Equation of state near the first order phase transition point of SU(3) gauge theory using gradient flow

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Introduction

Three methods for the Equation of State.

1. Integral method

$$\frac{p}{T^4} = \frac{N_t^3}{N_s^3} \int_0^{\beta} \frac{\partial \ln Z(\beta')}{\partial \beta} d\beta'$$

2. Derivative method

$$\frac{\epsilon}{T^4} = T^{-1} \frac{\partial \ln Z}{\partial T^{-1}} \qquad \frac{p}{T^4} = V \frac{\partial \ln Z}{\partial V} \qquad \begin{aligned} T^{-1} &= a_t N_t \\ V &= (a_s N_s)^3 \end{aligned}$$

a: lattice space

3. Gradient flow method

New method!

In this talk, we focus on the gradient flow method.

Gradient flow method for EoS

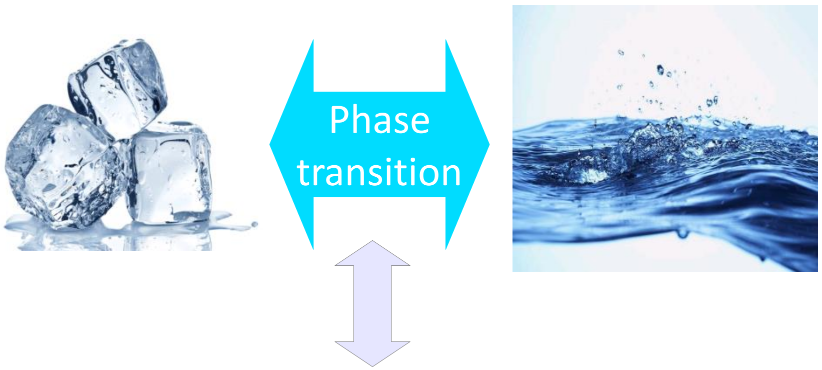
- Gradient flow [Narayanan-Neuberger (2006), Lüscher (2009-)]
- Energy Momentum Tensor by gradient flow [H. Suzuki (2013)]
- Test in quenched-QCD [Flow QCD (2014,2016)]
  - Pressure and energy density are consistent with those by the integral method.
- Application to Full QCD EoS by GF [Suzuki & Makino (2014)]
- Test in Full QCD [WHOT QCD (2016-)]

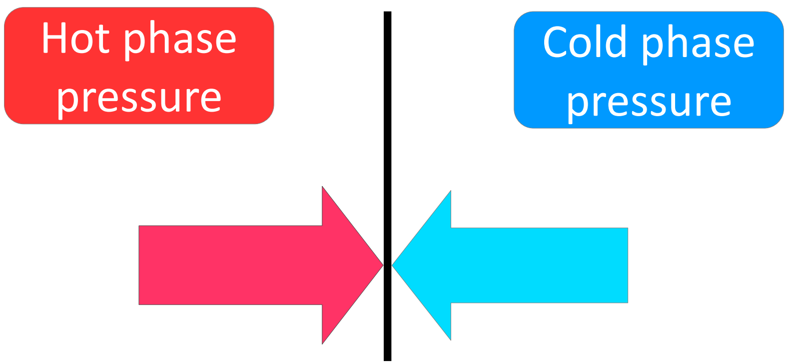
Gradient flow method for EoS is in the stage of development.
In this study, we want to confirm the usability and reliability of the gradient flow method.

Latent heat and pressure gap

To test the gradient flow method.

We calculate gaps of energy density and pressure
at the first order phase transition of SU (3) lattice gauge theory.





Latent heat

The latent heat is the most important and well-defined quantity at T_c.

Compare: results by the derivative method.

Pressure: balance

Because two phases coexist at T_c, the pressure in the hot and cold phases must be balanced.

The lattice spacing is given by $T_c = \frac{1}{a_t N_t}$.

Very good quantities
to test the new method.

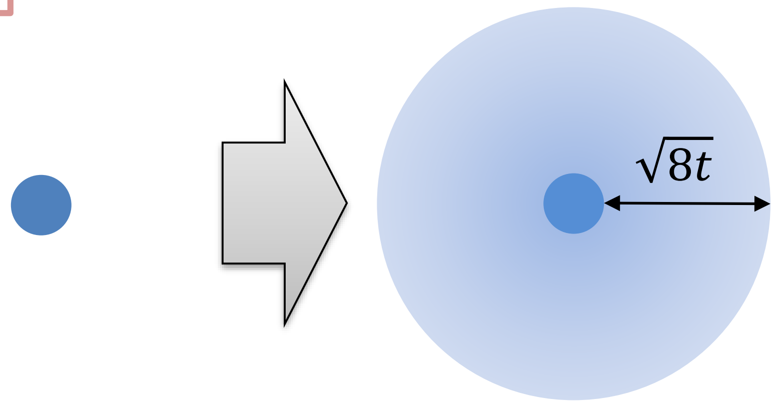
Gradient flow

Introduce fictitious time t.

Solving the flow equation, the gauge fields are smeared.

Flow equation (Diffusion equation)

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)$$



Flowed gauge field : $B_\mu(t,x)$

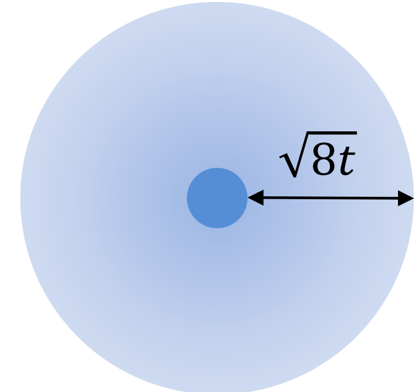
Flowed field strength : $G_{\mu\nu}(t,x)$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)]$$

Initial condition

$$B_\mu(t=0,x) = A_\mu(x)$$ (original gauge field)

Gradient flow



- We may view the flowed field B_μ as a smeared A_μ over a physical range $\sqrt{8t}$.
- It was shown that expectation values of operators in terms of B_μ have no UV divergence at finite t and these are independent of the lattice regularization in $a \to 0$.
- The gradient flow defines a physical renormalization scheme, which can be calculated directly on the lattice.
- When flow time t increases, $a/\sqrt{t}$ decreases for each fixed t. Thus, the discretization error may become smaller.

