π-K scattering in the maximal isospin channel

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Introduction, Motivation

- πK scattering length in the maximal isospin channel
- Testing SU(3) ChPT
- Dirac collaboration: πK atom Adeva et al(2016)
- Strong energy splitting Schweizer(2004):

$$\Delta E_{h,1} = -2\alpha^3_{\mathrm{QED}}\mu^2_{\pi K}\left(a_0^- + a_0^+\right)$$

  - a_0^-, a_0^+: isospin-odd(even) scattering length

$$\mathscr{A}^+ = \frac{1}{3}\left(\mathscr{A}^{1/2}(s,t,u) + 2\mathscr{A}^{3/2}(s,t,u)\right)$$
$$\mathscr{A}^- = \frac{1}{3}\left(\mathscr{A}^{1/2}(s,t,u) - \mathscr{A}^{3/2}(s,t,u)\right)$$

  - $\mu_{\pi K}$: reduced mass of the πK system (109 MeV)

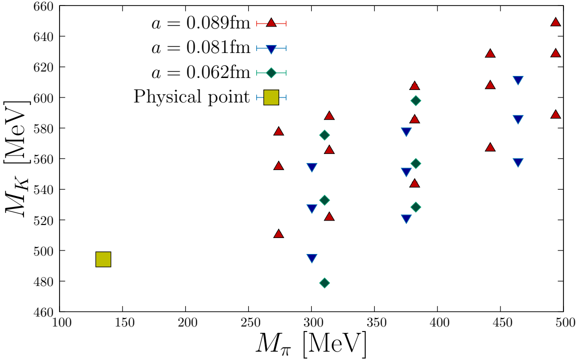
LQCD status for the scattering length a_0

- Staggered sea, domain wall valence Beane et al.(2006)
- Full 2+1 asqtad dynamical staggered Zu(2012)
- 2+1 O(a) improved Wilson with Iwasaki gauge Sasaki et al.(2014)
- Möbius Domain Wall Janowski et al.(2014)
- This conference: (2+1) dynamical Wilson Clover Brett et al.(2018)

This talk:
Chiral+Continuum extrapolation

Numerical setup

- 2+1+1 twisted mass configurations with Iwasaki gauge (*ETMC*)
- Mixed action: Osterwalder-Seiler valence quarks



Hadron Interactions from Lattice QCD

Lüscher's finite volume method (M.Lüscher(1991))

- Energy shift of two-particle system in "finite box"
  $\Delta E_L = E_{\pi K} - M_\pi - M_K$
- Infinite volume scattering length: a_0
- Using finite range expansion to the energy shift one determines the scattering length

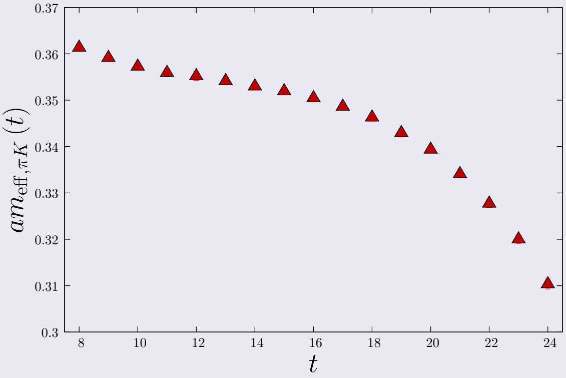
$$\Delta E_L = -\frac{2\pi a_0}{\mu_{\pi K} L^3}\left(1 + c_1 \frac{a_0}{L} + c_2 \frac{{a_0}^2}{L^2}\right) + \mathcal{O}(L^{-6}).$$

- High precision in obtaining ΔE_L: S-LapH smearing
  Peardon et al.(2009)

πK interacting system

$$C_{\pi K}\left(t-t'\right)=\langle\mathscr{O}\left(\pi K\right)\left(t\right)\mathscr{O}\left(\pi K\right)^{\dagger}\left(t'\right)\rangle$$

Problem: Thermal states due to finite T



πK interacting system

$$C_{\pi K}(t-t') = \langle \mathcal{O}(\pi K)(t)\,\mathcal{O}(\pi K)^{\dagger}(t')\rangle$$

Problem: Thermal states due to finite T

- Spectral decomposition of $C_{\pi K}(t)$

$$C_{\pi K}(t) = |\langle 0|\pi^{+}K^{+}|\pi K\rangle|^{2}\left(e^{-E_{\pi K}t} + e^{-E_{\pi K}(T-t)}\right) +$$
$$\left(|\langle K|\pi^{+}K^{+}|\pi\rangle|^{2} + |\langle \pi|\pi^{+}K^{+}|K\rangle|^{2}\right)\left(e^{-M_{\pi}T}e^{(M_{\pi}-M_{K})t} + e^{-M_{K}T}e^{(M_{K}-M_{\pi})t}\right)$$
$$+\cdots$$

- Signal distorted (Especially for large t)
- Pollution is t dependent
- Cannot be removed by computing $\Delta_t C_{\pi K}(t)$
- However: Determine M_{π}, M_{K} using $C_{\pi}(t), C_{K}(t)$

