

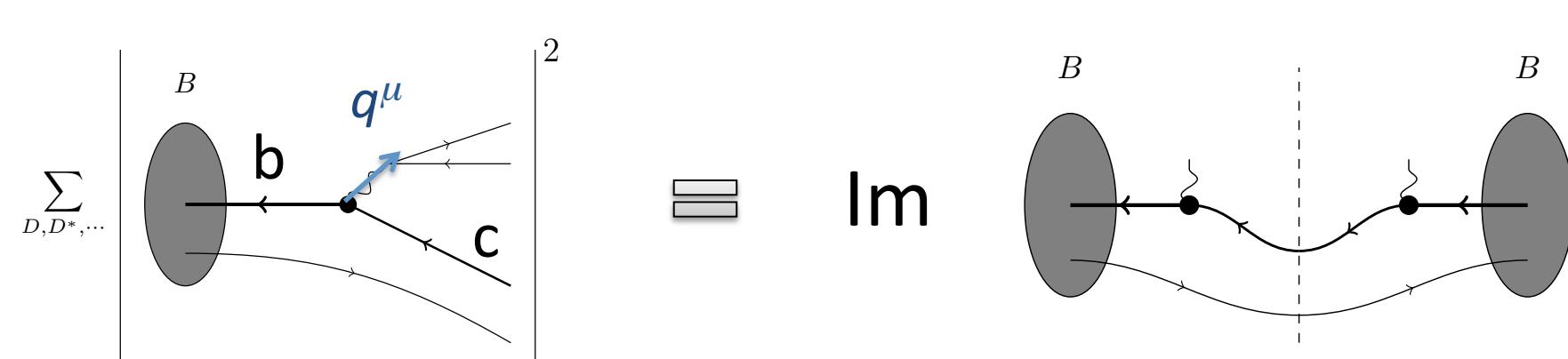
# Inclusive decay structure function for $B \rightarrow X_c \ell \nu$ : comparison of a lattice calculation with the heavy quark expansion

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## Introduction

“Inclusive” = sum over all final states

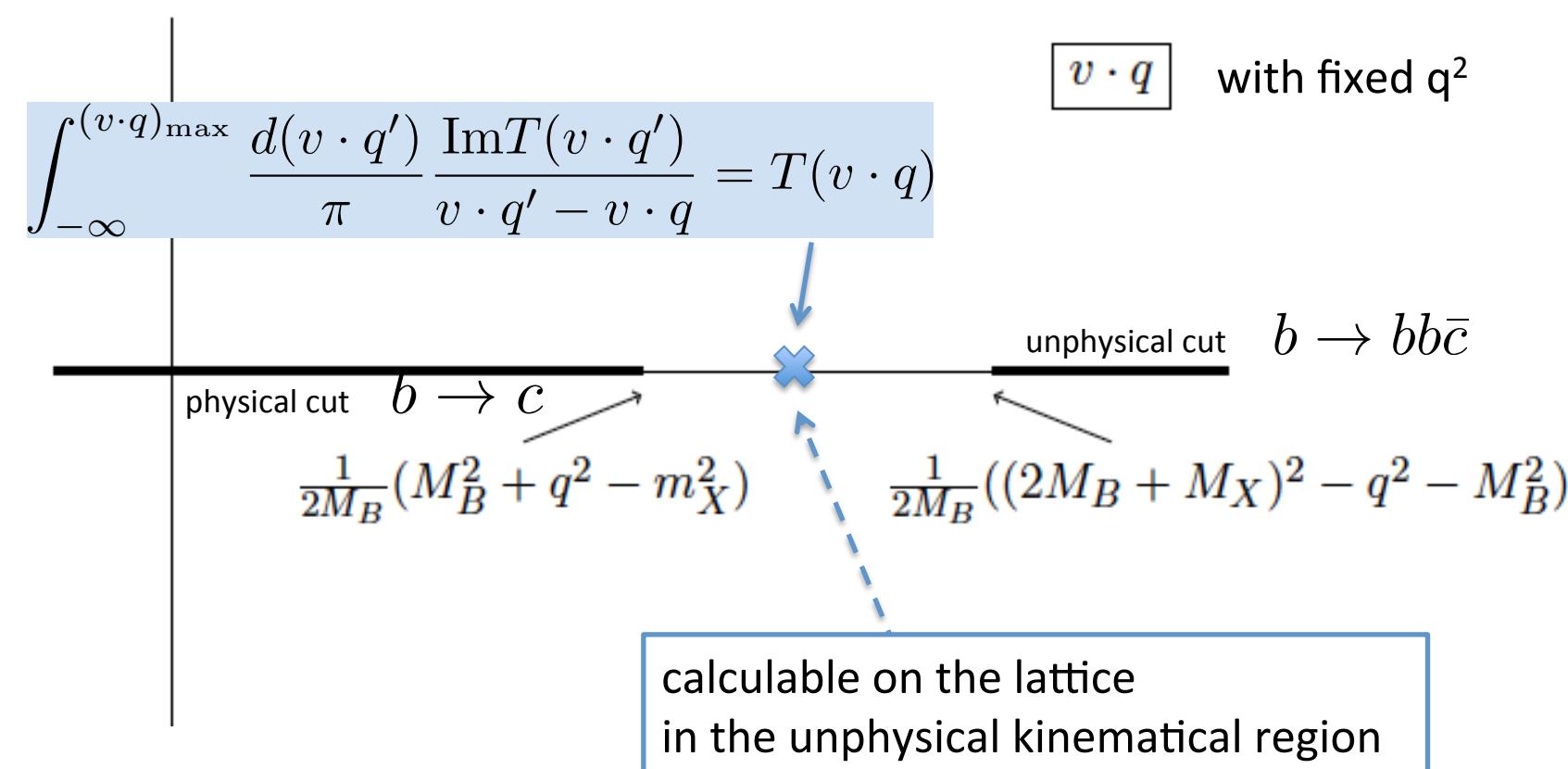
Use dispersion relation:



Forward scattering matrix element:

$$T_{\mu\nu} = i \int d^4x e^{-iqx} \frac{1}{2M_B} \langle B | T\{J_\mu^\dagger(x) J_\nu(0)\} | B \rangle$$

Analytic continuation:

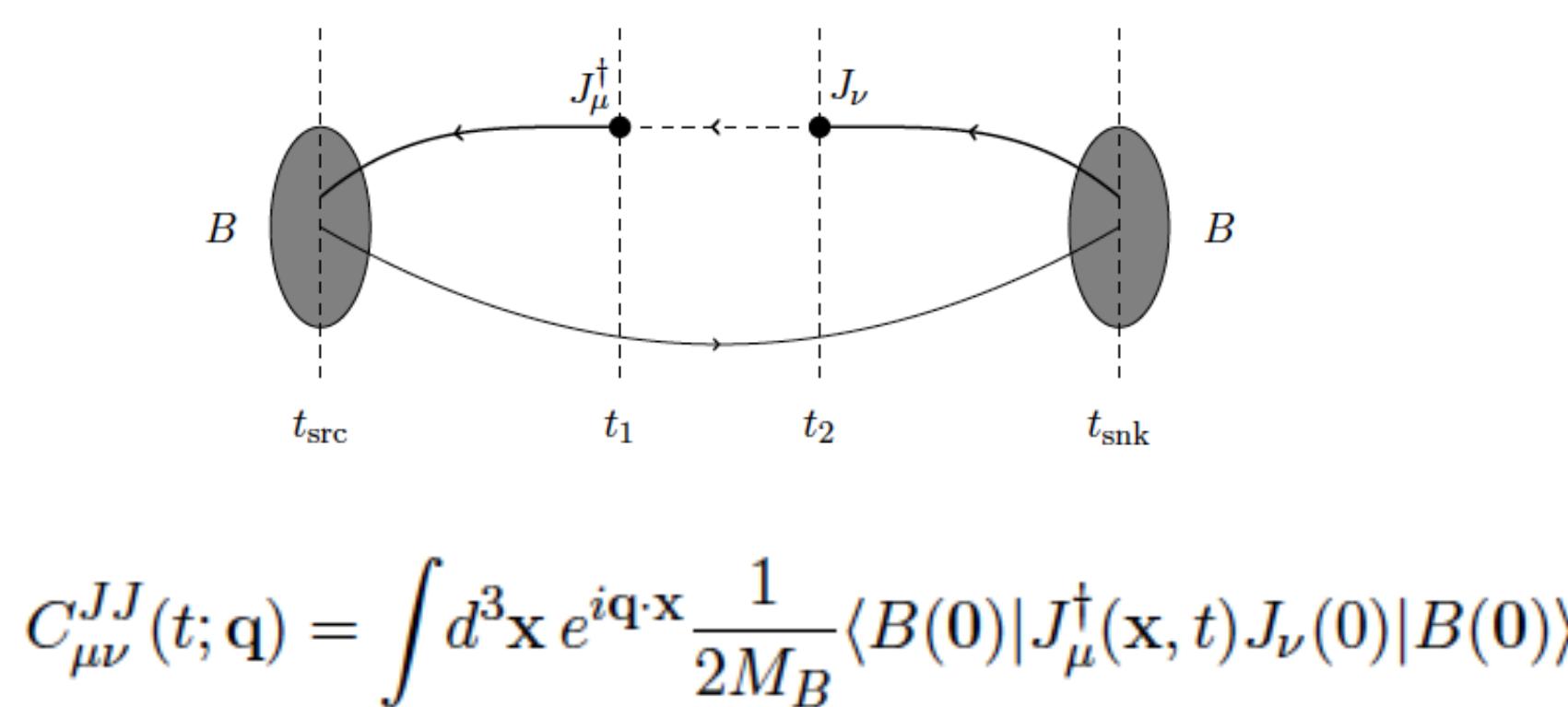


