The large-mass regime of confining but nearly conformal gauge theories

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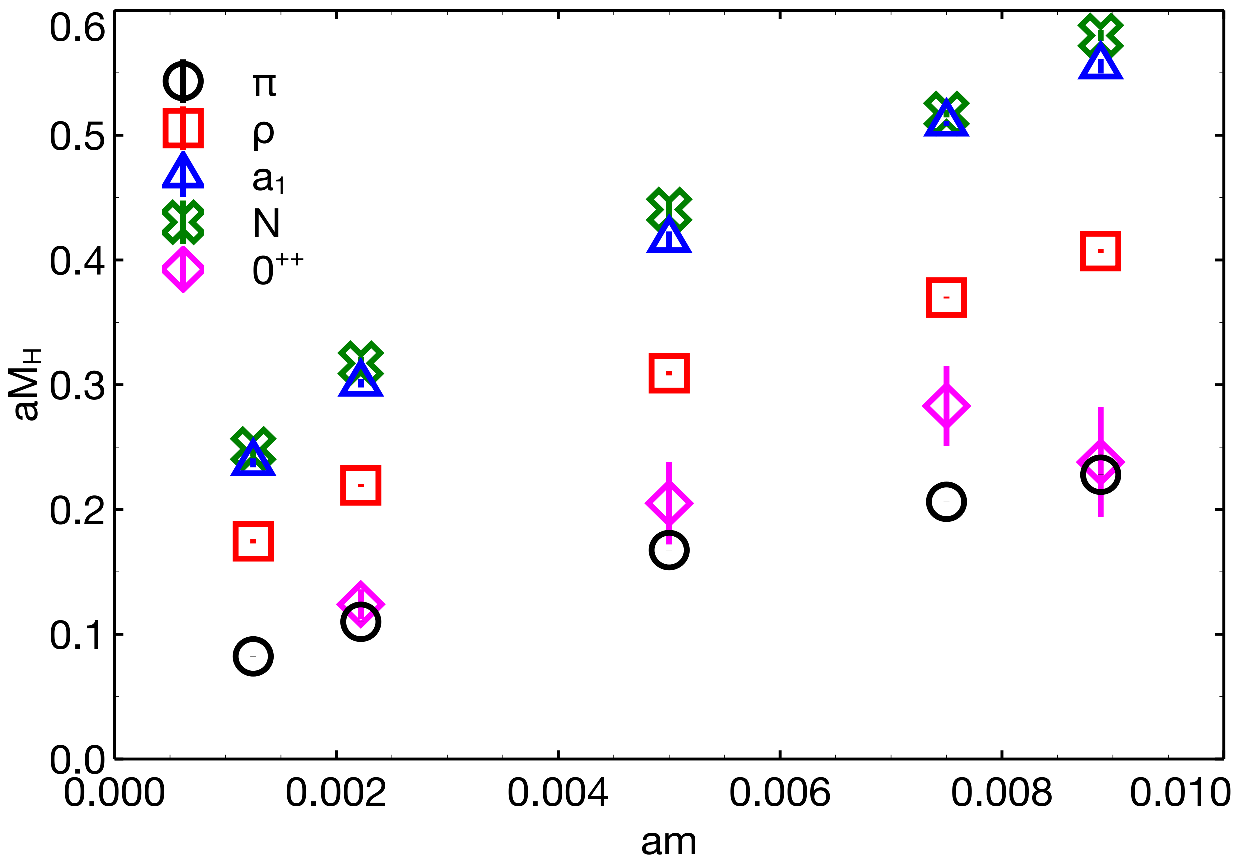
Claims:

- Even if the fermion mass of a near-conformal theory is large compared to the chiral-symmetry breaking/confinement scale, it can be described systematically by an EFT
- In this large-mass regime, it is very hard to distinguish such a theory from a mass-perturbed theory which is conformal in the infrared
- In the case of the SU(3), $N_f = 8$ theory, either much smaller fermion masses or a much higher precision will be needed in current simulations

A light flavor-singlet scalar: $SU(3)$ $N_f = 8$ spectrum



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The EFT for dilatons and pions

Assumptions:

- Scale invariance gets restored as we take the theory closer to the conformal window. For N_f fundamental flavors in an $SU(N_c)$ theory this happens when N_f crosses into the conformal region. Technically, $n_f \equiv N_f/N_c \uparrow n_f^*$ in the limit $N_f\ ,\ N_c \to \infty$.
- The theory contains pions associated with chiral symmetry breaking, and a dilaton associated with breaking of scale symmetry, which becomes massless for $n_f \to n_f^*$ (and $m \to 0$).
- The dilaton potential has a zero, as a function of the dilaton field τ .

