Form factors for moments of correlation functions

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Motivation

Lepton universality

Charge radius puzzle

μ vs *e* Hydrogen radius in 5σ tension @ 4%.

B-meson semileptonic decay

4σ tension from Standard Model prediction.

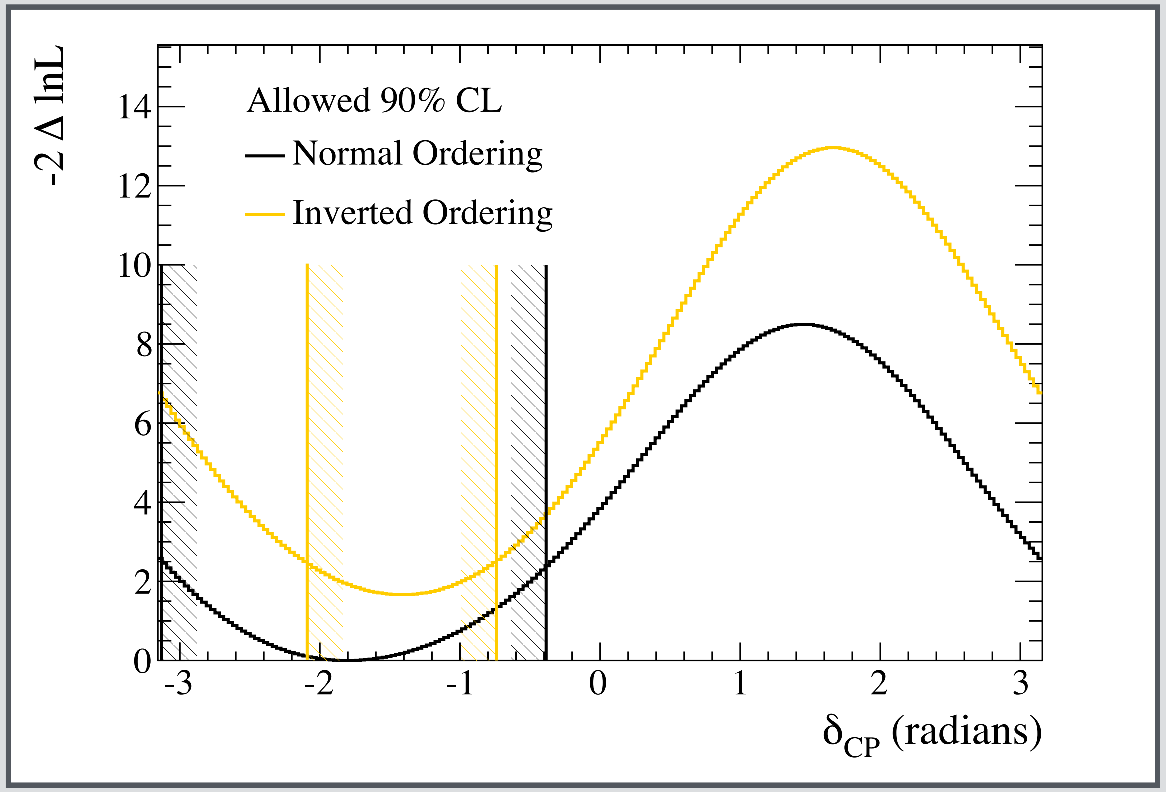
Origin of matter

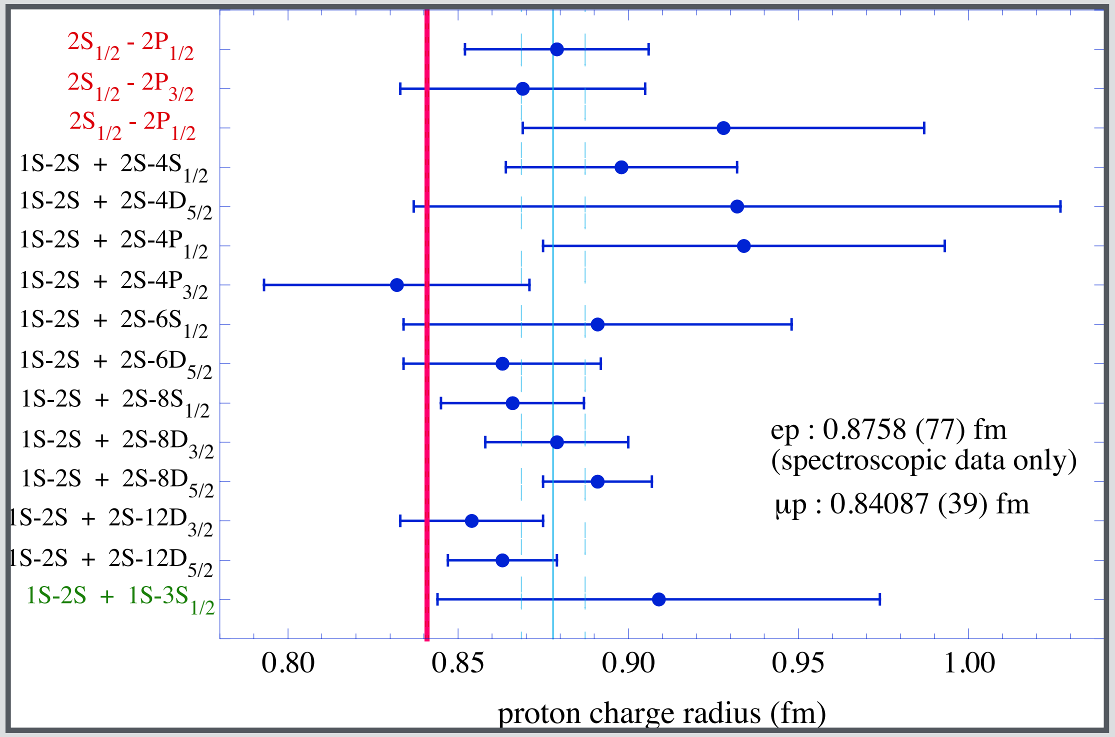
CP-violation from neutrinos

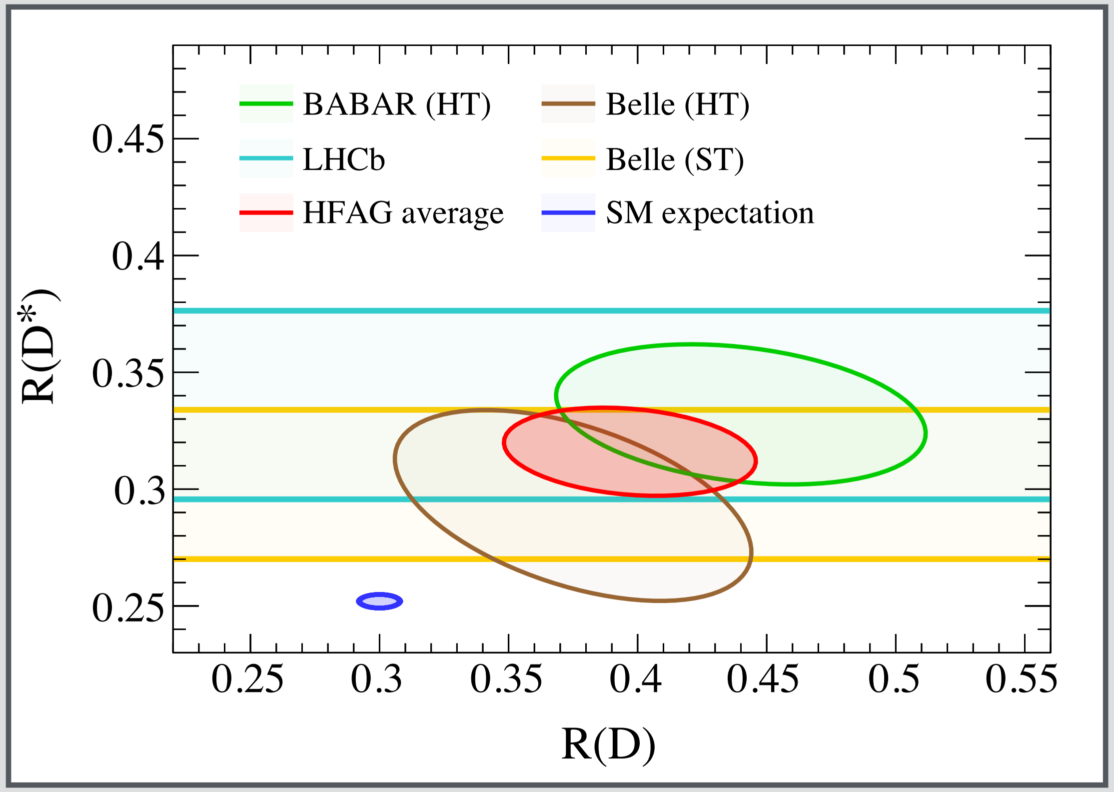
T2K measures maximal CPV in PMNS @ 2σ.

Future experiments aim to improve this.

Axial form factor @10% improves constraint.







Direct calculation of slope is interesting for a wide range of applications.

Overview of moment methods

Issues with moment methods:

Odd moments on lattice yields wrong ground state.

PRD 66 017502 (2002)

Some existing related works:

Slope of the Isgur-Wise function — Nucl. Phys. B444 401 (1995)

HVP - temporal moments — PRD 89 114501 (2014), PRD 96 034516 (2017)

Radii - expansion of lattice operators — Phys. Lett. B718 (2012), PRD 90 054508 (2014), PRD 97 034504 (2018)

ETMC - position space method — PRD 94 074508 (2016)

Existing methods take derivatives at $q^2 = 0$

Proposed method:

Method takes $\partial/\partial q^2$ generalized to all momenta

Ensemble and correlator overview

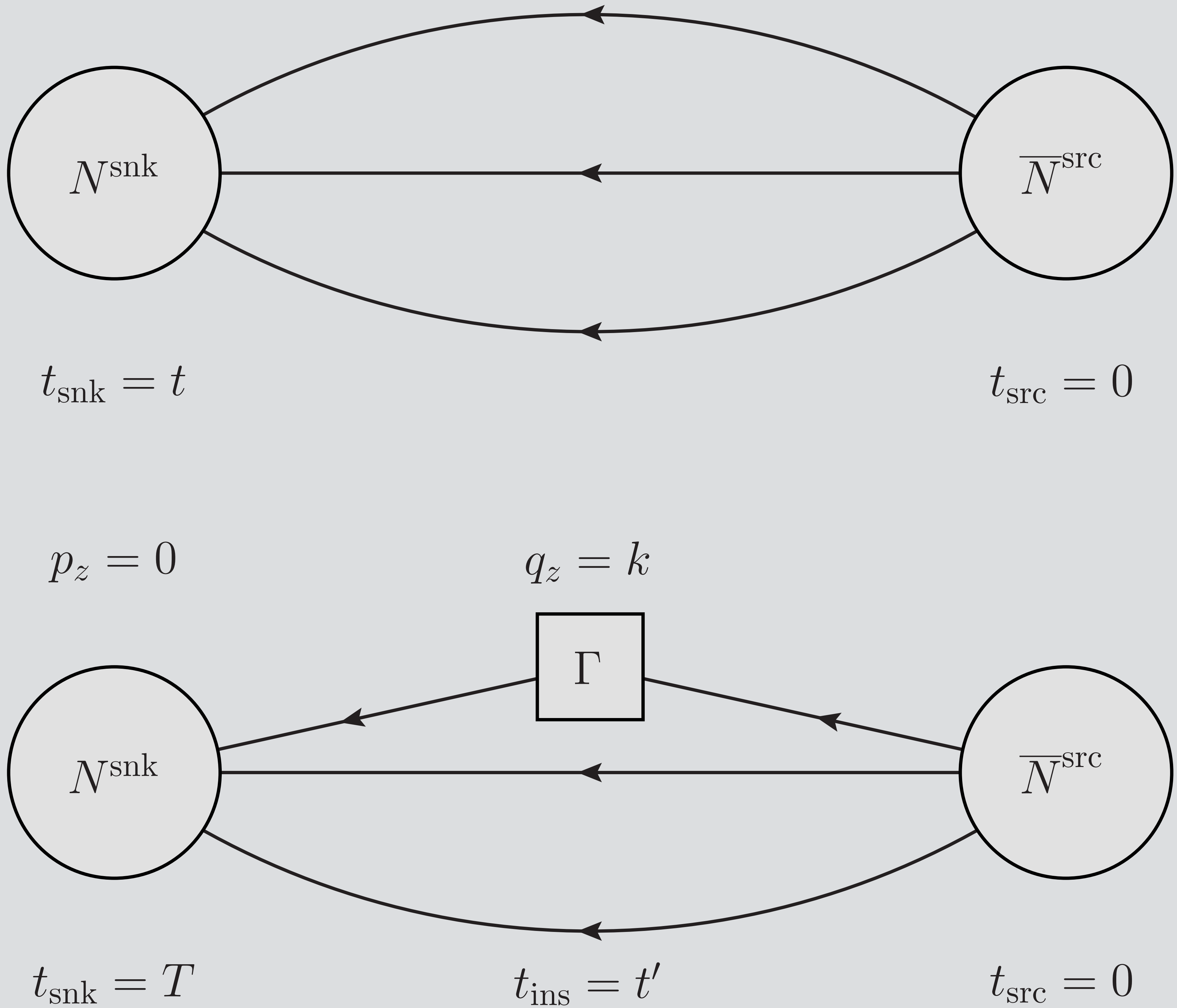
2+1 flavor JLab isotropic clover ensemble
$a \sim 0.12$ fm
$m_\pi \sim 400$ MeV
$N_x^3 \times N_t = 32^3 \times 96$ and $48^3 \times 96$

~ 200 configurations x 2 sources [0, 48]
3-point correlator $T_{snk} = 8$ and 10
Isovector vector form factor is studied

Goal of this study
Low statistics demonstration of the proposed method.
Study finite volume effects.

