Quark orbital angular momentum in the proton evaluated using a direct derivative method

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LHPC

Acknowledgments:
M. Burkardt, S. Liuti

Gauge ensembles provided by:
R. Edwards, B. Joó and K. Orginos

Direct evaluation of quark orbital angular momentum

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T \, (r_T \times k_T)_3 \, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

n : Number of valence quarks

$p' = P + \Delta_T/2$, $p = P - \Delta_T/2$, P, S in 3-direction, $P \to \infty$

This is the same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information

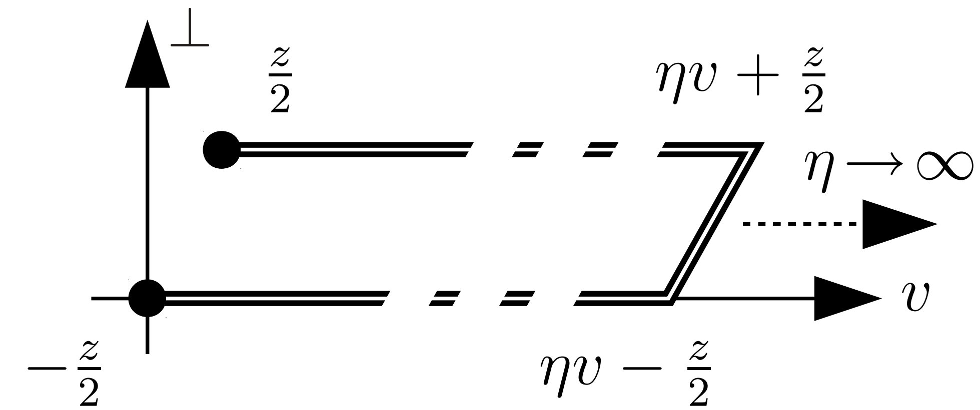
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\,\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Y. Hatta, M. Burkardt:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



Direct evaluation of quark orbital angular momentum

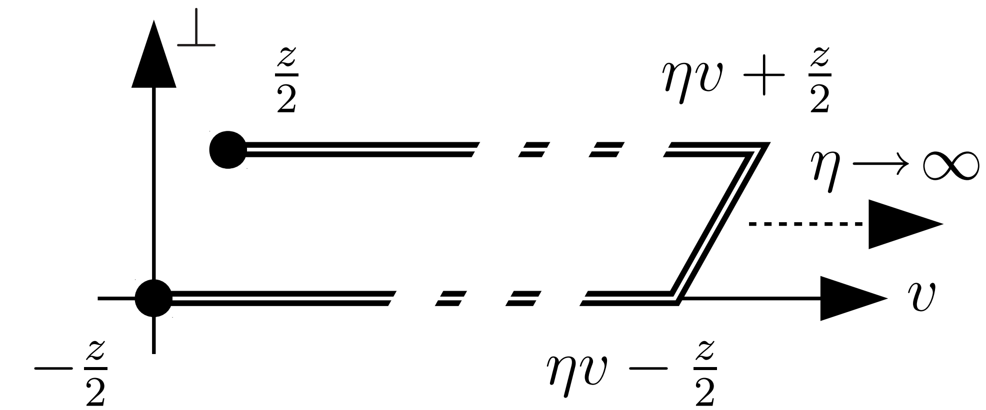
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



Ensemble details

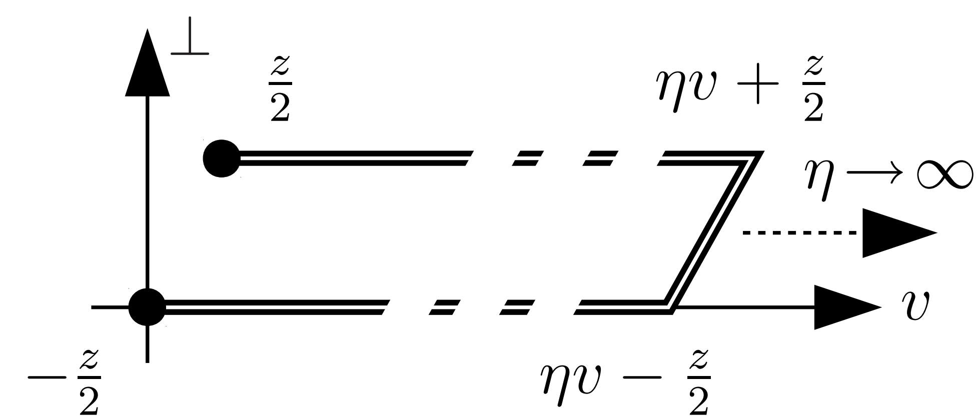
Clover ensemble provided by R. Edwards, B. Joó and K. Orginos (JLab / W&M Collaboration)

$L^3 \times T$	a(fm)	m_π (MeV)	m_N (GeV)	#conf.	#meas.
$32^3 \times 96$	0.11403(77)	317(2)(2)	1.077(8)	967	23208

Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Parameters to consider: Δ, z, $\hat{\zeta}$, η



Direct evaluation of quark orbital angular momentum

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Perform Δ_T-derivative using Rome method (Phys. Lett. B718 (2012) 589)

