

Systematics in nucleon matrix element calculations

Jeremy Green

NIC, DESY, Zeuthen

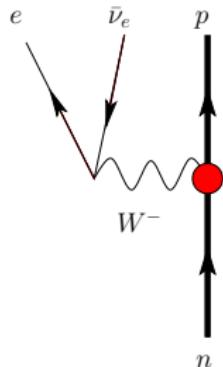
The 36th International Symposium on Lattice Field Theory
East Lansing, MI, USA
July 22–28, 2018

Motivation

Three reasons for studying structure of protons and neutrons:

- ▶ Understanding the quark and gluon substructure of a hadron.
- ▶ Understanding nucleons as tools in experiments.
- ▶ Validation of lattice QCD using “benchmark” observables.

Neutron beta decay



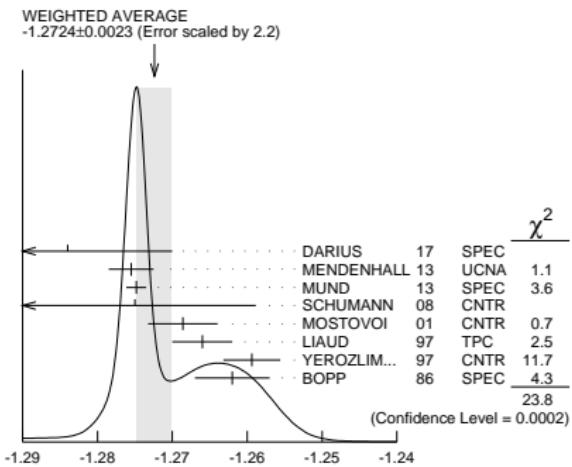
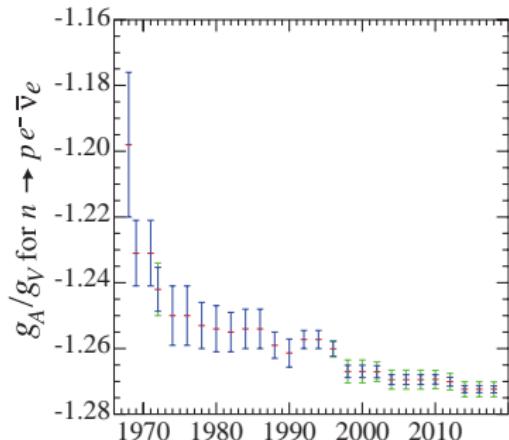
Coupling to W boson via axial current depends on nucleon axial charge,

$$\langle p(P, s') | \bar{u} \gamma^\mu \gamma_5 d | n(P, s) \rangle = g_A \bar{u}_p(P, s') \gamma^\mu \gamma_5 u_n(P, s).$$

Interpreted as isovector quark spin contribution.

→ Yi-Bo Yang plenary

PDG 2018: $g_A = 1.2724(23)$.



Other observables

Precision β -decay experiments may be sensitive to BSM physics; leading contributions are controlled by the scalar and tensor charges:

T. Bhattacharya *et al.*, Phys. Rev. D 85, 054512 (2012) [1110.6448]

$$\langle p(P, s') | \bar{u}d | n(P, s) \rangle = \textcolor{orange}{g}_S \bar{u}_p(P, s') u_n(P, s),$$

$$\langle p(P, s') | \bar{u} \sigma^{\mu\nu} d | n(P, s) \rangle = \textcolor{orange}{g}_T \bar{u}_p(P, s') \sigma^{\mu\nu} u_n(P, s).$$

Sigma terms control the sensitivity of direct-detection dark matter searches to WIMPs that interact via Higgs exchange.

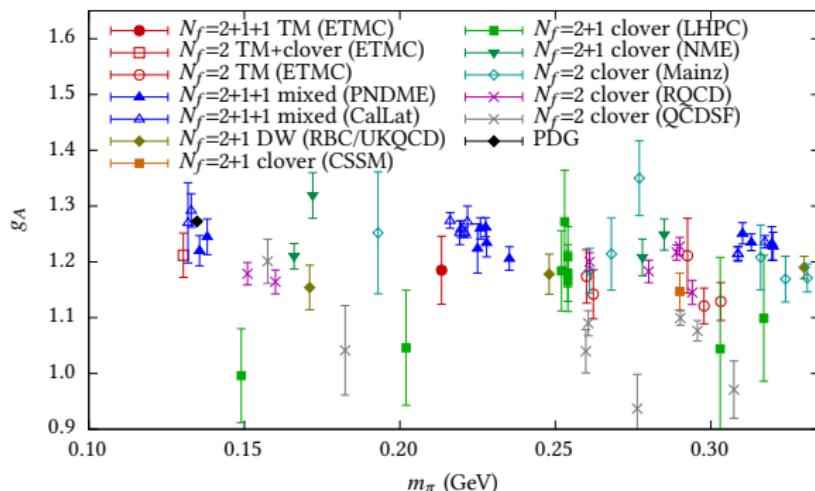
$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle \quad \sigma_q = m_q \langle N | \bar{q}q | N \rangle$$

Toward precision nucleon structure

Trustworthy lattice calculations need control over all systematics:

- ▶ Physical quark masses
- ▶ Isolation of ground state
- ▶ Infinite volume
- ▶ Continuum limit

Want to transition from this ...

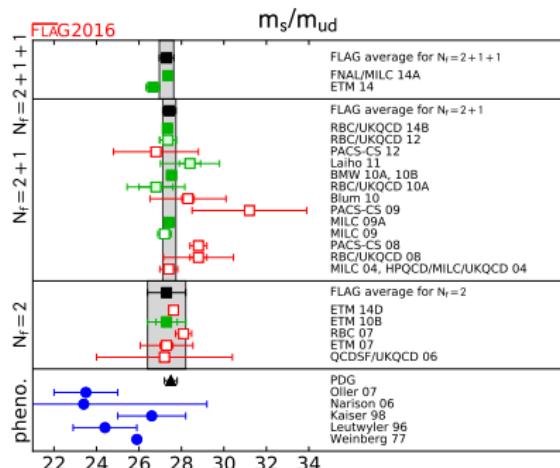


Toward precision nucleon structure

Trustworthy lattice calculations need control over all systematics:

- ▶ Physical quark masses
- ▶ Isolation of ground state
- ▶ Infinite volume
- ▶ Continuum limit

Want to transition from this ... to something like this.



Outline

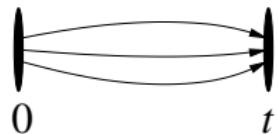
1. Methodologies for nucleon matrix elements
2. Excited-state effects: theory and practice
3. Other systematics: L , (a) , m_q
4. Outlook

Hadron correlation functions

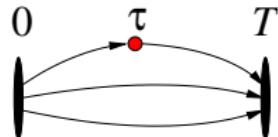
Compute two-point and three-point functions, using interpolator χ and operator insertion O . In simplest case:

$$\begin{aligned} C_{2\text{pt}}(t) &\equiv \langle \chi(t) \chi^\dagger(0) \rangle \\ &= \sum_n |Z_n|^2 e^{-E_n t} \\ &\rightarrow |Z_0|^2 e^{-E_0 t} \left(1 + O(e^{-\Delta E t}) \right), \end{aligned}$$

where $Z_n = \langle \Omega | \chi | n \rangle$,



$$\begin{aligned} C_{3\text{pt}}(\tau, T) &\equiv \langle \chi(T) O(\tau) \chi^\dagger(0) \rangle \\ &= \sum_{n, n'} Z_{n'} Z_n^* \langle n' | O | n \rangle e^{-E_n \tau} e^{-E_{n'}(T-\tau)} \\ &\rightarrow |Z_0|^2 \langle 0 | O | 0 \rangle e^{-E_0 T} \left(1 + O(e^{-\Delta E \tau}) + O(e^{-\Delta E(T-\tau)}) \right) \end{aligned}$$



Hadron matrix elements

Ratio method

$$R(\tau, T) \equiv \frac{C_{3\text{pt}}(\tau, T)}{C_{2\text{pt}}(T)} = \langle 0|O|0\rangle + O(e^{-\Delta E\tau}) + O(e^{-\Delta E(T-\tau)})$$

Midpoint yields $R(\frac{T}{2}, T) = \langle 0|O|0\rangle + O(e^{-\Delta ET/2})$.

Summation method

L. Maiani *et al.*, Nucl. Phys. B 293, 420 (1987); g_A in S. Güsken *et al.*, Phys. Lett. B 227, 266 (1989)

$$S(T) \equiv \sum_{\tau} R(\tau, T), \quad \frac{d}{dT} S(T) = \langle 0|O|0\rangle + O(T e^{-\Delta ET})$$

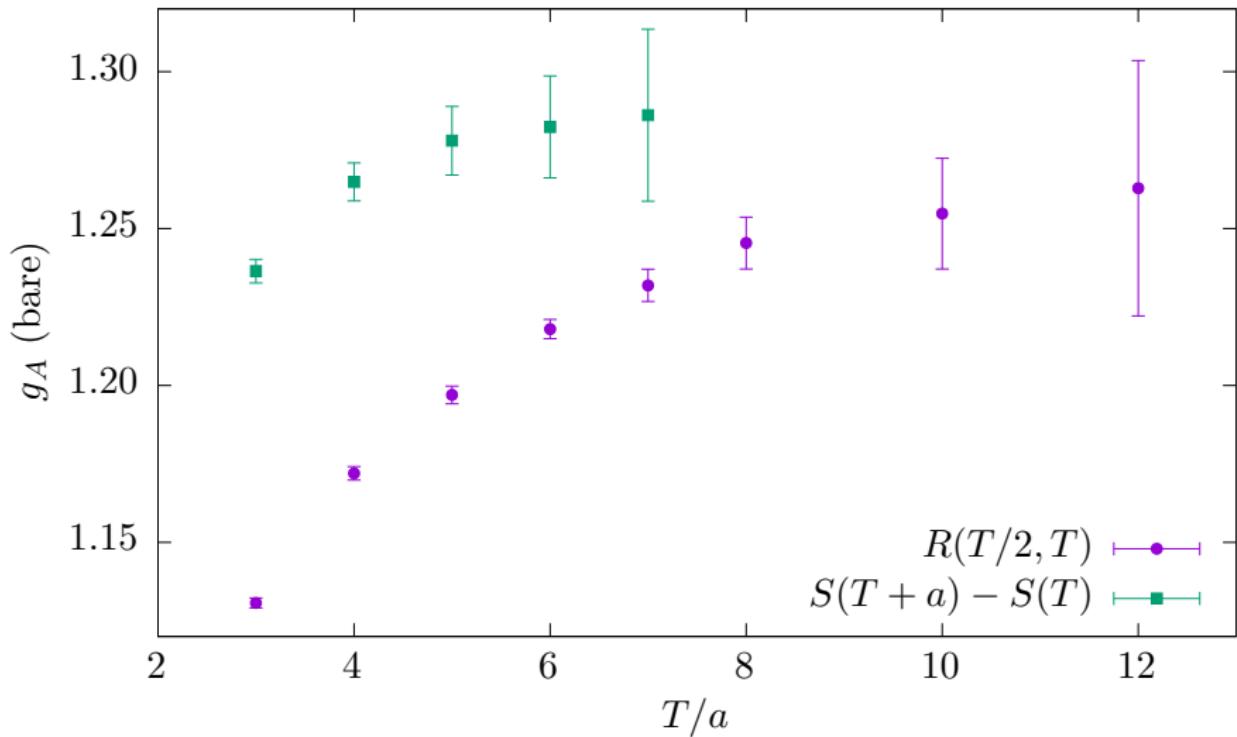
Sum can be over all timeslices or from τ_0 to $T - \tau_0$.

Improved asymptotic behaviour noted in talks at Lattice 2010.

S. Capitani *et al.*, PoS LATTICE2010 147 [1011.1358]; J. Bulava *et al.*, *ibid.* 303 [1011.4393]

In practice noisier than ratio method at same T .

Ratio and summation method



physical m_π , $a = 0.116$ fm

N. Hasan, JG, *et al.*, in preparation

Feynman-Hellmann approach

If we add a term to the Lagrangian $\mathcal{L}(\lambda) \equiv \mathcal{L} + \lambda O$, then

$$\frac{\partial}{\partial \lambda} E_n(\lambda) \Big|_{\lambda=0} = \langle n | O | n \rangle.$$

Discrete derivatives sometimes used, particularly for sigma terms:

$$\sigma_q \equiv m_q \langle N | \bar{q}q | N \rangle = m_q \frac{\partial m_N}{\partial m_q}.$$

Exact derivatives lead directly to summation method (neglecting $\langle \Omega | O | \Omega \rangle$):

$$-\frac{\partial}{\partial \lambda} \log C_{2\text{pt}}(t) \Big|_{\lambda=0} = S(t).$$

L. Maiani *et al.*, Nucl. Phys. B **293**, 420 (1987)

A. J. Chambers *et al.* (CSSM and QCDSF/UKQCD), Phys. Rev. D **90**, 014510 (2014) [1405.3019]

C. Bouchard *et al.*, Phys. Rev. D **96**, 014504 (2017) [1612.06963]

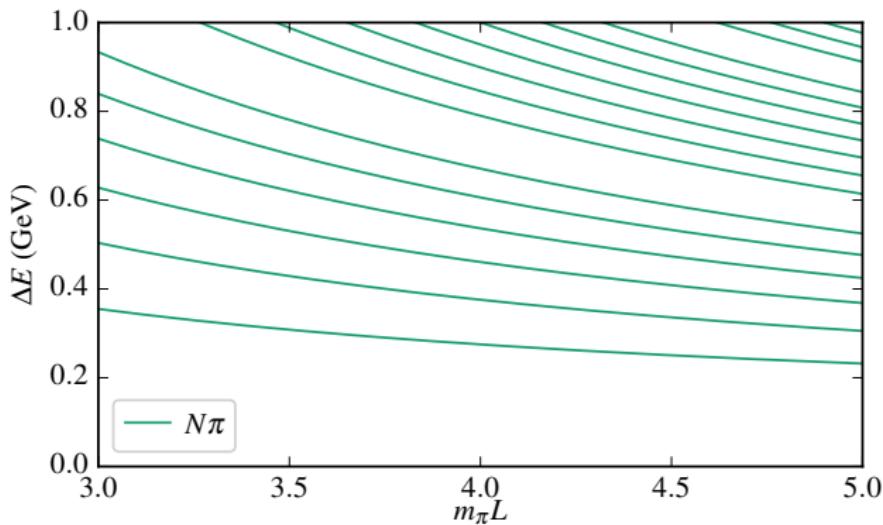
Controlling excited states

Three approaches:

- ▶ Go to large T . Need $\Delta ET \gg 1$.
But signal-to-noise decays as $e^{-(E_0 - \frac{3}{2}m_\pi)T}$.
- ▶ Improve the interpolating operator.
- ▶ Fit correlators to remove excited states.

Excited-state energies

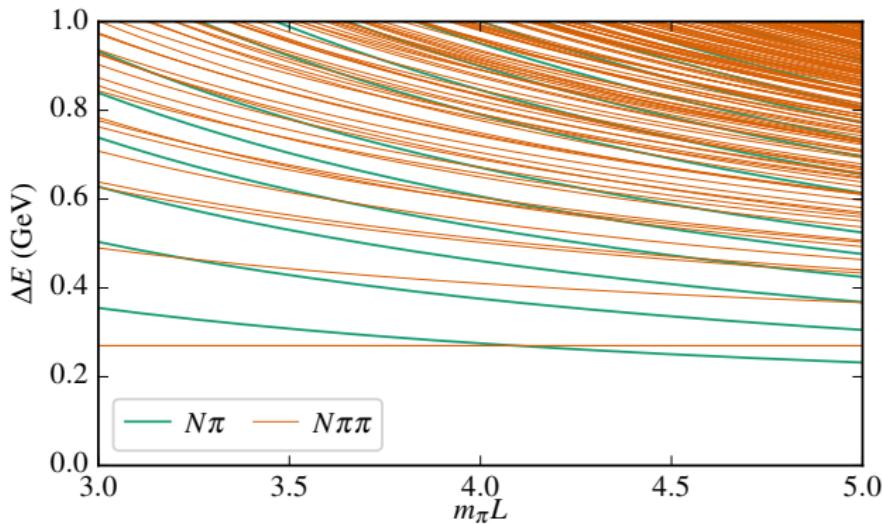
First approximation: noninteracting stable hadrons in a box. $N\pi, N\pi\pi, \dots$



Physical m_π .

Excited-state energies

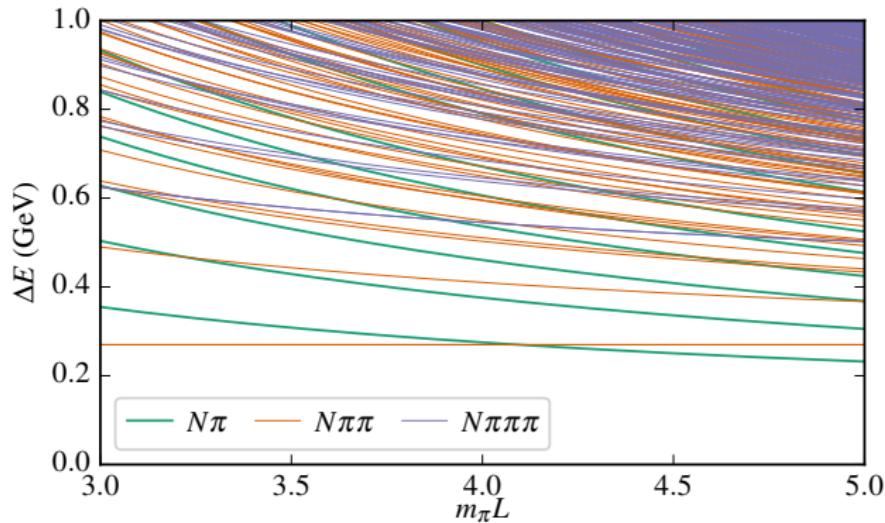
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Physical m_π .

