Investigation of the 1+1 dimensional Thirring model using the method of matrix product states

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Outline

- Preliminaries
- Lattice simulations, the MPS and DMRG
- Phase structure of the Thirring model
- Remarks and outlook

Preliminaries

Motivation

- Tensor network for lattice field theory
- Topological phase transitions
- Real-time dynamics (long-term goal)

The 1+1 dimensional Thirring model and its duality to the sine-Gordon model

$$S_{\mathrm{Th}}\left[\psi,\bar{\psi}\right] = \int d^2x \left[\bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi - m_0\bar{\psi}\psi - \frac{g}{2}\left(\bar{\psi}\gamma_{\mu}\psi\right)^2\right]$$

$\boxed{\text{strong-weak duality } g \leftrightarrow \kappa}$

$$S_{\mathrm{SG}}\left[\phi\right] = \int d^2x \left[\frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x) + \frac{\alpha_0}{\kappa^2}\cos\left(\kappa\phi(x)\right)\right]$$

$$\xrightarrow{\phi\rightarrow\phi/\kappa,\ \text{and}\ \kappa^2=t} \frac{1}{t}\int d^2x \left[\frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}\phi(x) + \alpha_0\cos\left(\phi(x)\right)\right]$$

Works in the zero-charge sector

Dualities and phase structure

Thirring	sine-Gordon	XY
g	$\frac{4\pi^2}{t} - \pi$	$\frac{T}{K} - \pi$

★ The K-T phase transition at $T \sim K\pi/2$ in the XY model.

$g \sim -\pi/2$, Coleman's instability point

★ The phase boundary at $t \sim 8\pi$ in the sine-Gordon theory.

➡ The cosine term becomes relevant or irrelevant.

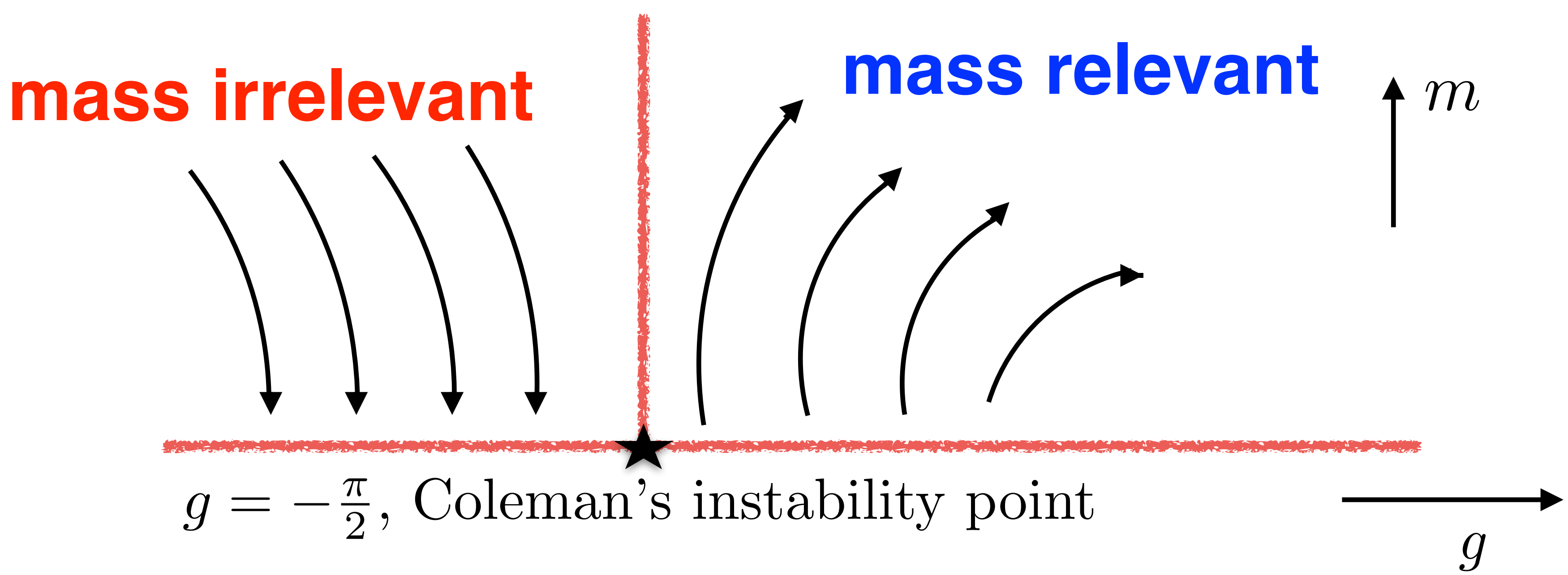
Thirring	sine-Gordon
$\bar{\psi}\gamma_\mu\psi$	$\frac{1}{2\pi}\epsilon_{\mu\nu}\partial_\nu\phi$
$\bar{\psi}\psi$	$\frac{\Lambda}{\pi}cos\phi$

RG flows of the Thirring model

$$\beta_g \equiv \mu \frac{dg}{d\mu} = -64\pi \frac{m^2}{\Lambda^2},$$

$$\beta_m \equiv \mu \frac{dm}{d\mu} = \frac{-2(g+\frac{\pi}{2})}{g+\pi} m - \frac{256\pi^3}{(g+\pi)^2 \Lambda^2} m^3$$

★ Massless Thirring model is a conformal field theory



Lattice simulations, the MPS and DMRG

Operator formalism and the Hamiltonian

- Operator formaliam of the Thirring model Hamiltonian

C.R. Hagen, 1967

$$H_{\mathrm{Th}} = \int dx \left[-i\bar{\psi}\gamma^1\partial_1\psi + m_0\bar{\psi}\psi + \frac{g}{4}\left(\bar{\psi}\gamma^0\psi\right)^2 - \frac{g}{4}\left(1+\frac{2g}{\pi}\right)^{-1}\left(\bar{\psi}\gamma^1\psi\right)^2 \right]$$

- Staggering, J-W transformation ($S_j^{\pm} = S_j^x \pm iS_j^y$):

J. Kogut and L. Susskind, 1975; A. Luther, 1976

$$\bar{H}_{XXZ} = \nu(g)\left[-\frac{1}{2}\sum_n^{N-2}\left(S_n^+S_{n+1}^- + S_{n+1}^+S_n^-\right) + a\tilde{m}_0\sum_n^{N-1}(-1)^n\left(S_n^z+\frac{1}{2}\right) + \Delta(g)\sum_n^{N-1}\left(S_n^z+\frac{1}{2}\right)\left(S_{n+1}^z+\frac{1}{2}\right) \right]$$

$$\nu(g) = \frac{2\gamma}{\pi\sin(\gamma)},\quad \tilde{m}_0 = \frac{m_0}{\nu(g)},\quad \Delta(g) = \cos(\gamma),\quad \text{with } \gamma = \frac{\pi - g}{2}$$

$$\boxed{\bar{H}_{XXZ}^{(\mathrm{penalty})} = \bar{H}_{XXZ} + \lambda\left(\sum_{n=0}^{N-1} S_n^z - \boxed{S_{\mathrm{target}}}\right)^2}$$

projected to a sector of total spin

JW-trans of the total fermion number,
Bosonise to topological index in the SG theory.