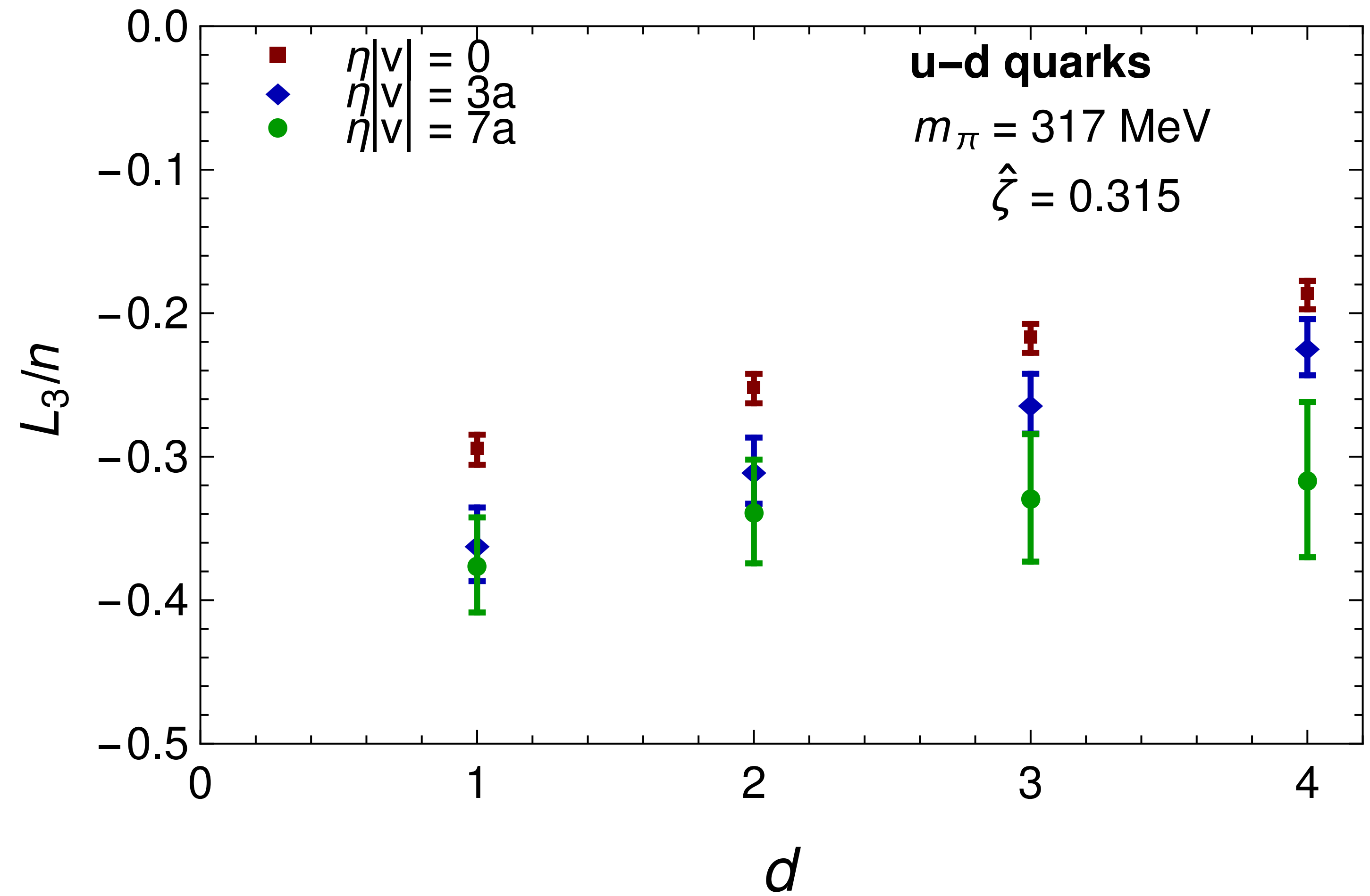
$$G(x, y; \vec{p}) = e^{-i\vec{p}(\vec{x}-\vec{y})} G(x, y)$$

$$\frac{\partial}{\partial p_j} G(x, y; \vec{p}) \Big|_{\vec{p}=0} = -i \sum_z G(x, z) \Gamma_V^j G(z, y)$$

Clover fermions: $$\Gamma_V^j G(z, y) = U_j^\dagger(z - \hat{j}) \frac{1 + \gamma^j}{2} G(z - \hat{j}, y) - U_j(z) \frac{1 - \gamma^j}{2} G(z + \hat{j}, y)$$

(further contributions from derivatives of source/sink smearings)

Direct evaluation of quark orbital angular momentum

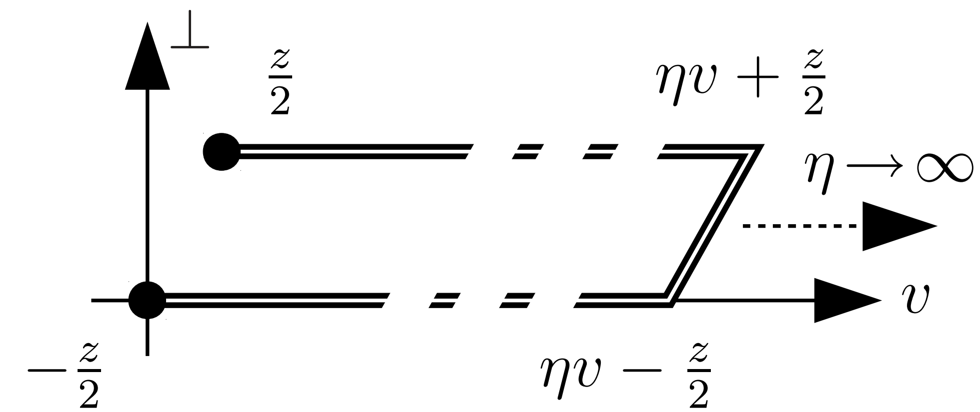


$$\left.\frac{\partial f}{\partial z_{T,i}}\right|_{z_{T,i}=0} = \frac{1}{2da}(f(dae_i) - f(-dae_i))$$