Calculating Energy Momentum Tensor

$$T^R_{\mu\nu}(x) \equiv c_1(t)\left[G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)\right]$$
$$+c_2(t)\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) + \cdots$$

H. Suzuki [PTEP 2013, 083B03 (2013)]

Perturbative matching coefficients

$$c_1(t) = \frac{1}{\bar{g}(\mu)^2} - b_0 \log \pi - \frac{1}{(4\pi)^2}\left[\frac{7}{3}C_2(G) - \frac{3}{2}T(R)N_f\right]$$

$$c_2(t) = \frac{1}{8}\frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2(G) + \frac{11}{3}T(R)N_f\right]$$

b_0 : coefficient of beta function.

$$f^{acd}f^{bcd} = C_2(G)\delta^{ab} \qquad \mathrm{tr}\left(T^aT^b\right) = -T(R)\delta^{ab}$$

Calculating Energy Momentum Tensor

$$T^R_{\mu\nu}(x) \equiv c_1(t)\left[G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)\right]$$
$$+c_2(t)\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x) + \cdots$$

H. Suzuki [PTEP 2013, 083B03 (2013)]

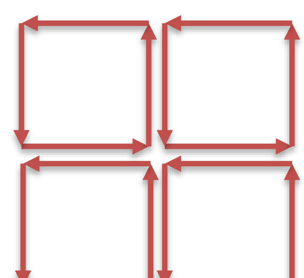
Energy density and pressure are given by

$$\langle T_{00}\rangle = \epsilon \qquad \frac{\langle T_{11}\rangle + \langle T_{22}\rangle + \langle T_{33}\rangle}{3} = p$$

Conventional combinations of ϵ and p: $\dfrac{(\epsilon-3p)}{T^4}, \dfrac{(\epsilon+p)}{T^4}.$

$G_{\mu\nu}G_{\mu\nu}$ is calculated by two alternative operators of clover and plaquette.

$G^2_{\mu\nu} \simeq$



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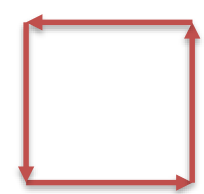


Three limits

1. Continuum limit $a \to 0$.

   Important: taking $a \to 0$ with keeping t finite.

2. Flow time zero limit $t \to 0$.

   In general, higher dimensional operators are mixed.

   To remove the Irrelevant operators, we need $t \to 0$.

$$\{T_{\mu\nu}\}(x,t,a) = \{T_{\mu\nu}\}_{\mathrm{WT}}(x) + \cancel{t(\text{dim6 operators})}$$

3. Volume infinity limit $V \to \infty$.

Check list

1. Pressure balance at T_c.
2. Compare the latent heat with derivative method.
3. Lattice regularization dependence in $G_{\mu\nu}^2$
   (clover operator and plaquette)
4. Order of the limiting procedures.
   (Lattice spacing a, Flow time t, Volume V)

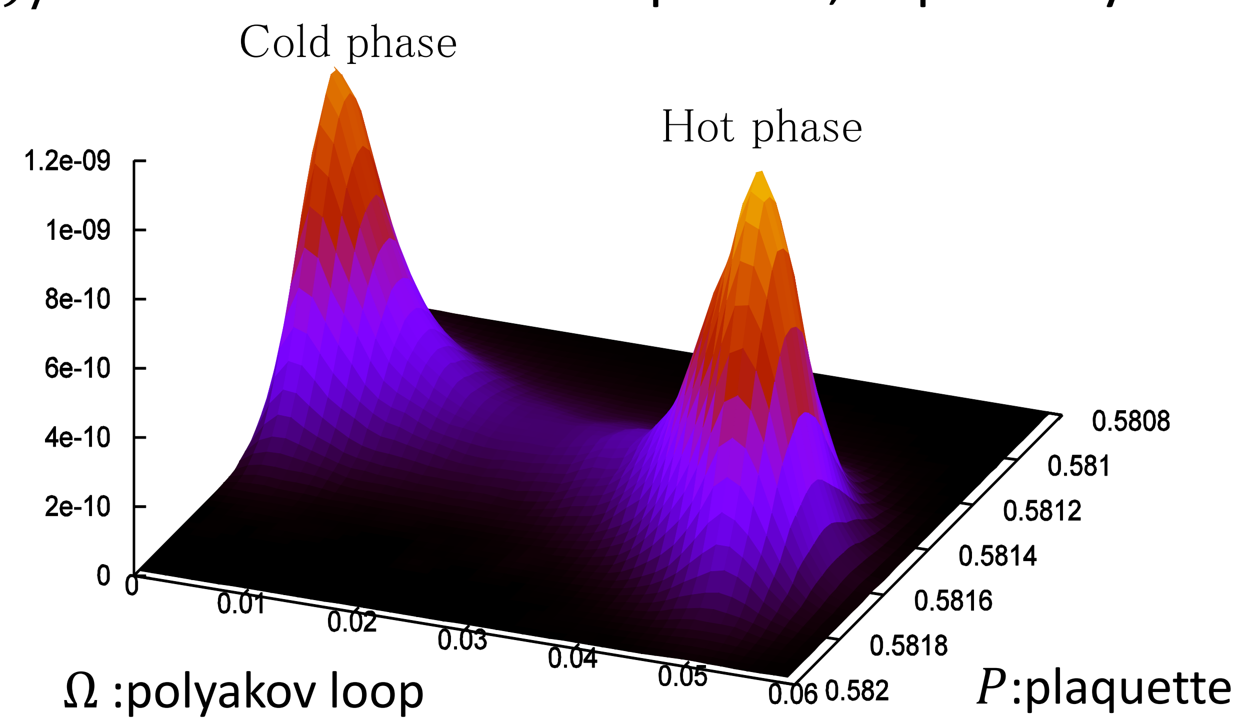
Two phases at the first order phase transition

$$\frac{\Delta(\epsilon+p)}{T^4} = \langle\frac{\epsilon+p}{T^4}\rangle_{\text{hot}} - \langle\frac{\epsilon+p}{T^4}\rangle_{\text{cold}}$$
$$\frac{\Delta(\epsilon-3p)}{T^4} = \langle\frac{\epsilon-3p}{T^4}\rangle_{\text{hot}} - \langle\frac{\epsilon-3p}{T^4}\rangle_{\text{cold}}$$

We classify the configurations into hot and cold phases by the value of Polyakov loop on the first order phase transition point.