Simulation details

- Matrix product operator for the Hamiltonian

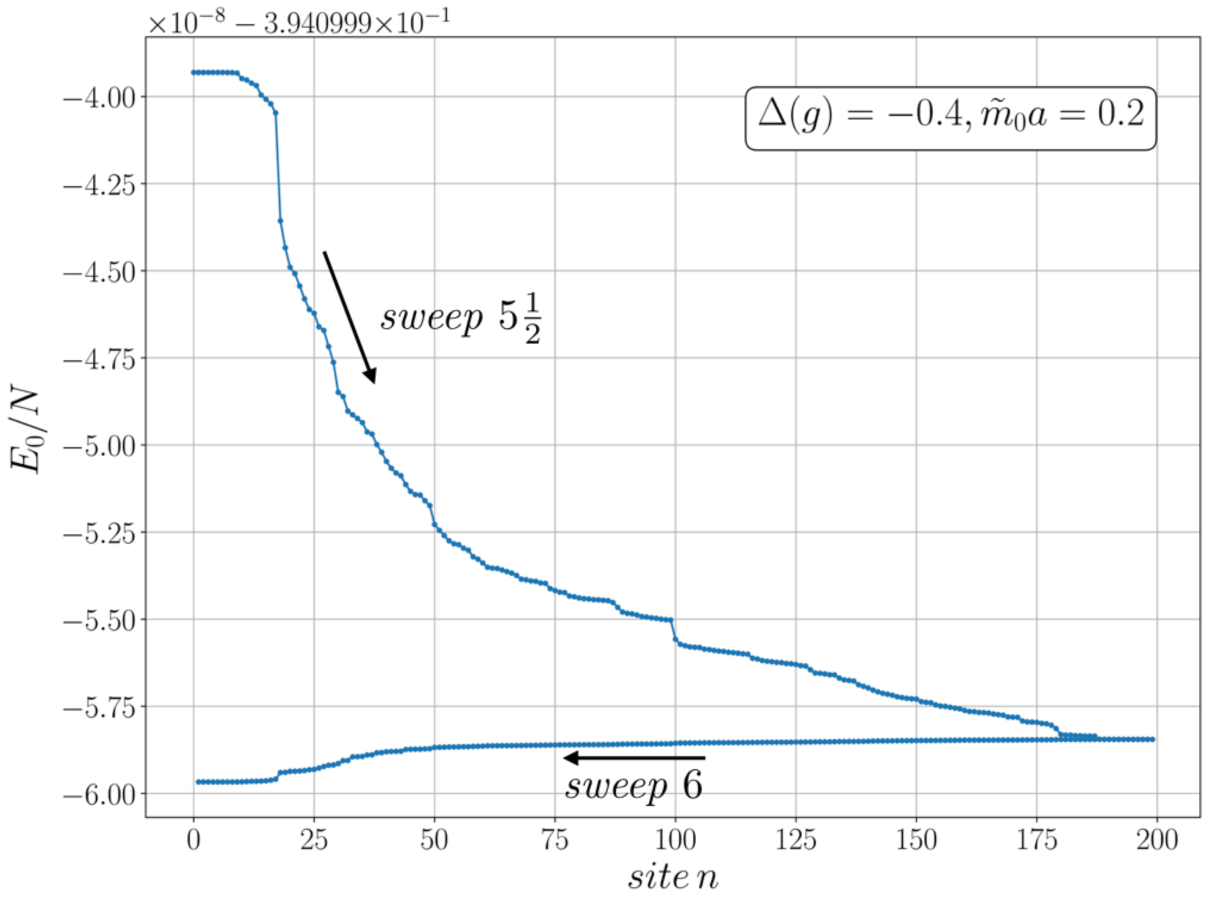
$$W^{[n]} = \begin{pmatrix} 1_{2\times2} & -\frac{1}{2}S^{+} & -\frac{1}{2}S^{-} & 2\lambda S^{z} & \Delta S^{z} & \beta_n S^{z} + \alpha 1_{2\times2} \\ 0 & 0 & 0 & 0 & 0 & S^{-} \\ 0 & 0 & 0 & 0 & 0 & S^{+} \\ 0 & 0 & 0 & 1 & 0 & S^{z} \\ 0 & 0 & 0 & 0 & 0 & S^{z} \\ 0 & 0 & 0 & 0 & 0 & 1_{2\times2} \end{pmatrix}$$

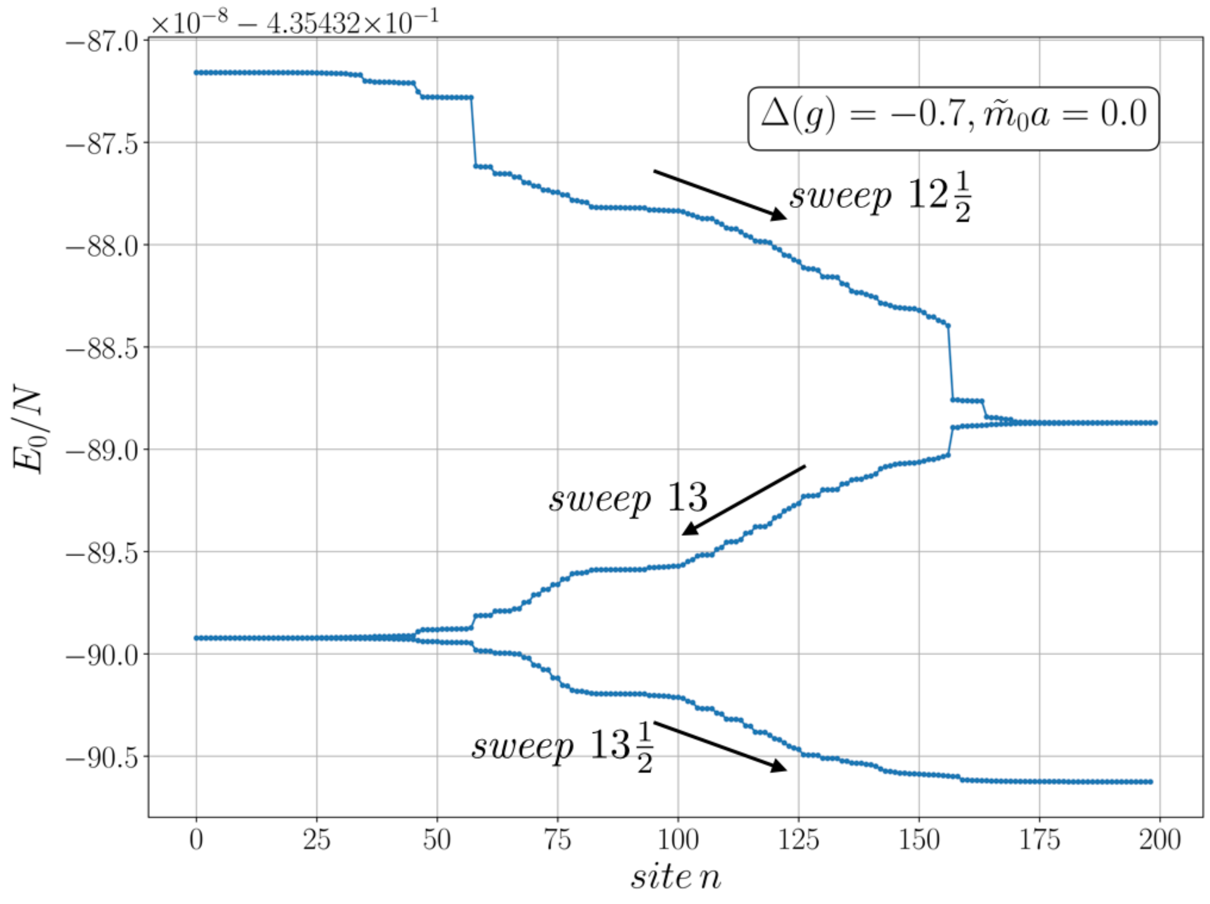
$$\beta_n = \Delta + (-1)^n\, \tilde{m}_0 a - 2\lambda\, S_{\text{target}}\,,\ \alpha = \lambda\left(\frac{1}{4} + \frac{S^2_{\text{target}}}{N}\right) + \frac{\Delta}{4}$$

