Results for the mass difference between the long- and short-lived K mesons for physical quark masses

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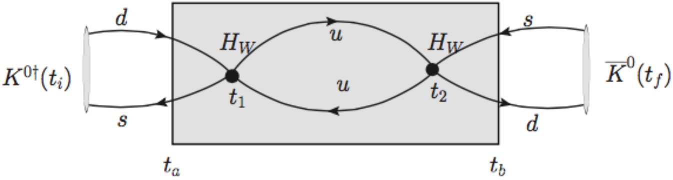
Motivation

- Physics:
  - $\Delta m_K = m_{K_L} - m_{K_S}$ is generated by K meson mixing through weak interaction
  - $\Delta m_{K,exp} = m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12} MeV$
    A discrepancy between the Standard Model prediction for this quantity and its experimental value will imply the existence of new physics
- Calculation:
  - This highly non-perturbative quantity is suitable for using Lattice QCD
  - Δm_K is one of RBC-UKQCD collaboration's calculations of long-distance contributions in kaon physics. Therefore, it is closely related to other kaon physics calculations like ϵ_K and rare kaon decays

From Integrated Correlator to Δm_K^{lat}



- Δm_K is given by:

$$\begin{aligned}\Delta m_K &\equiv m_{K_L} - m_{K_S} \\ &= 2\mathcal{P}\sum_n \frac{\langle K^0|H_W|n\rangle\langle n|H_W|\bar{K}^0\rangle}{m_K - E_n}\end{aligned} \quad (1)$$

- The integrated correlator is defined as:

$$\mathcal{A} = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b}\langle 0|T\{\bar{K}^0(t_f)H_W(t_2)H_W(t_1)K^0(t_i)\}|0\rangle \quad (2)$$

From Integrated Correlator to Δm_K^{lat}

- If we insert a complete set of intermediate states, we find:

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle K^0|H_W|n\rangle\langle n|H_W|\bar{K}^0\rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\} \tag{3}$$

  with $T \equiv t_b - t_a + 1$.
- For $|n\rangle$(in our case $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$) with $E_n < m_K$ or $E_n \sim m_K$: the exponential terms will be significant. We can:
  - use the freedom of adding $c_s \bar{s} d$, $c_p \bar{s}\gamma^5 d$ operators to the weak Hamiltonian to remove two of the contributions. Here we choose:

$$\langle 0|H_W - c_p \bar{s}\gamma_5 d|K^0\rangle = 0, \langle \eta|H_W - c_s \bar{s} d|K^0\rangle = 0$$

  - subtract contributions from other states($|\pi\rangle$, $|\pi\pi\rangle$) explicitly
- Therefore, by fitting the coefficient of T from integrated correlators we can obtain:

$$\Delta m_K^{lat} \equiv 2 \sum_n \frac{\langle K^0|H_W|n\rangle\langle n|H_W|\bar{K}^0\rangle}{m_K - E_n} \tag{4}$$

Calculation of Δm_K^{lat}

- The $\Delta S = 1$ effective Weak Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \qquad (5)$$

where the $Q_i^{qq'}{}_{i=1,2}$ are current-current opeartors, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j)$$
$$Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)$$

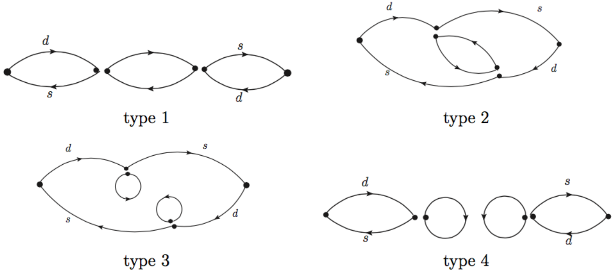
- There are four states need to subtracted: $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$. We add $c_s \bar{s} d$, $c_p \bar{s} \gamma^5 d$ operators to weak operators to make:

$$\langle 0|Q_i - c_{pi}\bar{s}\gamma_5 d|K^0\rangle = 0, \langle \eta|Q_i - c_{si}\bar{s}d|K^0\rangle = 0 \qquad (6)$$

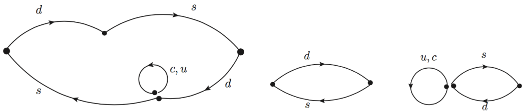
$$Q'_i = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s}d \qquad (7)$$

Calculation of Δm_K^{lat}

- For contractions among Q_i, there are four types of diagrams to be evaluated.