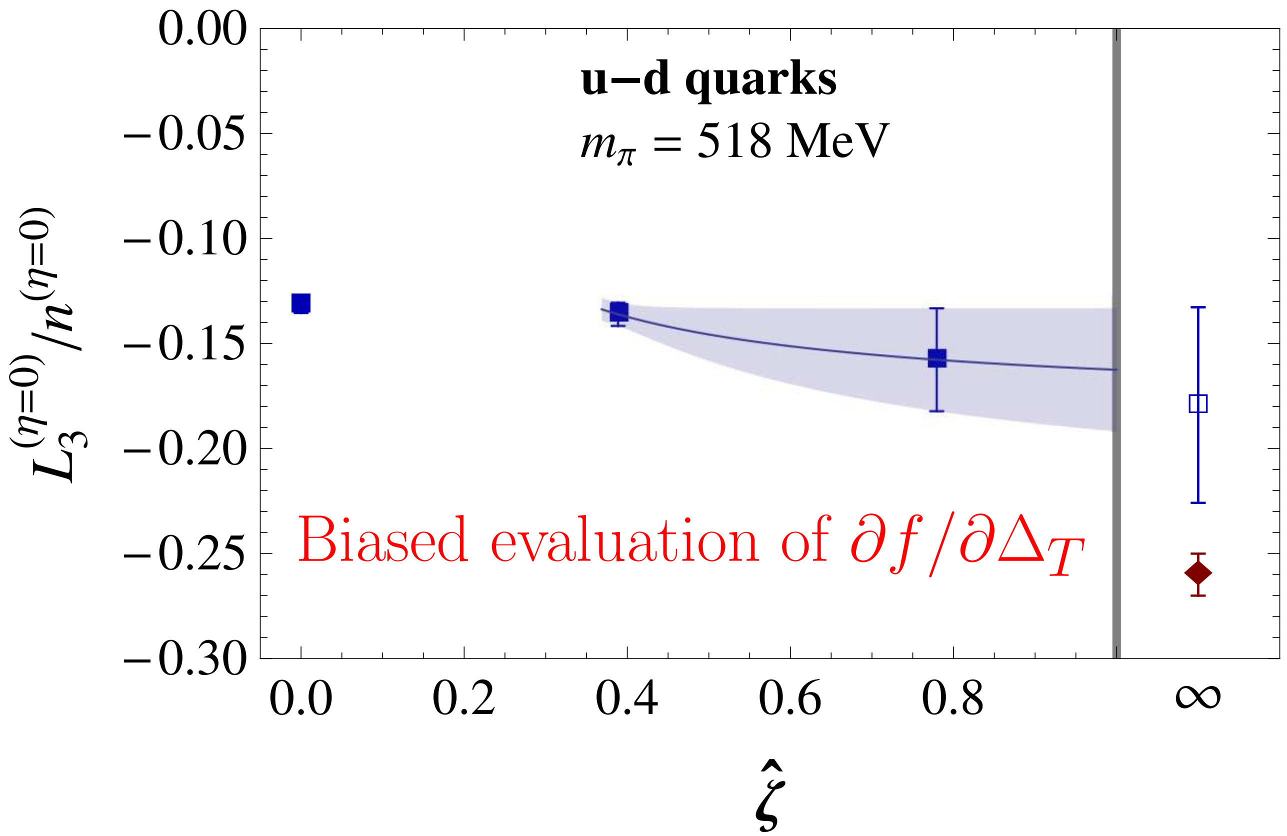
Direct evaluation of quark orbital angular momentum

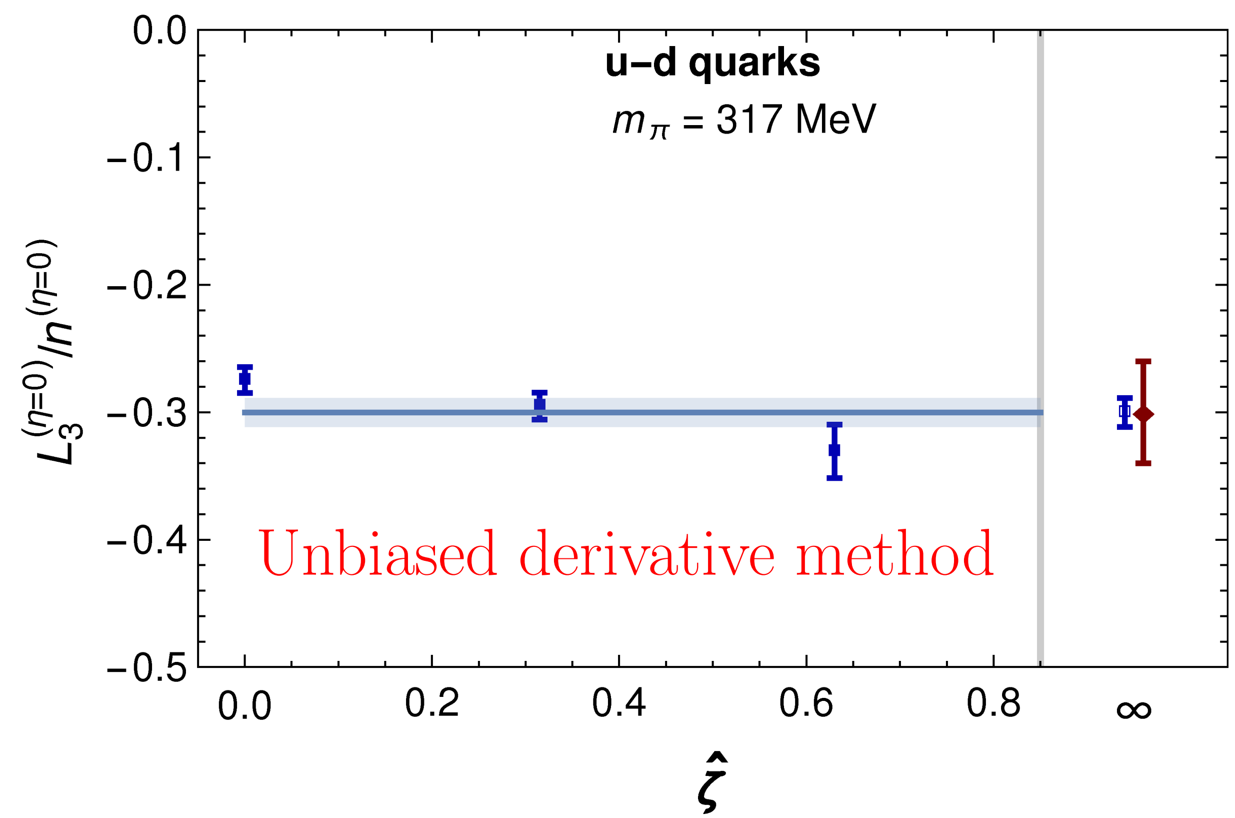
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Remaining parameters to consider: $\hat{\zeta}$, η