- Choices of parameters
  - ★ Twenty values of $\Delta(g)$, ranging from -0.9 to 1.0
  - ★ $\tilde{m}_0 a = 0.0, 0.1, 0.2, 0.3, 0.4$ (run 1)
  - ★ $\tilde{m}_0 a = 0.005, 0.01, 0.02, 0.03, 0.04, 0.06, 0.08, 0.13, 0.16$ (run 2)
  - ★ Bond dimension $D = 50, 100, 200, 300, 400, 500, 600$
  - ★ System size $N = 400, 600, 800, 1000$

Convergence of DMRG

- Start from random tensors at D=50, then go up in D
- DMRG converges fast at $\tilde{m}_0 a \neq 0$ and $\Delta(g) \gtrsim -0.7$



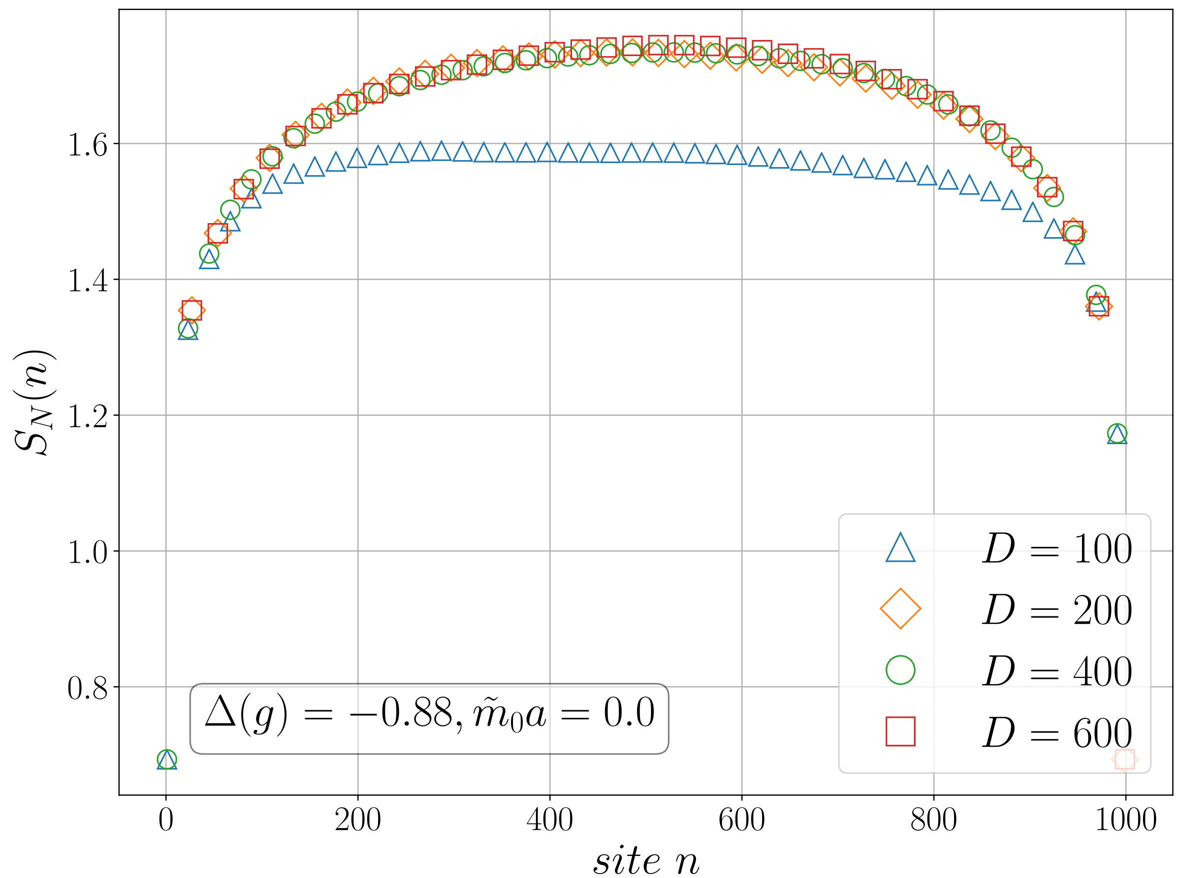


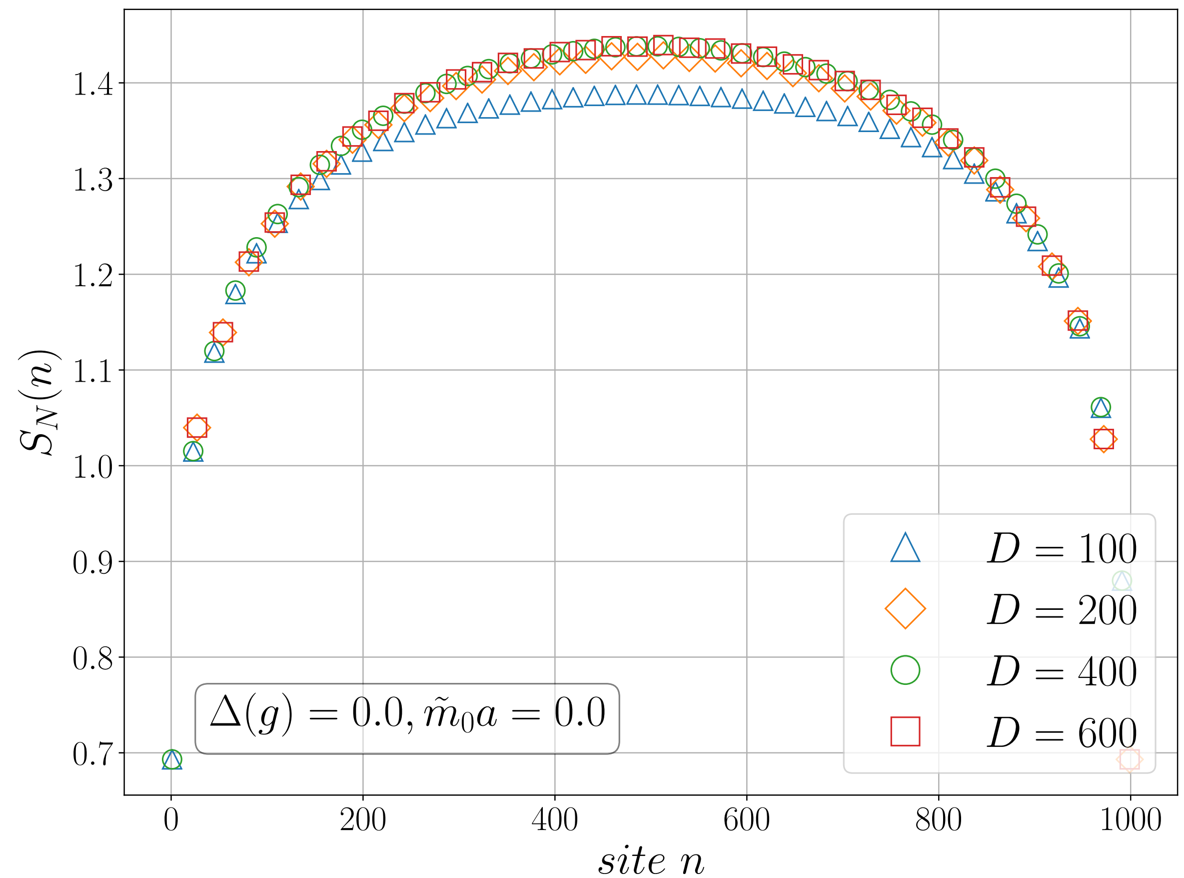
