Studies of I=0 and 2 pi-pi scattering at kaon mass with physical pion mass in GPBC

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Outline

Introduction

Why $\pi\pi$ scattering?

- Need $\pi\pi$ erengy and amplitude in $K->\pi\pi$ calculation, see C.Kelly
- We start first lattice calculation on $\pi\pi$ scattering with physical pion mass at kaon mass
- Consistency check

Lattice:

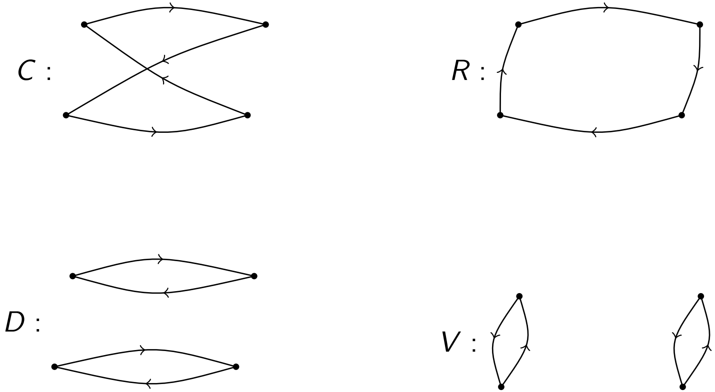
- 2+1 flavor Mobius DWF, $m_s = 0.045$, $m_l = 0.0001$
- Iwasaki+DSDR gauge action with $\beta = 1.75$
- $a^{-1} = 1.3784(68) GeV$
- $32^3 \times 64$ space time volume with $Ls = 12$
- 216 confs(2015, preliminary) to 1386 confs(now), statistical error decrease by factor of 2.5

Technique

- G-parity boundary condition
  Pion ground state with momentum $(\frac{\pi}{L}, \frac{\pi}{L}, \frac{\pi}{L})$
  Stationary kaon ground state
  Helps with $K- > \pi\pi$ calculation
- All to all propagator
  900 low modes plus 1536 random modes from time/flavor/color/spin dilution, 1s hydrogen wave smearing function
- Time separated pipi operator
  Two pions are time separated by 4

Diagram

- 4 types of diagrams

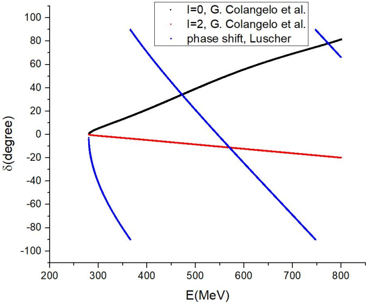


- $I = 0$ and $I = 2$ correlator

$$_t\langle 20|20\rangle_0 = 2D - 2C$$
$$_t\langle 00|00\rangle_0 = 2D + C - 6R + 3V$$

Dispersion

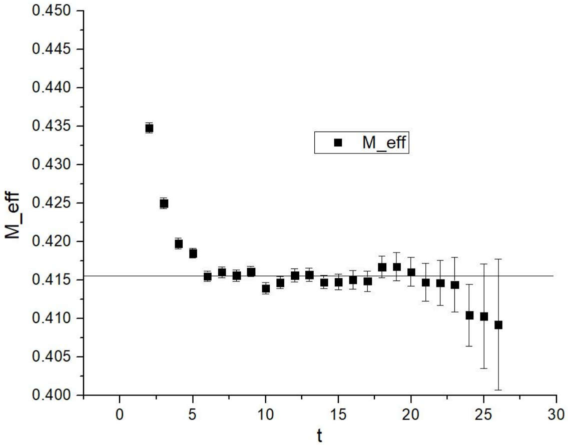
- Schenk's ansatz
  $tan\delta_I = \sqrt{1 - \frac{4M_\pi^2}{s}}(A_I + B_I q^2 + C_I q^4 + D_I q^6)(\frac{4M_\pi^2 - s_I}{s - s_I})$
- Luscher's formula (GPBC)
  $tan\delta = \frac{\pi^{3/2}\sqrt{\bar{m}}}{Z_{00}^{0,G}(1,\bar{m})}$



S wave phase shift and Luscher's formula.[1]

[1] G. Colangelo, Nuclear Physics B 603 (2001) 125 - 179

eff energy plot



Correlated 3 parameter fit, fit range: (6-25), $\chi^2/dof \sim 1.3$

$$C(t) = A \cdot (e^{-Et} + e^{-E \cdot (L_t - 2 \cdot t_{sep\pi\pi} - t)}) + C$$