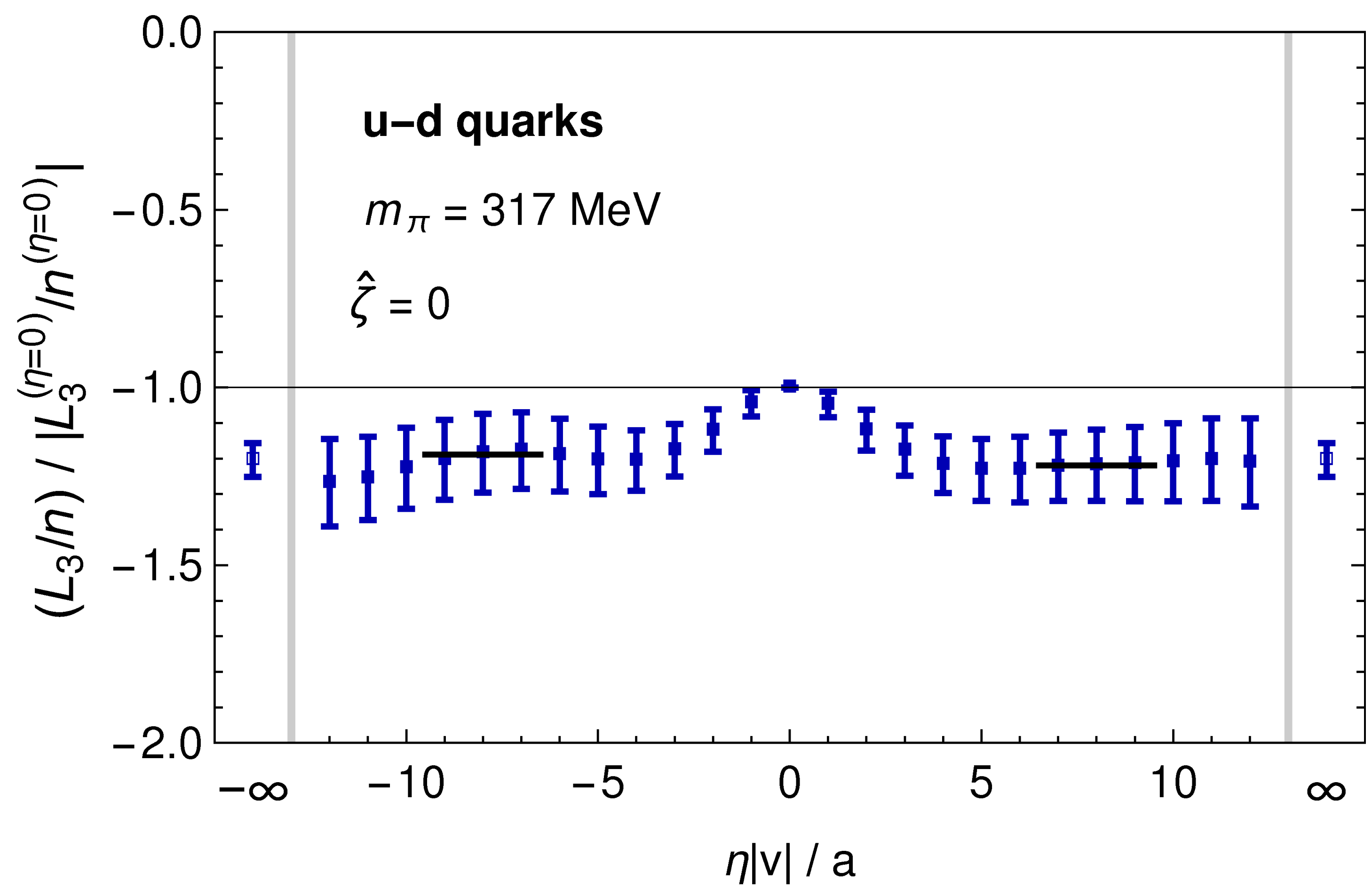


Ji quark orbital angular momentum: $\eta = 0$

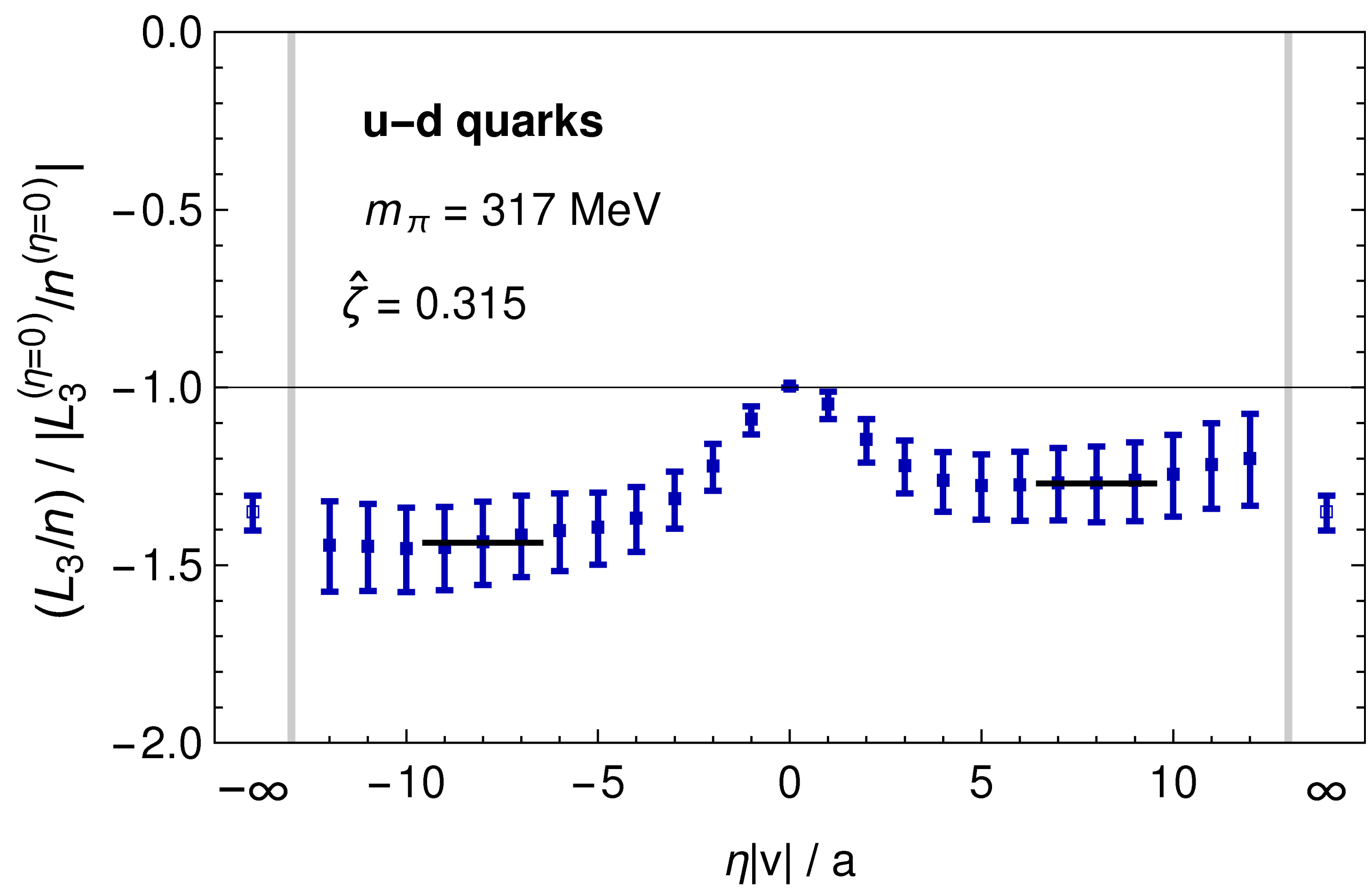




