The properties of $D1$-branes from lattice super Yang–Mills theory using gauge/gravity duality

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1. INTRODUCTION

- Maximally supersymmetric Yang–Mills (SYM) theory in $p+1$-dimensions provide a holographic description of string theories containing Dp-branes. The p = 3 corresponds to the case where the world-volume of $D3$-branes is described in terms of $\mathcal{N}=4$ SYM in four dimensions and yields the AdS/CFT correspondence. These holographic dualities mostly assume that the gauge theory is supersymmetric, however, some holographic properties have also been observed in QCD !
- Since this duality is expected to be valid across various space-time dimensions, we consider the case of $p=1$ corresponding to 1+1-dimensional supersymmetric Yang–Mills with sixteen supercharges (known as $\mathcal{N}=(8,8)$ SYM) which is dual to non-conformal D1-branes on the supergravity side. We have already explored this system on skewed geometry in [1], here we will restrict ourselves to square torus. The 0+1-dimensional theory has been well explored [2] using lattice and momentum-cutoff methods.

2. TYPE IIA/B SUPERGRAVITY

- IIB supergravity $\leftrightarrow$ 'decoupling' limit of N coincident $D1$ branes when $1 \ll \lambda\beta^2 \ll N^{\frac{4}{3}}$
- At low temperatures ($t \approx 1/N$), one flows to a free orbifold CFT description.

From supergravity (SUGRA) calculations, the Hawking temperature, T_H is given by,

$$T_H = \frac{(7-p)U_0^{\frac{5-p}{2}}}{4\pi\sqrt{d_p\lambda}} \tag{1}$$

where, $d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\frac{7-p}{2}\right)$ The event horizon of the black hole geometry described by SUGRA metric is at $U=U_0$. In terms of field theory, one should think of U as the expectation value of the Higgs. The energy and entropy densities can be calculated using [3] as

$$\left.\frac{\mathcal{E}}{N^2}\right|_{\text{Dp}-\text{brane}} = \frac{9-p}{2^{11-2p}\pi^{\frac{13-3p}{2}}\Gamma\left(\frac{9-p}{2}\right)\lambda^2}U_0^{7-p} \tag{2}$$

$$\frac{\mathcal{E}}{\mathcal{S}} = \frac{9-p}{14-2p} \quad ; \quad c_s^2 = \frac{5-p}{9-p} \tag{3}$$

3. GAUGE THEORY

The lagrangian of the theory written in terms of bosons and fermions fields is,

$$\mathcal{L}_{\text{Bosons}} = \frac{N}{\lambda}\left(\text{Tr}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left(D_\mu X^I\right)^2 - \frac{1}{4}\left[X^I, X^J\right]^2\right]\right)$$

$$\mathcal{L}_{\text{Fermions}} = \frac{N}{4\lambda}\left(\text{Tr}\left[\Psi\left(\not{D} - \left[\Gamma^I X^I, \cdot\,\right]\right)\Psi\right]\right)$$

Here X^I with $I = 2,\ldots,9$ are the eight spacetime scalars. They are $N\times N$ hermitian matrices in the adjoint representation of the gauge group, with Ψ also transforming in the adjoint. The dimensionful 't Hooft coupling in $p<3$, $\lambda = Ng_{YM}^2$ can be used to construct dimensionless quantities : $r_\tau = \beta\lambda^{1/3-p}$ and $r_L = (\lambda^{1/3-p})L$. We define the dimensionless temperature $t = 1/r_\tau$. We wish to consider $N\to\infty$ with r_τ and r_L fixed.

4. PROCEDURE

- Topological twisted formulation [4] of $\mathcal{N}=4$ SYM in four dimensions on a A_4^* lattice followed by classical Kaluza-Klein reduction along the two spatial directions. No fine-tuning required because of four exact supercharges and the theory being super-renormalizable since $[g_{\text{YM}}]=1$
- Choice of expansion of the gauge links in the moduli space : $A_2^* \to$ square lattice. Deconfinement transition dual to topological transition between two different black hole solutions around $r_L^2 \approx 2.45 r_\tau$.
- We restrict ourselves to the $D1$-phase (guided by the deconfinement transition) and calculate the trace of the energy momentum tensor $\Delta = \mathcal{E} - \mathcal{P}$, equation of state and a transport coefficient which is fixed by thermodynamics, *i.e.* c_s^2.

5. EXPECTATIONS

The numerical calculations are being performed using parallel software `SUSY LATTICE` developed in [5]. After locating the phase transitions, we will be to able to check if the speed of sound is given by $c_s^2 = \frac{1}{2}$ over the entire region where $D1$-description is valid. This is a result from the gravity analysis. One would ideally expect that, $c_s^2 = \frac{1}{2}$, when one has a conformal fluid in (2+1)-dimensions. It is interesting that this result is obtained in a two dimensional SYM theory in a region where $D1$ description is valid which has no connection to conformal three-dimensional theory. In two dimensions, there is no shear viscosity, and when the description is in terms of CFT at very low temperatures, the bulk viscosity vanishes as well. We hope that the lattice studies can help in understanding the properties at finite couplings and strong couplings.

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7. REFERENCES

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