Assess whether this is a viable strategy for improving lattice calculations for form factors.

Kinematic setup



Two-point correlators

Two-point correlator

$$C_{2\mathrm{pt}}(t) = \int d^3x \langle N_{t,x} | N^\dagger_{0,0} \rangle e^{-ikx_i}$$

Two-point moment

$$\frac{\partial}{\partial k^2} C_{2\mathrm{pt}}(t) = \int d^3x \langle N_{t,x} | N^\dagger_{0,0} \rangle \frac{-ix_i}{2k} e^{-ikx_i}$$

$$\lim_{k^2 \to 0} C'_{2\mathrm{pt}}(t) = \int d^3x \langle N_{t,x} | N^\dagger_{0,0} \rangle \frac{-x_i^2}{2}$$

Only have even spatial moments

Three-point correlators

Three-point correlator

$$C_{3\mathrm{pt}}(T,t') = \int d^3x d^3x' \langle N_{t,x}|\Gamma_{t'x'}|N^\dagger_{0,0}\rangle e^{-ikx'_i}$$

Three-point moment

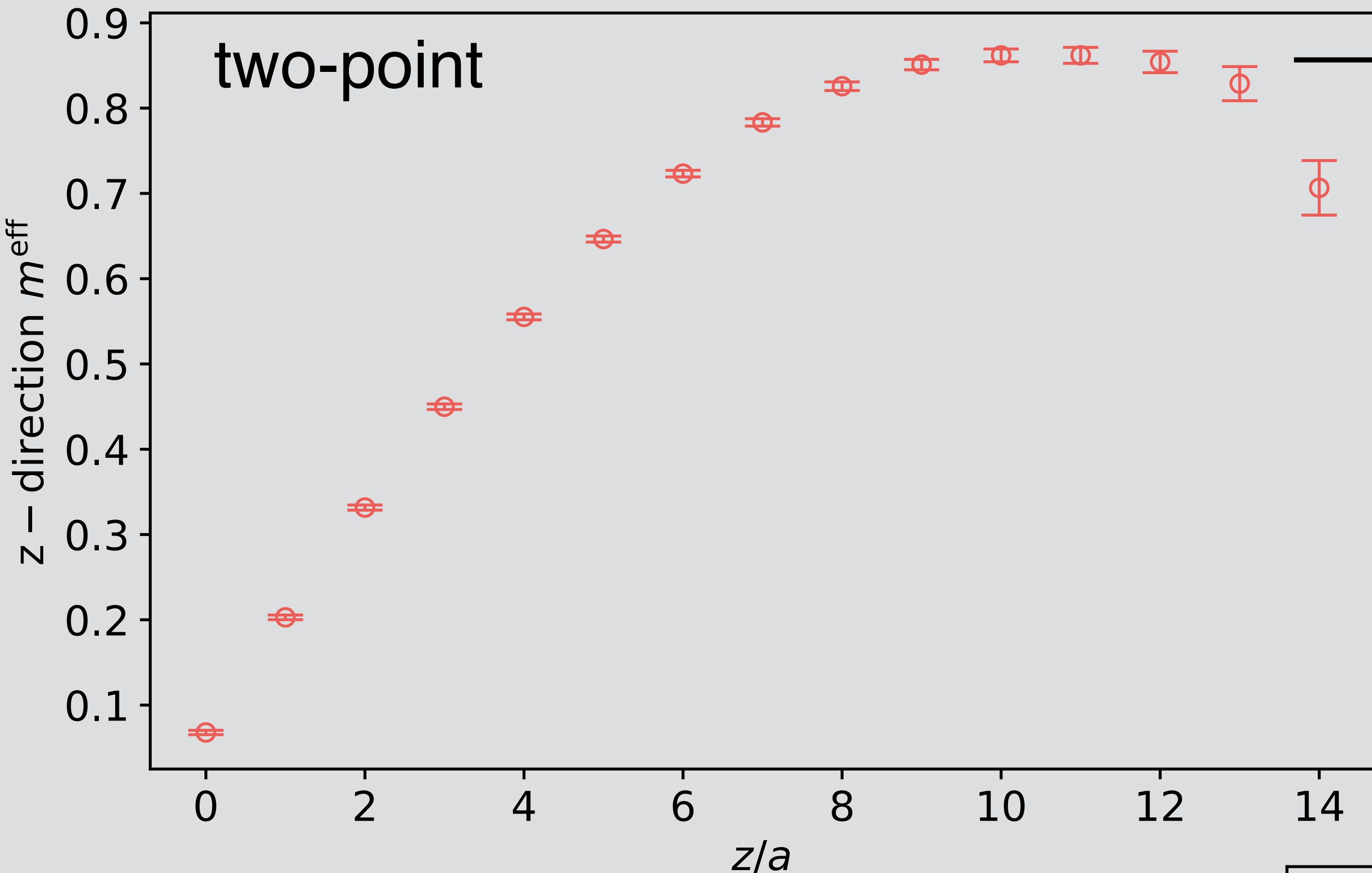
$$\frac{\partial}{\partial k^2} C_{3\mathrm{pt}}(T,t') = \int d^3x d^3x' \langle N_{T,x}|\Gamma_{t'x'}|N^\dagger_{0,0}\rangle \frac{-ix'}{2k} e^{-ikx'_i}$$

$$\lim_{k^2\to 0} C'_{3\mathrm{pt}}(T,t') = \int d^3x d^3x' \langle N_{T,x}|\Gamma_{t'x'}|N^\dagger_{0,0}\rangle \frac{-{x'}^2}{2}$$

Moments are with respect to current insertion

Given correlators, moments are computationally free

z-direction correlator plots



approx. 1400 MeV

Sum over *x, y, t.*

Plot z-direction effective masses.