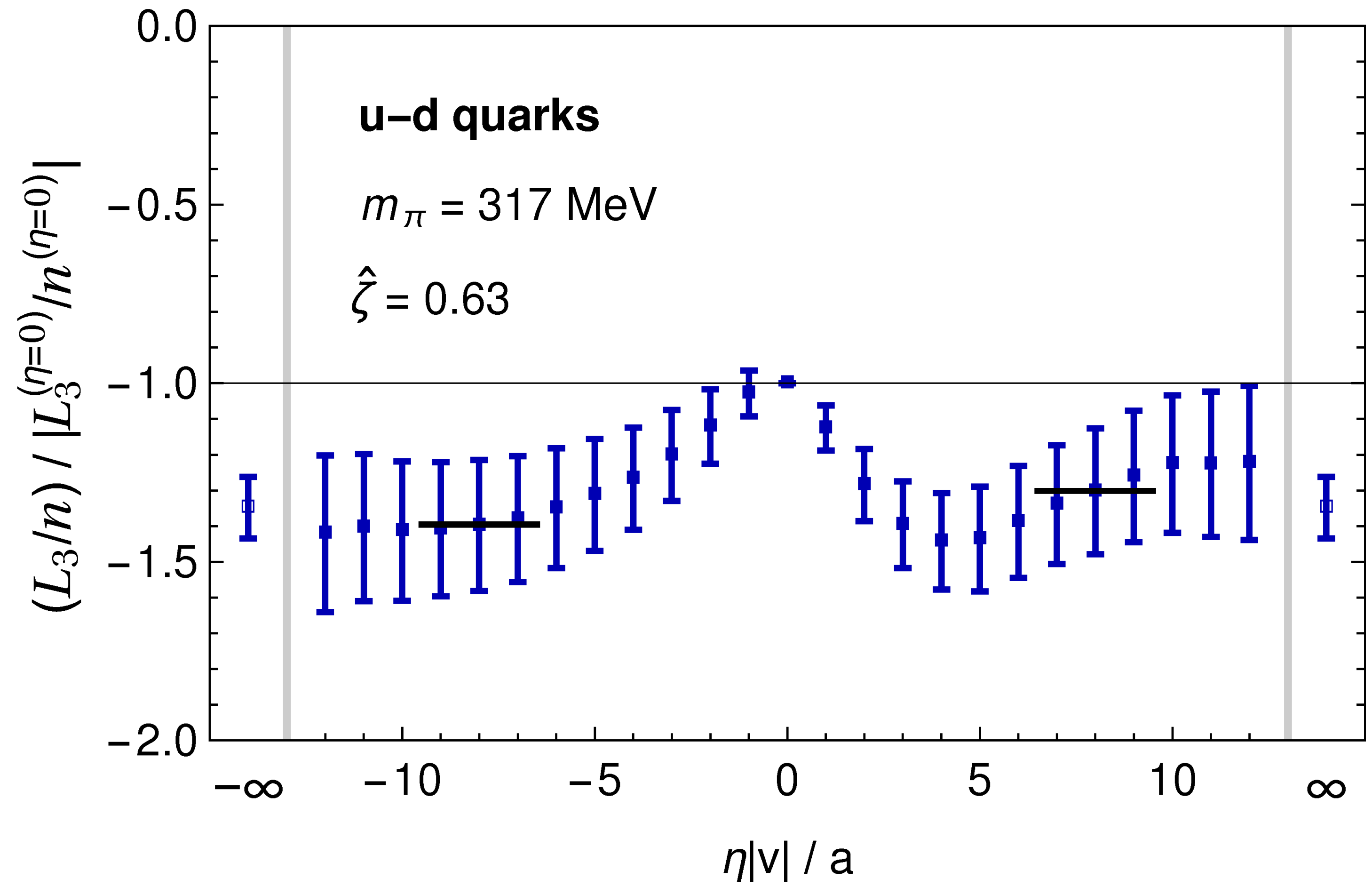
From Ji to Jaffe-Manohar quark orbital angular momentum