Results for the phase structure

Entanglement entropy

Calabrese-Cardy scaling and the central charge

$$S_N(n) = \frac{c}{6} \ln\left[\frac{N}{\pi}\sin\left(\frac{\pi n}{N}\right)\right] + k$$





★ Calabrese-Cardy scaling observed at all values of $\Delta(g)$ for $\tilde{m}_0 a = 0$

Entanglement entropy

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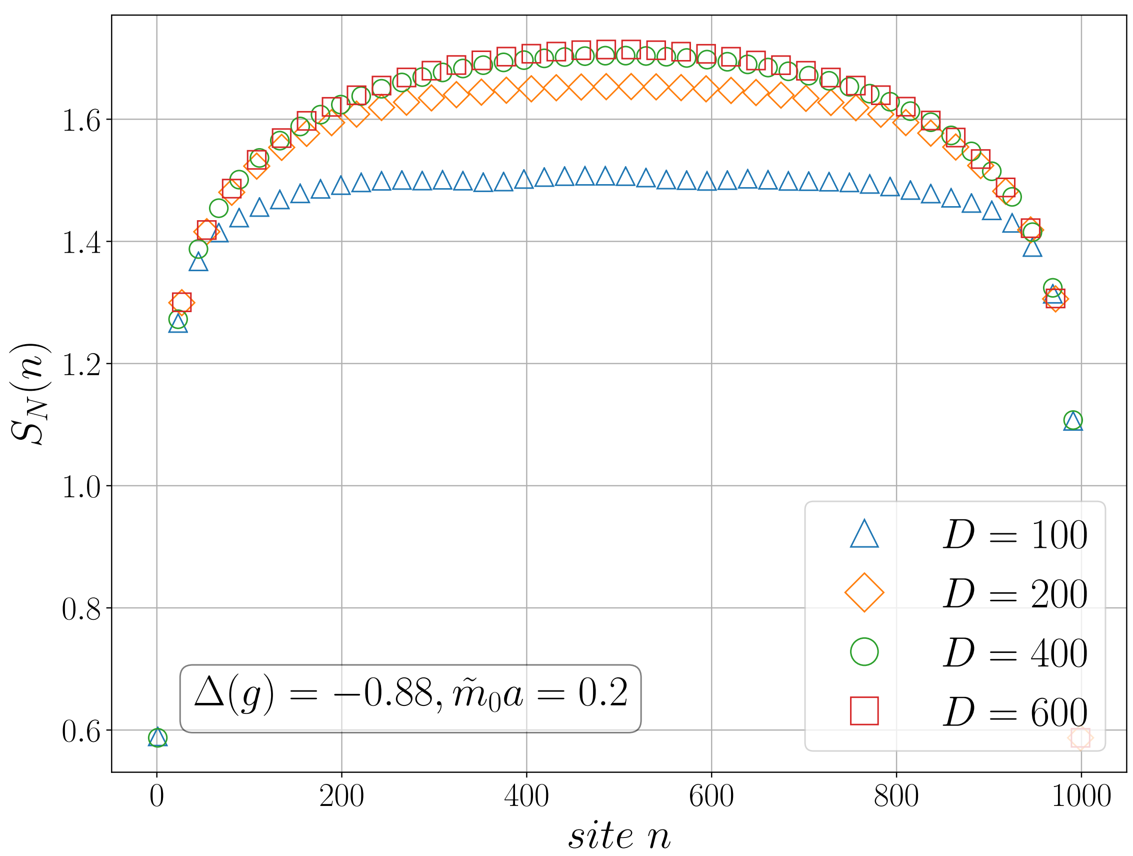

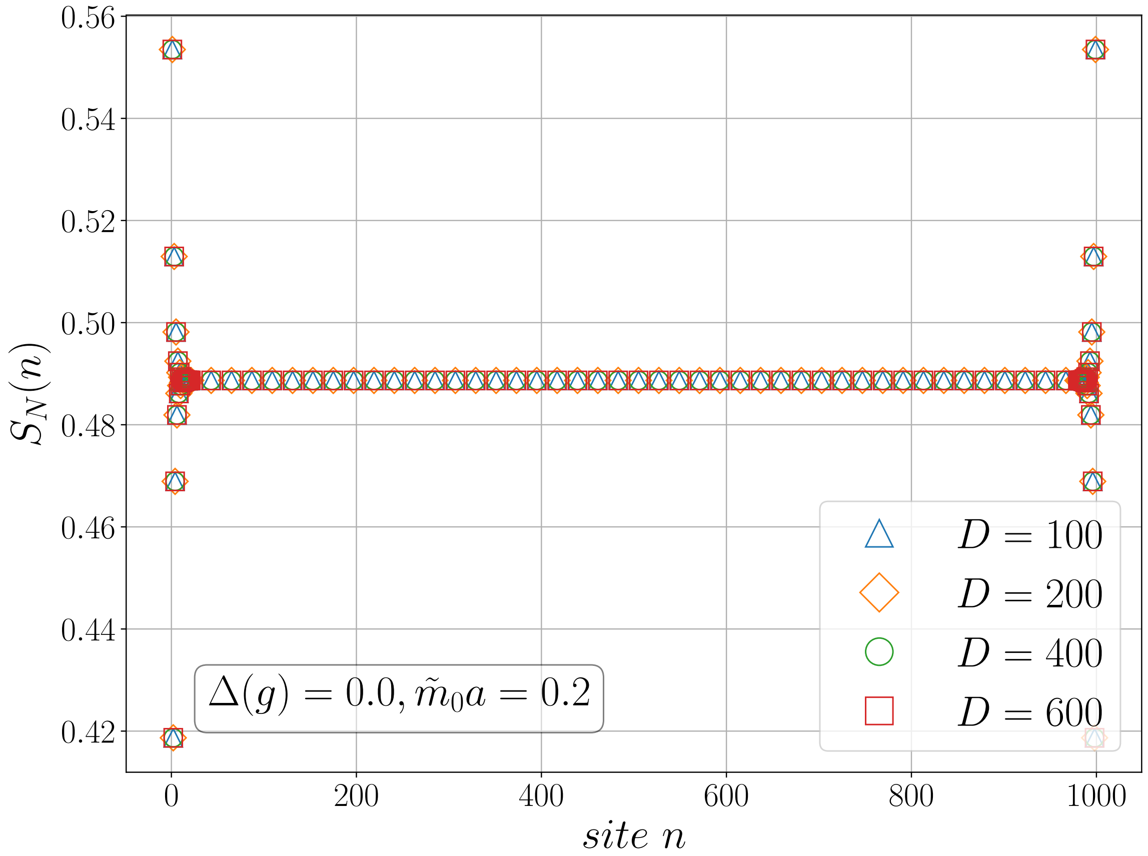

★ Calabrese-Cardy scaling observed at $\Delta(g) \lesssim -0.7$ for $\tilde{m}_0 a \neq 0$