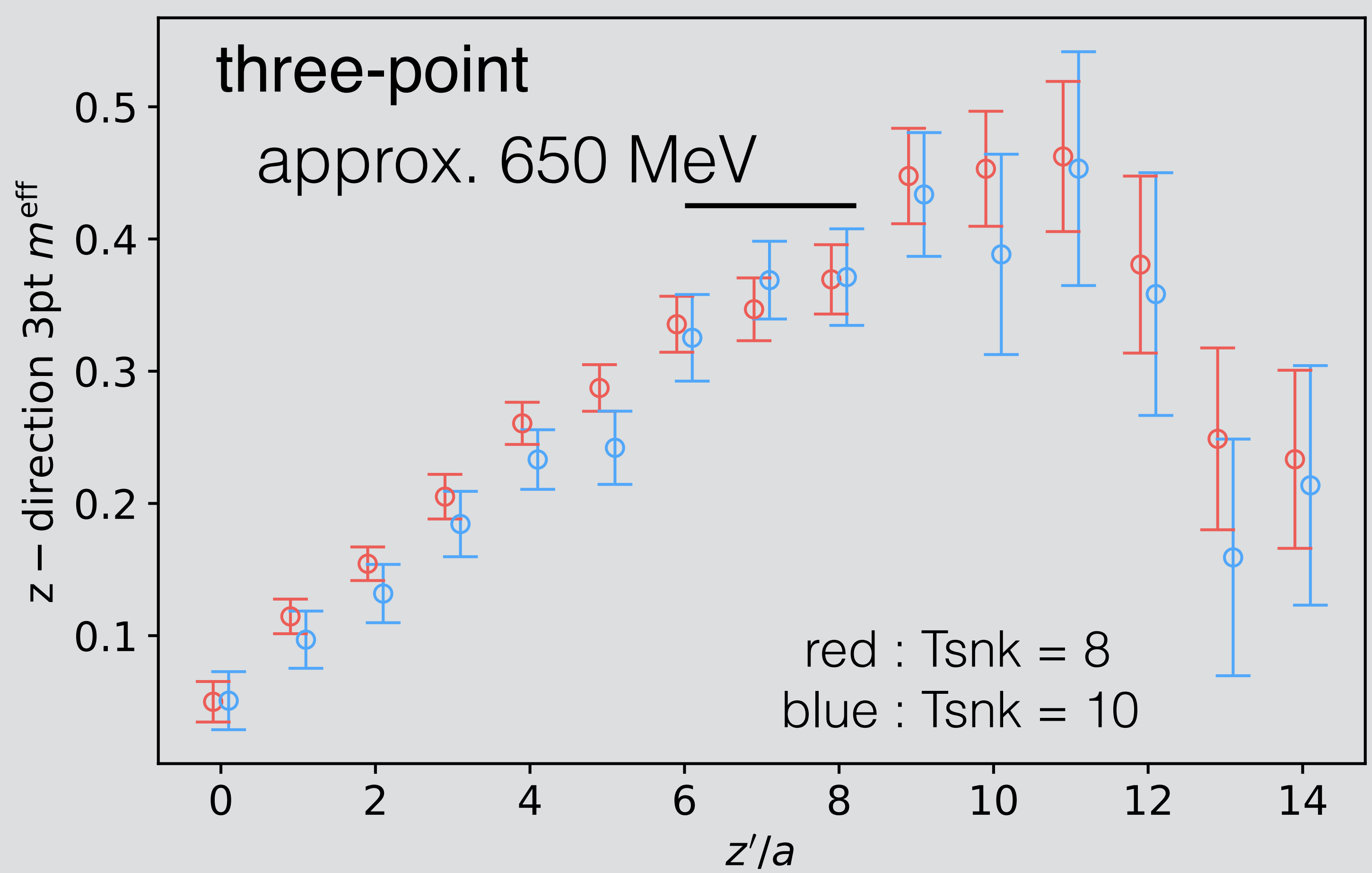
These effective mass have contamination from backward signal.

Key message
FV effects are exp. suppressed

Two-point FV corrections suppressed by mass of initial/final states.

Three-point FV suppressed by meson mass at the current.

For charge radius, 3pt has larger FV effects.



Finite volume correction

Correlator construction

Spatial moments are applied to X/2 (half the box size), due to periodic boundary.

What is the error for doing this? (Assume only ground state dominate past X/2)

1) Missing the spatial sum (in one dimension) from X/2 to infinite.
2) Wrong moment applied to the backward propagating signal from 0 to X.

Correction to 1) (This piece is missing)

$$\int_{X/2}^{\infty} x^2 e^{-Ex} dx = \left(\frac{X^2}{4E} + \frac{X}{E^2} + \frac{2}{E^2}\right) e^{-EX/2}$$

Correction to 2) (This piece is extra)

$$\int_{0}^{X/2} x^2 e^{E(x-X)} dx = \left(\frac{X^2}{4E} - \frac{X}{E^2} + \frac{2}{E^2}\right) e^{-EX/2} - \frac{2}{E^3} e^{-EX}$$

Total correction

$$\delta_{\mathrm{FV}}(E,X) = \frac{2X}{E^2} e^{-EX/2} + \frac{2}{E^3} e^{-EX}$$

Relative correction

$$R_{\mathrm{FV}}(E,X) = EXe^{-EX/2} + e^{-EX}$$

For a 1% relative correction (some numbers for reference)

if E ~ 800 MeV a 3.6 fm box is needed, if E ~ 600 MeV a 4.8 fm box is needed.

A similar approach can be found for HVP @ PRD 96 034516 (2017)

Two-point fit functions

Two-point fit function

$$C_{2\mathrm{pt}}(t) = \sum_n \frac{Z_n Z_n^\dagger}{2E_n} e^{-E_n t}$$

Two-point moment fit function

$$C'_{2\mathrm{pt}}(t) = \sum_n C^n_{2\mathrm{pt}}(t) \left(\frac{2Z'_n}{Z_n} - \frac{1}{2E_n^2} - \frac{t}{2E_n} \right)$$

Definitions

$$Z_n = \langle N | n \rangle \qquad E_n = \sqrt{M_n^2 + k^2}$$

Two-point constrains all parameters except Z'_n

Three-point fit functions

Three-point fit function

$$C_{3\mathrm{pt}}(t,t') = \sum_{n,m} \frac{Z_n(0)\Gamma_{nm}(k^2)Z_m^\dagger(k^2)}{4E_n(0)E_m(k^2)} e^{-E_n(0)(t-t')} e^{-E_m(k^2)t'}$$