The EFT for dilatons and pions

Leading-order Lagrangian:

$$\mathcal{L} = \frac{\hat{f}_\pi^2}{4}\, e^{2\tau} \mathrm{tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) + \frac{\hat{f}_\tau^2}{2}\, e^{2\tau} (\partial_\mu \tau)^2$$
$$- \frac{\hat{f}_\pi^2 \hat{B}_\pi m}{2}\, e^{(3-\gamma_*)\tau} \mathrm{tr}(\Sigma + \Sigma^\dagger)$$
$$+ \hat{f}_\tau^2 \hat{B}_\tau\, e^{4\tau} c_1 (\tau - 1/4)$$

- m is the fermion mass, γ_* is its anom. dim. at $n_f = n_f^*$, at the IRFP
- The parameters $c_1 \propto n_f - n_f^*$ and m are assumed small
- $v \equiv \langle \tau \rangle$ vanishes (at tree level) for $m = 0$

Lowest-order predictions:

At nonzero mass: $$\frac{m}{c_1 \hat{\mathcal{M}}} = v\, e^{(1+\gamma_*)v}$$

$$\hat{\mathcal{M}} = \frac{4\hat{f}_\tau^2 \hat{B}_\tau}{\hat{f}_\pi^2 \hat{B}_\pi N_f (3-\gamma_*)}$$

This is an $O(1)$ relation – both m and c_1 are small

If the LHS is much larger than 1 we find $e^v \approx \left(\frac{m}{c_1 \hat{\mathcal{M}}}\right)^{1/(1+\gamma_*)}$

while all masses and decay constants scale as $M_\pi \propto F_\pi \propto \cdots \propto m^{1/(1+\gamma_*)}$

⇒ Hyperscaling as in mass-perturbed conformal theory!

"Large-mass regime"

In more detail:

$$F_\pi = \hat{f}_\pi e^v \propto \hat{f}_\pi \left(\frac{m}{c_1}\right)^{1/(1+\gamma_*)}$$

$$M_\pi^2 = 2\hat{B}_\pi m e^{(1-\gamma_*)v} \propto 2\hat{B}_\pi m^{2/(1+\gamma_*)}$$

$$M_{\rm nucleon} = M_0 e^v \propto M_0 \left(\frac{m}{c_1}\right)^{1/(1+\gamma_*)}$$