Calculate $(\epsilon+p)/T^4, (\epsilon-3p)/T^4$ in the hot and cold phases, separately.

There are few intermediate states. We omit the intermediate states by choosing a large V in this calculation.



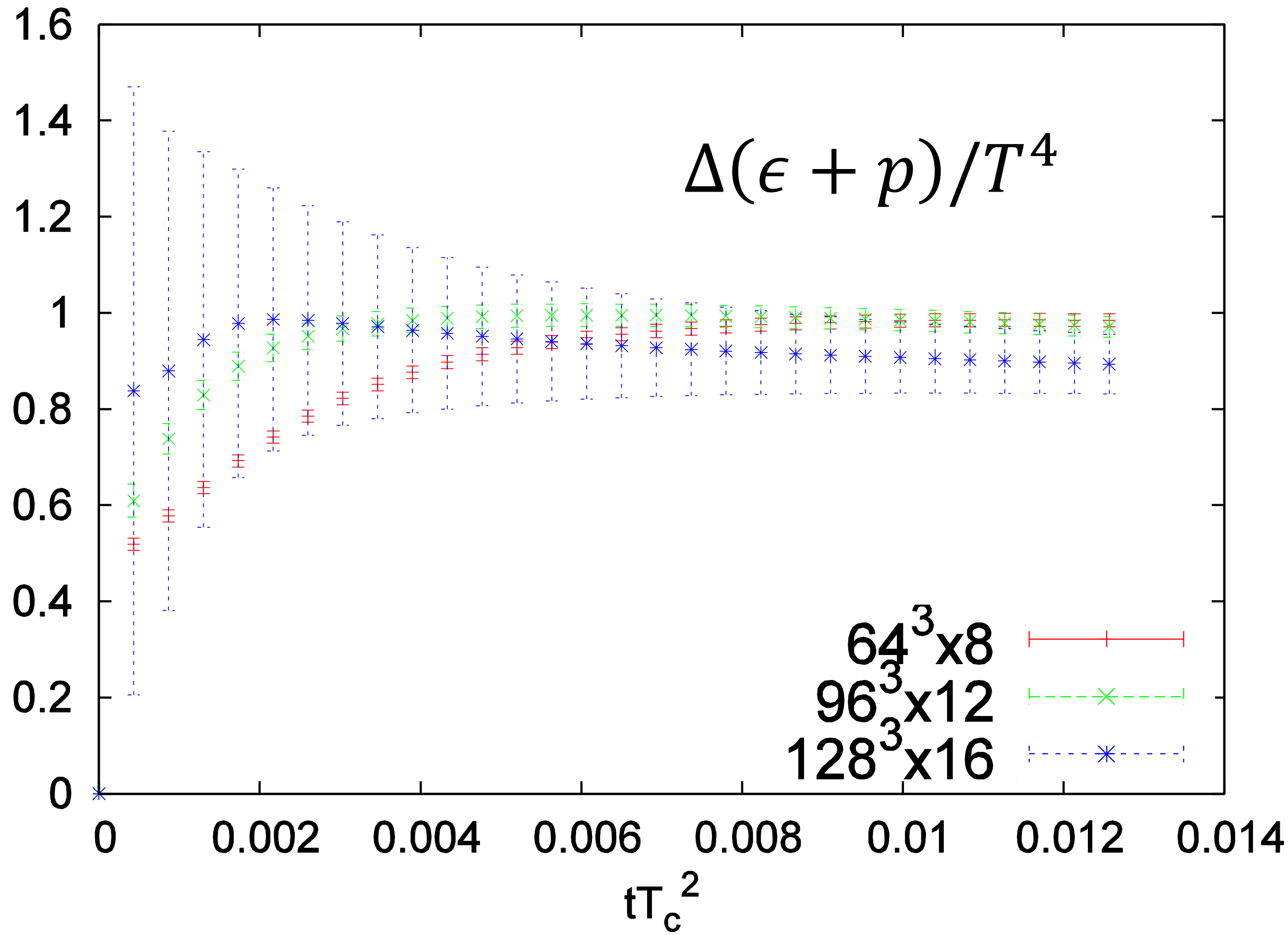
Simulation parameters

Simulation of SU(3) gauge theory

- Pseudo-heat bath method.
- Generate configurations at 3 to 5 points near T_c.
- We performed gradient flow observables every 20-100.

Lattice size $N_s^3 \times N_t$	Range of β	Total number of configurations	Measurement of flowed operator
$48^3 \times 8$	6.056-6.067	1,200,000	60,000
$64^3 \times 8$	6.0585-6.065	4,600,000	230,000
$64^3 \times 12$	6.332-6.339	690,000	23,000
$96^3 \times 12$	6.33-6.339	1,500,000	50,000
$96^3 \times 16$	6.543-6.547	1,500,000	15,000
$128^3 \times 16$	6.543-6.547	570,000	5,700