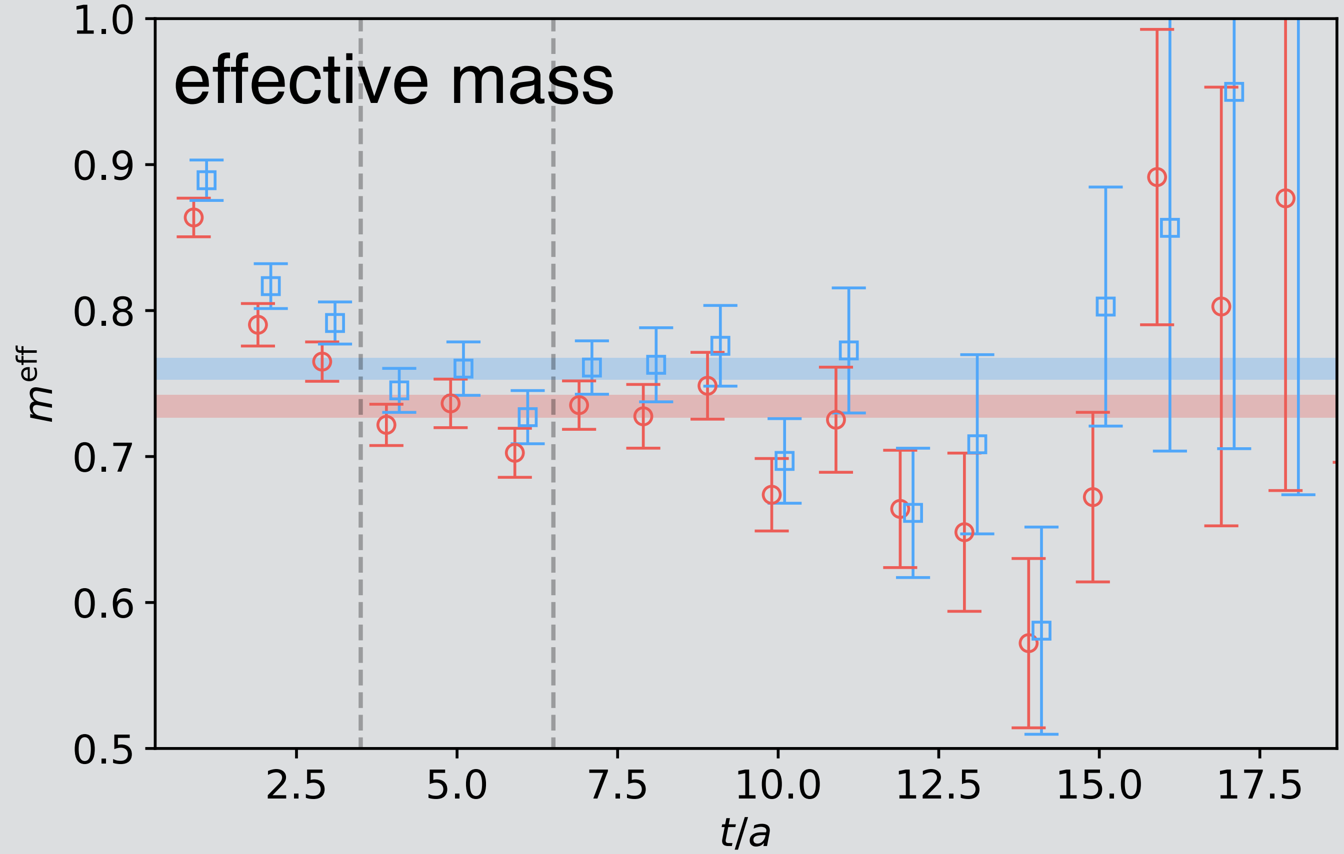
Three-point moment fit function

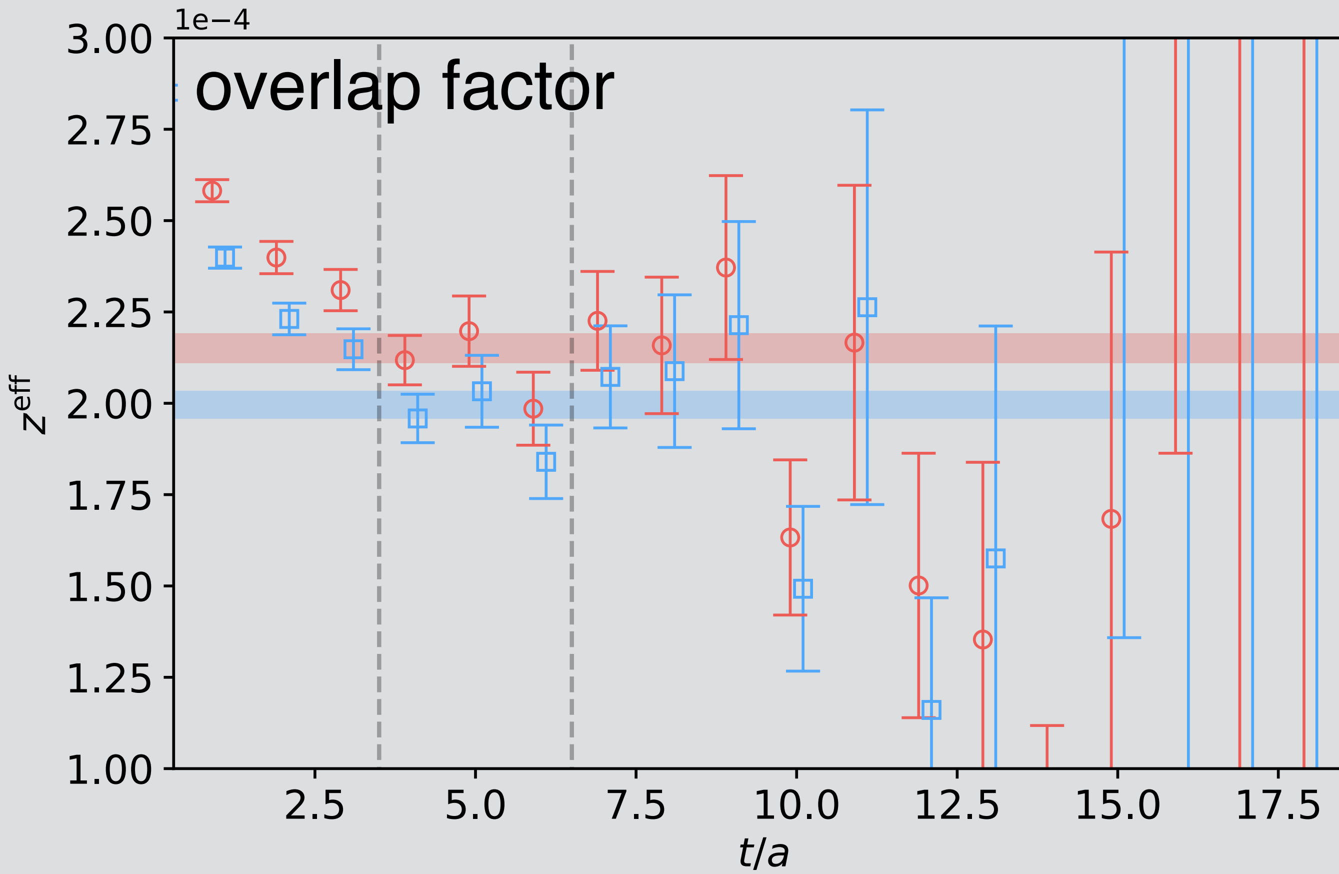
$$C'_{3\mathrm{pt}}(t,t') = \sum_{n,m} C_{3\mathrm{pt}}^{nm}(t,t') \left(\frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z'_m}{Z_m} - \frac{1}{2E_m^2} - \frac{t'}{2E_m} \right)$$