Entanglement entropy

Calabrese-Cardy scaling and the central charge

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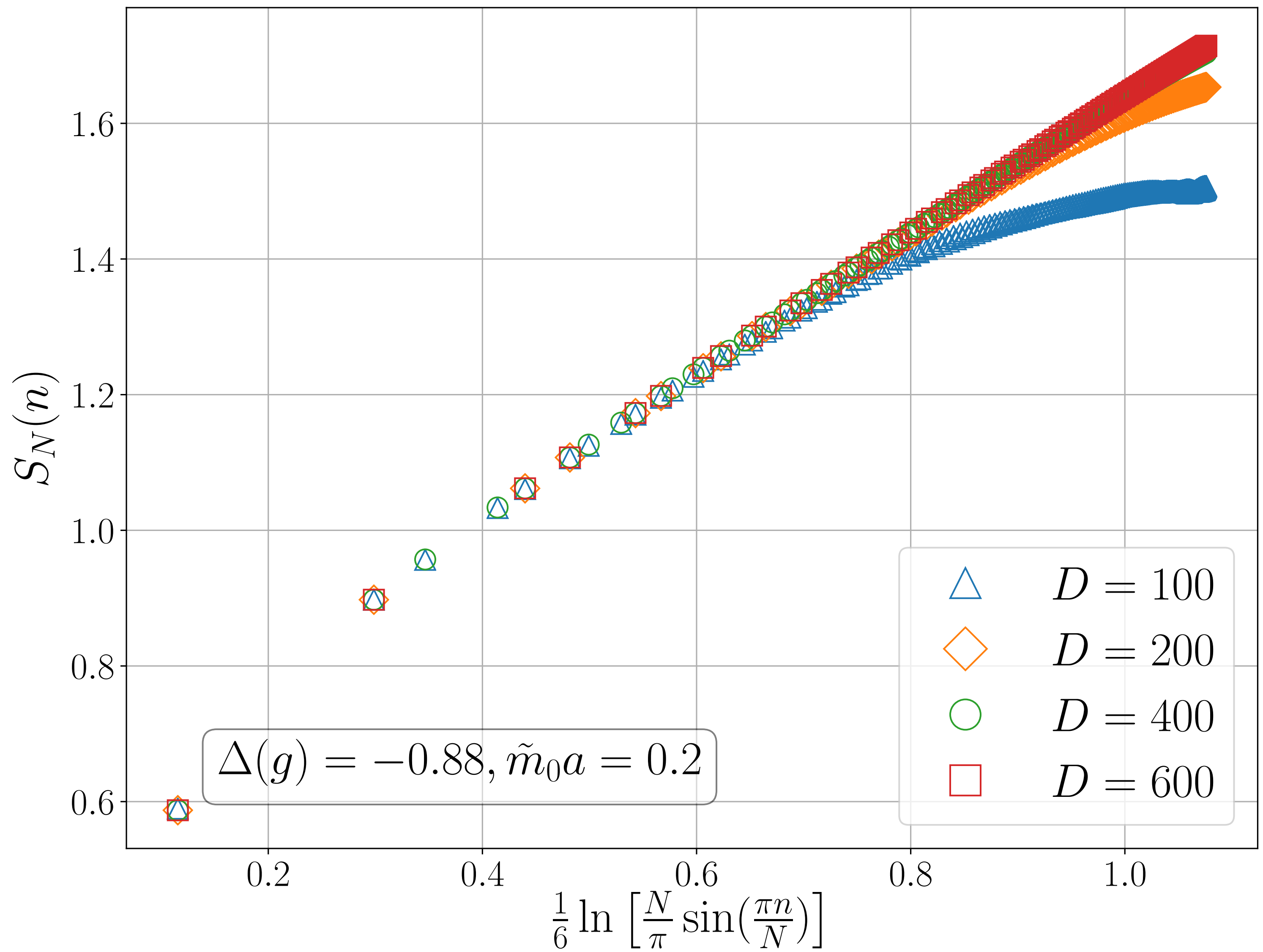


★ Central charge is unity in the critical phase

Soliton correlators

S. Mandelstam, 1975

$$\underbrace{\psi^\dagger_\alpha(x)\psi_\alpha(y)}_{\alpha=\pm} = \mp\, i|2\pi(x-y)|^{-1}|c\mu(x-y)|^{-\beta^2 g^2/(2\pi)^3}$$
$$\times : \exp\left\{ -2\pi i\beta^{-1}\int_x^y d\xi\,\dot{\phi}(\xi) \mp \frac{1}{2} i\beta\left[\phi(y)-\phi(x)\right] + O(x-y)^2 \right\} :$$