Excited-state energies

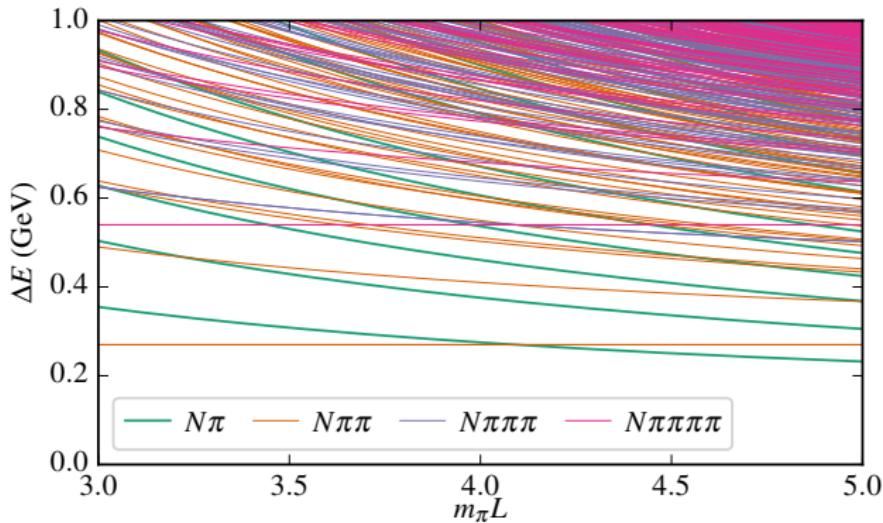
First approximation: noninteracting stable hadrons in a box. $N\pi, N\pi\pi, \dots$



Physical m_π .

Excited-state energies

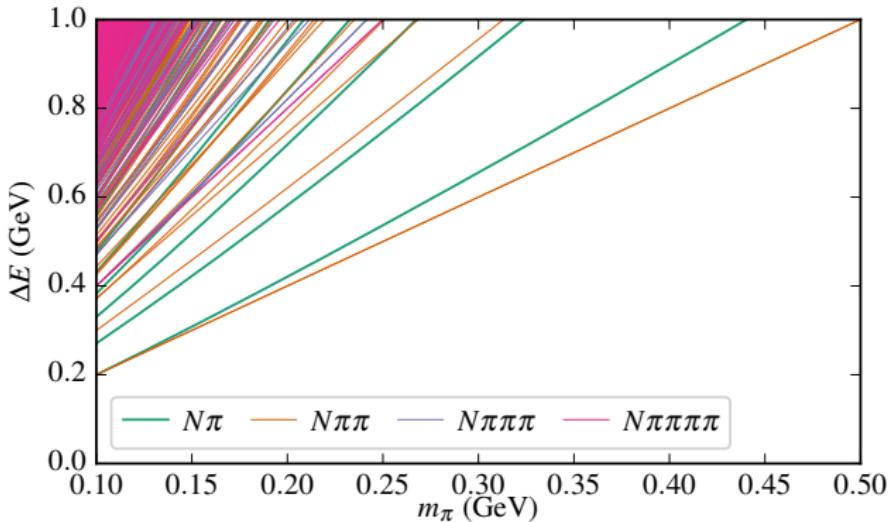
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Physical m_π .

Excited-state energies

First approximation: noninteracting stable hadrons in a box. $N\pi, N\pi\pi, \dots$

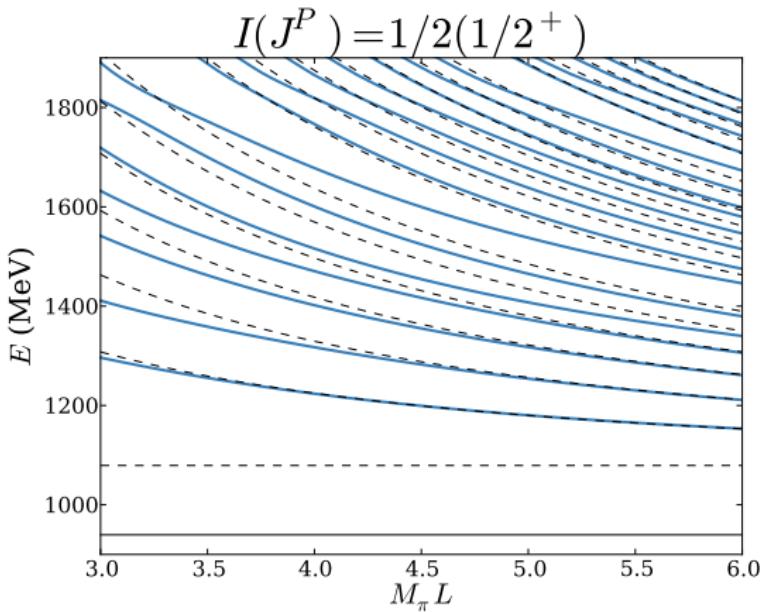


$$m_\pi L = 4.$$

Excited-state energies

M. T. Hansen and H. B. Meyer, Nucl. Phys. B 923, 558 (2017) [1610.03843]

Focus on $N\pi$ sector and apply finite-volume quantization using scattering phase shift from experiment.



Going to large T

Statistical errors:

$$\delta_{\text{stat}} \sim N^{-1/2} e^{(E_0 - \frac{3}{2}m_\pi)T}$$

Excited state systematics:

$$\delta_{\text{exc}} \sim e^{-\Delta ET/2} \text{ (ratio method)}$$

Suppose we want these to be equal, i.e. $\delta_{\text{stat}} = \delta_{\text{exc}} \equiv \delta$.

Required statistics are given by

$$N \propto \delta^{-\left(2 + \frac{4E_0 - 6m_\pi}{\Delta E}\right)}.$$

At the physical point with $\Delta E = 2m_\pi$, the exponent is ≈ -13 .

Multi-level methods could potentially improve this.

Predicting excited-state contributions

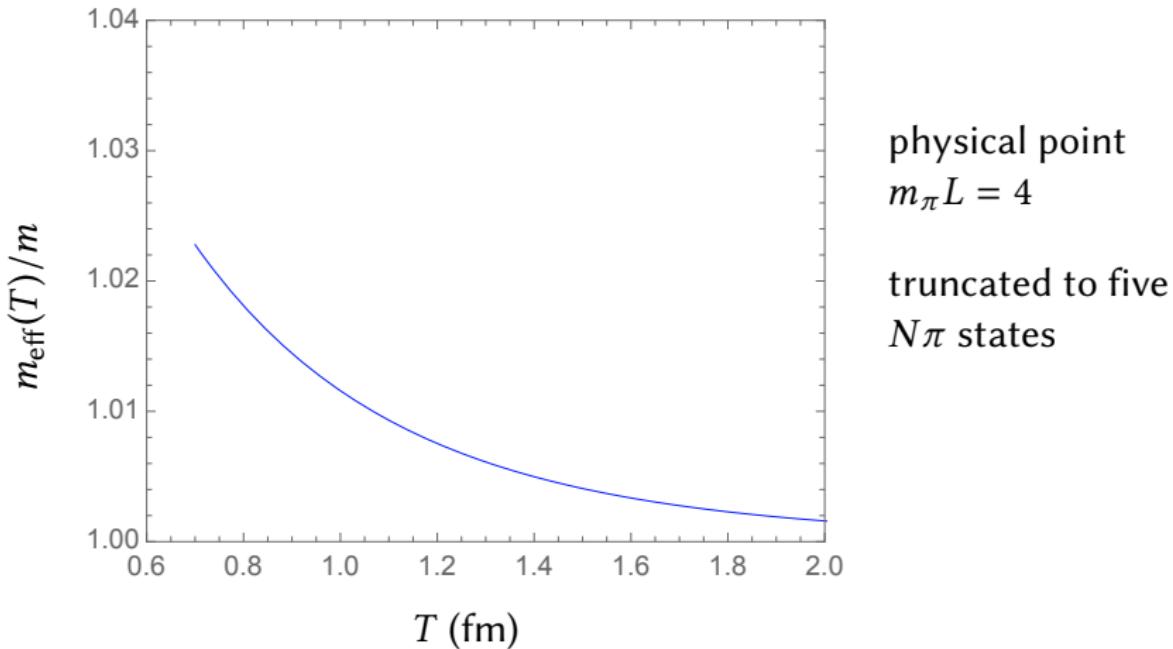
Need to know:

1. energies E_n
2. matrix elements $\langle n' | O | n \rangle$
3. overlaps Z_n

Have been studied in ChPT. [B. C. Tiburzi; O. Bär](#)

Key insight: at leading order a single LEC controls coupling of local interpolator to N and $N\pi$ states.

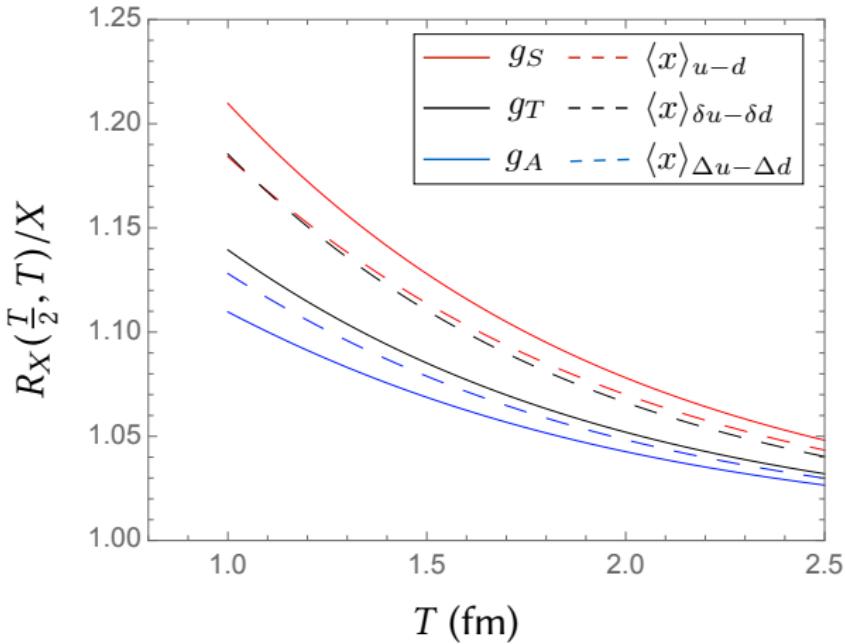
ChPT prediction of excited-state effects



O. Bär, Int. J. Mod. Phys. A 32 no. 15, 1730011 (2017) [1705.02806]

generalization to axial FFs → Oliver Bär parallel, Wed. 14:00

ChPT prediction of excited-state effects



physical point
 $m_\pi L = 4$

truncated to five
 $N\pi$ states

need $T \gtrsim 2$ fm for
ChPT to be reliable

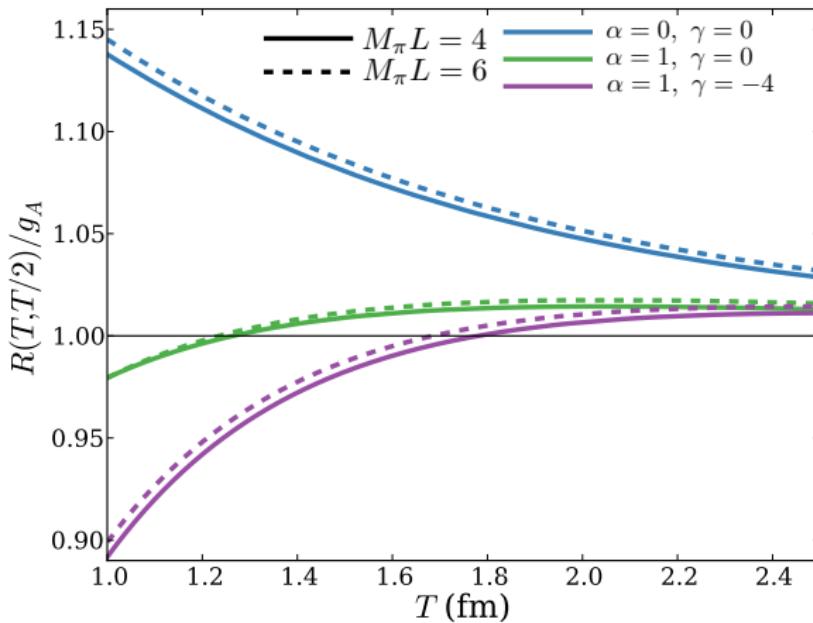
O. Bär, Int. J. Mod. Phys. A 32 no. 15, 1730011 (2017) [1705.02806]

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Going beyond ChPT

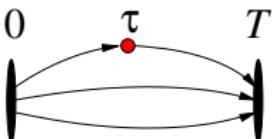
M. T. Hansen and H. B. Meyer, Nucl. Phys. B 923, 558 (2017) [1610.03843]

Deviations of $\langle \Omega | \chi | N\pi \rangle$ and $\langle N\pi | A_\mu | N \rangle$ from ChPT in the resonance regime modeled with parameters α and γ .



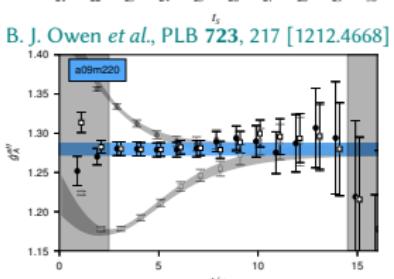
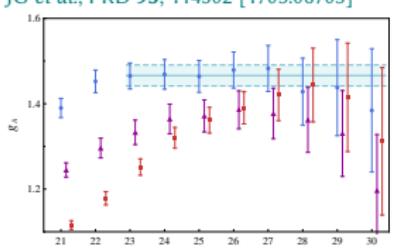
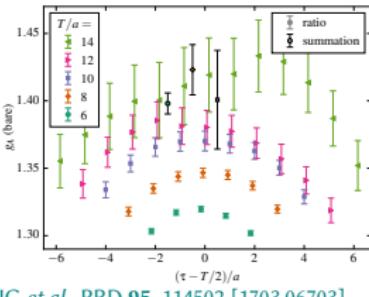
Numerical studies

Available data depends on how C_{3pt} is computed.



For connected diagrams:

- ▶ Fixed sink: most common. Get data at all τ and any operator insertion. Cost increases with each value of T .
- ▶ Fixed operator and τ . Get data at all T and any set of interpolators. [variational studies by CSSM](#)
- ▶ Fixed operator and summed τ . Get summation data at all T and any set of interpolators. [used by CalLat, NPLQCD](#)

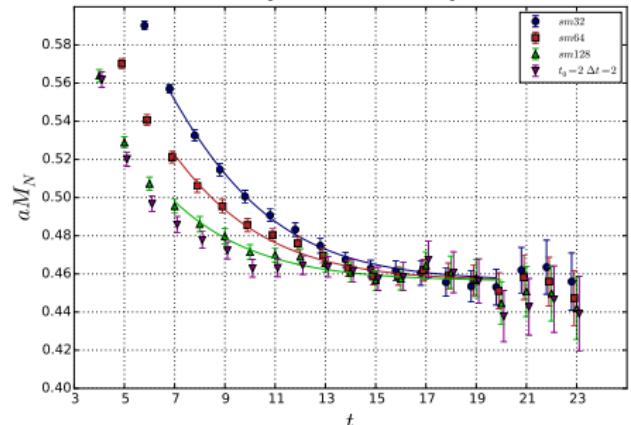


Improving the nucleon interpolator

- ▶ Standard operator: $\chi = \epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c$, with smeared quark fields.
- ▶ Smearing width typically tuned so that m_{eff} has early plateau.
- ▶ Variational approach: $\chi = \sum_i c_i \chi_i$ with optimized c_i .
 - ▶ Simple basis: use varying smearing widths for χ_i .
 - ▶ Can be extended with more local structures (derivatives and $G_{\mu\nu}$).
 - ▶ More expensive: include nonlocal operators for better isolation of $N\pi$ and $N\pi\pi$ states.

Variational approach: different smearings – effective mass

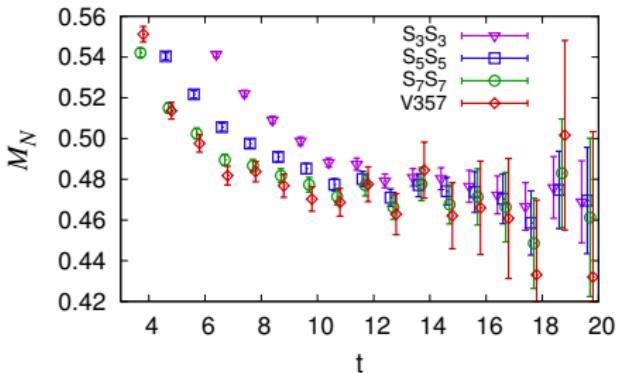
Mass 2exp Variational Comparison



$m_\pi = 460$ MeV, $a = 0.074$ fm

J. Dragos *et al.*, PRD 94, 074505 (2016) [1606.03195]

Small improvement over widest smearing.