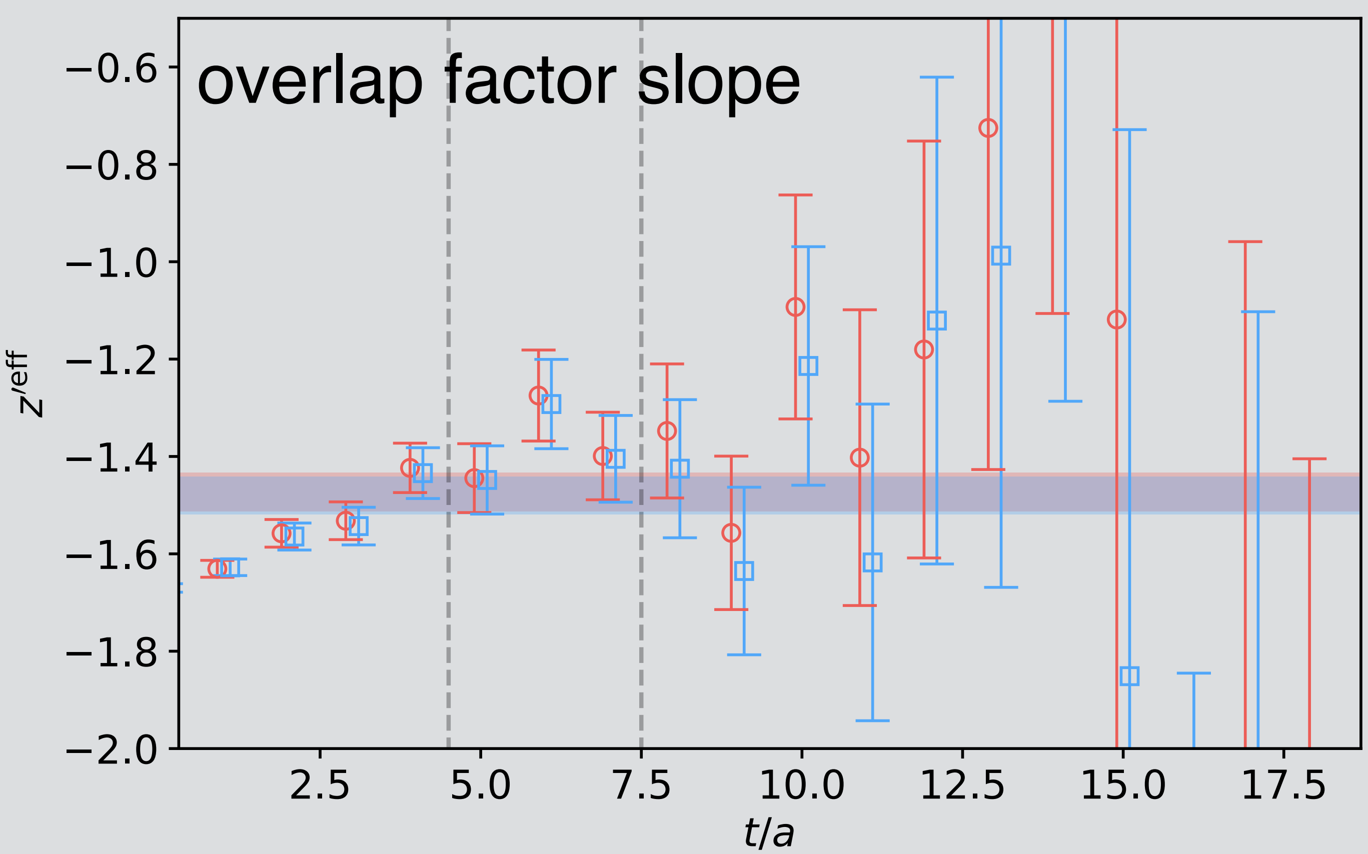
2pt and 3pt constraints all params. except slopes
2pt moment needed for smeared source/sink
3pt moment constrains slope of form factor

Two-point related fits

preliminary







Fit strategy

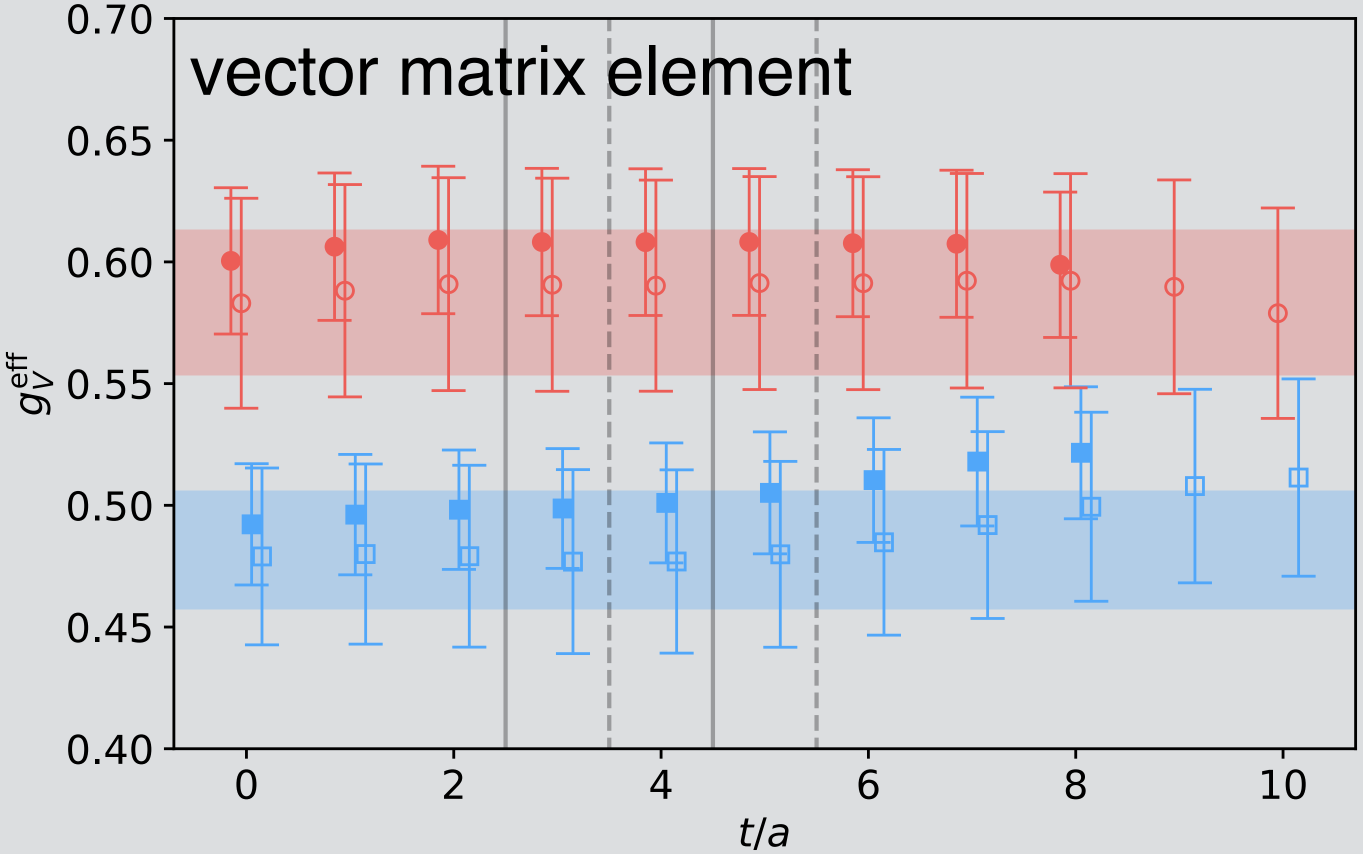
2 state Bayesian simultaneous fit
2pt, 2pt deriv., 3pt, 3pt deriv. @ q=[0,1]