Soliton operators

connecting vortex and anti-vortex

★ Power-law in the critical phase
★ Exponential-law in the gapped phase

Vertex operators

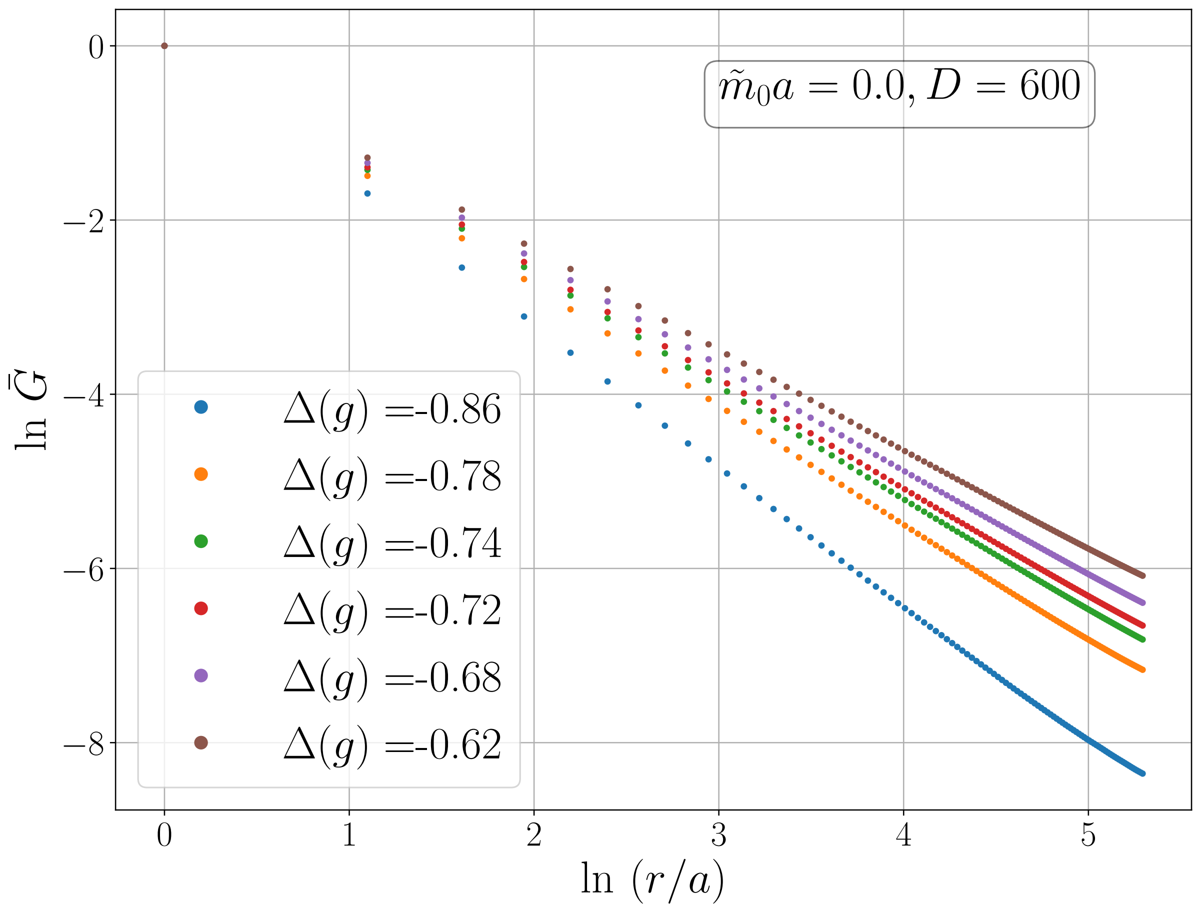
Power-law

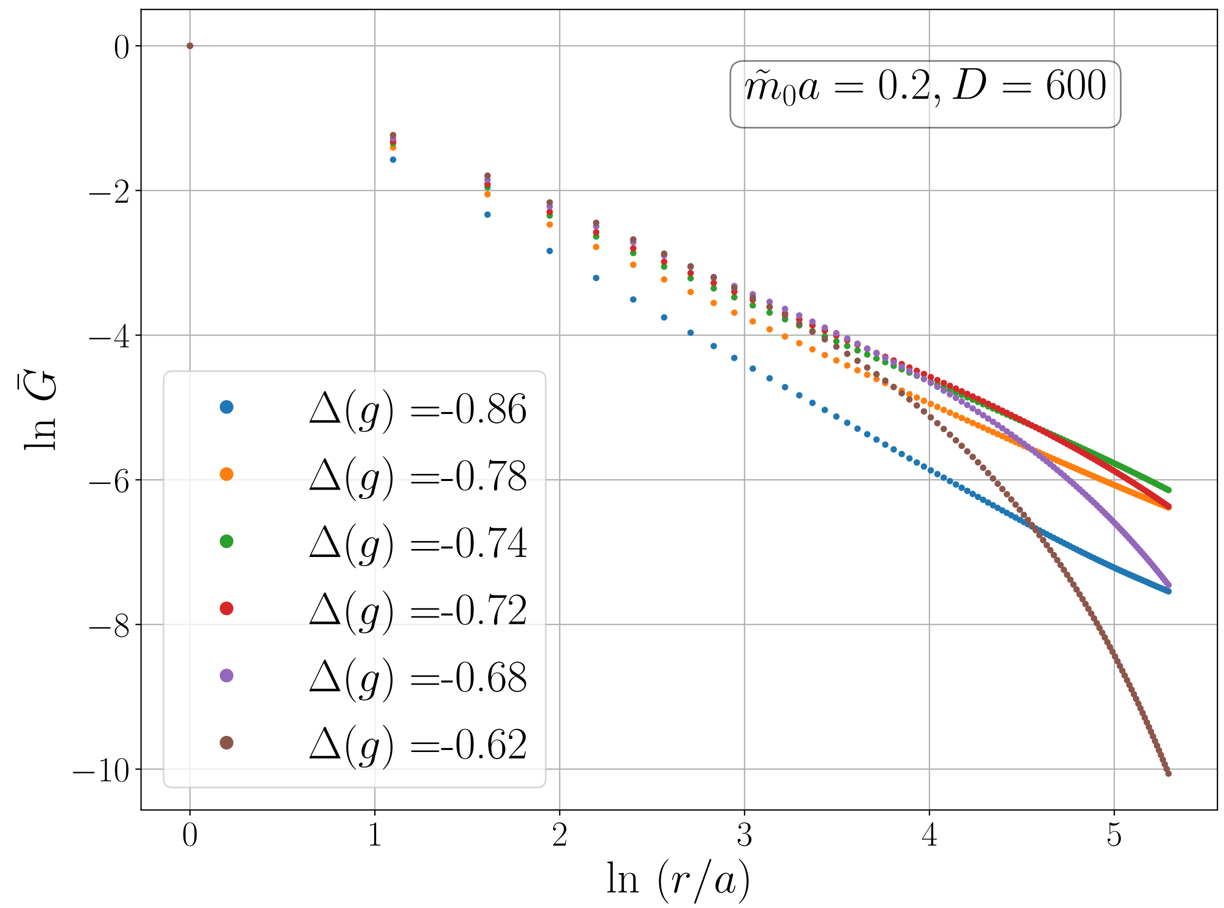
Jordan-Wigner transformation

$$S^+_m e^{i\pi\sum_{j=m+1}^{n-1} S^z_j} S^-_n$$

Soliton correlators

$$G(r) = \langle \psi_+^\dagger(r)\psi_+(0)\rangle, \;\; \bar{G}(r) = G(r)/G(0)$$