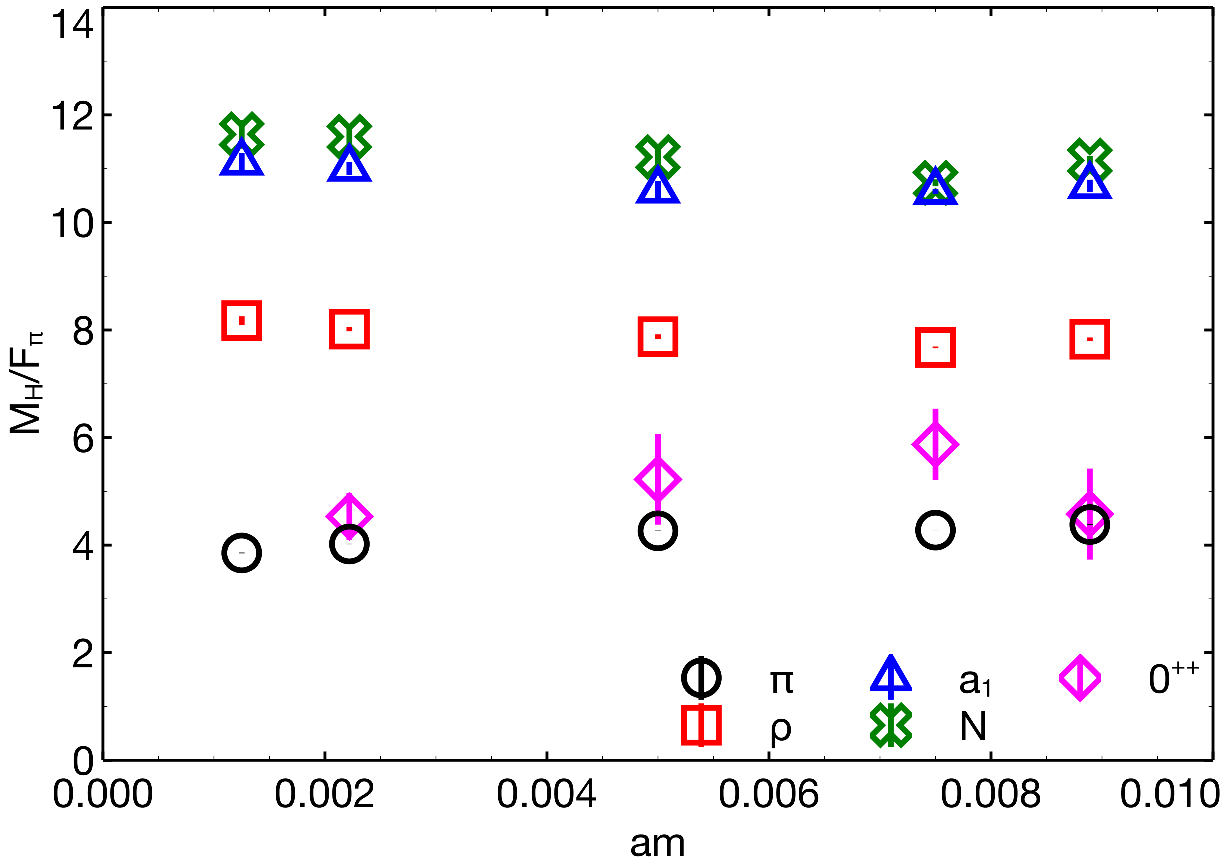
etc.

Hence the loop-expansion parameter

$$\frac{M_\pi^2}{(4\pi F_\pi)^2} = \frac{2c_1 \hat{\mathcal{M}} \hat{B}_\pi}{(4\pi \hat{f}_\pi)^2}\, v$$

is small as long as $c_1 \log m \ll 1$ is small, where we used $v \sim \log m$.

Numerical results for ratios ($SU(3)$, $N_f = 8$):



$$\frac{M_\pi^2}{(4\pi F_\pi)^2} \approx 0.1$$

still, possibly sizeable NLO corrections

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Numerical estimates ($SU(3)$, $N_f = 8$)

- From $M_\pi^2 F_\pi^{-1+\gamma_*} = 2\hat{B}_\pi \hat{f}_\pi^{-1+\gamma_*} m$ estimate that $\gamma_* \approx 1$ (Appelquist *et al.*)

- Then, using nucleon mass values from LSD, we find at the lightest fermion mass

$$am = 0.00125 \quad \Rightarrow \quad \frac{m}{c_1 \hat{\mathcal{M}}} \sim 100$$

This implies that, to unambiguously determine whether this theory is conformal or chiral-symmetry breaking, we need either *much* smaller masses, or enough precision to disentangle subleading effects to the large-mass behavior.

- Also find $\hat{B}_\pi / \hat{f}_\pi \sim 10^3$: condensate enhancement?

Conclusions

- In near-conformal theories, there exists a ``large-mass'' regime, defined by $\dfrac{m}{c_1\hat{\mathcal{M}}} \gg 1$; region of EFT applicability is $c_1 \log m \ll 1$.

- Current simulations in the $SU(3)\ N_f = 8$ theory are in the large-mass regime, $m/(c_1\hat{\mathcal{M}}) \geq \sim 100$.

- In this theory, need either much smaller fermion mass
  or enough precision to disentangle subleading effects.

- Explanation why it is so hard to distinguish a conformal theory from a chiral-symmetry breaking theory near the conformal sill?