Both lattice and continuum methods may apply in the unphysical kinematical region. Are they consistent?

## Lattice calculation

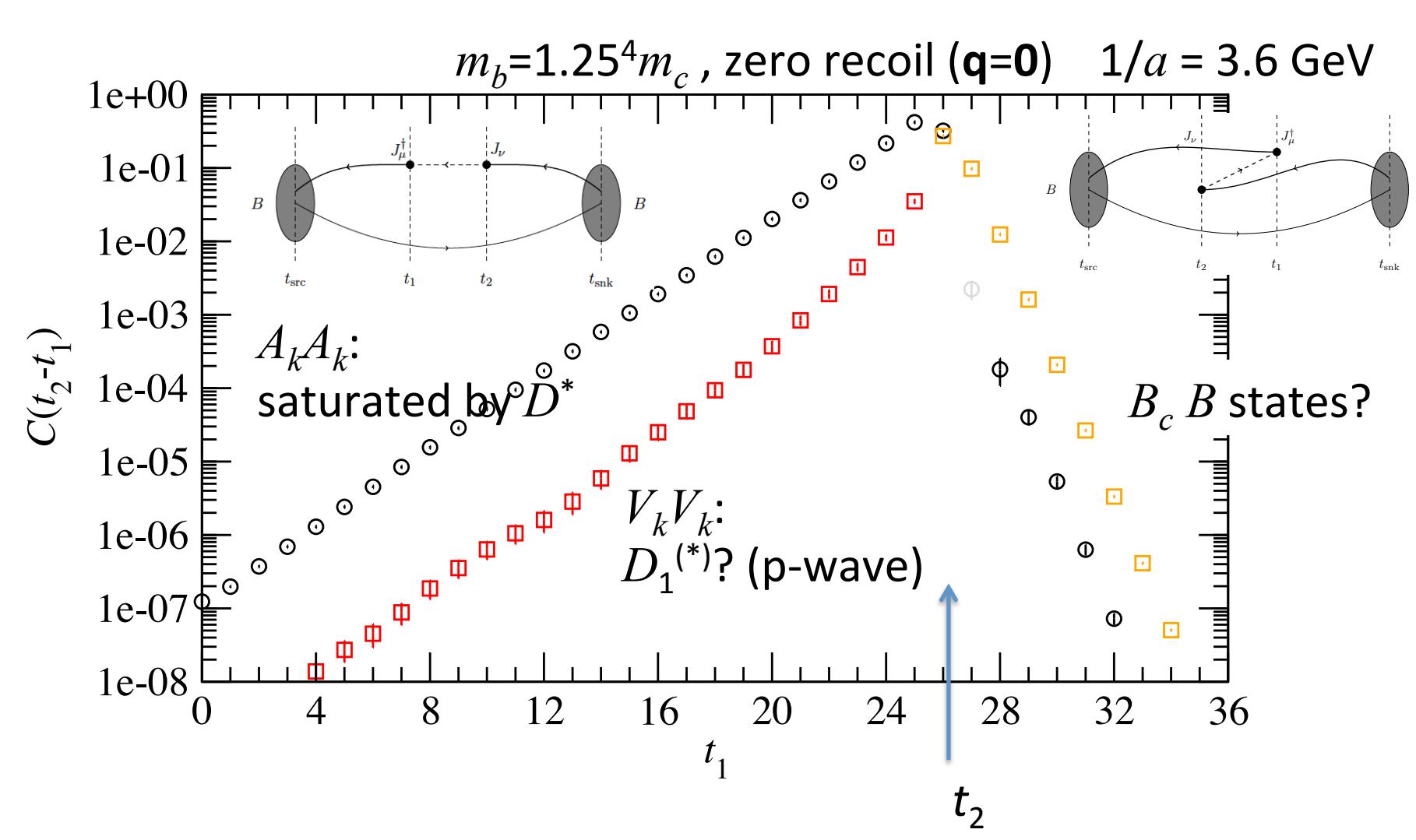
See, SH, arXiv:1703.01881

Four-point function:



$$C_{\mu\nu}^{JJ}(t; q) = \int d^3x e^{iqx} \frac{1}{2M_B} \langle B(0) | J_\mu^\dagger(x, t) J_\nu(0) | B(0) \rangle$$

may be obtained by the standard sequential source method.



Lattice ensembles:

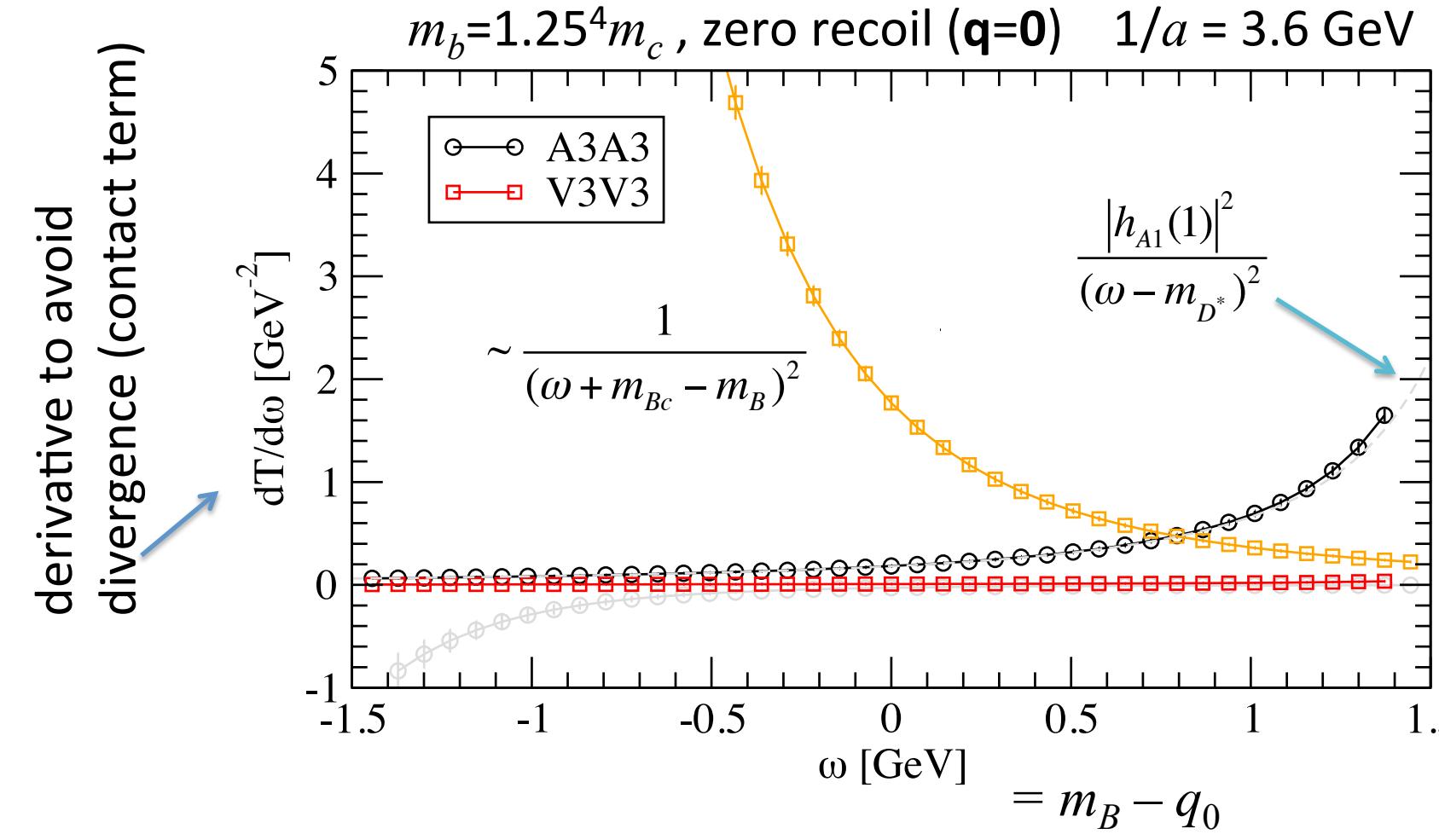
- Möbius domain-wall (sea and valence)
- $\alpha^{-1} = 3.6$  GeV
- $L = 2.7$  fm ( $48^3 \times 96$  lattice)
- $m_\pi \sim 300$  MeV
- 200 measurements
- charm tuned; bottom  $m_b = (1.25)^4 m_c$