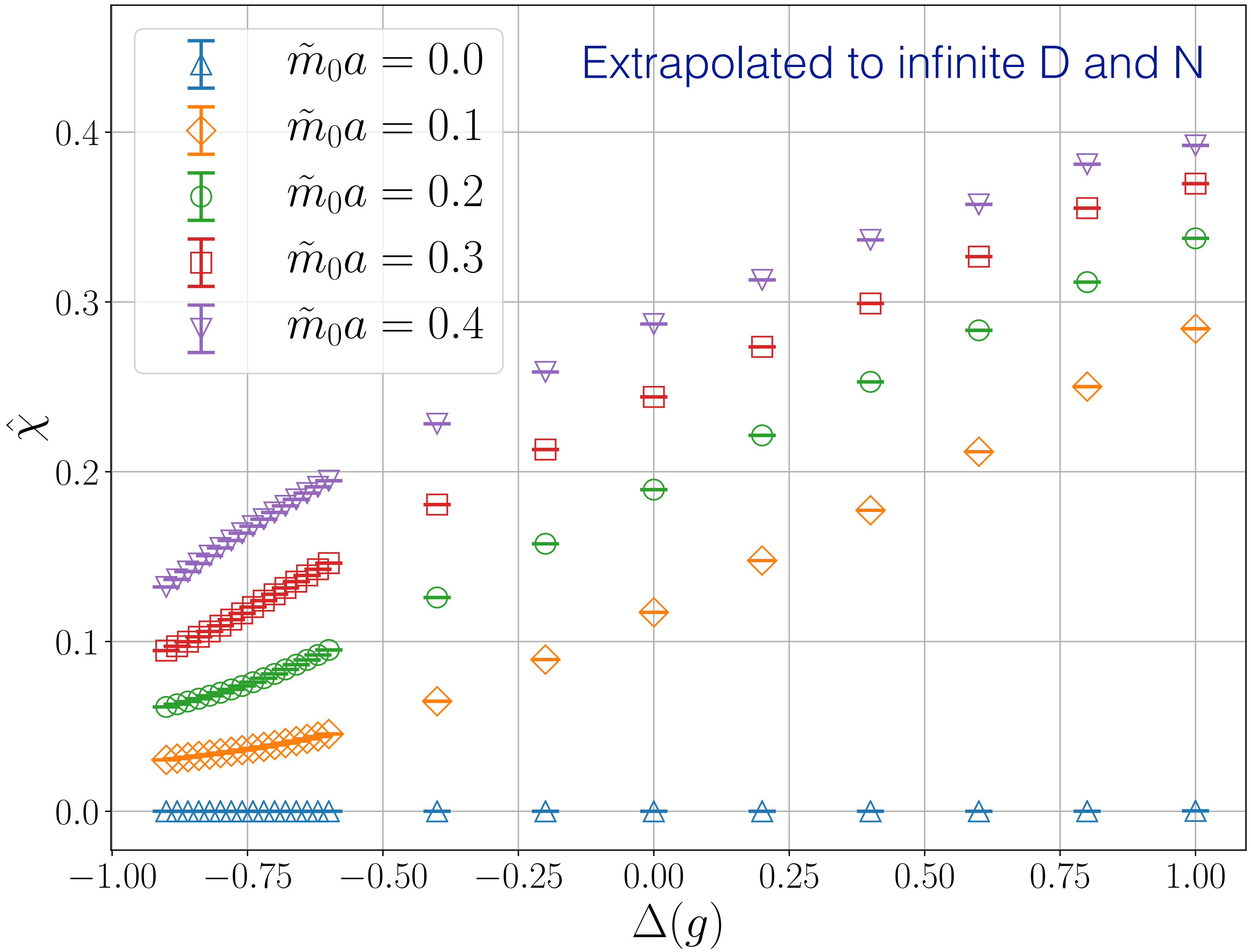




★ Evidence for BKT phase transition

Chiral condensate

$$\hat{\chi} = a\left|\langle\bar{\psi}\psi\rangle\right| = \frac{1}{N}\left|\sum_n (-1)^n S_n^z\right|$$



★ Zero-mass results reproduced using uMPS

Chiral condensate

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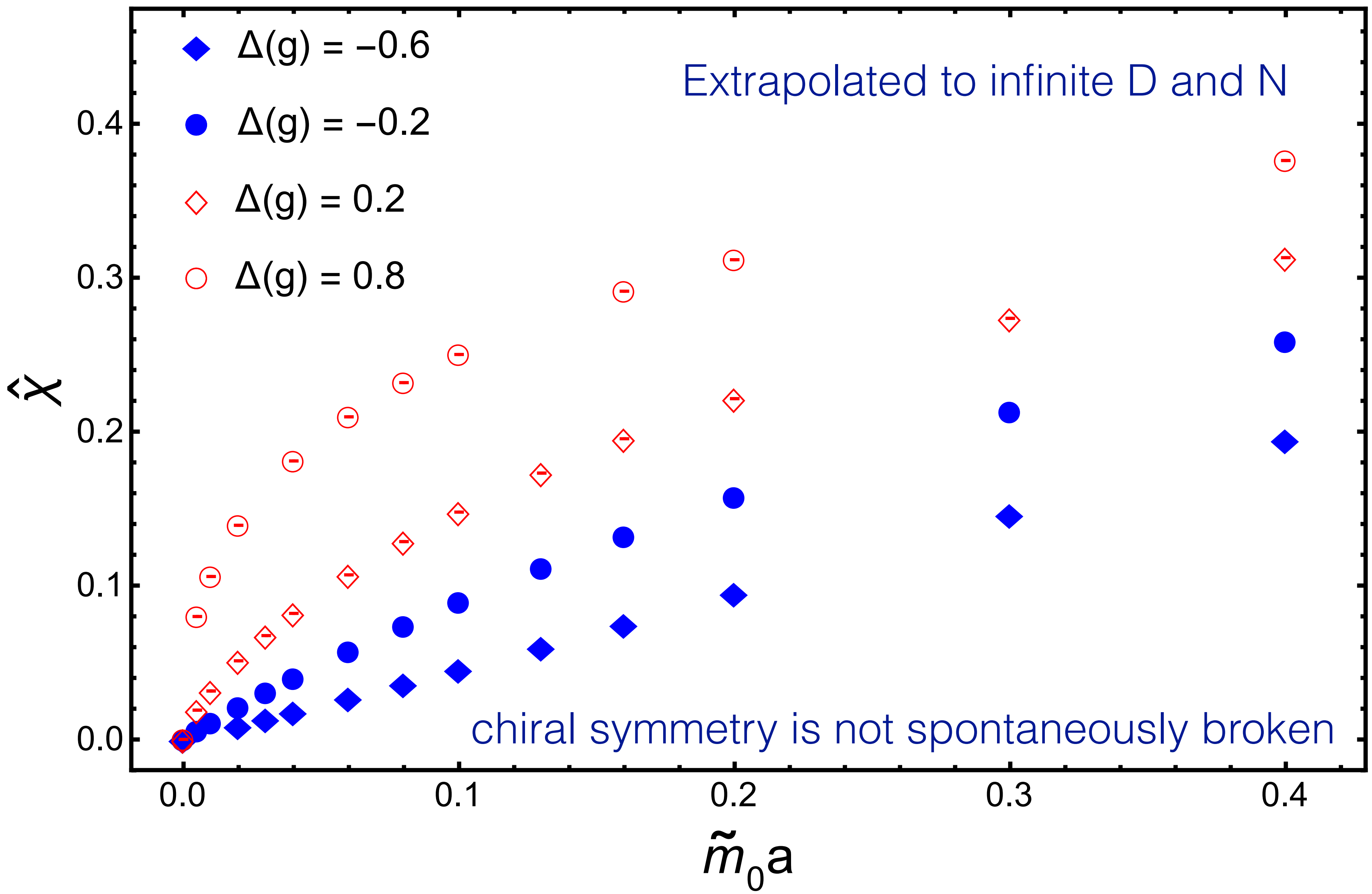
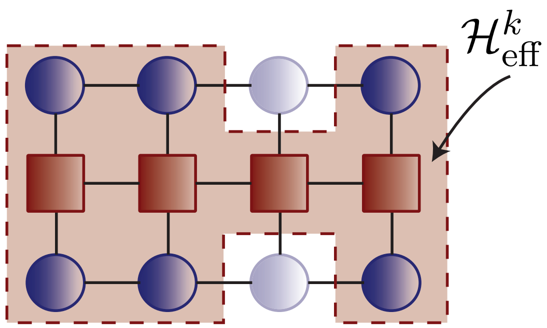
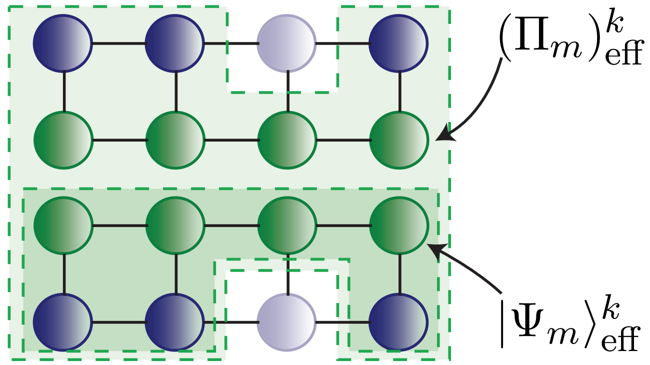


