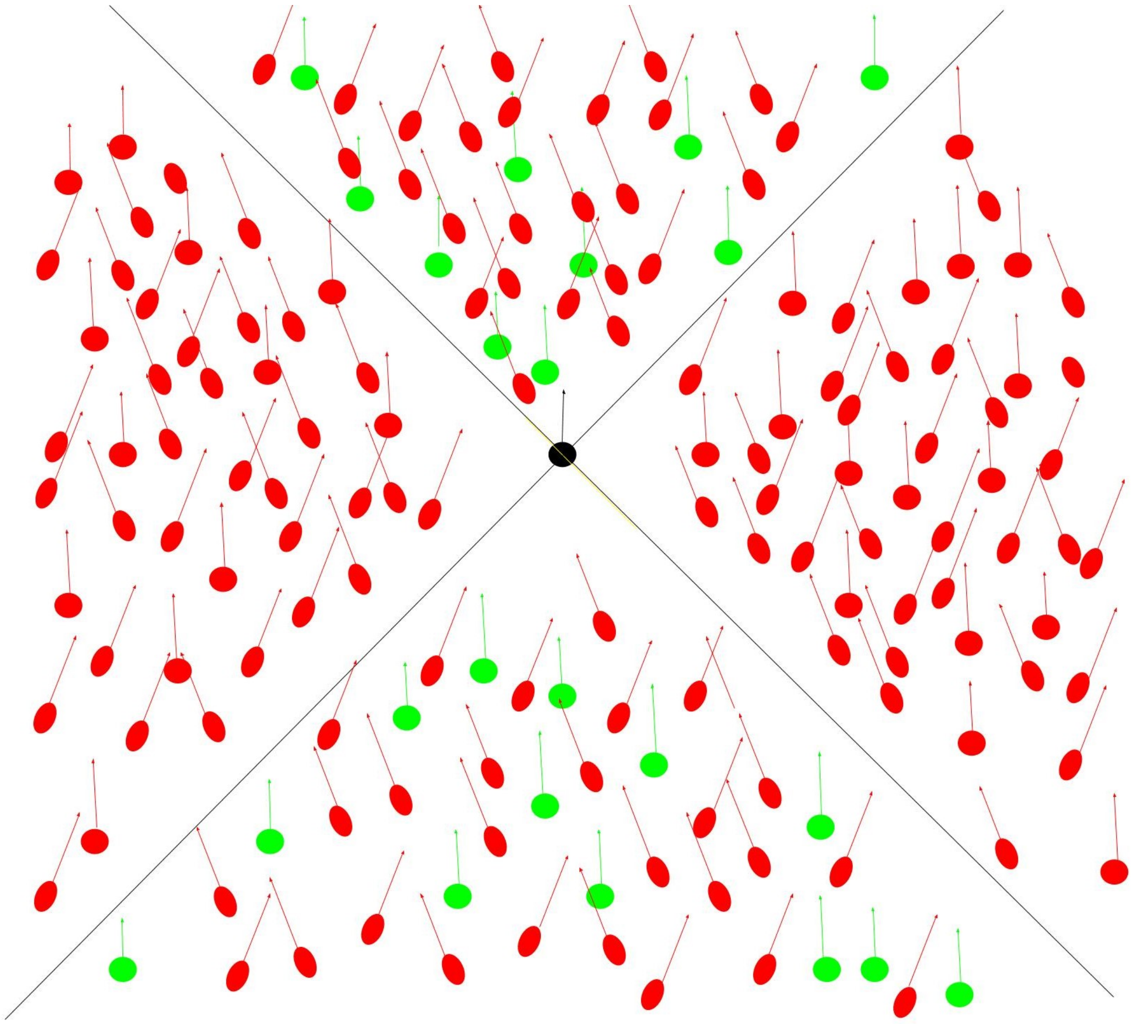
The density can become finite again IF the sprinkling on the tangent bundle is replaced with the following process:

a) Sprinkle random points on a manifold

b) On a tangent plane to each sprinkled point, sprinkle timelike tangent vectors



A point on a manifold is defined IN TERMS OF a construction involving tangent vectors (see above)



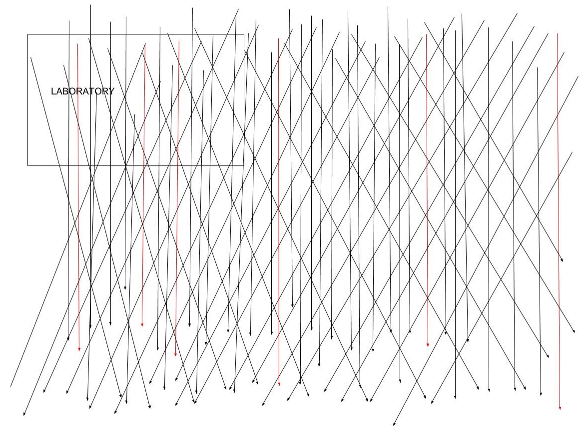
Instead of using bounded acceleration, use **parallel transport**

Due to Poisson nature, parallel transport is **almost parallel**, not exactly parallel

Still, upper bound on shift from parallelism $\ll$ upper bound on horizontal shift

NOTE: The shape of light cone is, once again, an exact cone

Long edges (arXiv:1805.11420)



— Kinetic term only has **"parallel" component** $(\partial_{\|}\phi)^2$

— In order for "parallel" component not to INADVERTEDLY produce "orthogonal" term, the **CONSTRAINT** $\partial_{\perp}^2\phi = -\epsilon(R)\phi$ is needed

— In order to impose that constraint, we need to DEFINE **orthogonal derivative** $\partial_{\perp}^2\phi$