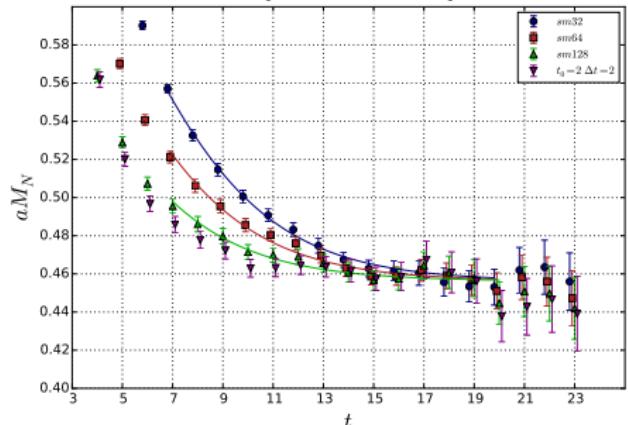
$m_\pi = 312$ MeV, $a = 0.081$ fm

B. Yoon *et al.* (NME), PRD 93, 114506 (2016)

[1602.07737]

Variational approach: different smearings – effective mass

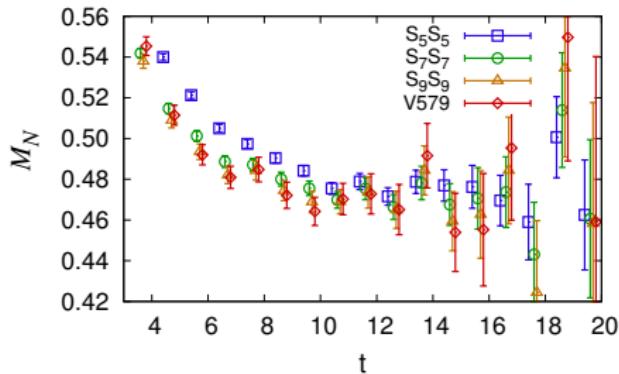
Mass 2exp Variational Comparison



$$m_\pi = 460 \text{ MeV}, a = 0.074 \text{ fm}$$

J. Dragos *et al.*, PRD 94, 074505 (2016) [1606.03195]

Small improvement over widest smearing... so use even wider smearings.

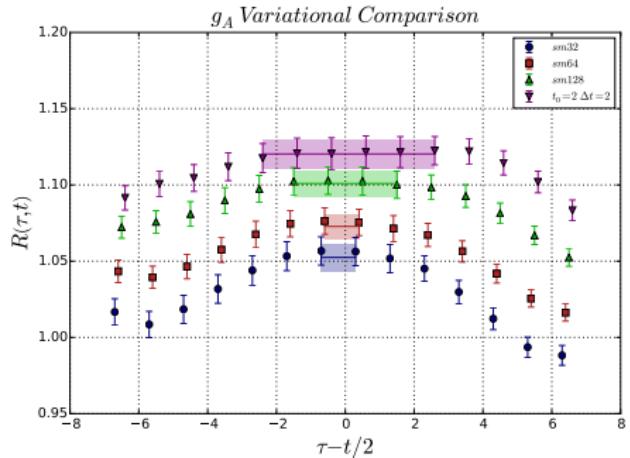


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B. Yoon *et al.* (NME), PRD 93, 114506 (2016)

[1602.07737]

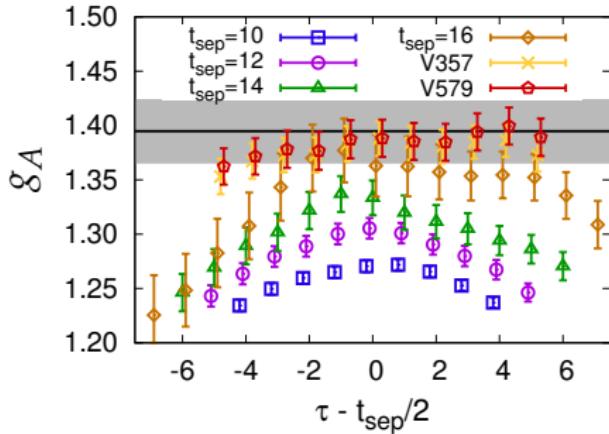
Variational approach: different smearings – axial charge



$m_\pi = 460$ MeV, $a = 0.074$ fm

$T = 0.96$ fm

J. Dragos *et al.*, PRD 94, 074505 (2016) [1606.03195]



$m_\pi = 312$ MeV, $a = 0.081$ fm

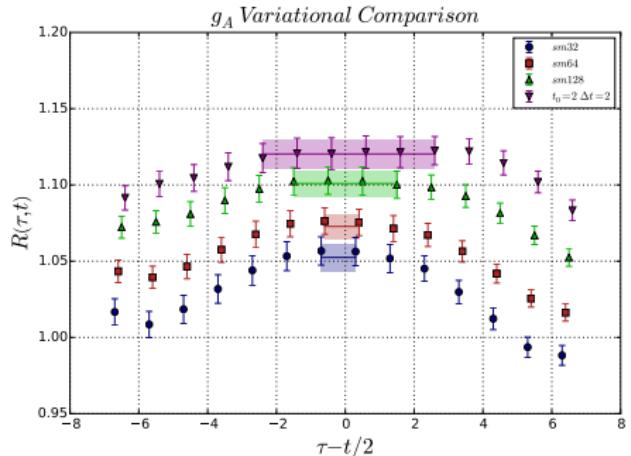
variational at $T = 0.97$ fm versus S5

B. Yoon *et al.* (NME), PRD 93, 114506 (2016)

[1602.07737]

Improvement over best tuned single smearing seems small.

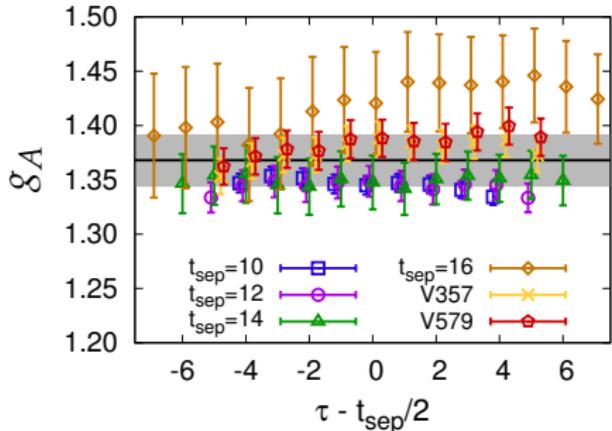
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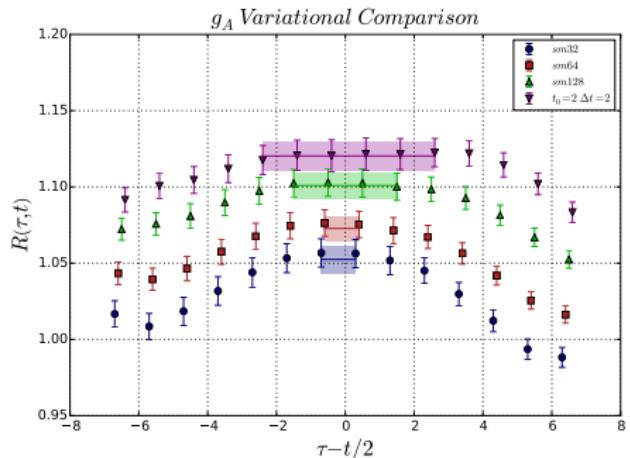
variational at $T = 0.97$ fm versus S9

B. Yoon *et al.* (NME), PRD 93, 114506 (2016)

[1602.07737]

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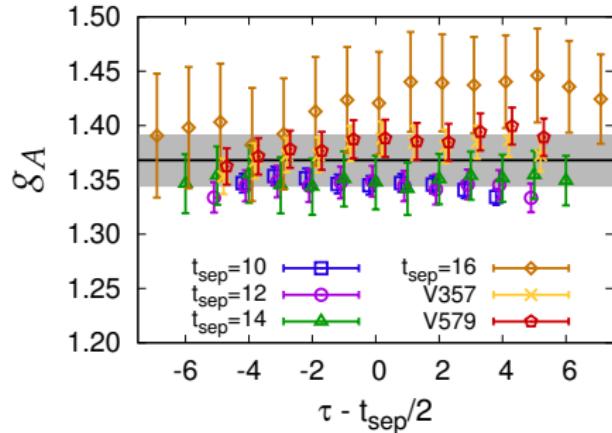
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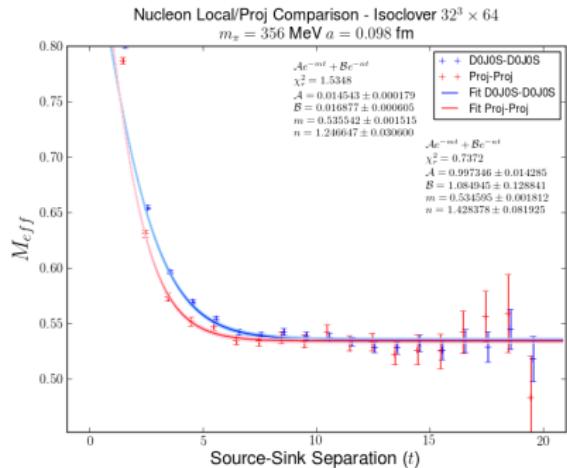
[1602.07737]

Improvement over best tuned single smearing seems small.

See also: including negative parity operators in basis when $\vec{p} \neq 0$.

F. Stokes *et al.*, in preparation

Variational approach: larger basis – effective mass



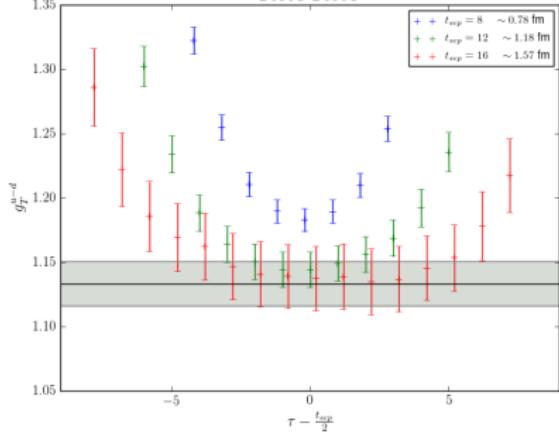
Calculation performed using distillation.

Operators with derivatives and hybrid operators with $G_{\mu\nu}$ included in basis.

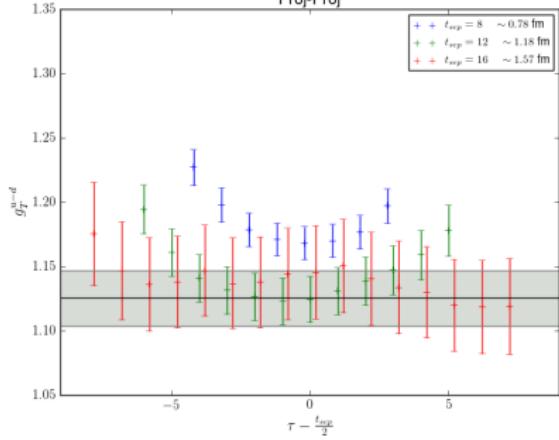
→ C. Egerer parallel, Wed. 14:40

Variational approach: larger basis – tensor charge

Nucleon effective tensor Charge - Isoclover $32^3 \times 64$ $m_\pi = 356$ MeV $a = 0.098$ fm
DOJOS-DOJOS



Nucleon effective tensor Charge - Isoclover $32^3 \times 64$ $m_\pi = 356$ MeV $a = 0.098$ fm
Proj-Proj



Standard operator

Calculation performed using distillation.

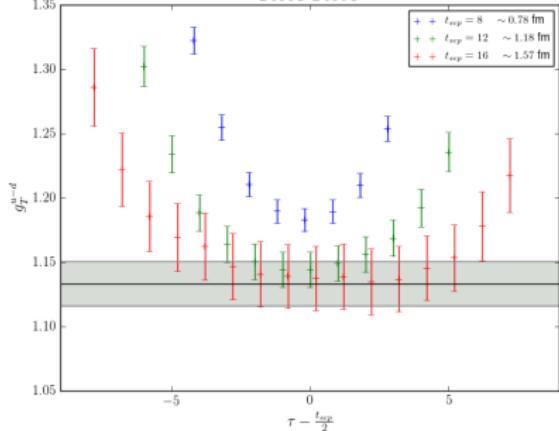
Operators with derivatives and hybrid operators with $G_{\mu\nu}$ included in basis.

→ C. Egerer parallel, Wed. 14:40

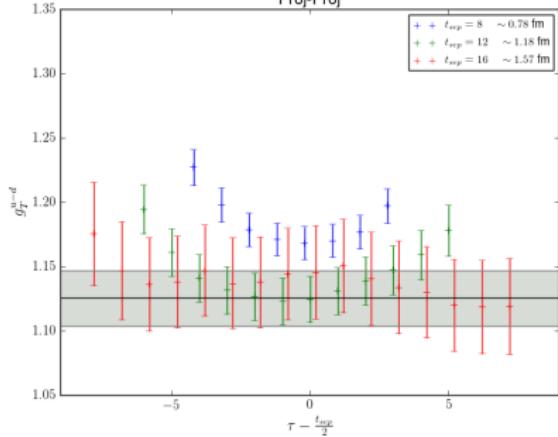
Variational

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Nucleon effective tensor Charge - Isoclover $32^3 \times 64$ $m_\pi = 356$ MeV $a = 0.098$ fm
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Standard operator

Calculation performed using distillation.

Operators with derivatives and hybrid operators with $G_{\mu\nu}$ included in basis.

→ C. Egerer parallel, Wed. 14:40

Variational

How does this compare with a standard calculation with tuned smearing?

Fitting approaches

Fit correlators using ansatz based on N -state model.

- ▶ E_n and Z_n generally determined from $C_{2\text{pt}}$.
- ▶ $\langle n'|\mathcal{O}|n\rangle$ determined from $C_{3\text{pt}}$, either in combined or separate fit.

Typical energy gap:

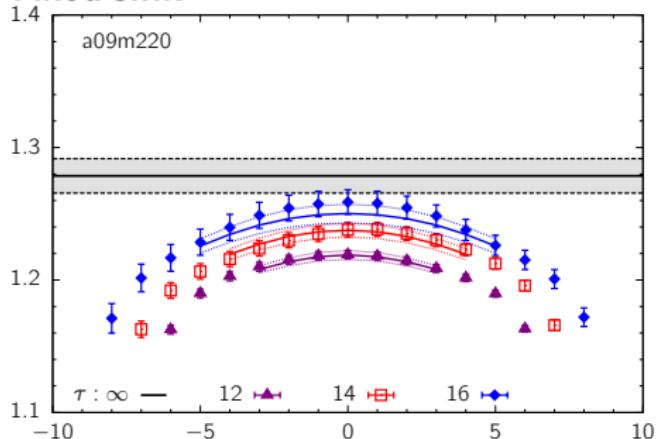
$$0.5 \text{ GeV} < E_1 - E_0 < 1 \text{ GeV},$$

usually greater than expected $2m_\pi$.

- ▶ Each model state approximates contribution from several states.
- ▶ Fit ansatz should be considered a somewhat uncontrolled model.

Fits for axial charge

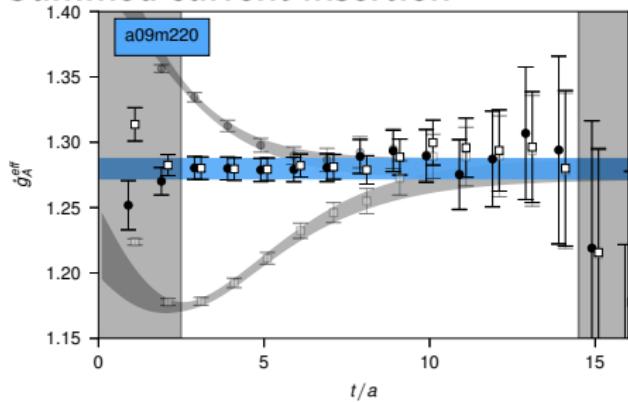
Fixed sink



R. Gupta *et al.* (PNDME), 1806.09006

constrained 3-state fit: $\tau, T - \tau \geq 3a$

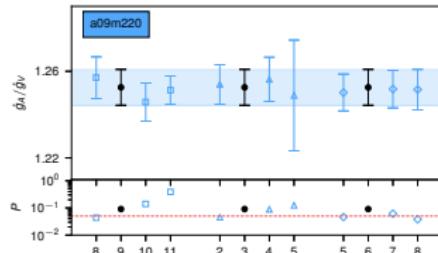
Summed current insertion



C. C. Chang *et al.*, Nature 558, 91 [1805.12130]

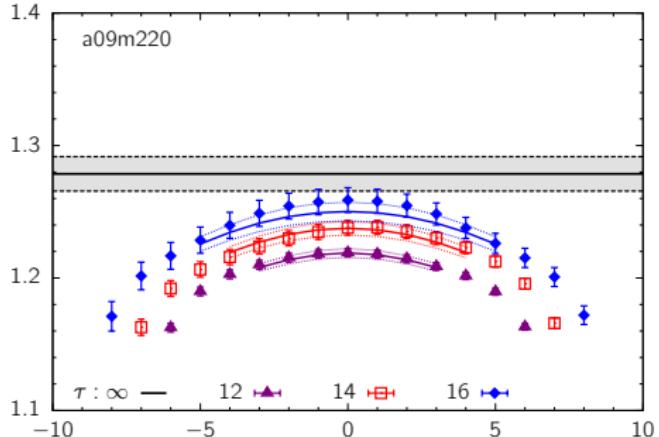
data with smeared and point sinks

unconstrained 2-state fit: $T \geq 3a$



Fits for axial charge

Fixed sink

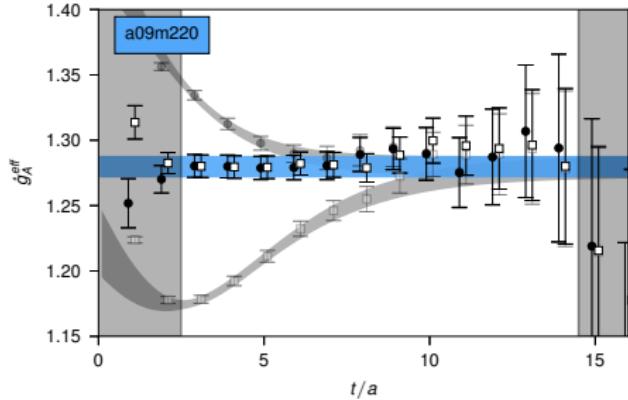


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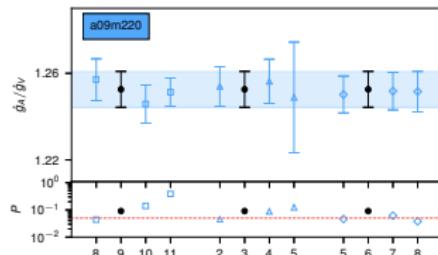
Note: more than 10 states with
 $\Delta E < 1 \text{ GeV} \implies e^{-3a\Delta E} > 0.25$.