PiPi I=2

result

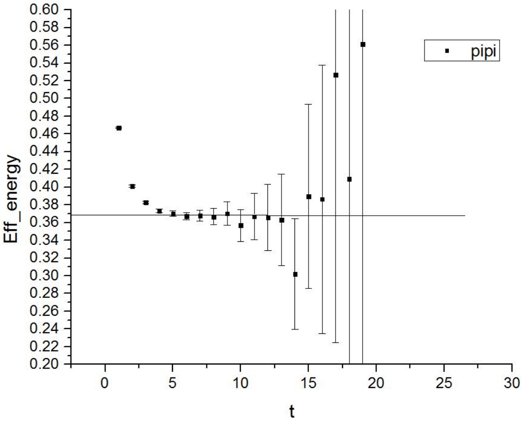
	E(MeV)(Old)	δ(Old)	E(MeV)(New)	δ(New)
S-wave	573.2(0.6)(2.8)	-11.0(2.9)(1.2)	573.9(0.2)(2.8)	-11.4(2.8)(1.2)
Dispersion	574.1	-11.4	574.1	-11.4
$2E_\pi$	549.2(0.8)(2.8)		549.0(0.3)(2.8)	
D-wave	549.4(0.4)(2.8)		549.9(0.2)(2.8)	

$$E : ()_{stat}()_{a^{-1}}$$
$$\delta : ()_{stat}()_{sys}$$

- calculate system error based on comparing 2 state fit and 1 state fit
- statistical error reduction in energy
- perfectly consistent phase shift
- D-wave energy close to $2E_\pi$

PiPi I=0

Single operator, eff energy plot



Correlated 3 parameter fit, fit range: (6-25), $\chi^2/dof \sim 1.6$

$$C(t) = A \cdot (e^{-Et} + e^{-E\cdot(L_t - 2\cdot t_{sep\pi\pi} - t)}) + C$$

PiPi I=0

Single operator, result

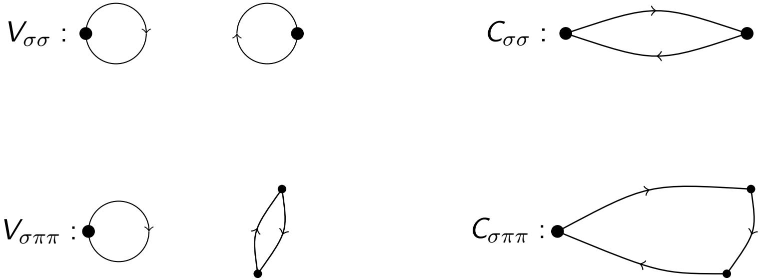
	E(MeV)(Old)	δ(Old)	E(MeV)(New)	δ(New)
S-wave	498(11)(3)	23.8(4.9)(1.2)	508(5)(3)	19.1(2.5)(1.2)
Dispersion	474.6	35.0	474.6	35.0
$2E_\pi$	549.2(0.8)(2.8)		549.0(0.3)(2.8)	
D-wave	548.6(0.9)(2.8)		548.1(0.4)(2.8)	

- error reduction
- energy different from dispersion by 5σ in latest result

2 cosh fit (3-25)	E0(MeV)	E1(MeV)
Lattice	507(6)	1729(376)
Dispersion	474.6	774.7

- Two cosh fit can't solve the problem
- D-wave energy close to $2E_\pi$

- $|\sigma >= \frac{i}{\sqrt{2}}(\bar{u}u + \bar{d}d)$
- Diagram.



- New correlators

  $_t\langle\sigma|\sigma\rangle_0 = 0.5V_{\sigma\sigma} - 0.5C_{\sigma\sigma}$

  $_t\langle\sigma|\pi\pi\rangle_0 = \frac{\sqrt{6}i}{4} \cdot V_{\sigma\pi\pi} - \frac{\sqrt{6}i}{2} \cdot C_{\sigma\pi\pi}$
- results based on 830 confs