Due to low statistics, fit to as little data as possible to control covariance matrix estimation.

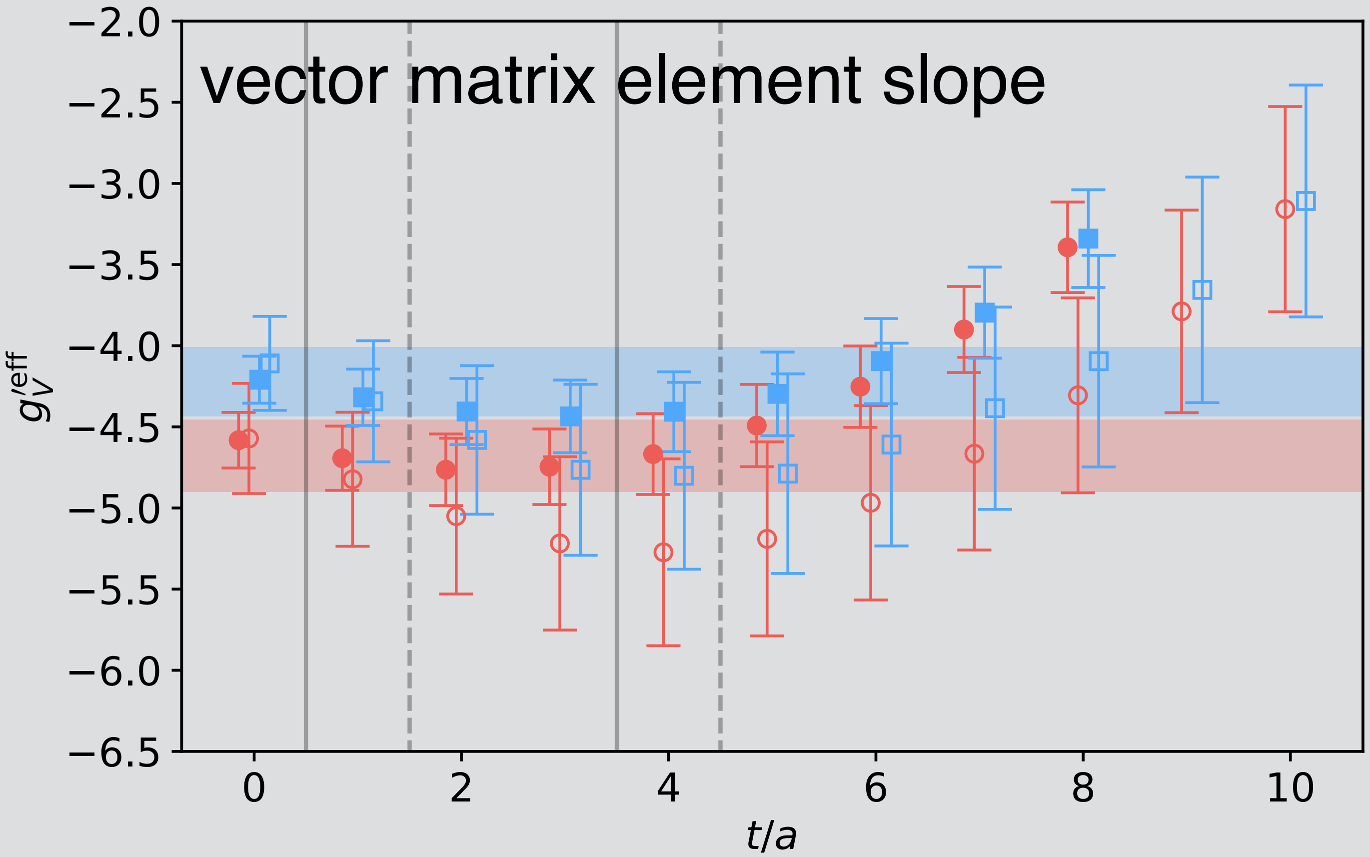
Red (q=0) Blue (q=1)
Red / Blue bands are the G.S. fit results.
Dotted gray line is fit region.

Three-point related fits

preliminary



Plot of the fit parameter.
Differ from gV by factors of energy.



Central result: Slope of form factors
Plot 3pt derivative / 3pt minus linear contaminations.

At *fixed* src-snk separation, the remaining contamination is linearly increasing.