Summed current insertion

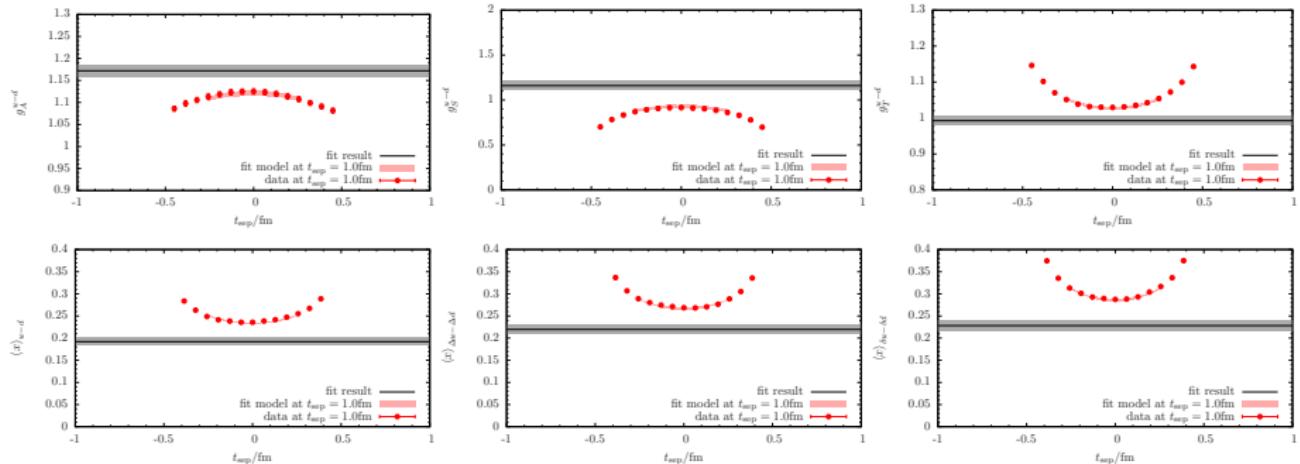


C. C. Chang *et al.*, Nature 558, 91 [1805.12130]

data with smeared and point sinks
 unconstrained 2-state fit: $T \geq 3a$



Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}, T = 16a$$

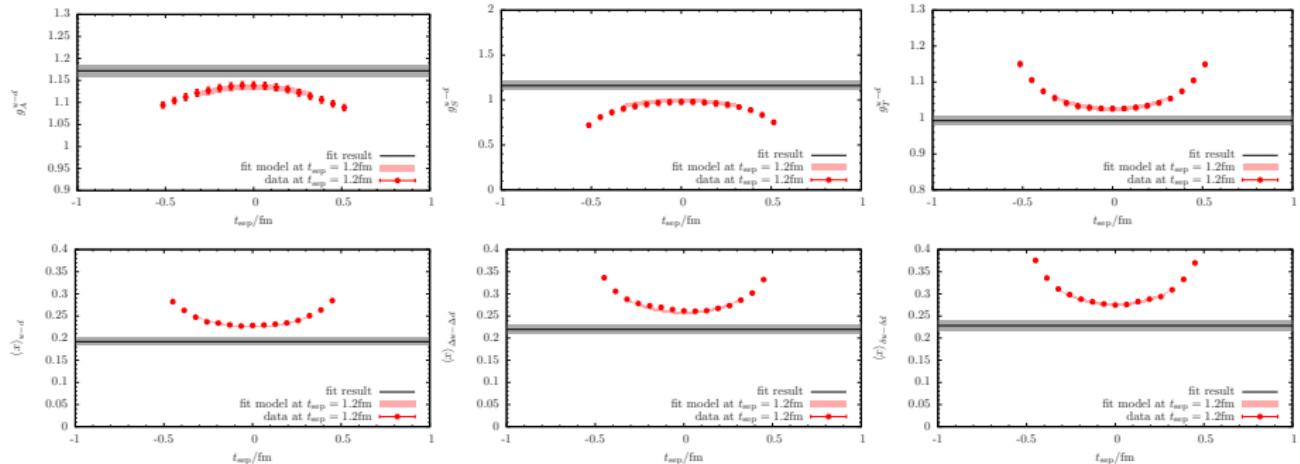
(all T included in fit).

Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{start}$ with $t_{start}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not C_{2pt} .

→ K. Ott nad parallel, Thu 12:00

Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}, T = 18a$$

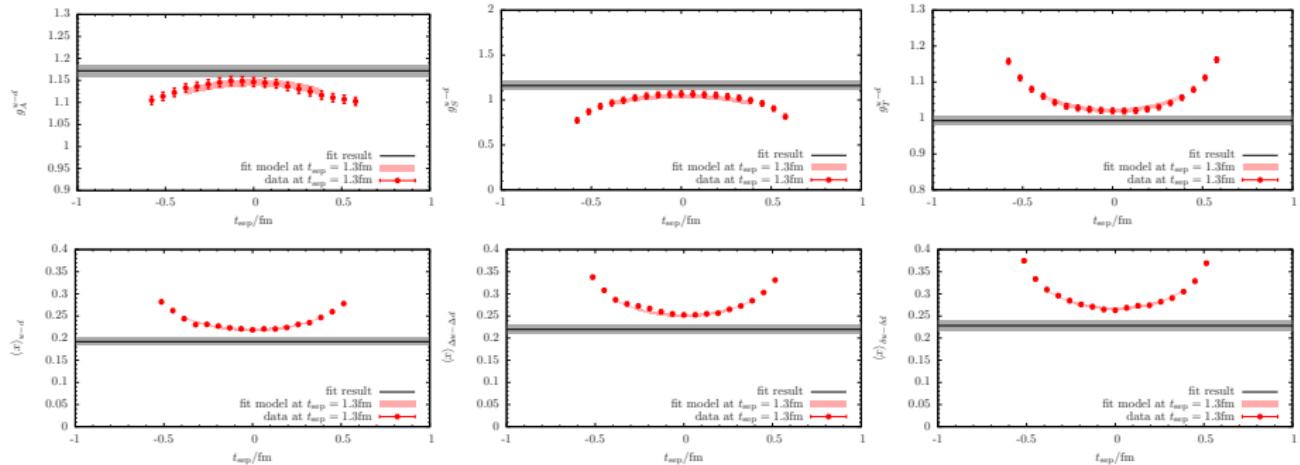
(all T included in fit).

Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{start}$ with $t_{start}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not C_{2pt} .

→ K. Ott nad parallel, Thu 12:00

Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}, T = 20a$$

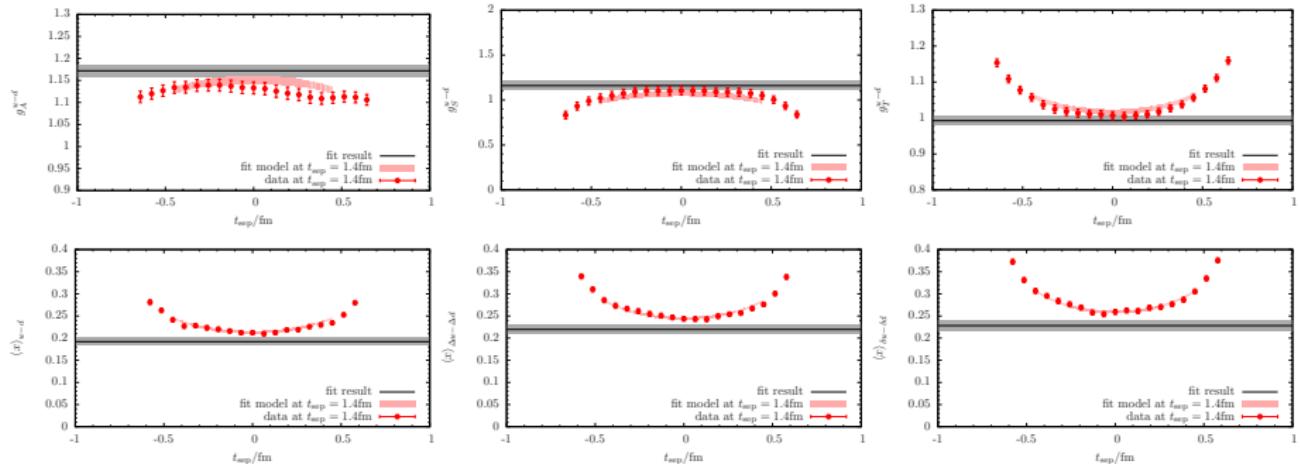
(all T included in fit).

Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{start}$ with $t_{start}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not $C_{2\text{pt}}$.

→ K. Ott nad parallel, Thu 12:00

Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}, T = 22a$$

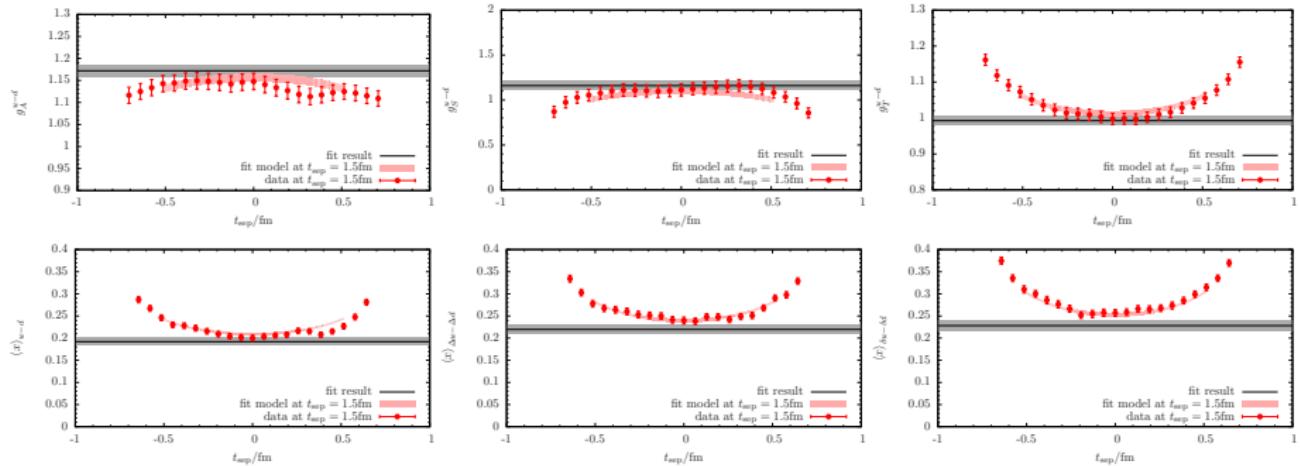
(all T included in fit).

Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{start}$ with $t_{start}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not $C_{2\text{pt}}$.

→ K. Ott nad parallel, Thu 12:00

Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}, T = 24a$$

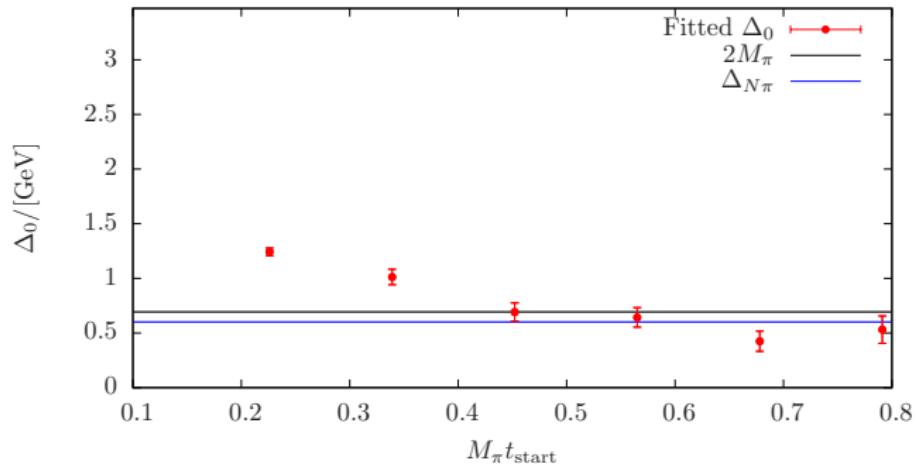
(all T included in fit).

Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{start}$ with $t_{start}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not C_{2pt} .

→ K. Ott nad parallel, Thu 12:00

Simultaneous fits



$$m_\pi = 347 \text{ MeV}, a = 0.064 \text{ fm}$$

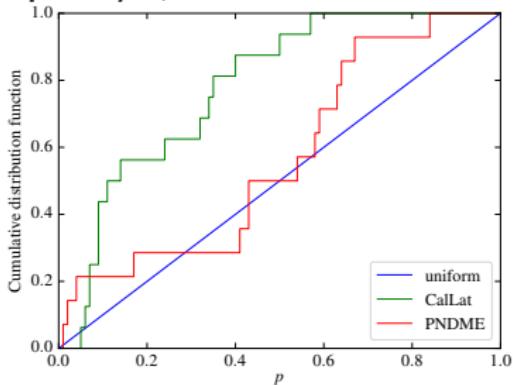
Two-state fit to ratios for six observables in range $\tau, T - \tau \geq t_{\text{start}}$ with $t_{\text{start}}M_\pi = 0.4$ fixed globally for all ensembles.

ΔE determined only from ratios, not C_{2pt} .

→ K. Ott nad parallel, Thu 12:00

Fit quality

Can we trust these fits? Ideally we would require that they have good fit quality. (Not a sufficient condition!)



Should expect uniformly distributed p -values.
Following

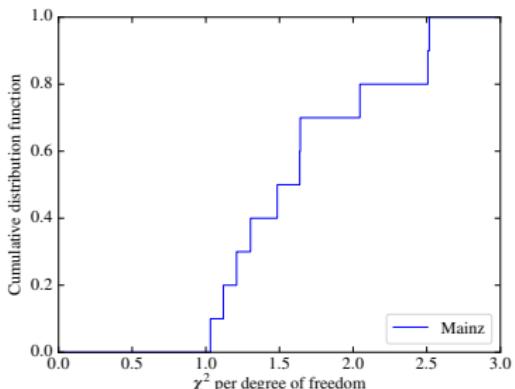
Sz. Borsanyi *et al.*, Science 347, 1452 (2015) [1406.4088]

can use Kolmogorov-Smirnov test.

PNDME: $p = 0.26$

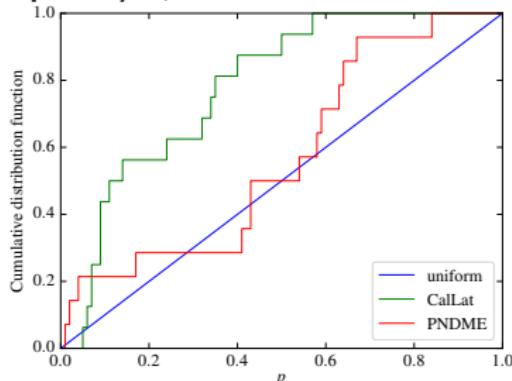
CalLat: $p = 0.00077$

p -values not provided by Mainz,
but reduced χ^2 are large.



Fit quality

Can we trust these fits? Ideally we would require that they have good fit quality. (Not a sufficient condition!)



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Following

Sz. Borsanyi *et al.*, Science 347, 1452 (2015) [1406.4088]

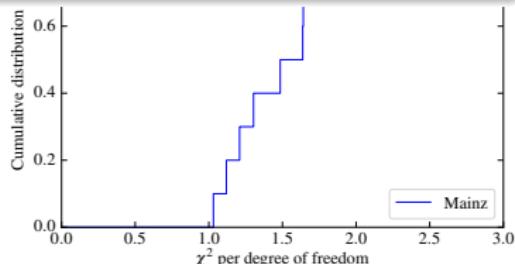
can use Kolmogorov-Smirnov test.

PNDME: $p = 0.26$

CalLat: $p = 0.00077$

May be difficult to estimate χ^2 due to difficulty inverting large covariance matrix.

p -values not provided by Mainz,
but reduced χ^2 are large.



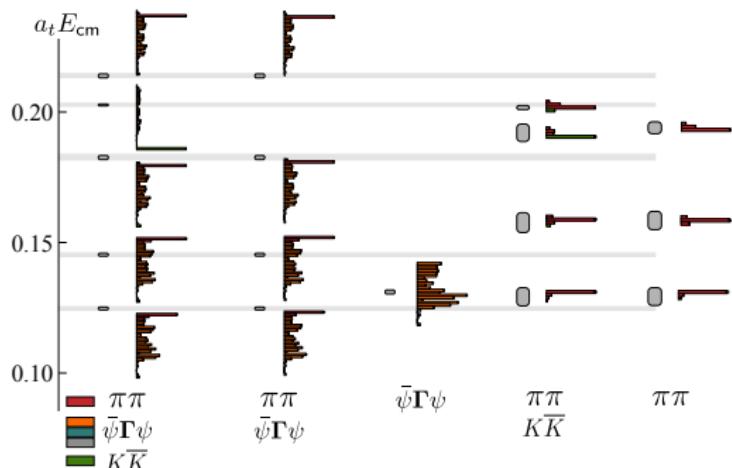
What about $N\pi$ states?