- In addition, there are "mixed" diagrams from the contractions between the $c_s \bar{s} d$ $c_p \bar{s}\gamma^5 d$ operators and Q_i operators.



From Δm_K^{lat} to Δm_K

To get Δm_k from Δm_K^{lat}, we need to consider:

- Ultraviolet divergences as the two H_W approach each other:
  GIM mechanism removes both quadratic and logarithmic divergences
- Renormalization of Lattice operator $Q_{1,2}$ in 3 steps:
  - Non-perturbative Renormalization: from lattice to RI-SMOM
  - Perturbation theory: from RI-SMOM to $\overline{MS}$

    C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 014001
  - Use Wilson coefficients in the $\overline{MS}$ scheme

    G. Buchalla, A.J. Buras and M.E. Lautenbacher, arXiv:hep-ph/9512380

Status of RBC-UKQCD calculations of Δm_k

- **"Long-distance contribution ot the $K_L - K_S$ mass difference"**, **N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu**

  Phys. Rev. D 88(2013), 014508

  Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses($m_\pi = 421 MeV$) including only connected diagrams
- **"$K_L - K_S$ mass difference from Lattice QCD"** **Z. Bai, N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu**

  Phys. Rev. Lett. 113(2014), 112003

  All diagrams included on a $24^3 \times 64$ lattice with unphysical masses
- **"Neutral Kaon Mixing from Lattice QCD"**

  Z. Bai, Ph.D. thesis(2017),

  Presented by C. T. Sachrajda in Lattice 2017

  All diagrams included on a $64^3 \times 128$ lattice with **physical mass** on 59 configurations: $\Delta m_k = (5.5 \pm 1.7) \times 10^{-12} MeV$
- Here I present an update of the methods used and results extending Z. Bai's calculation from 59 to 129 configurations.

Details of the Calculation

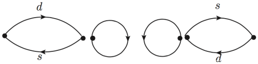
- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV)

  Input parameters are listed below:

a^{-1}/GeV	β	am_l	am_h	$\alpha = b + c$	L_s
2.36	2.25	0.0006203	0.02539	2.0	12

  We used $am_c \simeq 0.31$.

- Data and Data Analysis:
  - Sampling AMA Correction and Super-jackknife Method
  - Disconnected Type4 diagrams:
    save left- and right-pieces separately and use multiple source-sink separation for fitting



type 4

Sampling AMA Correction

- We use Sampling All Mode Averaging (AMA) to reduce the computational cost.

  T. Blum, T. Izubuchi, and E. Shintani, Phys. Rev. D88(9), 094503 (2013)

data type	CG stop residual
sloppy	$1e-4$
exact	$1e-8$

  The difference between the "exact" and the "sloppy" result for a same quantity(e.g. a strange propagator) is used as a correction.
- Usually AMA correction is performed on each configuration, among different time slices
- Our Sampling AMA correction is applied among configurations
- We do only "sloppy" measurements on most configurations and do both "sloppy" and "exact" measurements on some other configurations to serve as corrections.

Super-jackknife Method

- The super-jackknife method is used to estimate the error when we have more than one set of measurement and would like to combine the data for fitting.
- For example, we have:

Y_1	Y_2	$\cdots$	Y_{N_1-1}	Y_{N_1}	Z_1	Z_2	$\cdots$	Z_{N_2-1}	Z_{N_2}

N_1-elements N_2-elements

$\tilde{Y}_1 + \bar{Z}$	$\tilde{Y}_2 + \bar{Z}$	$\cdots$	$\tilde{Y}_{N-1} + \bar{Z}$	$\tilde{Y}_N + \bar{Z}$	$\bar{Y} + \tilde{Z}_1$	$\bar{Y} + \tilde{Z}_2$	$\cdots$	$\bar{Y} + \tilde{Z}_{N_2-1}$	$\bar{Y} + \tilde{Z}_{N_2}$

$(N_1 + N_2)$-elements

- In our case of sampling AMA,
  Y_i's are "sloppy" correlators from most configurations with only "sloppy" measurements, while
  Z_i's are corrections of correlators from configurations with both "sloppy" and "exact" measurements.