Latent heat and pressure gap with the clover operator

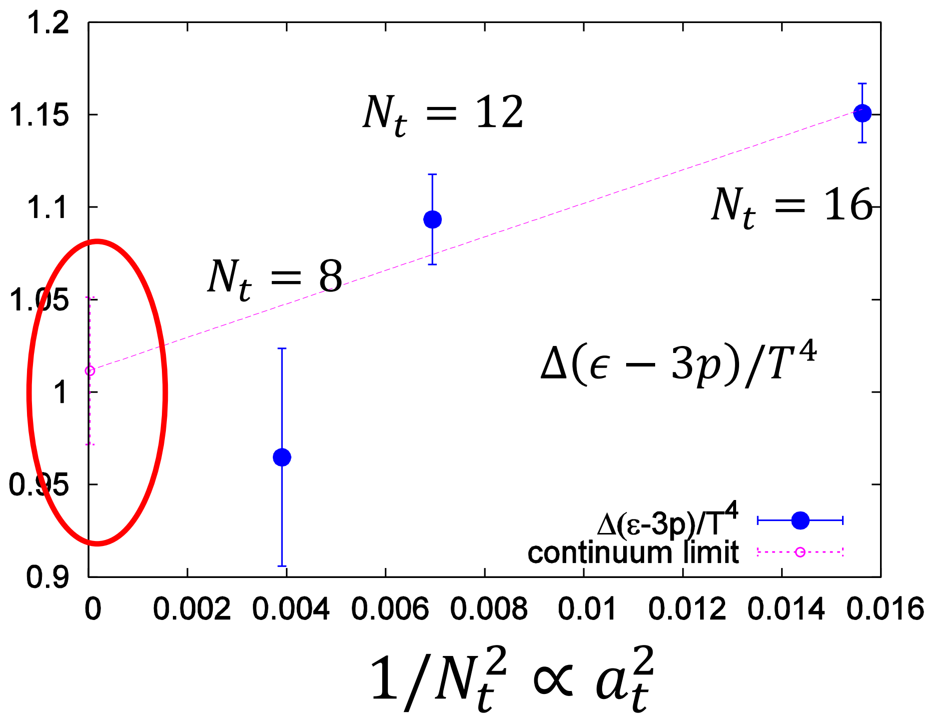


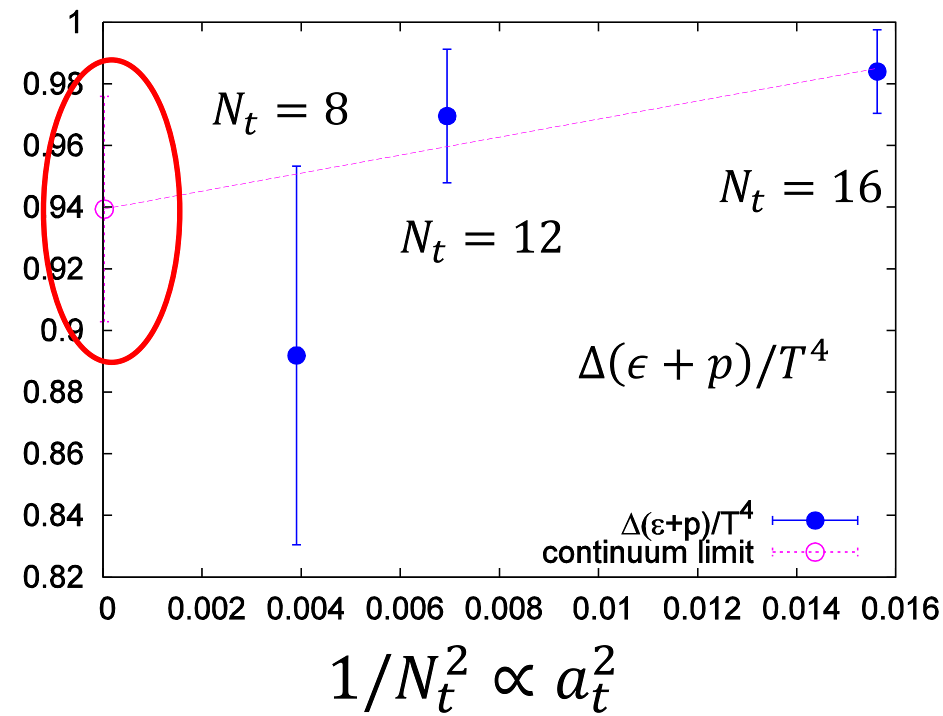
$N_t a = T_c^{-1}$
$N_s a = \sqrt[3]{V}$

- We plot results of three lattice spacings at a fixed aspect ratio $N_s/N_t = 8$.
- The physical volume V is the same because $N_s/N_t = \sqrt[3]{V}T_c$.
- When flow time increases, lattice discretization error decreases.
- In the region of $tT_c^2 >$ 0.008 the results are well linear.