Expected $N\pi$ states not seen in variational or fitting results. Why?

- ▶ Volume suppression? Contamination from each state $\propto L^{-3}$.
But density of states $\propto L^3$ compensates.
- ▶ At short T , higher states dominate?
 \implies not yet in asymptotic regime.

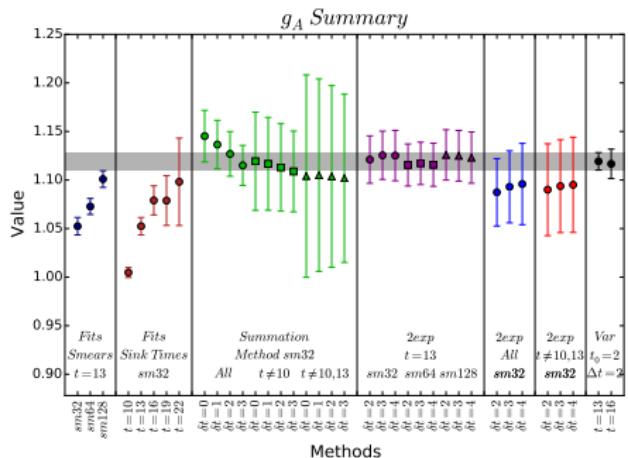
Familiar from meson spectroscopy.

Must include nonlocal operators in variational basis to identify complete spectrum.



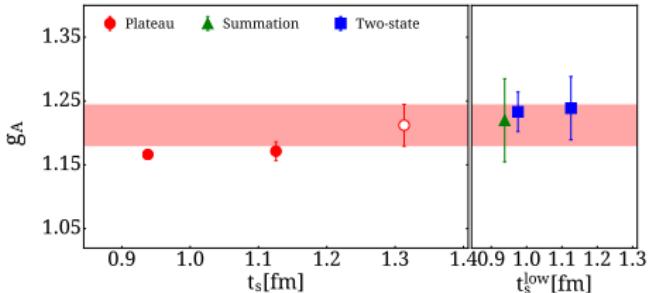
D. J. Wilson *et al.* (HadSpec), Phys. Rev. D 92, 094502 (2015)
[1507.02599]

Comparison of several approaches



$$m_\pi = 460 \text{ MeV}, a = 0.074 \text{ fm}$$

J. Dragos et al., PRD 94, 074505 (2016) [1606.03195]



$m_\pi = 130 \text{ MeV}, a = 0.093 \text{ fm}$

C. Alexandrou *et al.* (ETMC), PRD 96, 054507 (2017)

[1705.03399]

May be best to estimate systematic uncertainty using multiple different approaches.

Finite-volume effects

In general suppressed as $e^{-m_\pi L}$.

For g_A , computed in heavy baryon ChPT including Δ degrees of freedom:

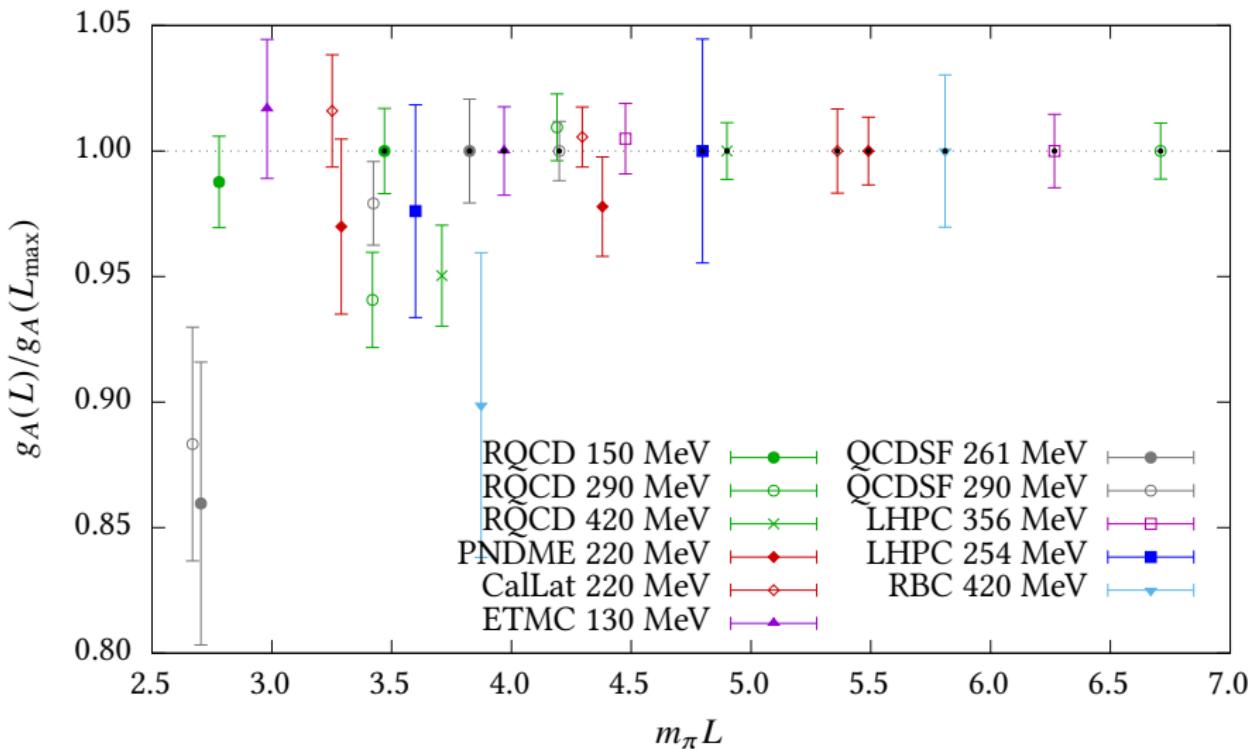
S. R. Beane and M. J. Savage, Phys. Rev. D **70** 074029 (2004) [[hep-ph/0404131](#)]

$$\frac{g_A(L) - g_A}{g_A} = \frac{m_\pi^2}{3\pi^2 f_\pi^2} \left[g_A^2 F_1(m_\pi L) + g_{\Delta N}^2 \left(1 + \frac{25g_{\Delta\Delta}}{81g_A} \right) F_2(m_\pi, \Delta, L) \right. \\ \left. + F_3(m_\pi L) + g_{\Delta N}^2 F_4(m_\pi, \Delta, L) \right],$$

Neglecting loops with Δ baryons, the leading contribution is

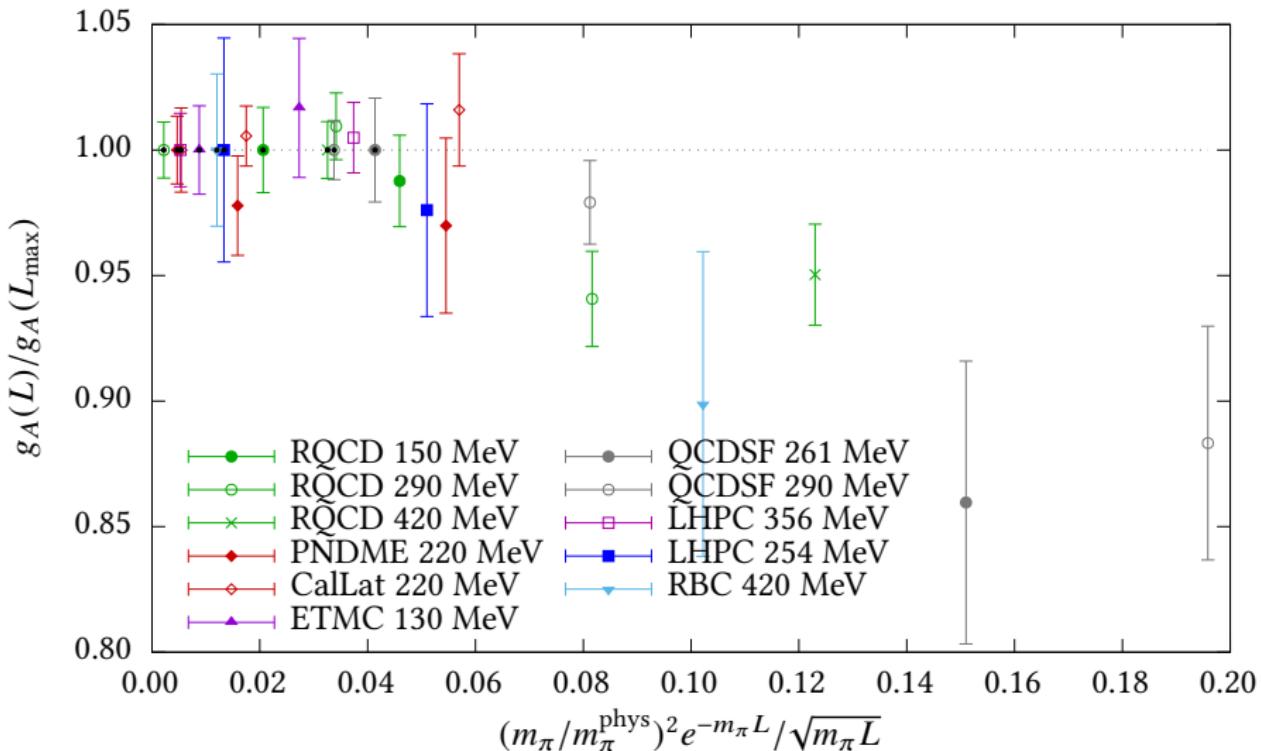
$$\frac{g_A(L) - g_A}{g_A} \sim \frac{m_\pi^2 g_A^2}{3\pi^2 f_\pi^2} \sqrt{\frac{\pi}{2m_\pi L}} e^{-m_\pi L}$$

Finite-volume effects: controlled studies



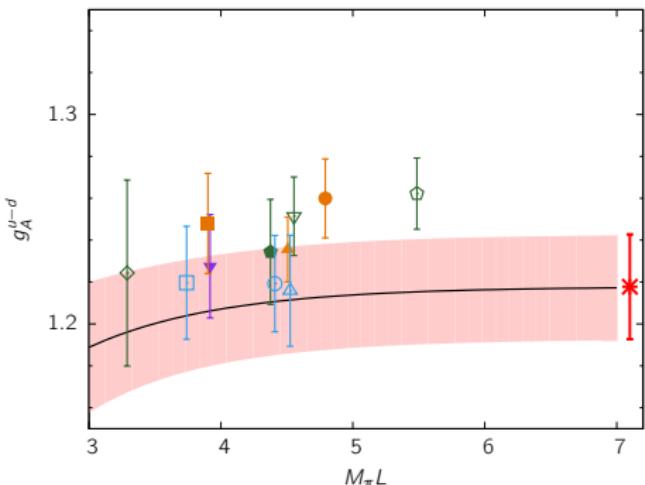
Reference data at L_{\max} indicated with black dot.

Finite-volume effects: controlled studies



Reference data at L_{\max} indicated with black dot.

Finite-volume effects: global fits

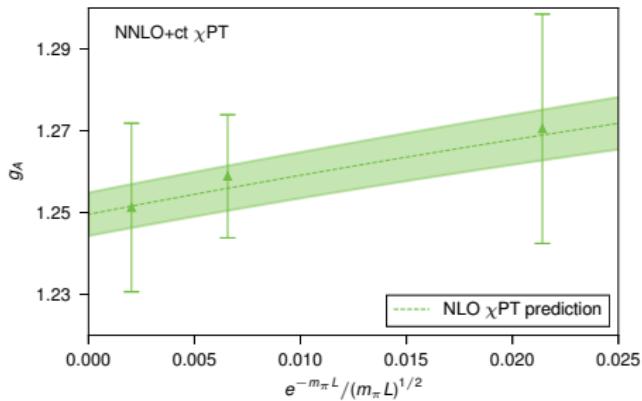


R. Gupta *et al.* (PNDME), 1806.09006

$$g_A(m_\pi, L, a) = f(m_\pi, a) + cm_\pi^2 e^{-m_\pi L}$$

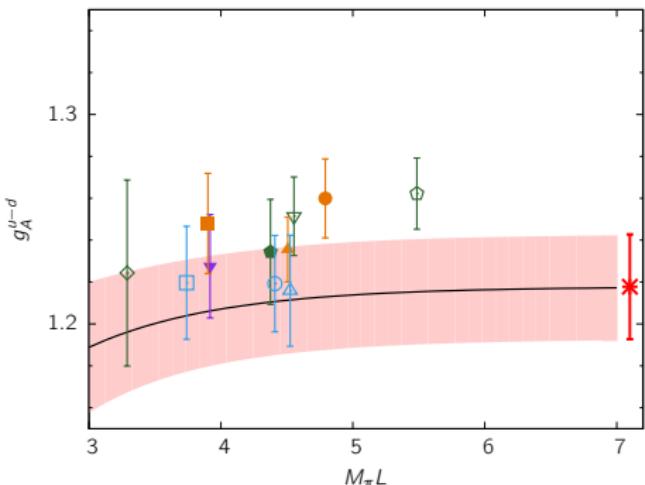
at m_π^{phys} , $m_\pi L = 4$: $-0.9(5)\%$ effect

Larger (few %) effect seen by Mainz group! K. Ott nad parallel, Thu 12:00



C. C. Chang *et al.*, Nature 558, 91 [1805.12130]
leading HBChPT expression (no Δ)

Finite-volume effects: global fits

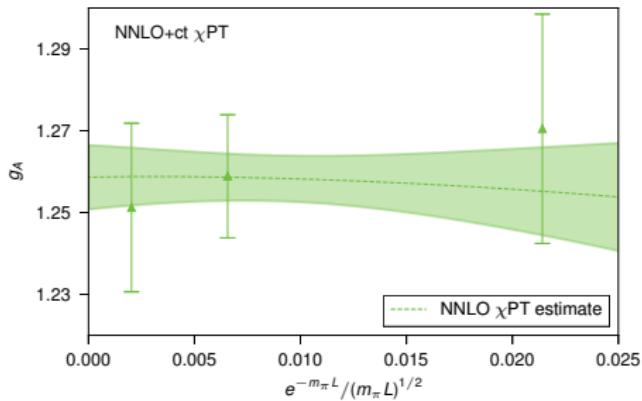


R. Gupta *et al.* (PNDME), 1806.09006

$$g_A(m_\pi, L, a) = f(m_\pi, a) + cm_\pi^2 e^{-m_\pi L}$$

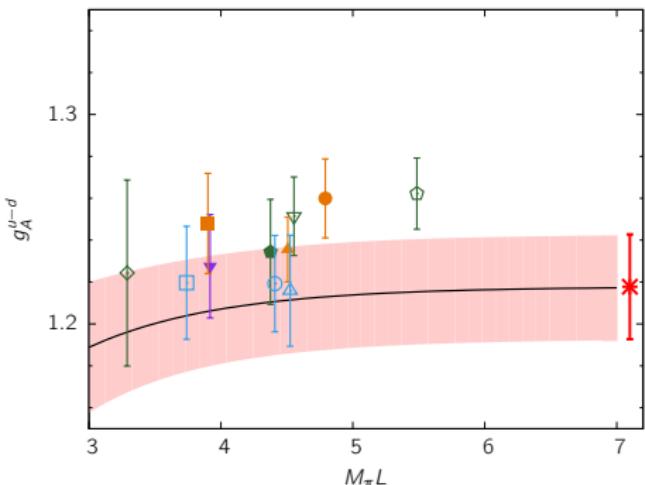
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C. C. Chang *et al.*, Nature 558, 91 [1805.12130]
 leading HBChPT expression (no Δ)
 $+c(m_\pi/f_\pi)^3 F_1(m_\pi L)$

Finite-volume effects: global fits

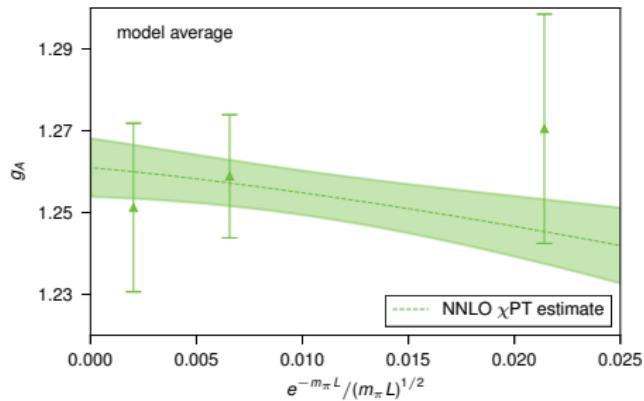


R. Gupta *et al.* (PNDME), 1806.09006

$$g_A(m_\pi, L, a) = f(m_\pi, a) + cm_\pi^2 e^{-m_\pi L}$$

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C. C. Chang *et al.*, Nature 558, 91 [1805.12130]
 leading HBChPT expression (no Δ)
 $+c(m_\pi/f_\pi)^3 F_1(m_\pi L)$

Chiral extrapolation

Increasing number of calculations at physical m_π .
Otherwise need extrapolation.