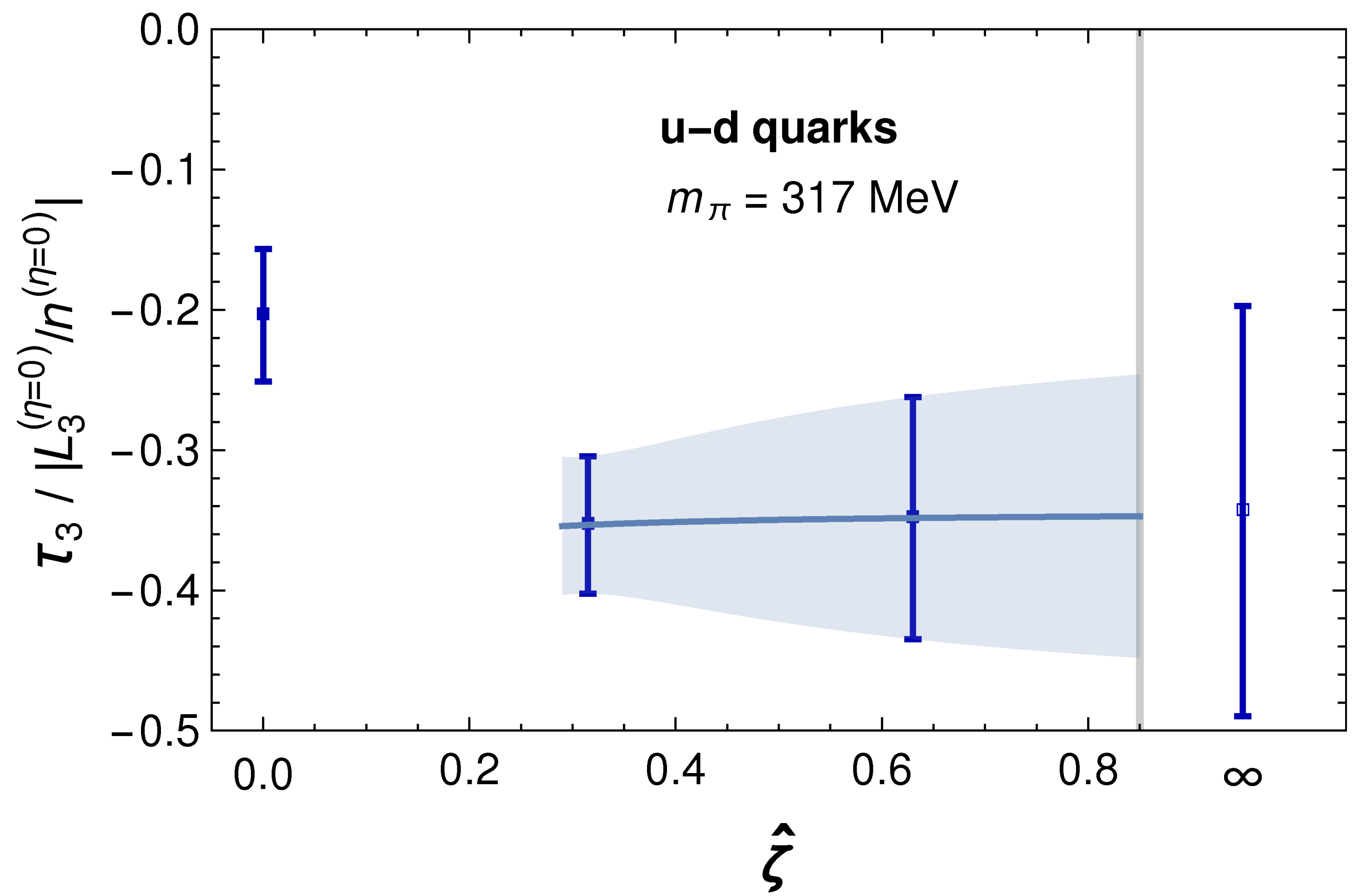
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