Extrapolation to the continuum limit

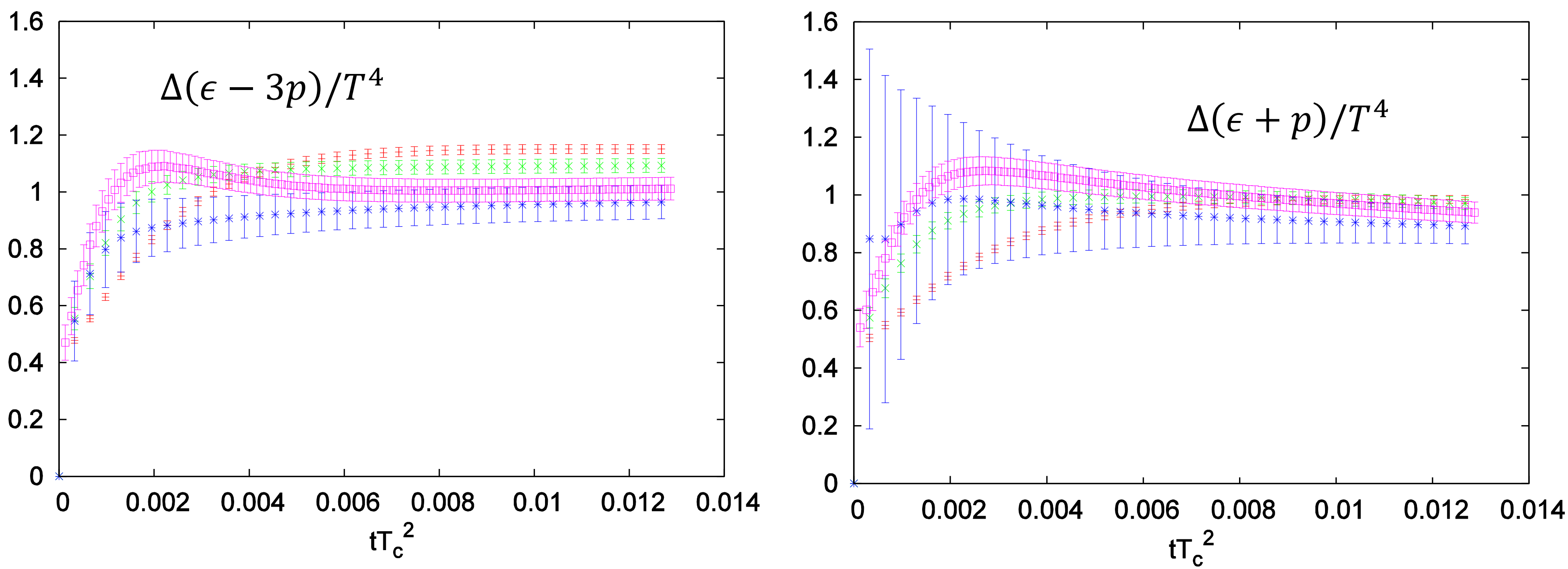
$tT_c^2 = 0.013$



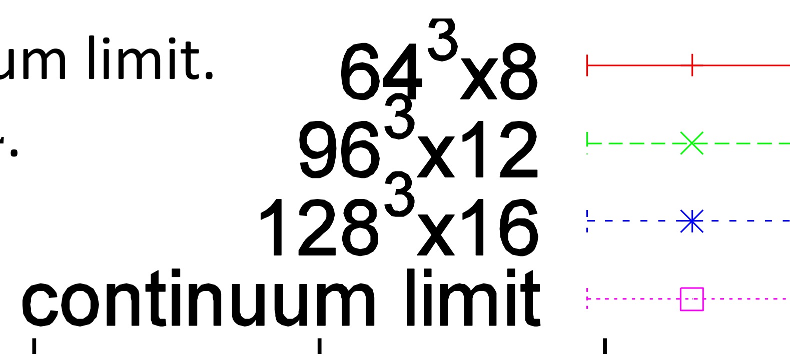


- Blue points are results of lattices with the same volume, at $N_t = 8{,}12{,}16$.
- We fit these data by a linear function of $1/N_t^2 \propto a_t^2$.
- Magenta line is the fit line.
- The symbol on the vertical axis is the result in the continuum limit.

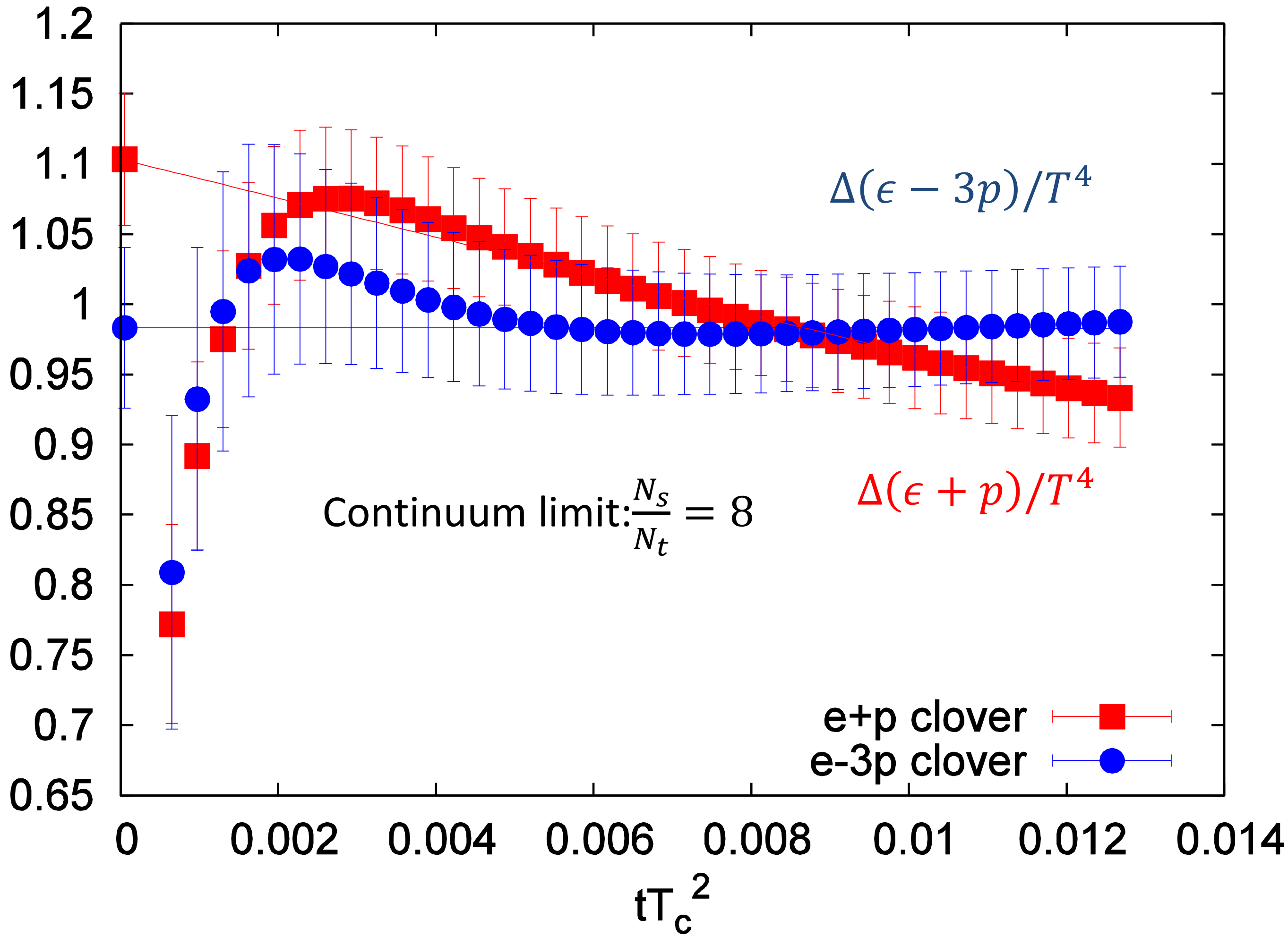
Extrapolation to the continuum limit



- Magenta lines are the values in the continuum limit.
- These results are well linear at $tT_c^2 > 0.004$.
- Lattice artifact remains at small t.