## Numerical tests

“Fourier (or Laplace) transform”:

$$T_{\mu\nu}^{JJ}(\omega, q) = \int_0^\infty dt e^{i\omega t} C_{\mu\nu}^{JJ}(t; q)$$

possible as long as  $\omega < m_{D^{(*)}}$



Saturation by the ground state ?

$$\langle D(0) | V^0 | B(0) \rangle = 2\sqrt{M_B M_D} h_+(1),$$

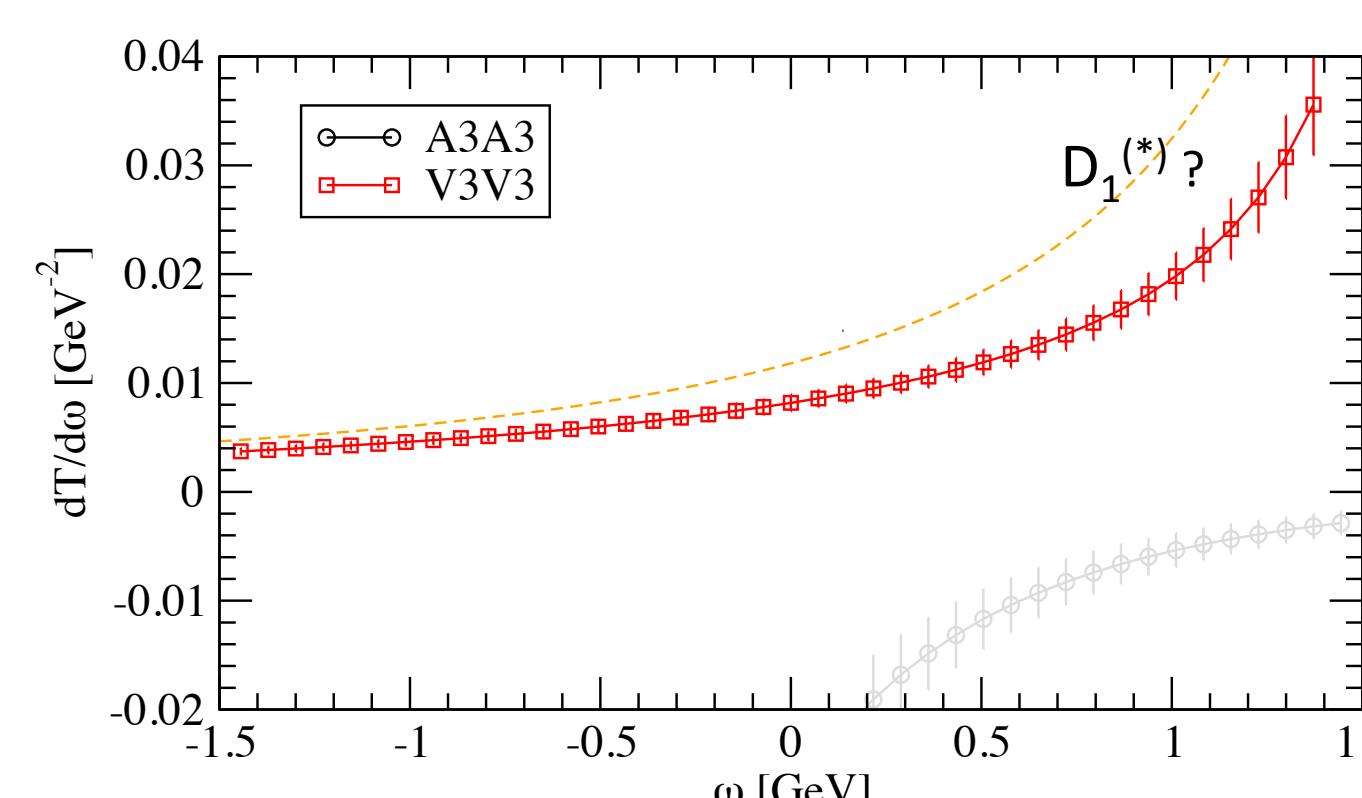
$$\langle D^*(0) | A^k | B(0) \rangle = 2\sqrt{M_B M_{D^*}} h_{A_1}(1) \varepsilon^k.$$

$$\begin{aligned} T_{00}^{VV}(\omega, 0) &= \frac{|h_+(1)|^2}{M_D - \omega}, \\ T_{kk}^{AA}(\omega, 0) &= \frac{|h_{A_1}(1)|^2}{M_{D^*} - \omega}. \end{aligned}$$