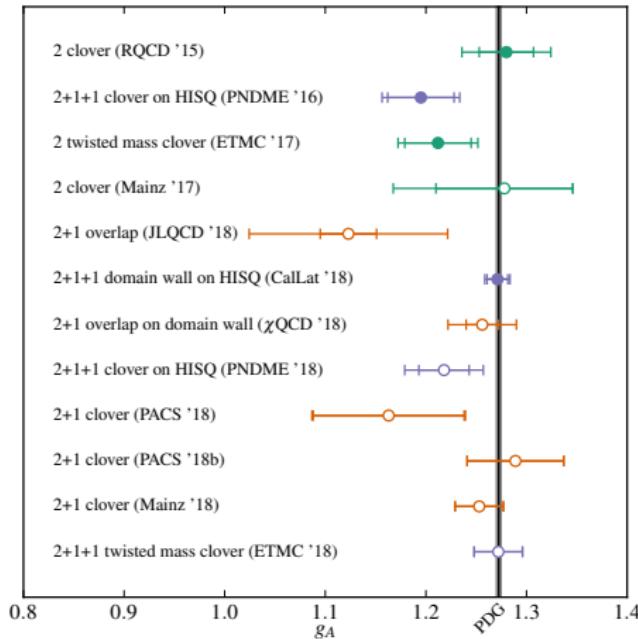
Heavy baryon ChPT for g_A :

$$g_A(m_\pi) = \textcolor{brown}{g}_0 - (\textcolor{brown}{g}_0 + 2\textcolor{brown}{g}_0^3) \left(\frac{m_\pi}{4\pi F} \right)^2 \log \frac{m_\pi^2}{\mu^2} + \textcolor{brown}{c}_1 m_\pi^2 + \textcolor{brown}{c}_2 m_\pi^3 + \dots$$

or use simple polynomials in m_π or m_π^2 .

Range of convergence is a priori unknown.

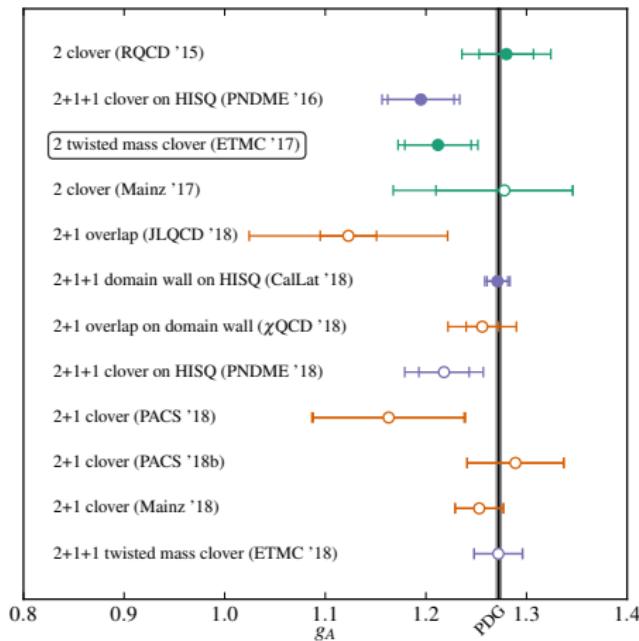
Axial charge: extrapolated and physical-point results



filled symbols: published

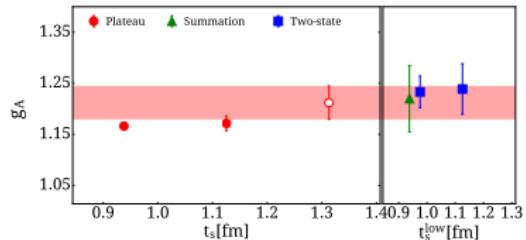
Selection based on quality criteria
still missing!

Axial charge: extrapolated and physical-point results

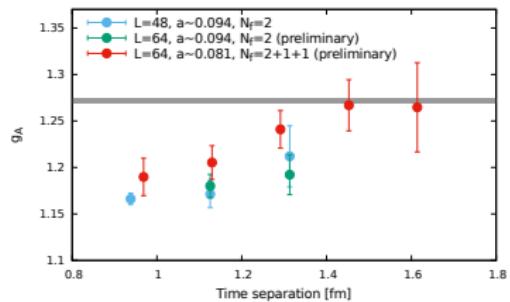


filled symbols: published

Selection based on quality criteria
still missing!



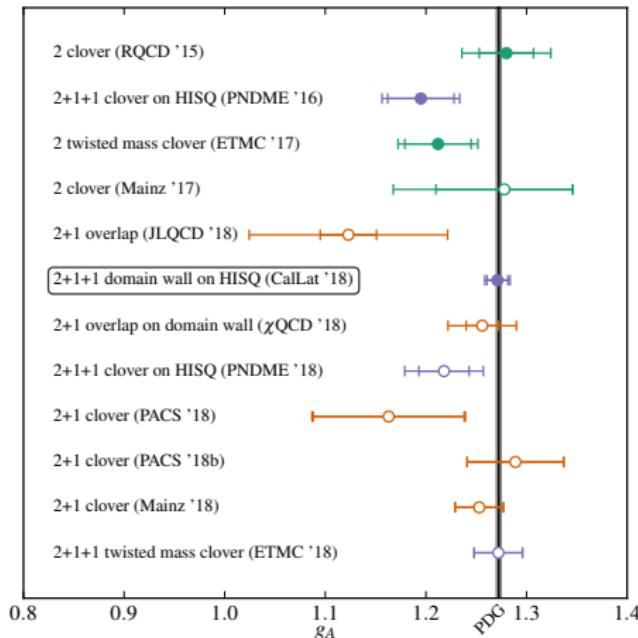
One ensemble: $m_\pi = 130$ MeV,
 $a = 0.093$ fm, $m_\pi L = 3.0$
 C. Alexandrou *et al.* (ETMC),
Phys. Rev. D **96**, 054507 (2017) [1705.03399]



New $N_f = 2$ ensemble: larger volume.

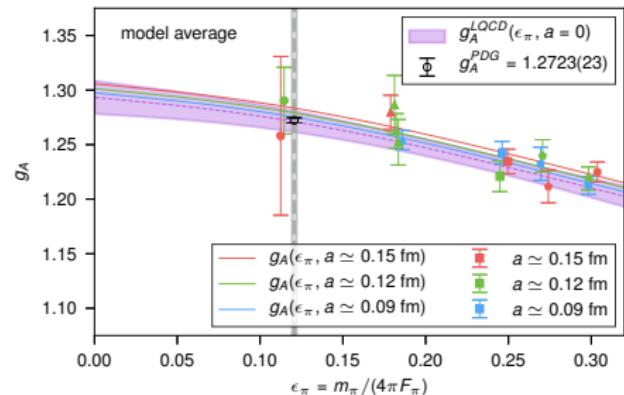
C. Lauer poster

Axial charge: extrapolated and physical-point results



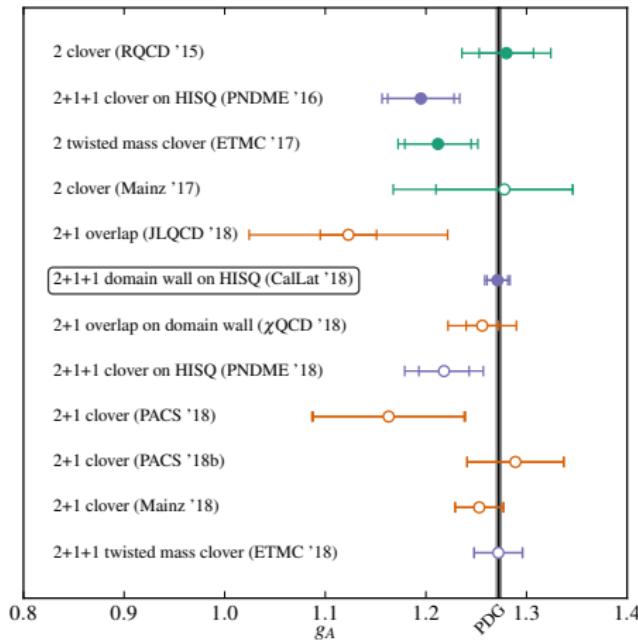
filled symbols: published

Selection based on quality criteria
still missing!



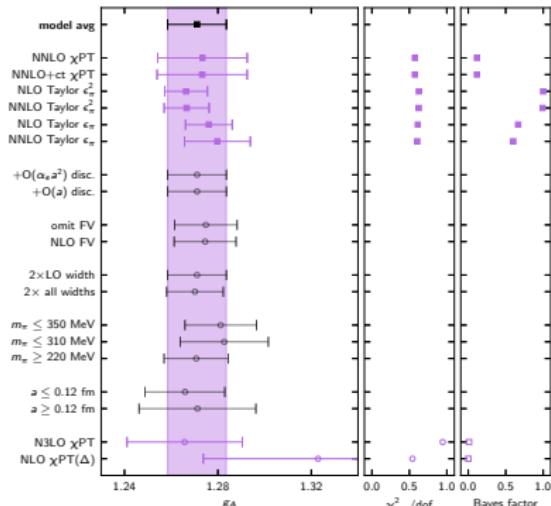
Bayesian average over fit models for
 $g_A(m_\pi/F_\pi, m_\pi L, a^2/w_0^2)$.
C. C. Chang *et al.*, Nature 558, 91 (2018)
[1805.12130]

Axial charge: extrapolated and physical-point results



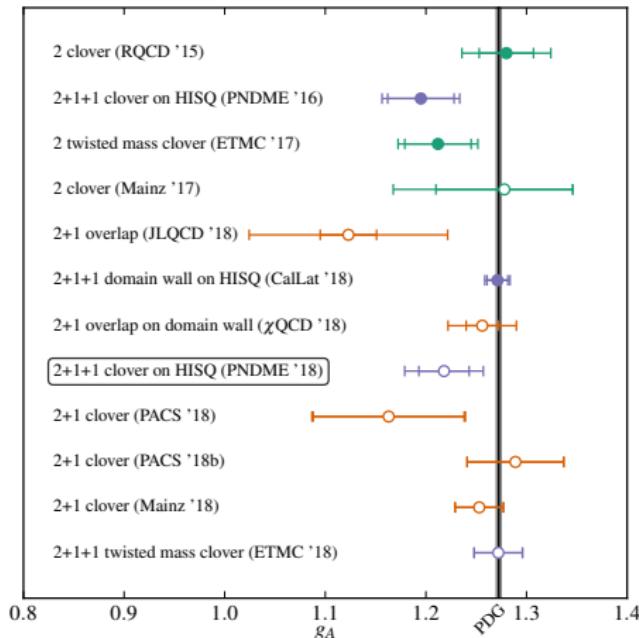
filled symbols: published

Selection based on quality criteria
still missing!



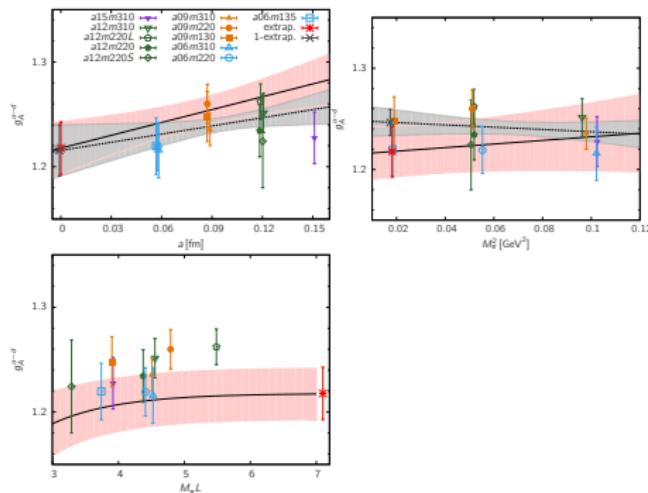
C. C. Chang *et al.*, Nature 558, 91 (2018)
[1805.12130]

Axial charge: extrapolated and physical-point results



filled symbols: published

Selection based on quality criteria
still missing!



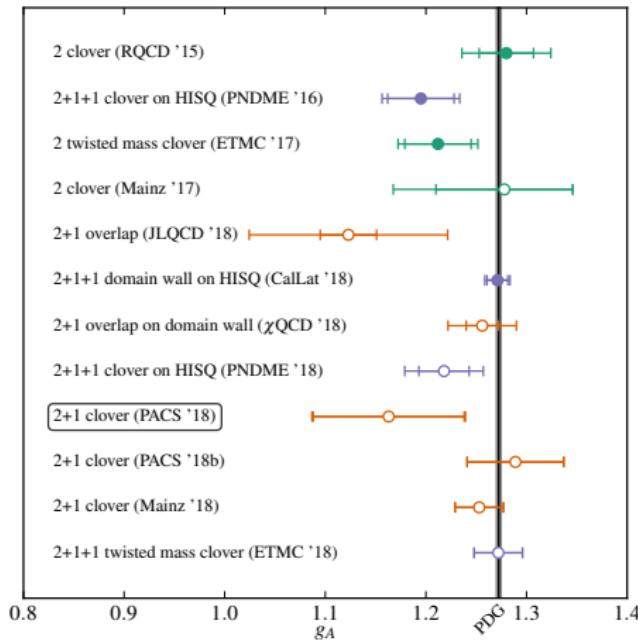
Main ansatz:

$$g_A(m_\pi, L, a) = c_1 + c_2 a + c_3 m_\pi^2 + c_4 m_\pi^2 e^{-m_\pi L}$$

R. Gupta *et al.* (PNDME), 1806.09006

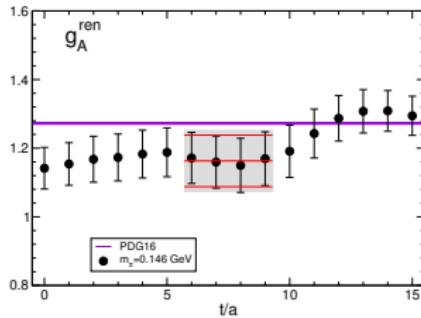
R. Gupta parallel, Thu 12:40

Axial charge: extrapolated and physical-point results



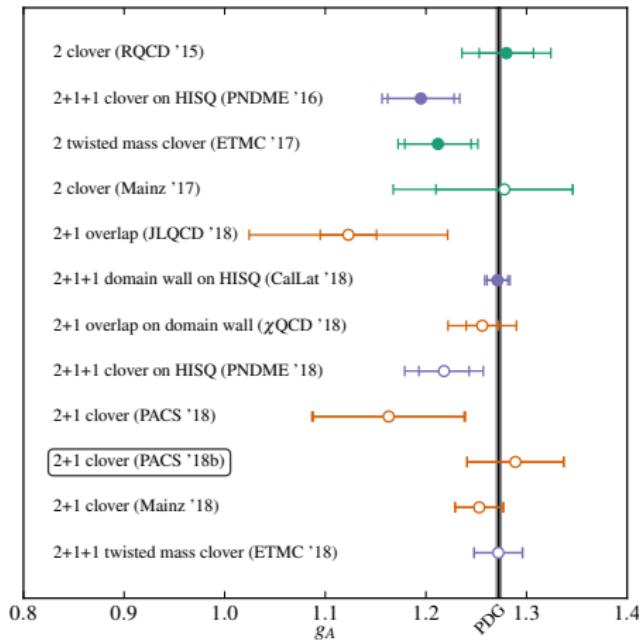
filled symbols: published

Selection based on quality criteria
still missing!



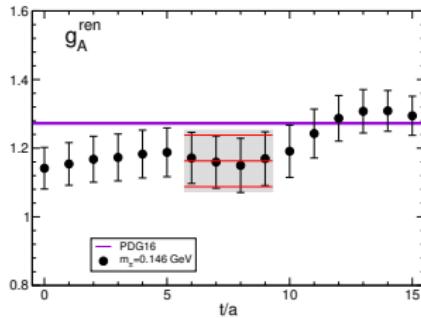
One ensemble: $m_\pi = 146 \text{ MeV}$,
 $a = 0.085 \text{ fm}$, $m_\pi L = 6.0$, $T = 1.3 \text{ fm}$.
K.-I. Ishikawa *et al.* (PACS), 1807.03974

Axial charge: extrapolated and physical-point results

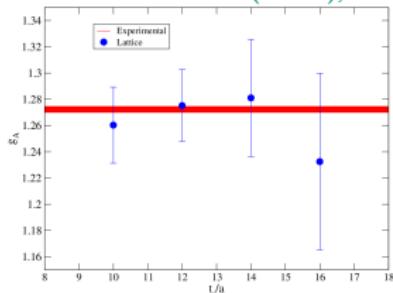


filled symbols: published

Selection based on quality criteria
still missing!

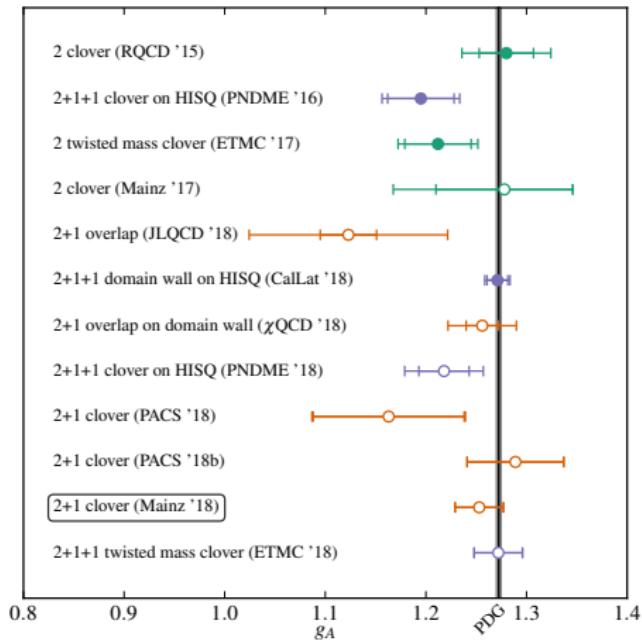


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 K.-I. Ishikawa *et al.* (PACS), 1807.03974



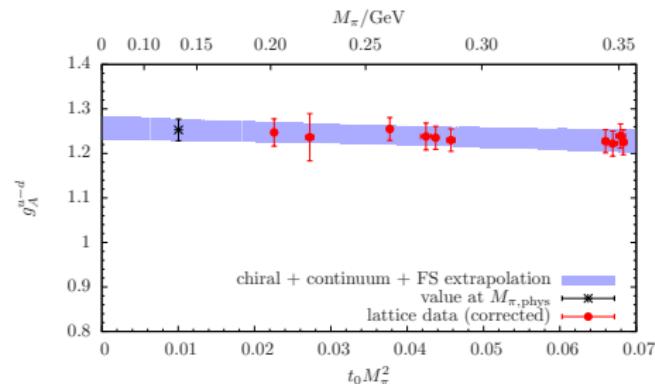
New ensemble: $m_\pi = 135 \text{ MeV}$,
 $L = 10.8 \text{ fm}$. Y. Kuramashi parallel, Thu 9:50

Axial charge: extrapolated and physical-point results



filled symbols: published

Selection based on quality criteria
still missing!