Update of the results

Number of Measurements

- In Lattice 2017, Prof. C. T. Sachrajda presented Z. Bai's preliminary results based on an analysis of 59 configurations:
  - type3 & 4 diagrams on 52 sloppy, and 7 correction configurations
  - type1 & 2 diagrams on 11 exact configurations
  - $\sigma_{total} \sim \sqrt{\sigma^2_{tp12} + \sigma^2_{tp34}}$
- Since August 2017, following the same routine, we finished more measurements to reduce statistical errors from both type12 and type34 contributions.

Data Set	# of Sloppy	# of Correction	# of Type12
Lattice 17	52	7	11
Since Aug. 2017	61	9	6
Total	113	16	17

Update of the results

Compare the errors from "sloppy" measurements

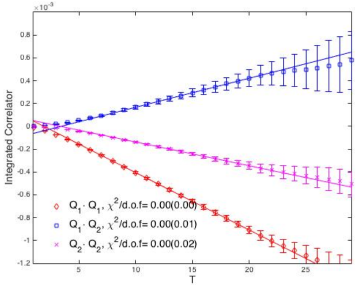
- Keep 7 AMA corrections and 11 Type 12 contribution averaged
- Compare fitting results of 2-point and 3-point functions:

Num	m_π	m_K	$\langle\pi\pi_{I=0}\|Q_1\|K^0\rangle$	$\langle\pi\pi_{I=0}\|Q_2\|K^0\rangle$
113	0.05733(9)	0.21041(14)	-7.9(13)$\times10^{-5}$	0.90(15) $\times10^{-4}$
52	0.05757(12)	0.21051(21)	-7.1(20)$\times10^{-5}$	0.90(20) $\times10^{-4}$
61	0.05713(12)	0.21033(20)	-9.2(26)$\times10^{-5}$	0.91(18)$\times10^{-4}$

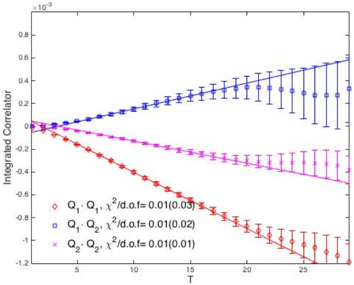
- The errors for these fitting results are reduced to $\frac{1}{\sqrt{113/52}} \sim 0.67$

Update of the results

Compare the errors from type1 & 2 diagrams, uncorrelated, preliminary



(a) 17 exact type1&2

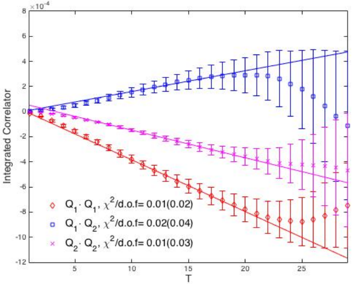


(b) 11 exact type1&2

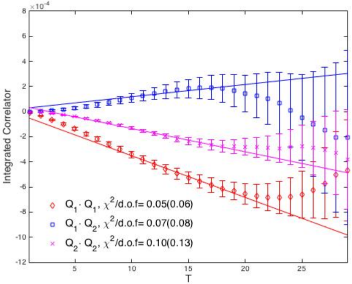
- type1&2 diagrams with fitting range 10:20
  17 configurations $\Delta m_{K,tp12} = 7.82(79)$
  11 configurations $\Delta m_{K,tp12} = 7.29(116)$
- Error from type12 is also reduced to $\frac{1}{\sqrt{17/11}} \sim 0.80$