πK interacting system

Thermal states removal with the help of M_π and M_K

$$A\left(e^{-M_\pi T} e^{(M_\pi - M_K)t} + e^{-M_K T} e^{(M_K - M_\pi)t}\right)$$

Weighting Shifting Dudek et al.(2012)

- Weighting the correlation function with one of the thermal pollutions: $C^w_{\pi K}(t) = e^{(M_K - M_\pi)t} C_{\pi K}(t)$
- Red part will be time independent
- $\tilde{C}^w_{\pi K}(t) = \Delta_t C^w_{\pi K}(t)$

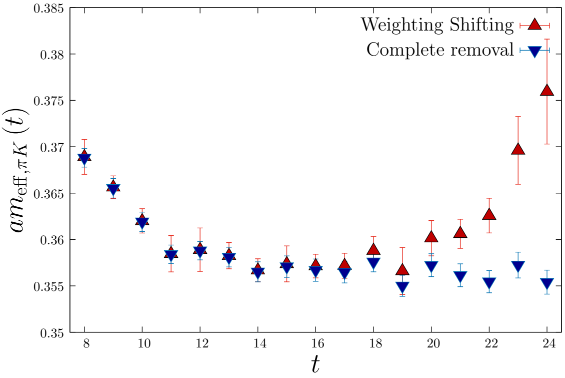
Complete removal

- Make the whole pollution time independent
- Weight with the inverse of Red+Blue:

$$C^w_{\pi K}(t) = \left(e^{-M_K T} e^{(M_K - M_\pi)t} + e^{-M\pi T} e^{(M_\pi - M_K)t}\right)^{-1} C_{\pi K}(t)$$

- Clear difference in terms of $m_{\text{eff}}(t)$ (it is not a simple log)

Example for effective mass plot



- For small t the two methods give consistent results
- Plateau for partial removal disappears, where the other pollution starts to contribute

Analysis methods

- Solving Lüscher's-formula we obtain the dimensionless scattering length: $\mu_{\pi K} a_0^{3/2}$
- Now we have to extrapolate to the physical point in the continuum limit

$$\mu_{\pi K} a_0^{I=\frac{3}{2}} = \frac{\mu_{\pi K}^2}{4\pi f_\pi^2} \Bigg\{ \overbrace{-1}^{\text{Leading order}} + \frac{32 M_\pi M_K}{f_\pi^2} L_{\pi K}(\Lambda_\chi) - \frac{16 M_\pi^2}{f_\pi^2} L_5(\Lambda_\chi) + \frac{1}{16\pi^2 f_\pi^2} \chi_{\rm NLO}^{3/2}(\Lambda_\chi, M_\pi, M_K, M_\eta) \Bigg\}, \quad a\Lambda_\chi = a f_\pi$$

- LO: $\mu_{\pi K} a_0^{I=\frac{3}{2}} = -\frac{\mu_{\pi K}^2}{4\pi f_\pi^2}$
- Parameter space $(r_0 m_\ell, r_0 m_s, a)$

Extrapolation to the physical point in the continuum

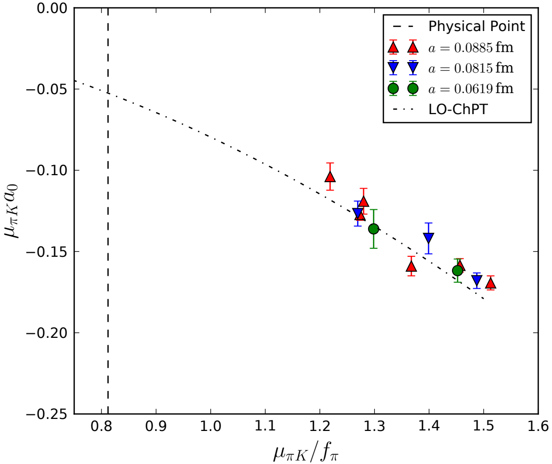
- Idea: Express the relevant meson parameters in terms of the renormalized quark masses and lattice spacing
  - $af_\pi\,(r_0 m_\ell, r_0 m_s, a)$
  - $(aM_\pi)^2\,(r_0 m_\ell, r_0 m_s, a)$
  - $(aM_K)^2\,(r_0 m_\ell, r_0 m_s, a)$
  - $(aM_\eta)^2\,(r_0 m_\ell, r_0 m_s, a)$
  - $\mu_{\pi K} a_0^{3/2}\,(r_0 m_\ell, r_0 m_s, a)$
- Fix $r_0 m_s$ for each a such that:

$$(r_0 M_K)^2 \left(r_0 m_s^{\text{fixed}}, r_0 m_\ell^{\text{physical}}, a\right) = (r_0 M_K)^2_{\text{physical}}$$