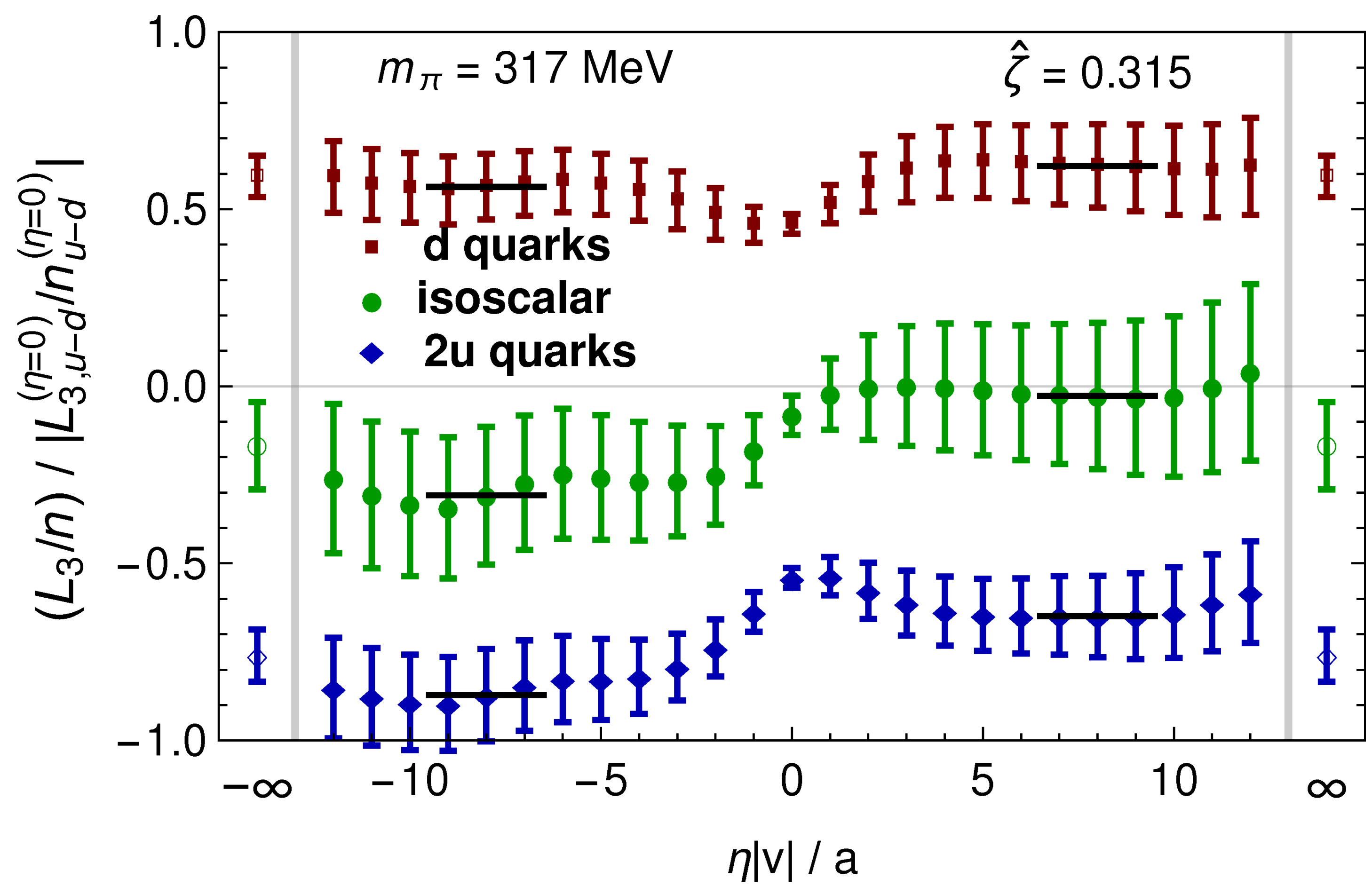
Burkardt's torque – extrapolation in $\hat{\zeta}$