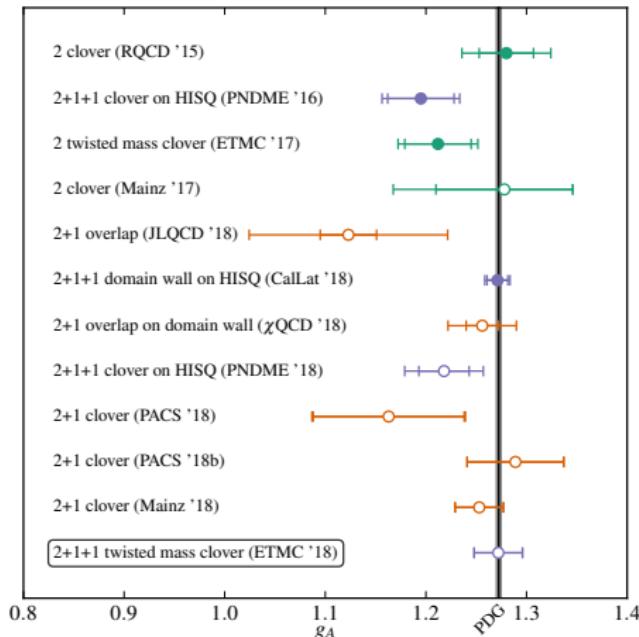
Ensembles have m_s varied such that
 $m_s + m_u + m_d = \text{const.}$

Fit ansatz:

$$g_A(m_\pi, L, a) = c_1 + c_2 a^2 + c_3 m_\pi^2 + c_4 m_\pi^2 e^{-m_\pi L}$$

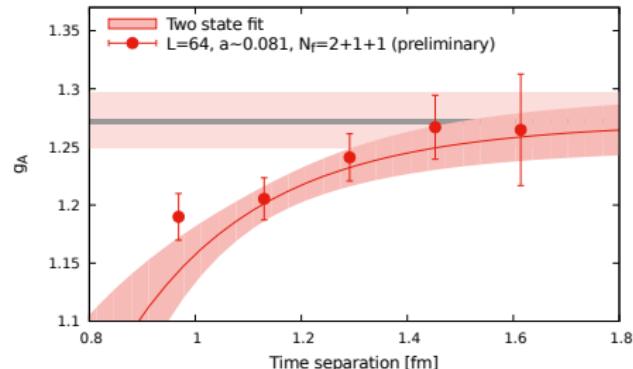
K. Ott nad parallel, Thu 12:00

Axial charge: extrapolated and physical-point results



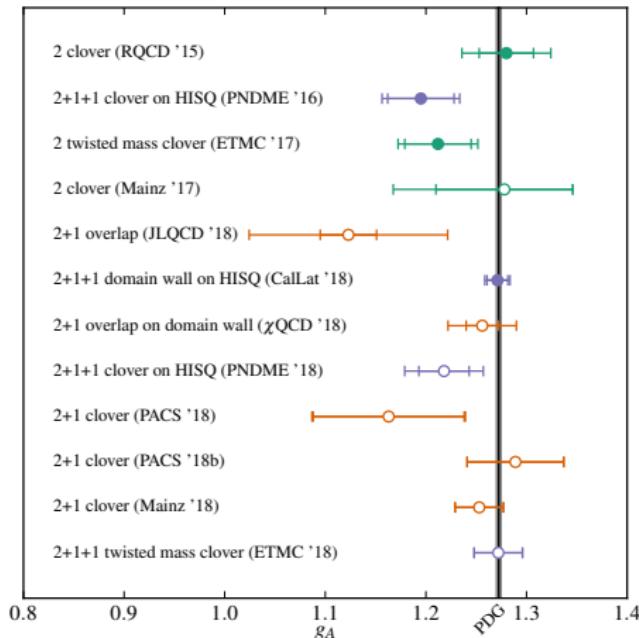
filled symbols: published

Selection based on quality criteria
still missing!



One ensemble: physical m_π ,
 $a = 0.081$ fm, $m_\pi L = 3.6$
C. Lauer poster
M. Constantinou parallel, Fri 17:50

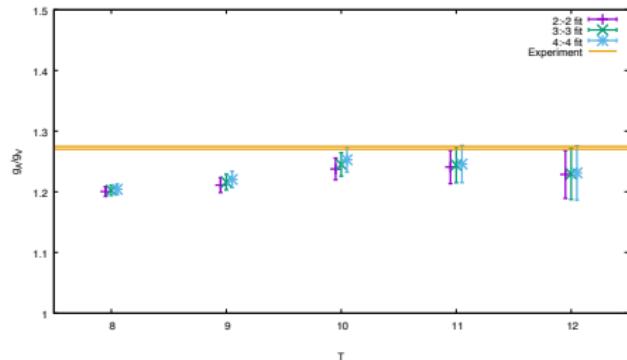
Axial charge: extrapolated and physical-point results



filled symbols: published

Selection based on quality criteria
still missing!

Other preliminary results:



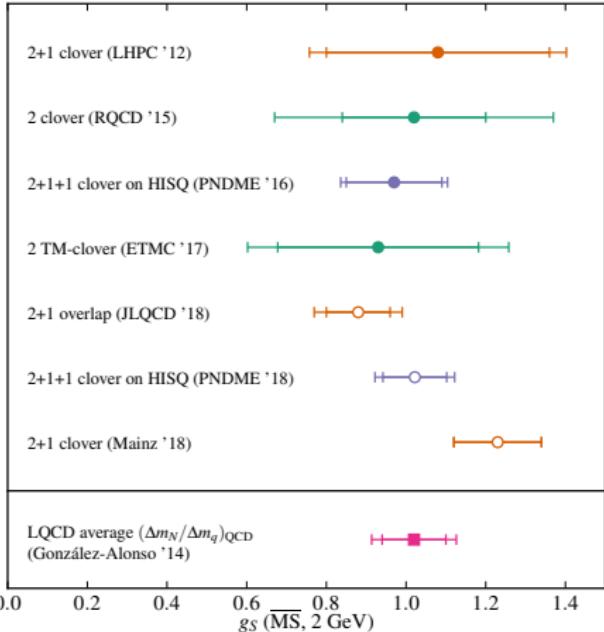
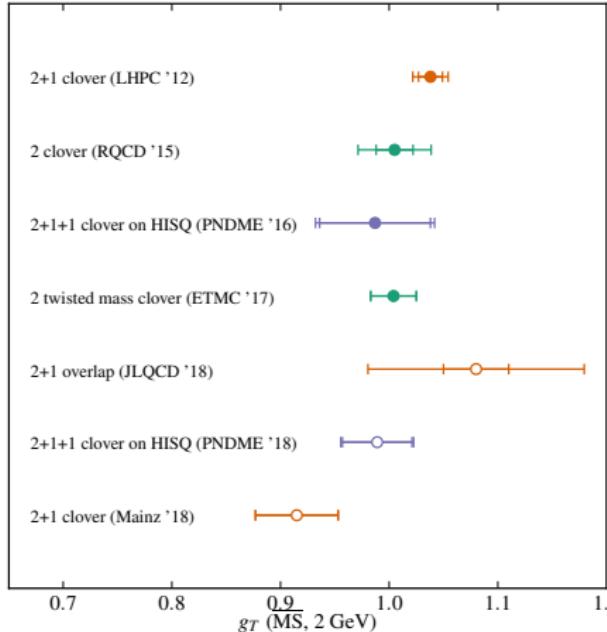
$N_f = 2 + 1$ domain wall (RBC+LHPC)
One ensemble: $m_\pi = 139$ MeV,
 $a = 0.11$ fm, $m_\pi L = 3.9$

S. Ohta parallel, Thu 11:40

$N_f = 2 + 1 + 1$ staggered
(Fermilab-MILC)
Blinded analysis.

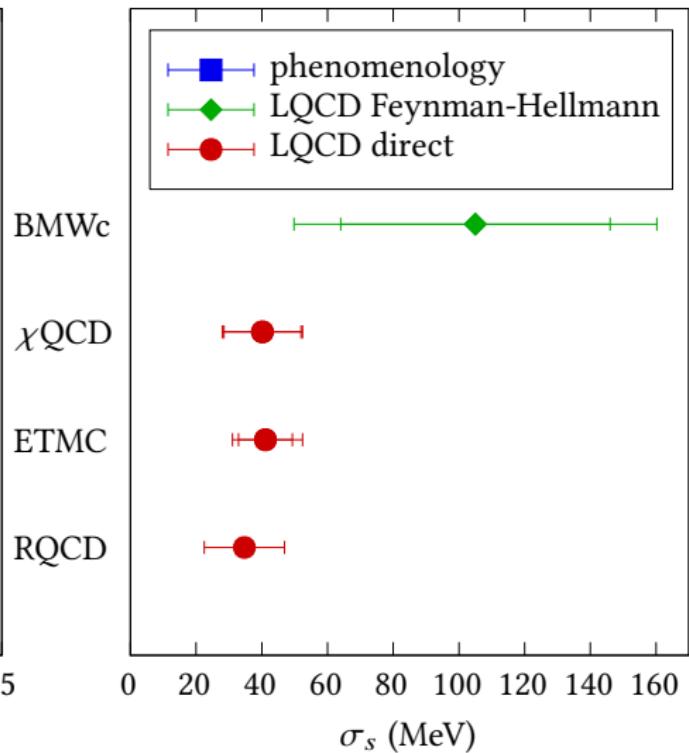
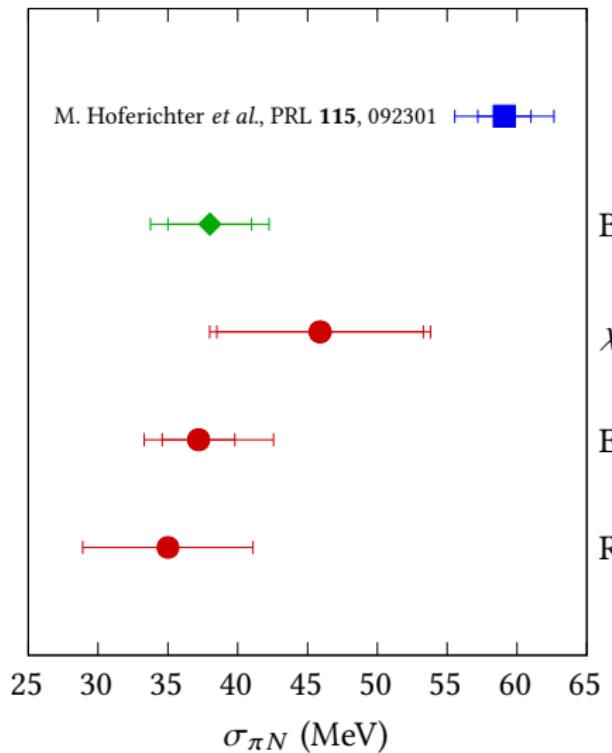
Y. Lin parallel, Thu 9:30

Tensor and scalar charges



Selection based on quality criteria still missing!

Sigma terms



Published results with (near-)physical m_π .

Update on BMWc $N_f = 1+1+1+1$ results: L. Varnhorst and C. Hoelbling, Wed. 16:10 and 16:30

Outlook

- ▶ Excited-state contamination is a major focus.
A full variational study including nonlocal $N\pi$ interpolators would help definitively resolve the issue.
- ▶ No sign of large finite-volume effect in fully-controlled studies at low pion mass.
- ▶ For LQCD averages, individual calculations must be selected using quality criteria. Fitting approaches for excited states vary; setting standards may be difficult.
 - ▶ One attempt: H.-W. Lin *et al.*, “Parton distributions and lattice QCD calculations: A community white paper” *Prog. Part. Nucl. Phys.* **100**, 107 (2018) [1711.07916]
 - ▶ Some nucleon structure to appear in next FLAG review.

Thanks to all who sent results and replied to my questions:

Constantia Alexandrou
Chia Cheng Chang
Colin Egerer
Rajan Gupta
Jian Liang
Shigemi Ohta
Konstantin Ottnad
Finn Stokes
André Walker-Loud
Takeshi Yamazaki

Scalar and tensor charges

Precision β -decay experiments may be sensitive to BSM physics; leading contributions are controlled by the scalar and tensor charges:

T. Bhattacharya *et al.*, Phys. Rev. D **85**, 054512 (2012) [1110.6448]

$$\langle p(P, s') | \bar{u}d | n(P, s) \rangle = \textcolor{brown}{g}_S \bar{u}_p(P, s') u_n(P, s),$$

$$\langle p(P, s') | \bar{u}\sigma^{\mu\nu}d | n(P, s) \rangle = \textcolor{brown}{g}_T \bar{u}_p(P, s') \sigma^{\mu\nu} u_n(P, s).$$

Tensor charges also control contribution from quark electric dipole moment to neutron EDM:

$$\mathcal{L}_{q\text{EDM}} = -\frac{1}{2} \sum_q d_q \bar{q} i \sigma^{\mu\nu} \gamma_5 q F_{\mu\nu} \implies d_n = d_u \textcolor{brown}{g}_T^d + d_d \textcolor{brown}{g}_T^u + d_s \textcolor{brown}{g}_T^s + \dots .$$

For scalar charge, conserved vector current relation implies

$$\textcolor{brown}{g}_S = \frac{(m_n - m_p)_{\text{QCD}}}{m_d - m_u},$$

up to second-order isospin breaking.

M. González-Alonso and J. Martin Camalich, Phys. Rev. Lett. **112**, 042501 (2014) [1309.4434]

Sigma terms

The contributions from quark masses to the nucleon mass.

$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle \quad \sigma_q = m_q \langle N | \bar{q}q | N \rangle$$

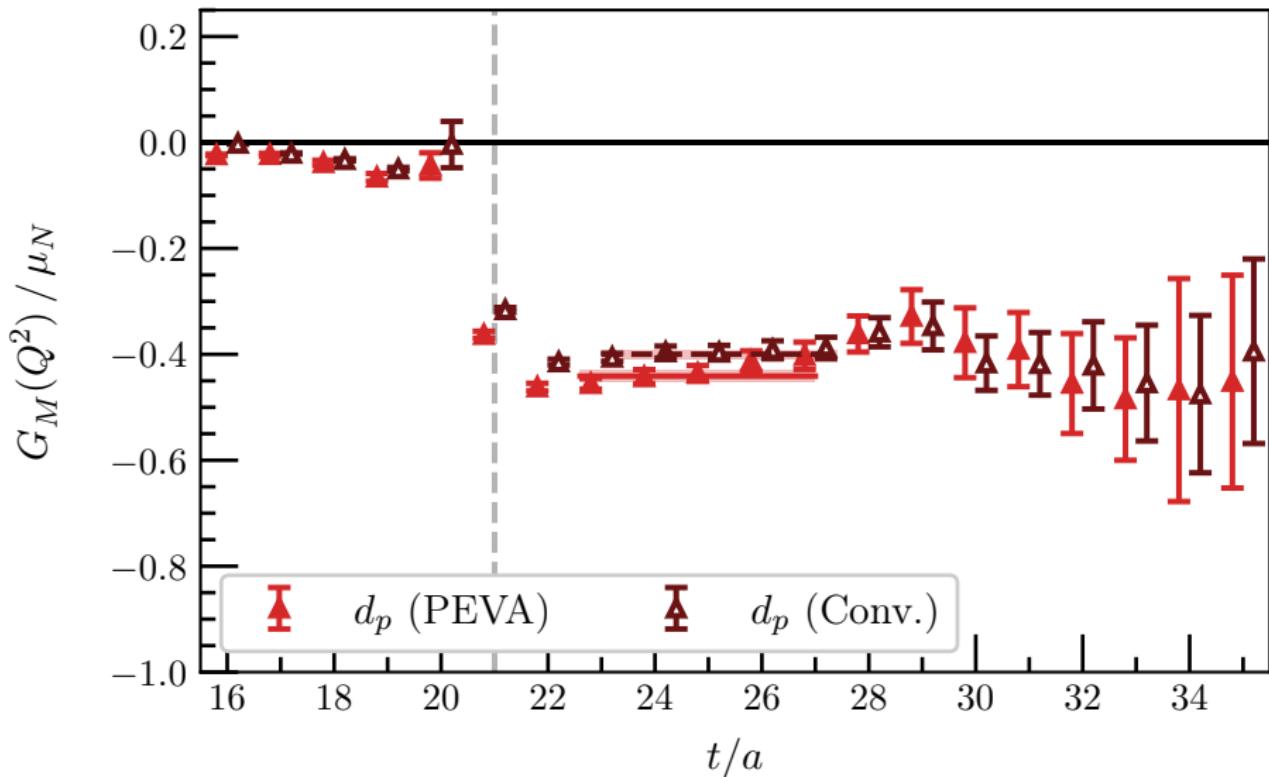
Control the sensitivity of direct-detection dark matter searches to WIMPs that interact via Higgs exchange.