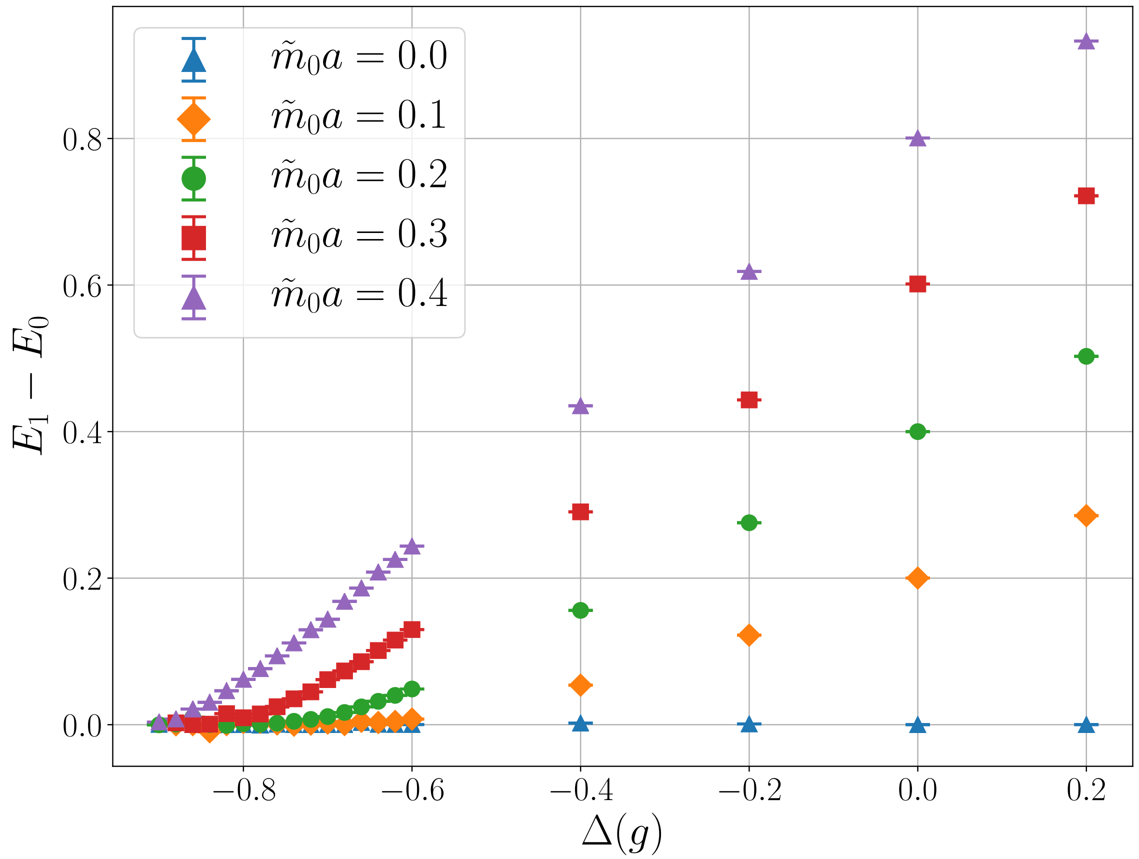
★ Curvature at small mass in the gapped phase

Mass gap

$$H_{\text{eff}}[M] = \Pi_{M-1} \dots \Pi_0 H \Pi_0 \dots \Pi_{M-1} = H - \sum_{k=0}^{M-1} E_k |\Psi_k\rangle\langle\Psi_k|$$

$\mathcal{H}^k_{\text{eff}}[\text{M}]=$ 

$+\sum_{m=0}^{M-1} E_m \times$ 



Phase structure of the Thirring model

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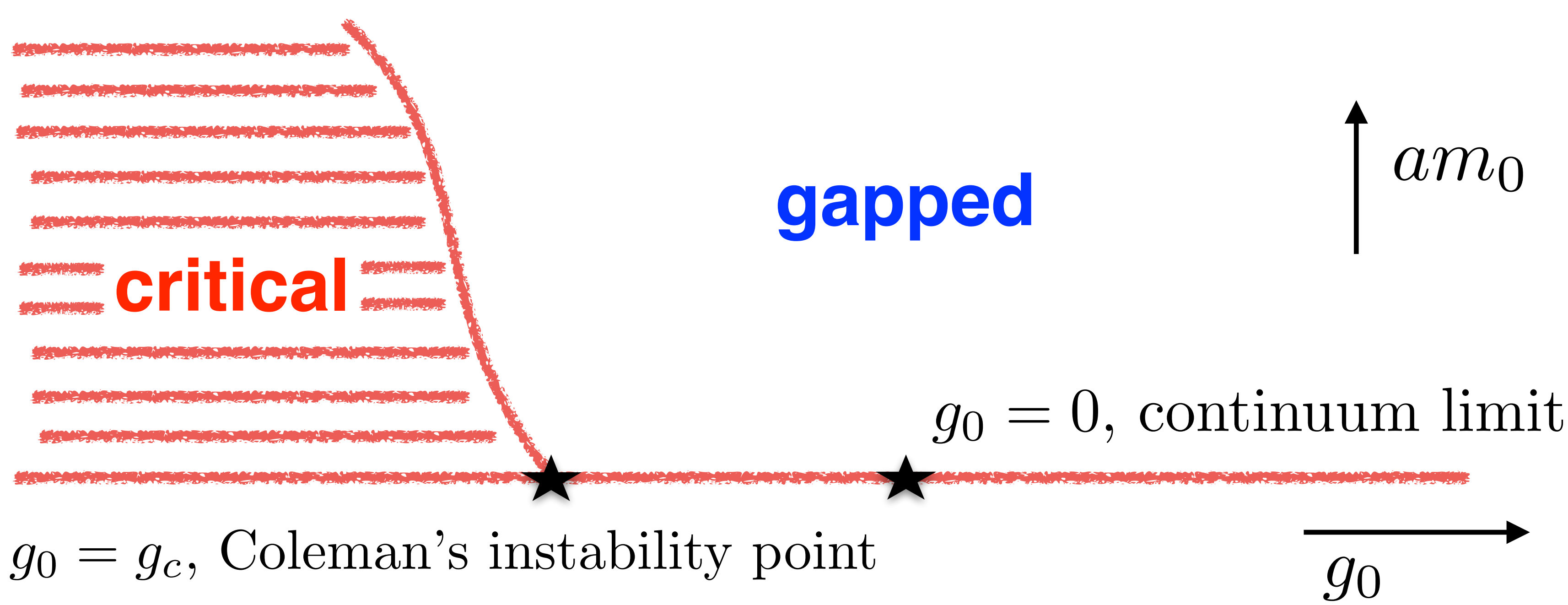
Massless Thirring model is a conformal field theory



Conclusion and outlook

- Evidence for BKT phase transition found using MPS
  - ★ Chiral symmetry is not spontaneously broken

- Current work for more detailed probe of the phase structure:
  - ★ More simulations at small fermion mass
  - ★ Eigenvalue spectrum of the transfer matrix

- Future projects:
  - ★ Chemical potential
  - ★ Real-time evolution with a quench