Update of the results

Compare the Integrate Correlator Fittings: All diagrams, uncorrelated, preliminary



(a) All diagrams fitting: 129 configurations



(b) All diagrams fitting: 59 configurations

- Fitting range: 10:20
- χ^2 get reduced from ~ 0.1 to ~ 0.01

Δm_k results preliminary

Data Set Info	tp12/ 10^{-12} MeV	tp34/10^{-12}MeV	$\Delta m_k/10^{-12}$MeV
113s+16c+17tp12	7.8(8)	-0.4(14)	7.3(17)
52s+7c+11tp12	7.3(12)	-1.1(12)	5.8(18)
61s+9c+6tp12	8.8(9)	0.2(20)	9.4(22)

- Error from the new type34 fitting is large, making the total error not reduced even with larger statistics.
- Possible reasons? Large error comes from type4 diagrams?

Update of the results preliminary

error from η

- We set $\langle\eta|Q_i - c_{si}\bar{s}d|K^0\rangle = 0$ because $\langle\eta|Q_i|K^0\rangle$ is noisy. However, the uncertainty of these $c_{si}=\frac{\langle\eta|Q_i|K^0\rangle}{\langle\eta|\bar{s}d|K^0\rangle}$'s still contribute to fitting results via "mixed" diagrams.
- Significant reduction of error from type34 when using central values of c_{s1} and c_{s2}
- For sloppy part only: 25% reduction of error

Data Set Info	tp3(xeta)	tp4(xeta)	tp34(xeta)	$\Delta M_K/10^{-12}$MeV
61s	2.03(61)	-2.50(113)	-0.95(163)	5.8(16)
61s*	1.91(41)	-2.49(107)	-0.67(117)	6.3(12)

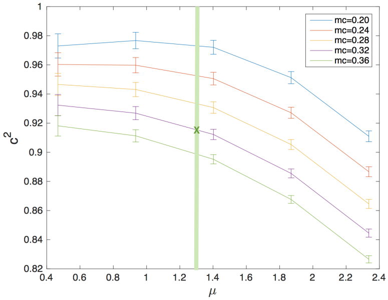
- For adding corrections and type12: 38% reduction of error

Data Set Info	σ_{slp}	σ_{corr}	σ_{tp12}	σ_{total}
61s+9e+6tp12	1.59	1.25	0.88	2.2
61s+9e+6tp12*	1.27	0.70	0.88	1.4

Update of the results

Systematic Errors

- Finite-volume corrections: small compared to statistical errors
  "Effects of finite volume on the $K_L - K_S$ mass difference"
  N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170
  Previous result gives: $\Delta m_K(FV) = 0.27(18) \times 10^{-12} MeV$
- Discretization effects are the largest source of systematic error:
  $\sim (m_c a)^2$ gives 25%
  Our preliminary estimate based on dispersion relation $c^2 = \frac{E^2 - m^2}{p^2}$ is $\leq 10\%$



Conclusion and Outlook

- By increasing number of total configurations from 59 to 129:
  - Errors from 2- and 3-point functions, reduced by $\sim 33\%$ as expected
  - Error from type 1&2 diagrams is reduced by $\sim 20\%$ as expected
  - Error from type 3&4 diagrams is only slightly reduced, probably due to large error contributions from η amplitudes.
    (Still in progress, $\sigma_{\Delta m_K} \sim 1.3 \times 10^{-12} MeV$, if $\overline{c_{si}}'s$ are used)
- Our preliminary result based on 129 configurations is

$$\Delta m_K = 7.0(17)_{stat} \times 10^{-12} MeV$$

to be compared to the physical value

$$(\Delta m_K)^{phys} = 3.483(6) \times 10^{-12} MeV$$

- Outlook
  - Expect to finish measurements on 160 configurations, aiming to reduce the statistical error to $\sim 1.0 \times 10^{-12} MeV$
  - Continue the calculation of Δm_K on finer lattice on Summit
  - Include other elements of our kaon physics program