Wrong parity channel, too

$B \rightarrow D_1^{(*)}$ : p-wave,  $1^+$  states  
 $D_1^{(*)}$ :  $s_l = 1/2$ , 2427 MeV (broad)  
 $D_1$ :  $s_l = 3/2$ , 2421 MeV (narrow)

$$\begin{aligned} \langle D_1^{(*)} | \bar{c} \gamma_\mu b | B \rangle &= \sqrt{m_{D_1} m_B} g_{V_1}(1) \epsilon_\mu^*, \\ \langle D_1 | \bar{c} \gamma_\mu b | B \rangle &= \sqrt{m_{D_1} m_B} f_{V_1}(1) \epsilon_\mu^*. \end{aligned} \quad \text{estimates in Bernlochner, Ligeti, Robinson, arXiv:1711.03110}$$



## Heavy quark expansion

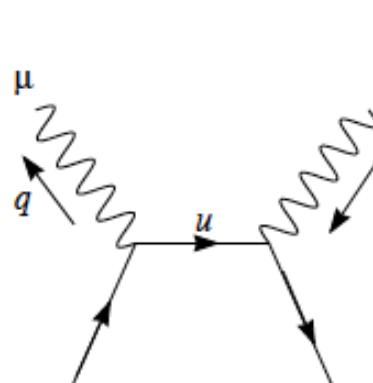
- Tree-level formulae:  
Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).  
Manohar, Wise, PRD49, 1310 (1993).

- Expand  $\frac{1}{m_b \not{p} - \not{q} + \not{k} - m_c}$  in small  $k$ .

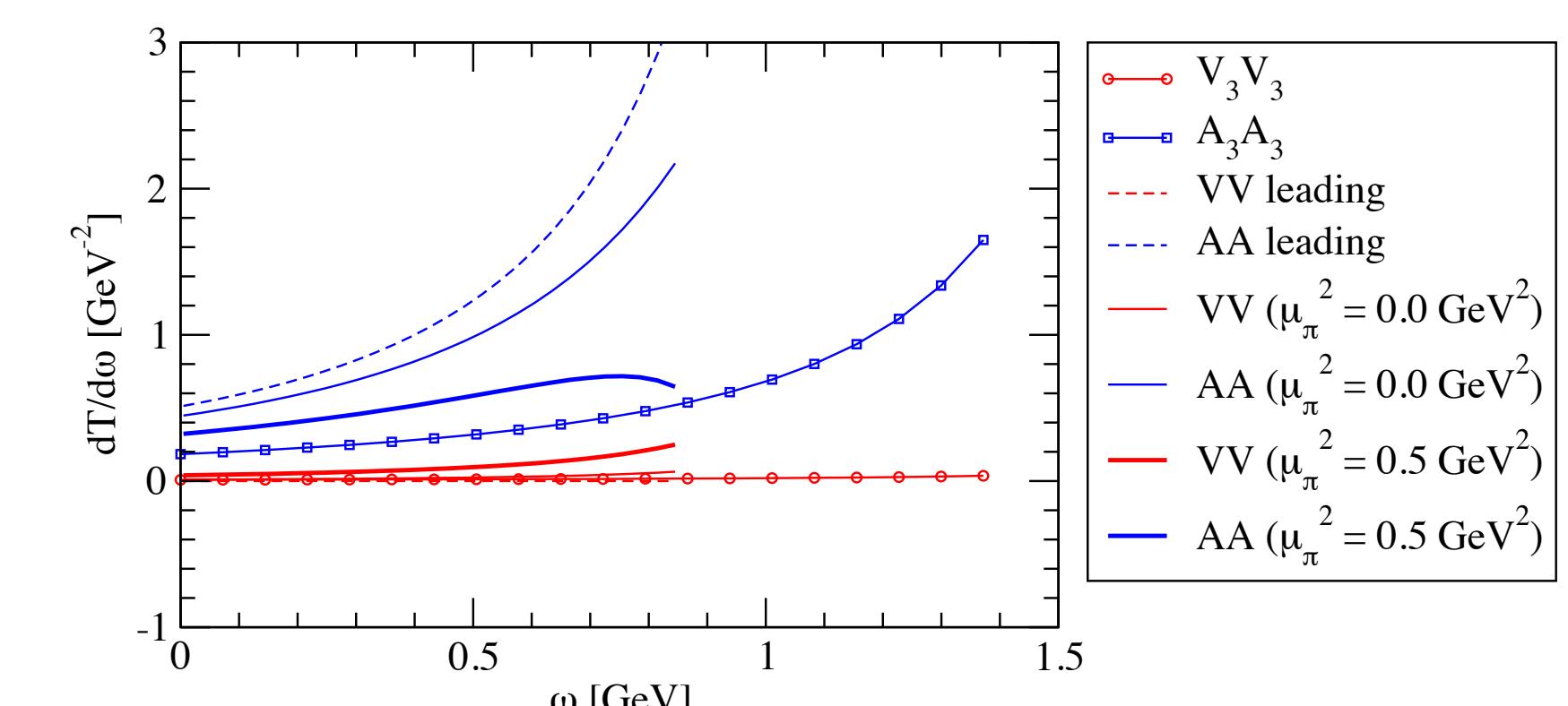
- Zero-recoil limit ( $V_k V_K$  or  $A_k A_k$  channel, leading order)