— In order for the definition of $\partial_{\perp}^2\phi$ not to INADVERTEDLY contain $\partial_{\|}^2\phi$ term, **ALMOST-EXACT** orthogonality is needed

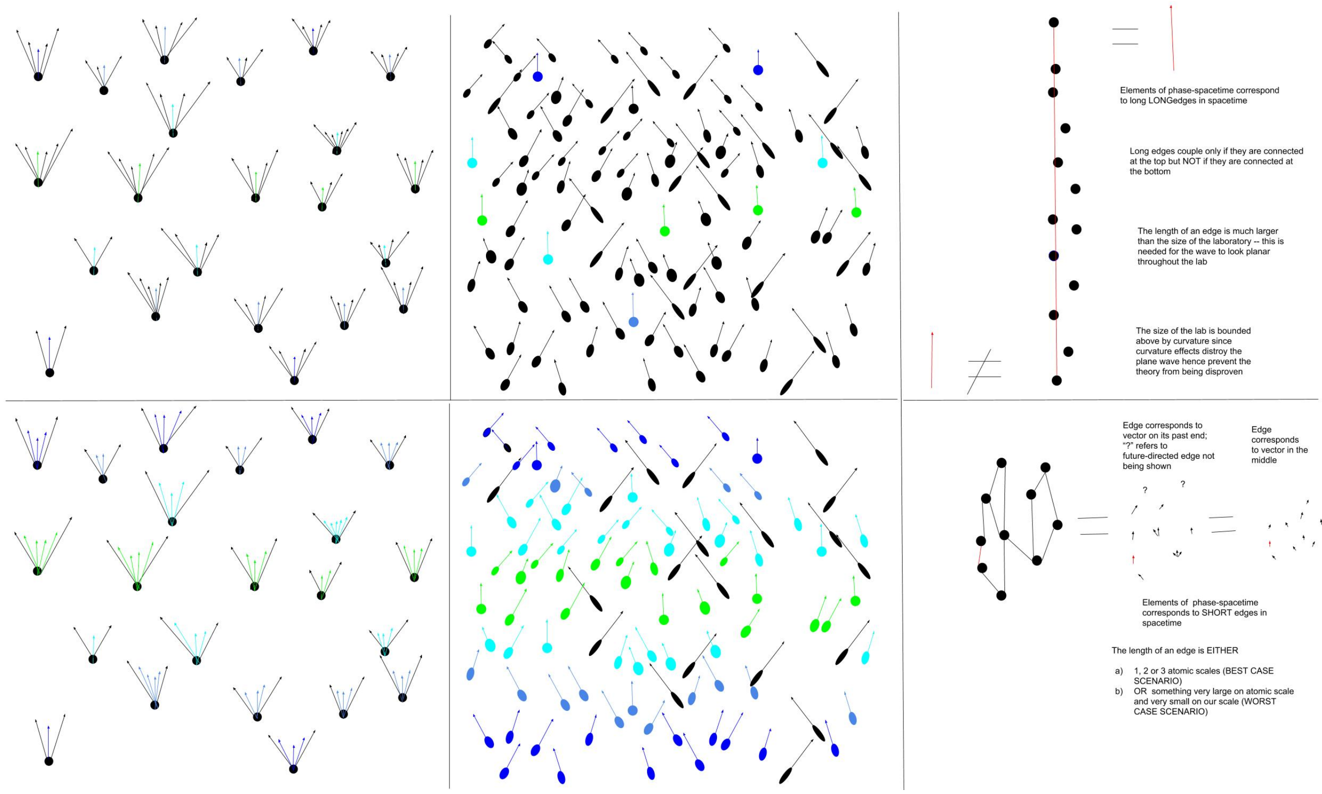
— In order to have almost-exact orthogonality DESPITE statistical fluctuations, we need

a) Very large length of edges

b) Several edges we "ignore" between any couple of edges we "pick"

$$\begin{gathered}\text{Distance between neighboring edges} \ll \text{Distance between edges we pick} \ll \\ \ll \text{size of visible objects} \ll \text{size of the laboratory} \ll \text{length of edges}\end{gathered} \tag{36}$$

$$\frac{\text{Distance between edges we count}}{\text{Distance between neighboring edges}} \gg \frac{\text{size of the laboratory}}{\text{distance between edges we count}} \tag{37}$$



How to read the above picture

— Colored edges designate edges affected by the wave

— Changing in color of the colored edges designate oscillations of the wave

— Black edges designate the edges that the wavee doesn't affect

Physical content

— In both cases the edges outside the cutoff aren't affected by the wave

— In one case I restricted it FURTHER so that only parallel edges are affected $\Longrightarrow$ no need to worry about $C(d)$ *BUT* things we *would* do might be artificial on their own right ("long edges", etc)

— In the other case, I didn't restrict it to parallel line $\Longrightarrow$ C(d) is still there $\Longrightarrow$ we can get rid of $C(d)$ by means of test functions

Causal sites (Christensen, Crane)

— Replace point by the region

— Subset relations defined AXIOMATICALLY

— See arXiv:gr-qc/0410104 for more detail

Connection between Christensen's idea and mine

— Shape of the region might determine momentum

— APART FROM momentum, their idea can also be applied to renormalization group

future work:

— Work something out more concretely on the level of position-momentum

— Generalize it to causal sites

NOTE: They haven't introduced Lagrangians (for all I know), so thats something for me to do

Conclusions

— Causal set theory prefers Poisson distributions to specific structures

— This comes at the cost of locality and other issues

— I try to address those issues by diverting from “traditional” causal set theory and inventing my own

— There might be several ways of filling those gaps and I am in the process of inventing new ones and comparing them to the other ones I invented