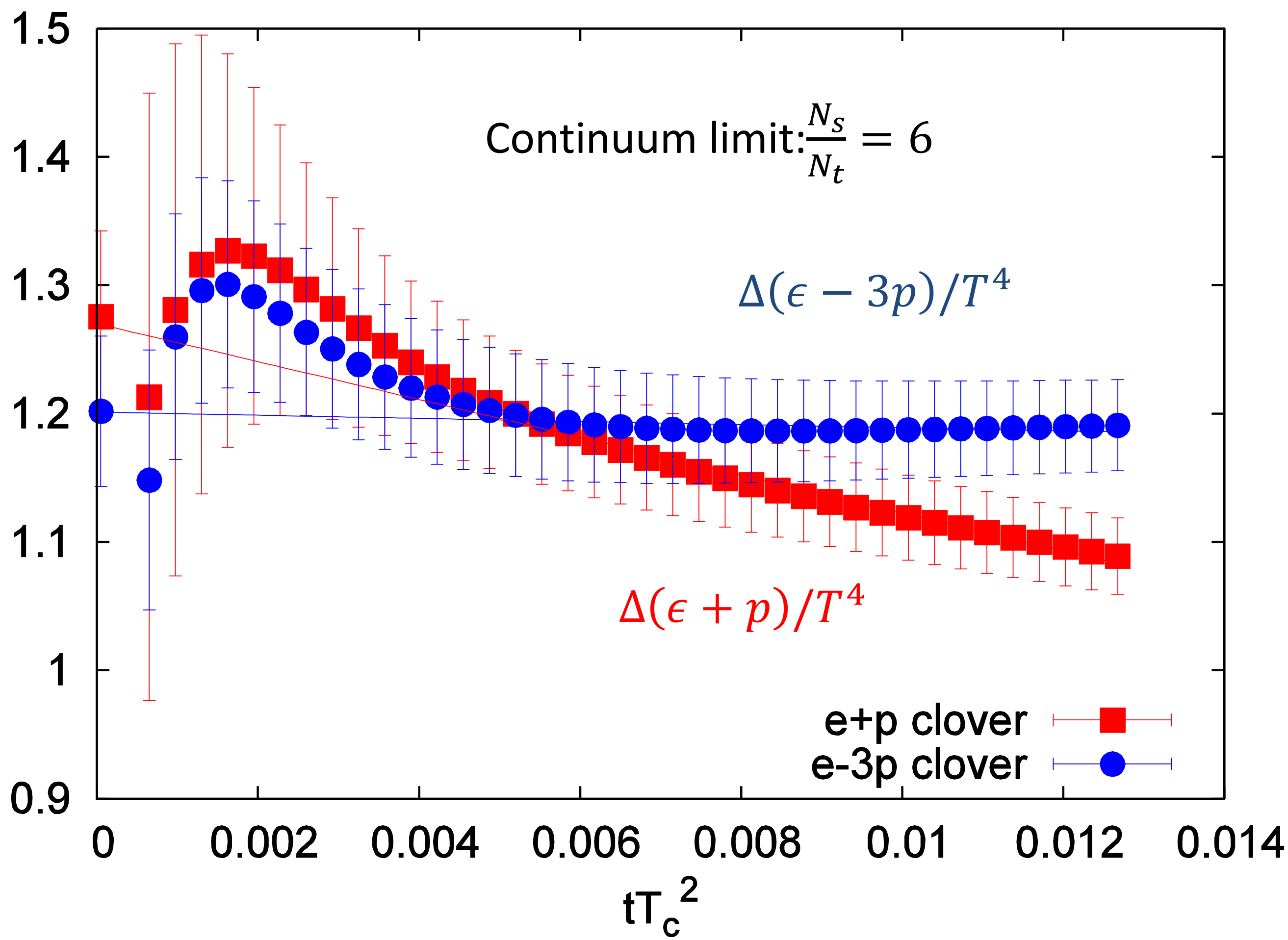


Vanishing pressure gap



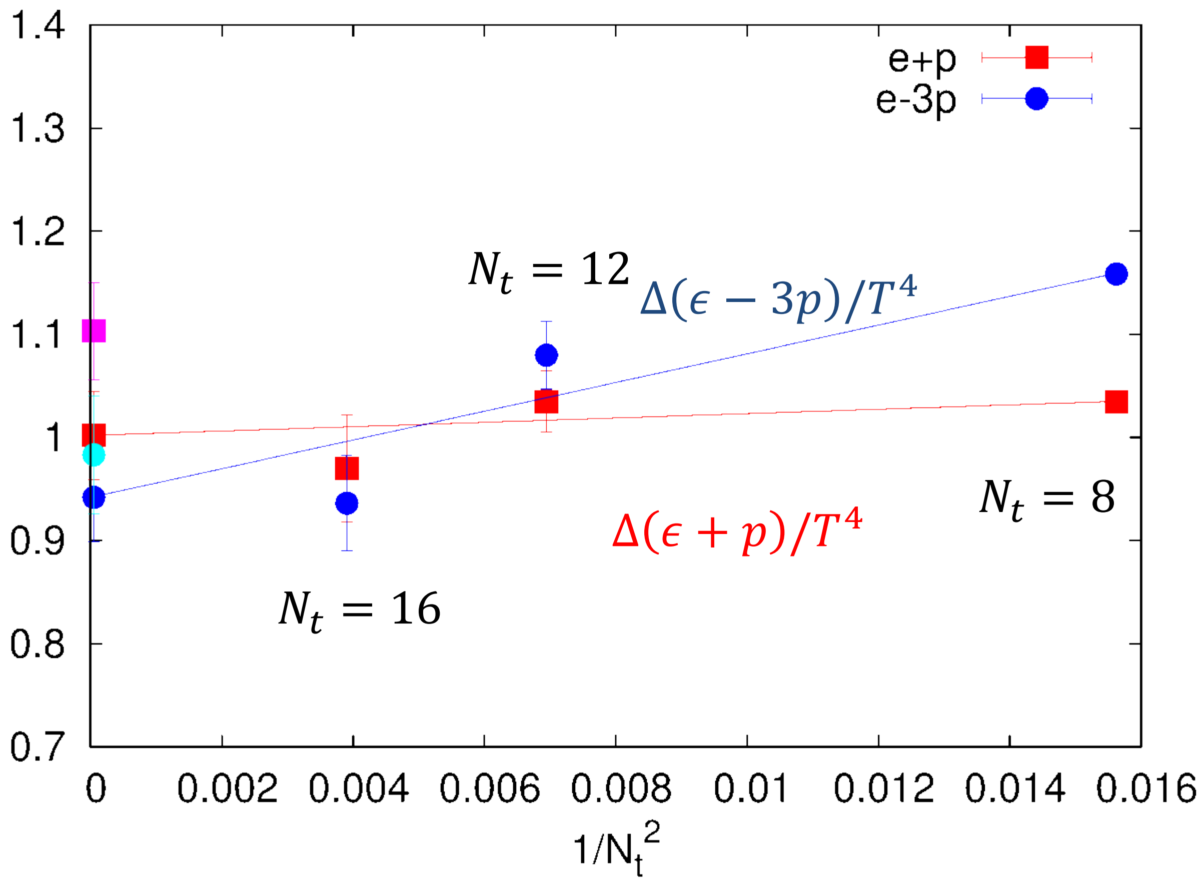
- We fit these data by linear function of t in $tT_c^2 > 0.004$.
- The values in the $t \to 0$ limit are roughly consistent within the statistical errors.
- It suggests that $\Delta p \to 0$.

Vanishing pressure gap



- The two values are consistent within the statistical errors.
- It suggests that $\Delta p \to 0$.