PiPi I=0

Sigma operator

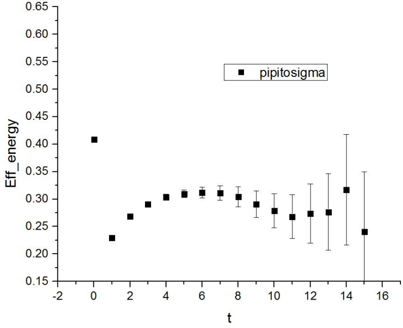


Figure: $\pi\pi->\sigma$

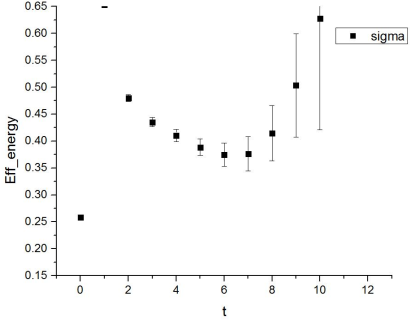


Figure: $\sigma->\sigma$

- We perform a correlated, two state(cosh) fit
  $C_{ij}=A_{i0}\cdot A_{j0}\cdot(e^{-m_0t}+e^{-m_0(Lt-t)})+A_{i1}\cdot A_{j1}\cdot(e^{-m_1t}+e^{-m_1(Lt-t)})$
- Tuning tmin for stable final energy

Range	E0(MeV)	$\delta 0$	E1(MeV)(New)	χ^2/dof
(4-10)	485.8(1.1)(2.7)	29.6(1.5)(3.0)	881(52)	2.2(0.8)
(5-10)	483.1(1.4)(2.7)	30.9(1.5)(3.0)	1005(109)	1.7(0.8)
(6-10)	482.0(2.2)(2.7)	31.5(1.7)(3.0)	1204(452)	2.0(1.0)
1 cosh (6-25)	508(5)(2.8)	19.1(2.5)(11.8)		1.6(0.7)
Dispersion	474.6	35.0	774.7	

- Huge statistical error reduction
- Ground state energy become much lower(Reduced excited state contamination)
- Poor excited state result
- Systematic error analysis based on GEVP

- Given n operator O_i, construct correlator matrix
  $C_{ij}(t) = \langle O_i(0)|O_j(t)\rangle$
- $C(t)v_n(t, t_0) = \lambda(t, t_0)C(t_0)v_n(t, t_0)$
- $E_n^{eff}(t, t_0) = log(\lambda(t, t_0)) - log(\lambda(t+1, t_0))$
  set $t_0 = \left\lceil \frac{t}{2} \right\rceil$ in this case
- Can also be used to calculate overlap between each operator and lattice eigenstate

PiPi I=0

GEVP, energy

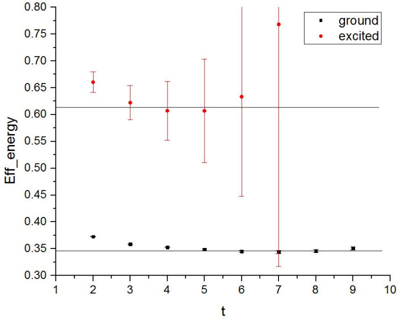


Figure: gevp energy

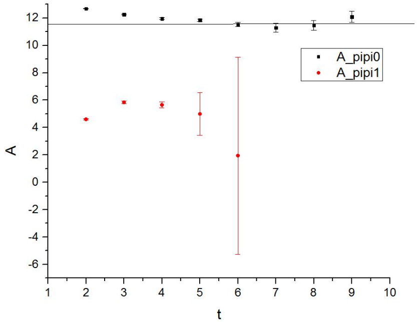


Figure: gevp overlap

- Consistency between GEVP and simultaneous fit

	E0(MeV)	δ	amplitude
sim-fit(5-10)	483.1(1.4)(2.7)	30.9(1.5)(3.0)	11.86(11)(12)
GEVP(6,3)	475.6(2.6)(2.7)	32.8(1.2)(3.0)	11.52(15)(12)
Dispersion	474.6	35.0	

- Systematic error goes down
  GEVP proves the systematic error for n-th state energy is proportional to $e^{-(E_{N+1}-E_n)\cdot t}$, in our case by including the second operator, we get a benefit of roughly a factor of 4 in decreasing of systematic error.

Conclusion

What do we get:

- Our earlier single-operator result, $\delta_0 = 23.8(4.9)(1.2)^\circ$, seriously underestimated the systematic error$(1.2- > 11.2)$.
- Good results for $\pi\pi_{I=2}$
- Improved $\pi\pi_{I=0}$ scattering result despite new noisy operator
- Big error in excited $\pi\pi$ state energy.

Outlook:

- Adding new operators ($\sim$ 20 confs now).
- Moving frame calculation.
- complete systematic error analysis