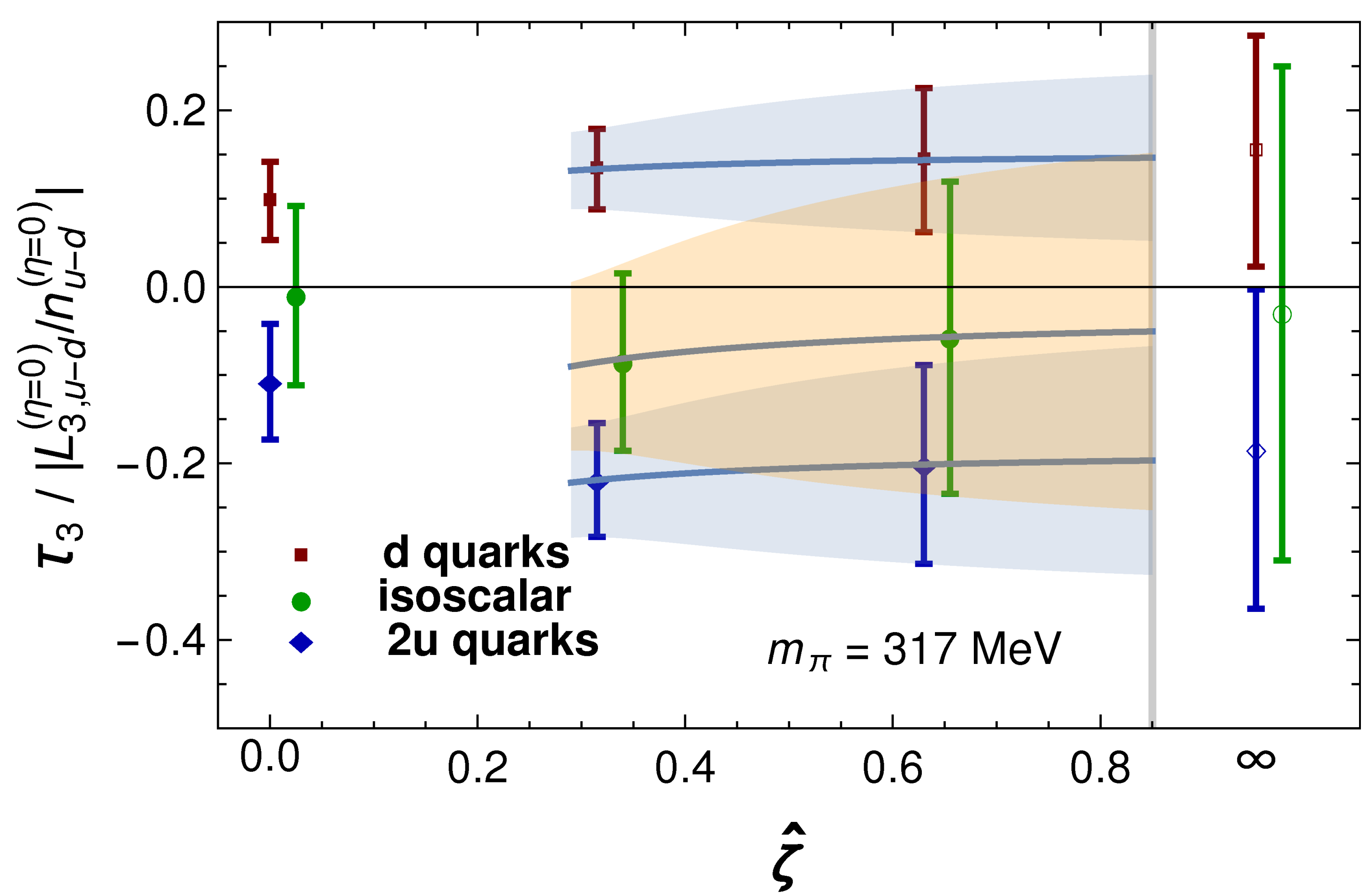
$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum



Flavor separation – Burkardt's torque



$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

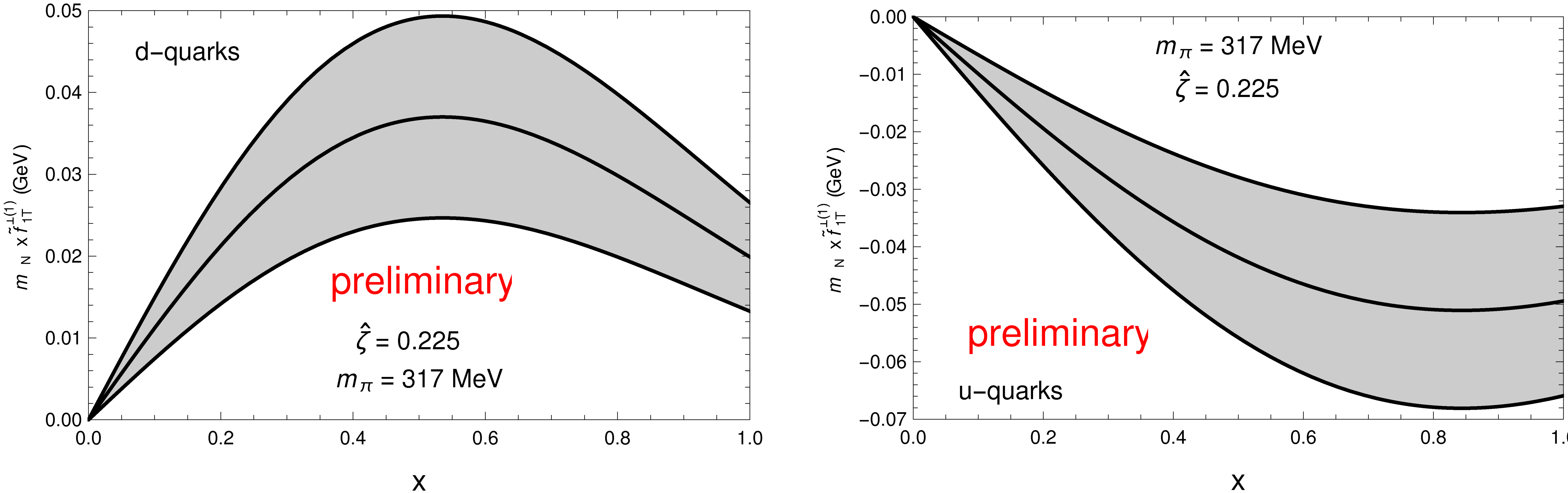
Integrated torque accumulated by struck quark leaving proton

Conclusions

- Quark orbital angular momentum can be accessed directly in Lattice QCD, continuously interpolating between the Ji and Jaffe-Manohar definitions.
- Direct evaluation of derivative w.r.t. momentum transfer (Rome method) rectifies bias affecting earlier exploration. Result obtained for Ji orbital angular momentum agrees with standard Ji sum rule result.
- Difference between the Ji and Jaffe-Manohar definitions (torque accumulated by struck quark leaving a proton) is clearly resolvable, sizeable ($\sim 1/3$ of the original Ji OAM, at $m_\pi = 317$ MeV), and leads to an enhancement of Jaffe-Manohar OAM relative to Ji OAM.
- Im future studies, explore lighter pion masses, higher longitudinal momenta.

And, as an outlook ...

Advertisement: x-dependence of Sivers transverse momentum shift



Average transverse momentum of unpolarized quarks in nucleon polarized in the other transverse direction