Alternative analysis: $t \to 0$ extrapolation at finite a

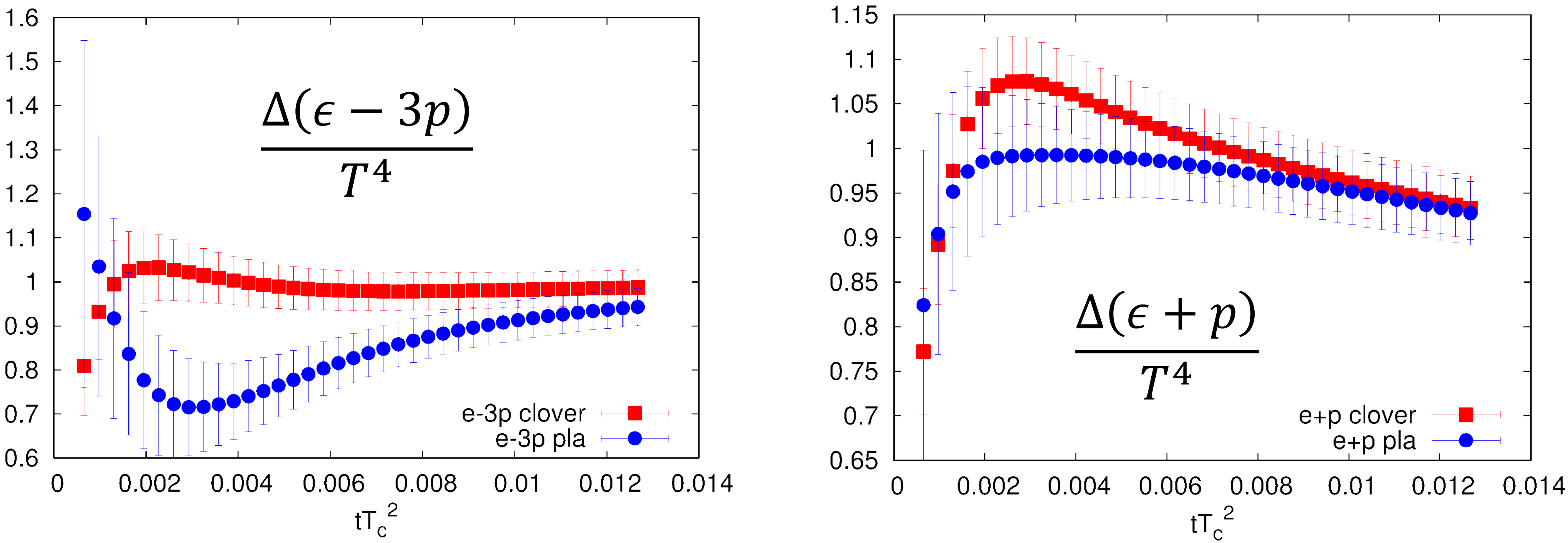


$N_s/N_t = 8$

- Cyan (e-3p) and magenta (e+p) symbols are the result of $t \to 0$ after $a \to 0$.
- $t \to 0$ extrapolation at each finite a. We then extrapolate them $a \to 0$ by blue and red fitting line.
- Two results of $a \to 0$ first and $t \to 0$ first are consistent within errors.
- The order of $a \to 0$ and $t \to 0$ are exchangeable.