$$T_1^{VV} = -\frac{\omega - m_c}{\omega^2 - m_c^2}, \quad T_1^{AA} = -\frac{\omega + m_c}{\omega^2 - m_c^2} \quad (\omega = m_B - q_0)$$

... poles at  $\omega = -m_c$  ( $V_k V_k$ ) or  $\omega = m_c$  ( $A_k A_k$ )



## At order $1/m^2$



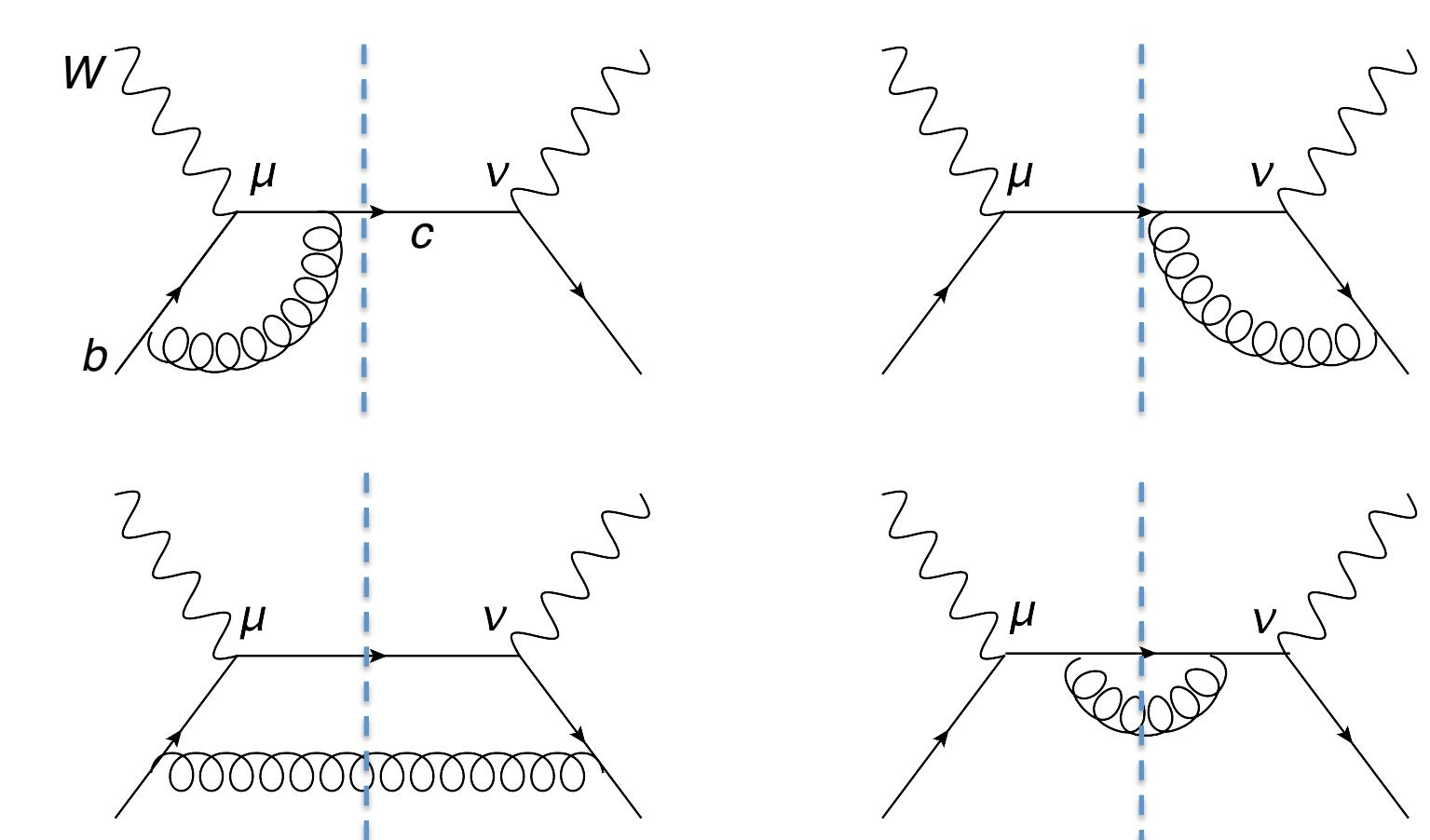
- HQE, written in terms of charm quark propagator (= pole at  $m_c$ , not  $m_{D^{(*)}}$ )
  - Large non-perturbative effect visible, that must be taken account of by higher orders in HQE and in  $\alpha_s$ .
- Better agreement by including the terms of  $1/m^2$ , that involve hadronic parameters:

$$\begin{aligned} \mu_\pi^2 &= \frac{1}{2M_B} \left\langle B \left| \bar{b} (i \vec{D})^2 b \right| B \right\rangle \sim 0.5 \text{ GeV}^2, \\ \mu_G^2 &= \frac{1}{2M_B} \left\langle B \left| \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle = 0.37 \text{ GeV}^2 \end{aligned}$$

## Going to higher orders

One-loop correction known only in the physical decay rates; need to carry out Cauchy's integral.

Trott, PRD70, 073003 (2004).  
Aquila, Gambino, Ridolfi, Uraltsev, NPB719, 77 (2005).



$1/m^3$  corrections: two new hadronic parameters appear:

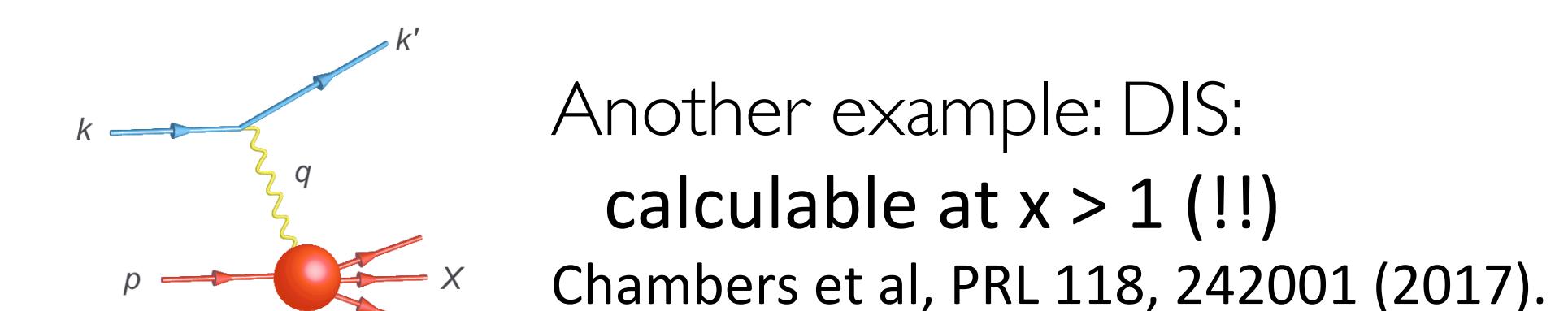
Gremm, Kapustin, PRD55 (1997) 6924.

Work in progress...

## Discussions

Lattice calculation for inclusive processes:

- Possible in the unphysical kinematical region.
- Short-distance physics accessible.



Another example: DIS:

calculable at  $x > 1$  (!!)

Chambers et al, PRL 118, 242001 (2017).

Comparison to HQE on-going, to test the methodology used in the previous analyses.