Phenomenological value $\sigma_{\pi N} = 59.1 \pm 3.5$ MeV determined based on combining:

- ▶ Cheng-Dashen low-energy theorem
- ▶ Roy-Steiner equations to constrain πN scattering amplitude
- ▶ Precise πN scattering lengths from pionic atoms

M. Hoferichter *et al.*, Phys. Rev. Lett. **115**, 092301 (2015) [1506.04142]

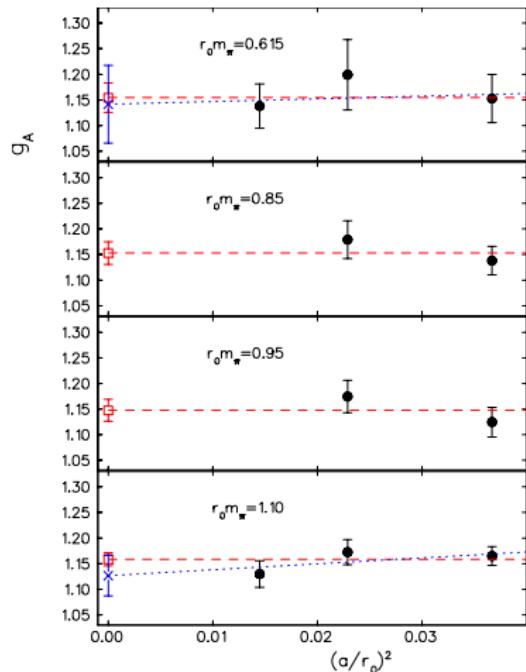
Variational approach: nonzero momentum



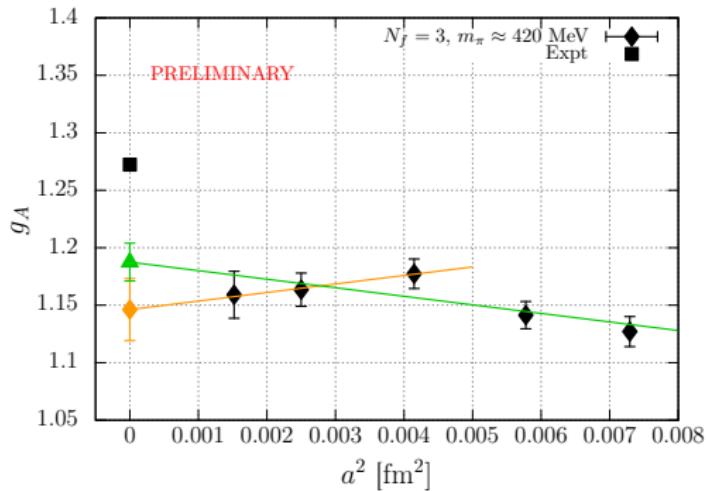
Include operators with negative parity in variational basis.

F. Stokes *et al.*, in preparation

Discretization effects: controlled studies



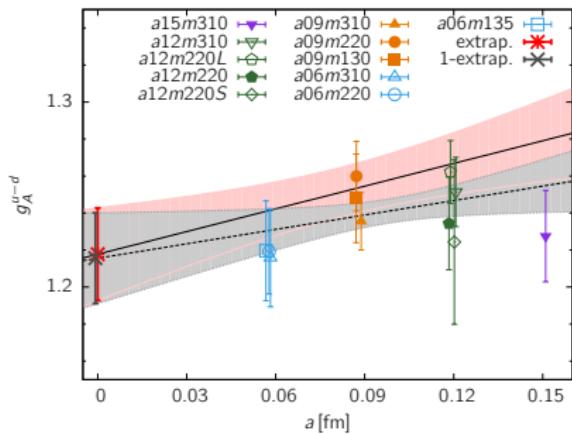
C. Alexandrou *et al.* (ETMC),
Phys. Rev. D 83, 045010 (2011) [1012.0857]
 $N_f = 2$ Wilson twisted mass.
No significant effect seen.



G. S. Bali *et al.* (RQCD), PoS LATTICE2016, 106
[1702.01035]

$N_f = 3$ clover.
Linear in a^2/t_0 extrapolations.

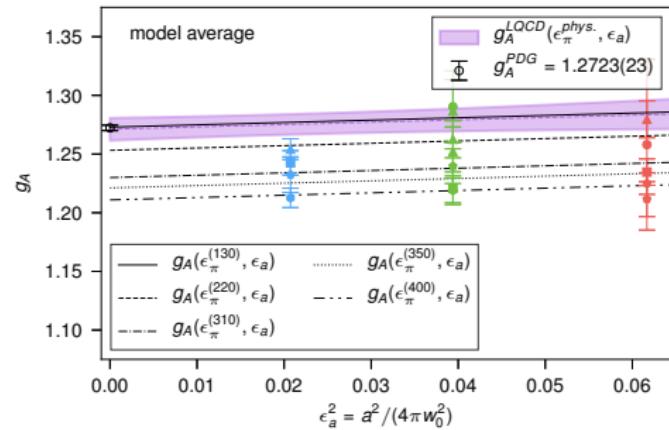
Discretization effects: global fits



R. Gupta *et al.* (PNDME), 1806.09006

$N_f = 2 + 1 + 1$ clover on HISQ.

pink: $g_A(m_\pi, L, a) = f(m_\pi, L) + ca/r_1$
 gray: $g_A(m_\pi, L, a) = g_A + ca/r_1$



C. C. Chang *et al.*, Nature 558, 91 [1805.12130]

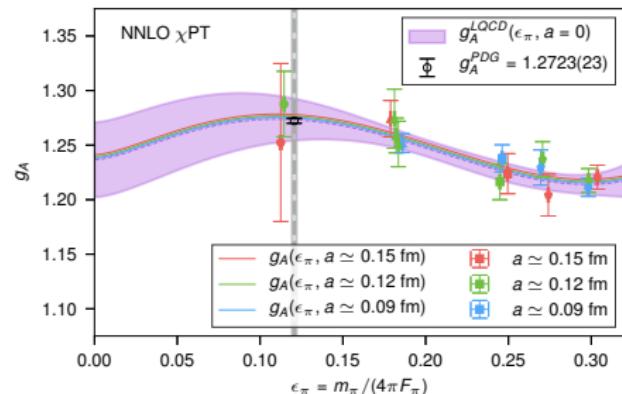
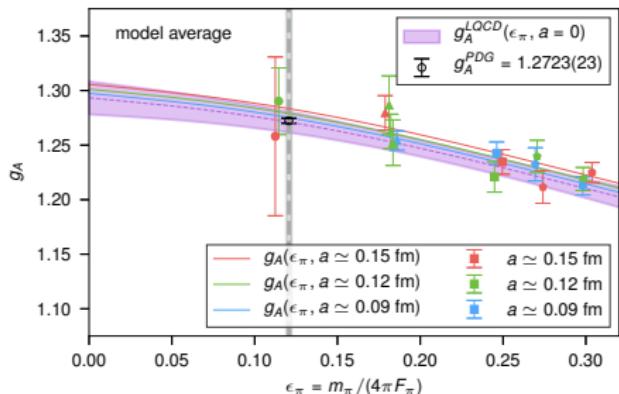
$N_f = 2 + 1 + 1$ domain wall on HISQ.

$g_A(m_\pi, L, a) = f(m_\pi, L) + ca^2/w_0^2$

Chiral log?

Preferred fit: $g_A(m_\pi) = c_1 + c_2 m_\pi^2$

$$\text{ChPT: } g_A(m_\pi) = g_0 - (g_0 + 2g_0^3) \left(\frac{m_\pi}{4\pi F} \right)^2 \log m_\pi^2 + cm_\pi^2 + \dots$$

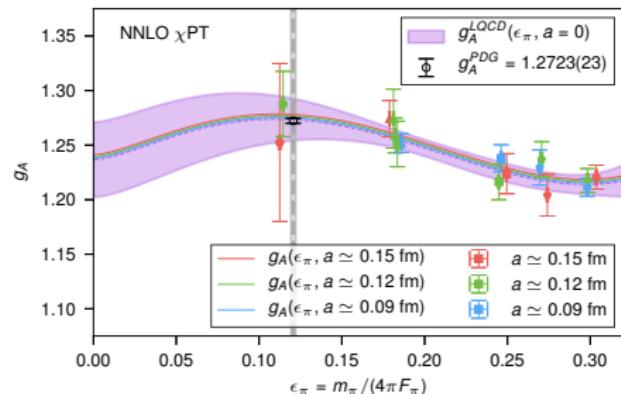
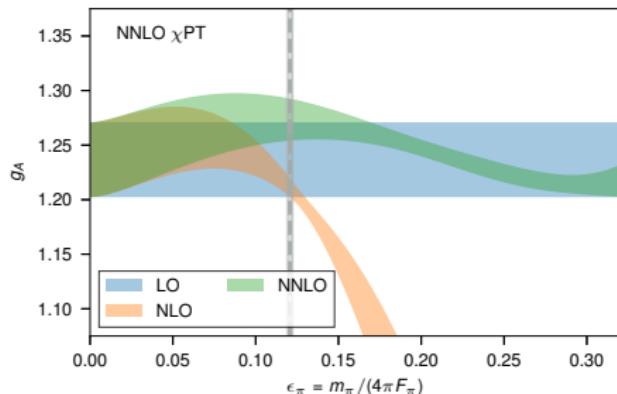


C. C. Chang *et al.*, Nature 558, 91 (2018) [1805.12130]

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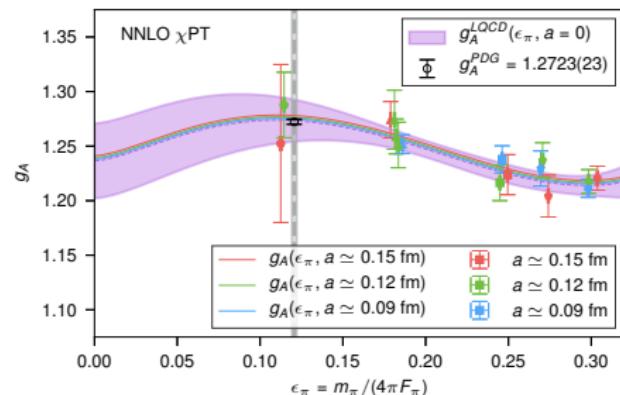
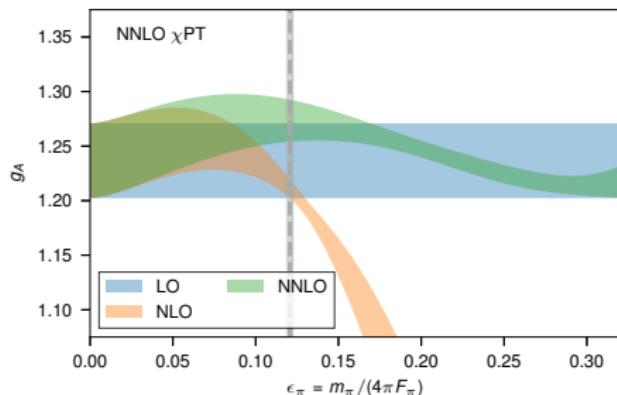


C. C. Chang *et al.*, Nature 558, 91 (2018) [1805.12130]

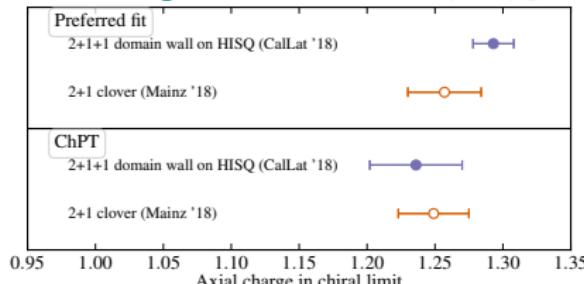
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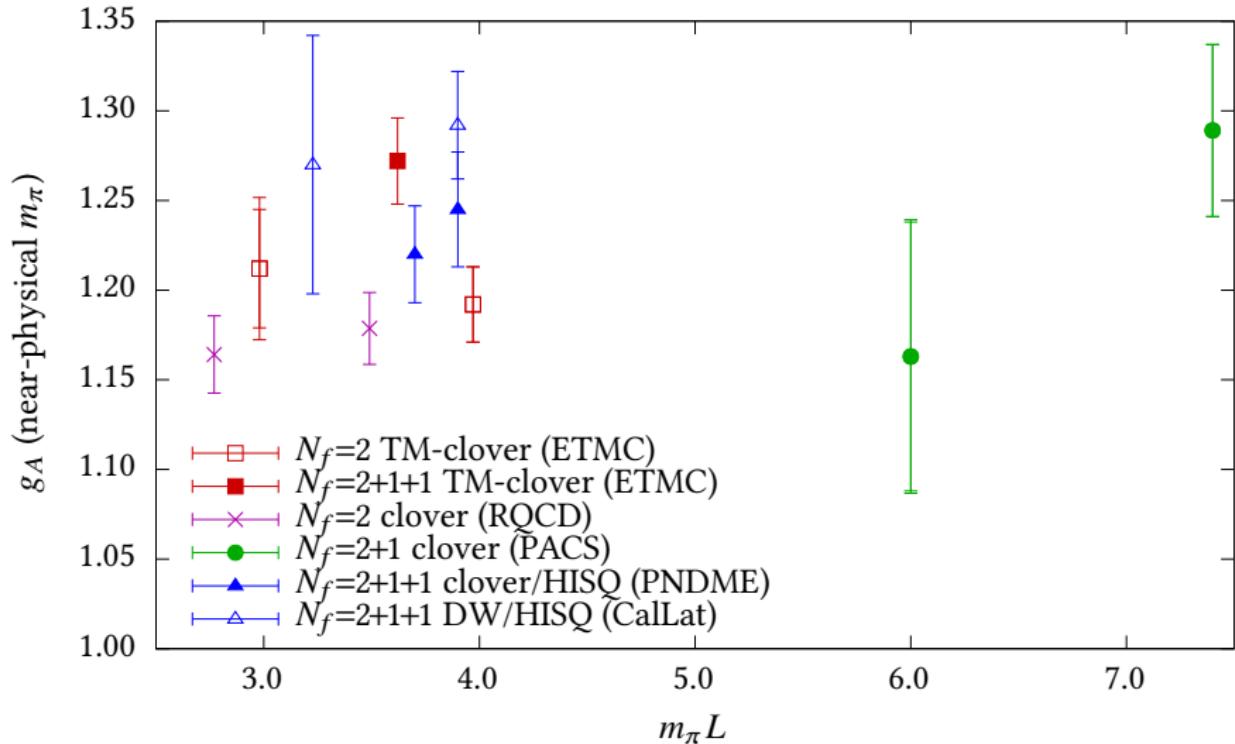


C. C. Chang *et al.*, Nature 558, 91 (2018) [1805.12130]



May need precise data below m_π^{phys} to see chiral log.

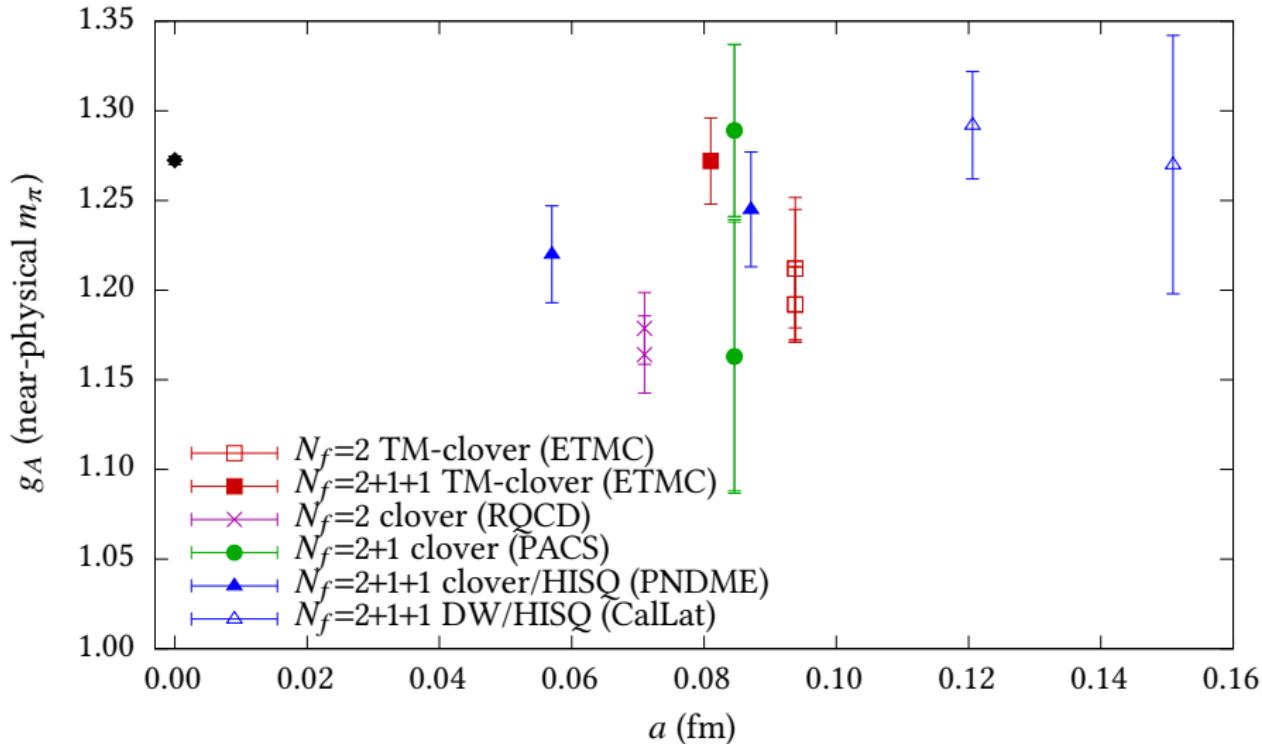
Calculations with near-physical m_π



see also: $N_f = 2 + 1$ domain wall (RBC+LHPC) → S. Ohta parallel, Thu 11:40

$N_f = 2 + 1 + 1$ HISQ staggered → Y. Lin parallel, Thu 9:30

Calculations with near-physical m_π



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