Extrapolation to the physical point in the continuum

Remaining parameters $r_0 m_\ell, a$

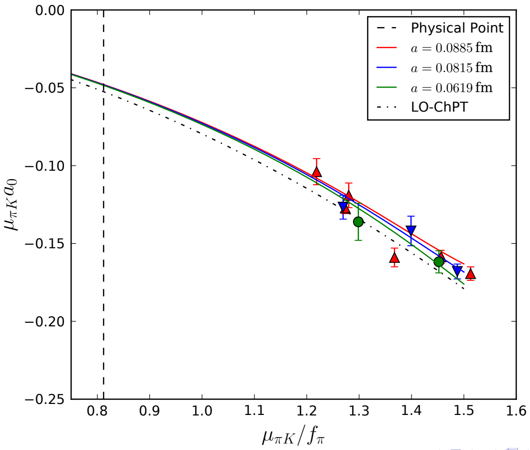
- Raw data interpolated in $r_0 m_s$: LO-ChPT is not a good approximation



Extrapolation to the physical point in the continuum

Remaining parameters $r_0 m_\ell, a$

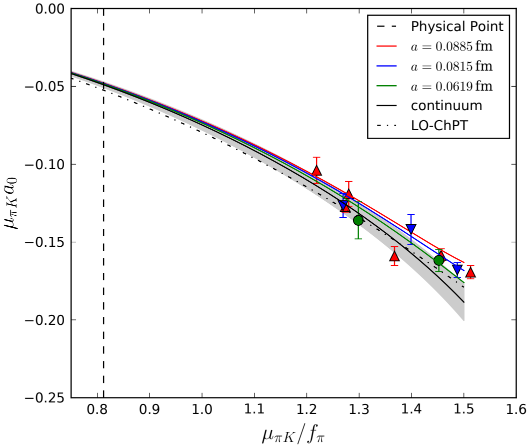
- Determine $\mu_{\pi K} a_0^{3/2}(r_0 m_\ell, a)$ and change variables: $r_0 m_\ell \to \mu_{\pi K}/f_\pi$



Extrapolation to the physical point in the continuum

Remaining parameters $r_0 m_\ell, a$

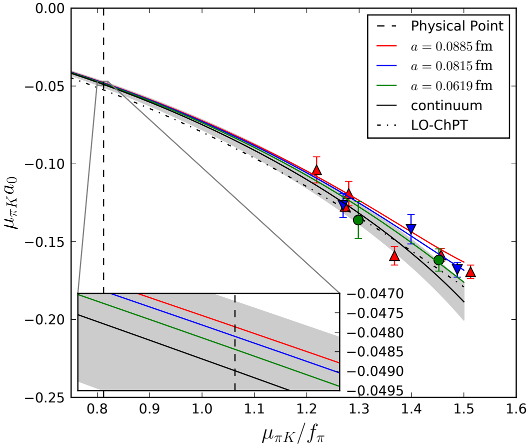
- Take the continuum limit at fixed $\mu_{\pi K}/f_\pi$



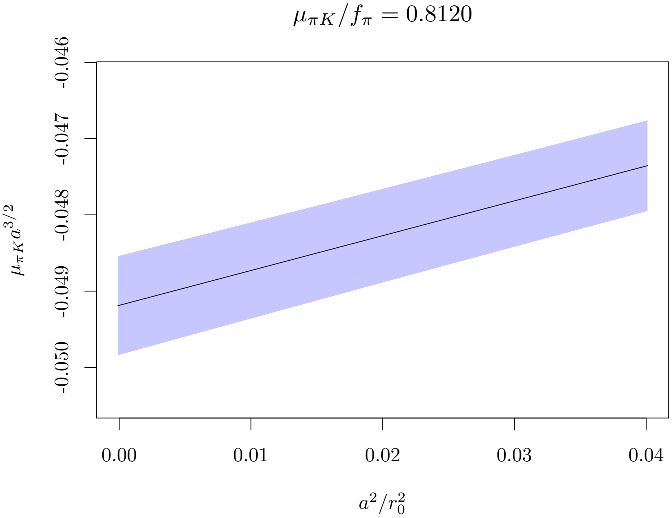
Extrapolation to the physical point in the continuum

Remaining parameters $r_0 m_\ell, a$

- Zoom to the physical point

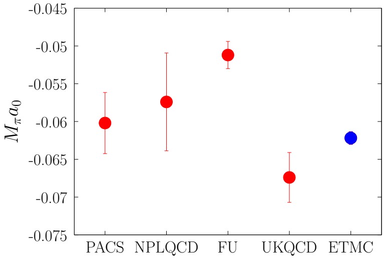


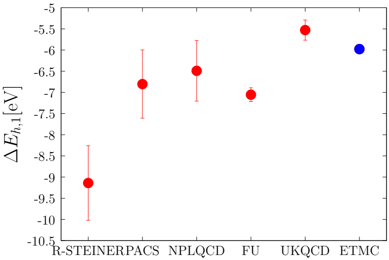
Scaling towards the continuum limit



Summary, outlook

Comparison with other collaborations





- For the first time we have done a continuum extrapolation for the π, K elastic scattering length in the maximal isospin channel
- To do: Determining also with these methods L_5

Thank you for your attention!

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