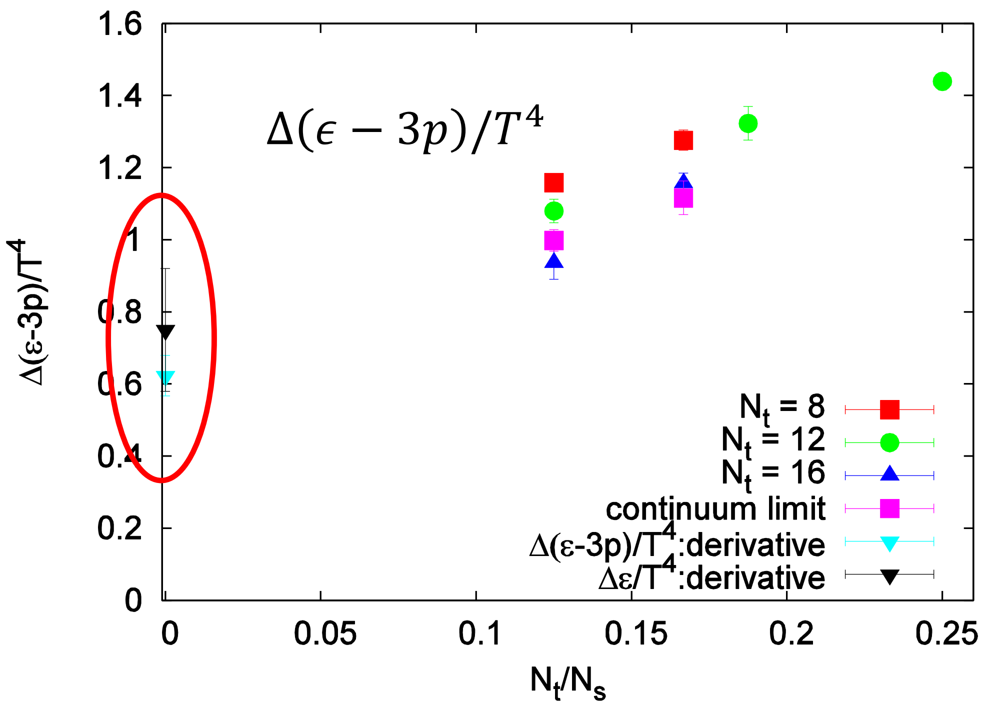
Comparison with the results of palquette operator

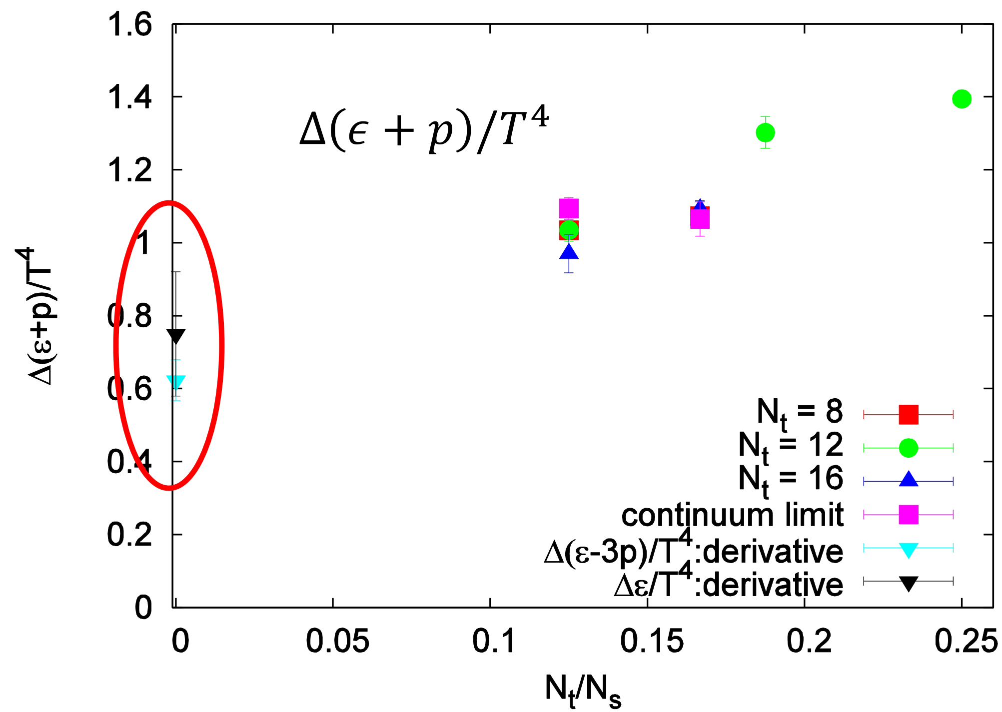
$$\frac{N_s}{N_t} = 8$$



- $G_{\mu\nu}$ is calculated by paquette (blue) and clover (red).
- The plaquette results approach the clover results at large tT_c^2 where lattice artifacts are expected to suppressed.
- The linear window is narrower when we adopt plaquette operator.

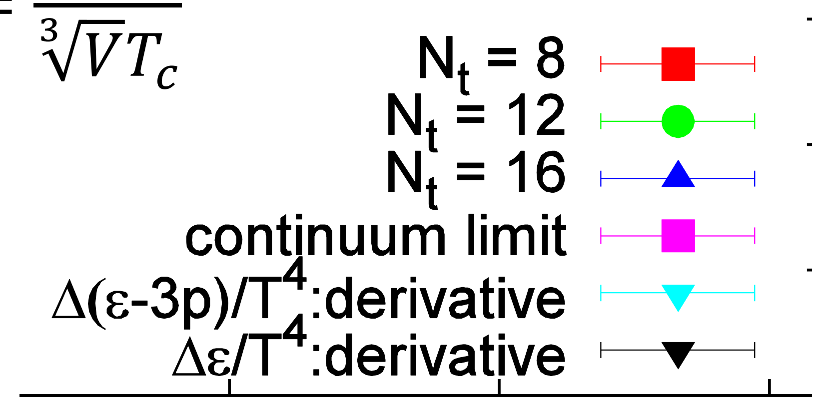
Thermodynamics quantities for each lattice





Result of derivative method

$$\frac{N_t}{N_s} = \frac{1}{\sqrt[3]{V}T_c}$$



・The result approaches the result of the derivative method as the volume increases

Conclusion & Outlook

- Using the gradient flow method we calculate the latent heat and the pressure gap at the first order phase transition of SU(3) gauge theory.
- The order of $a \to 0$ and $t \to 0$ are exchangeable.
- When lattice spacing is small, $\Delta p = 0$ at finite V.
- The statistical errors by gradient flow are very small in comparison with the results by the derivative method.
- The latent heat by the gradient flow method approaches that by the derivative method as the volume increases.

Gradient flow method works well.