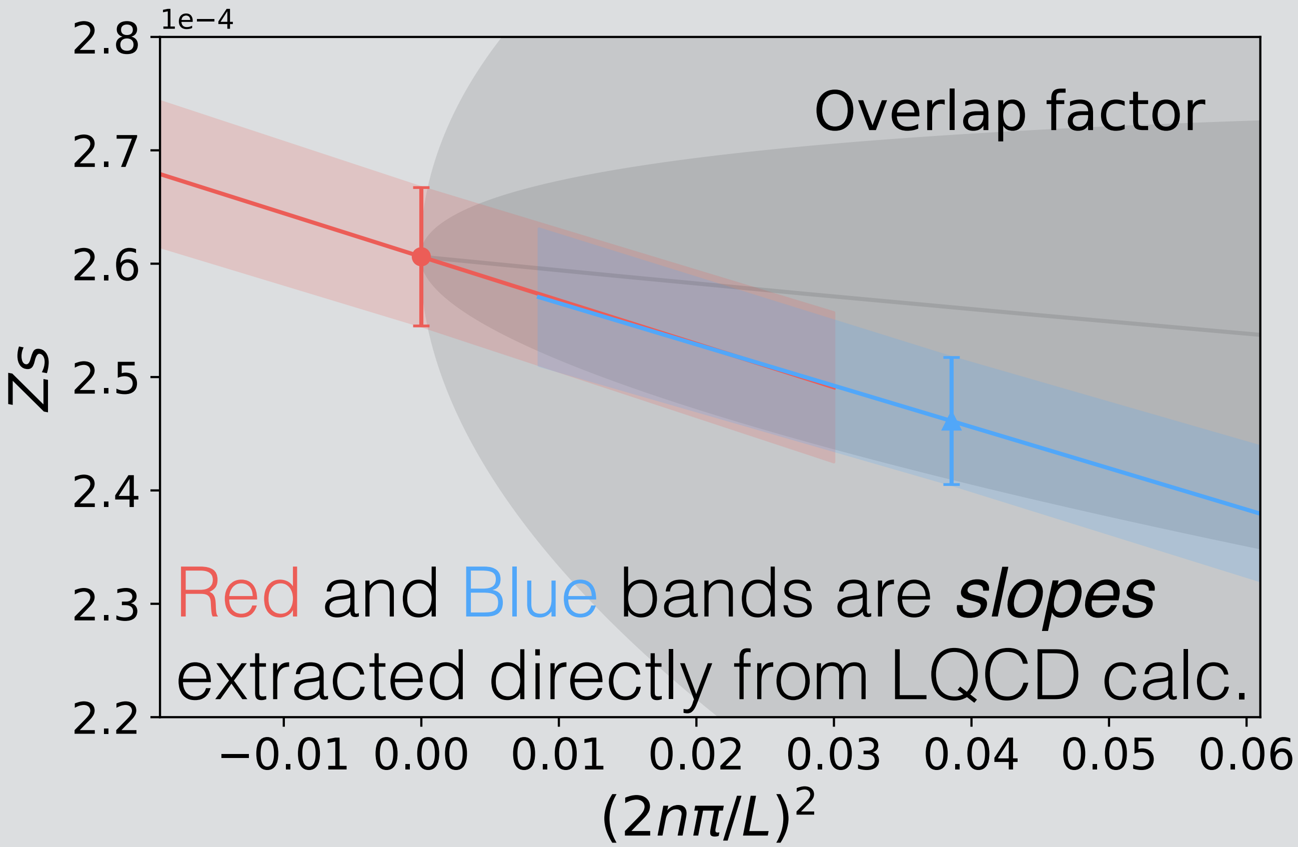
Therefore, this plot has *linearly increasing* excited state contamination.

Fit function reminder

$$C'_{\mathrm{3pt}}(t,t') = \sum_{n,m} C^{nm}_{\mathrm{3pt}}(t,t')\left(\frac{\Gamma'_{nm}}{\Gamma_{nm}} + \frac{Z'_m}{Z_m} - \frac{1}{2E_m^2} - \frac{t'}{2E_m}\right)$$

Some preliminary results

preliminary

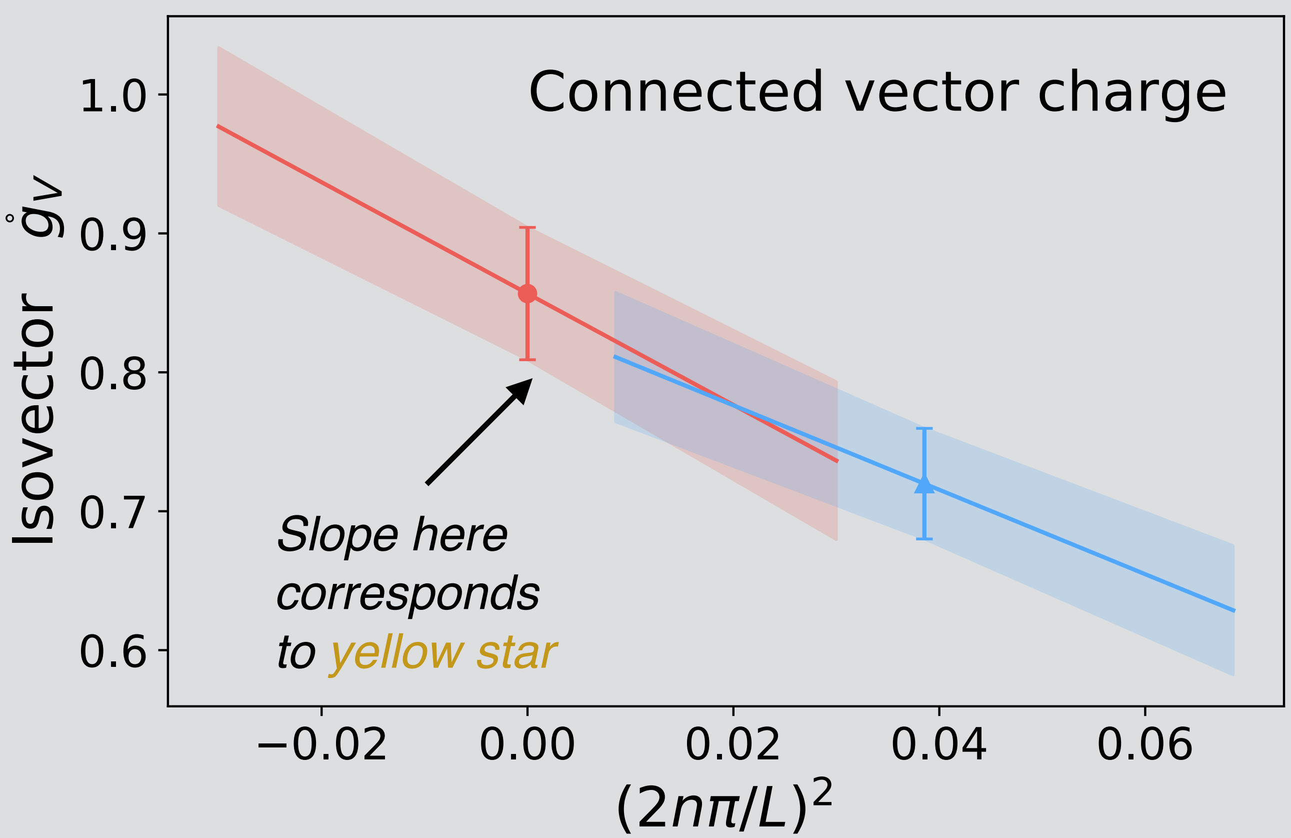


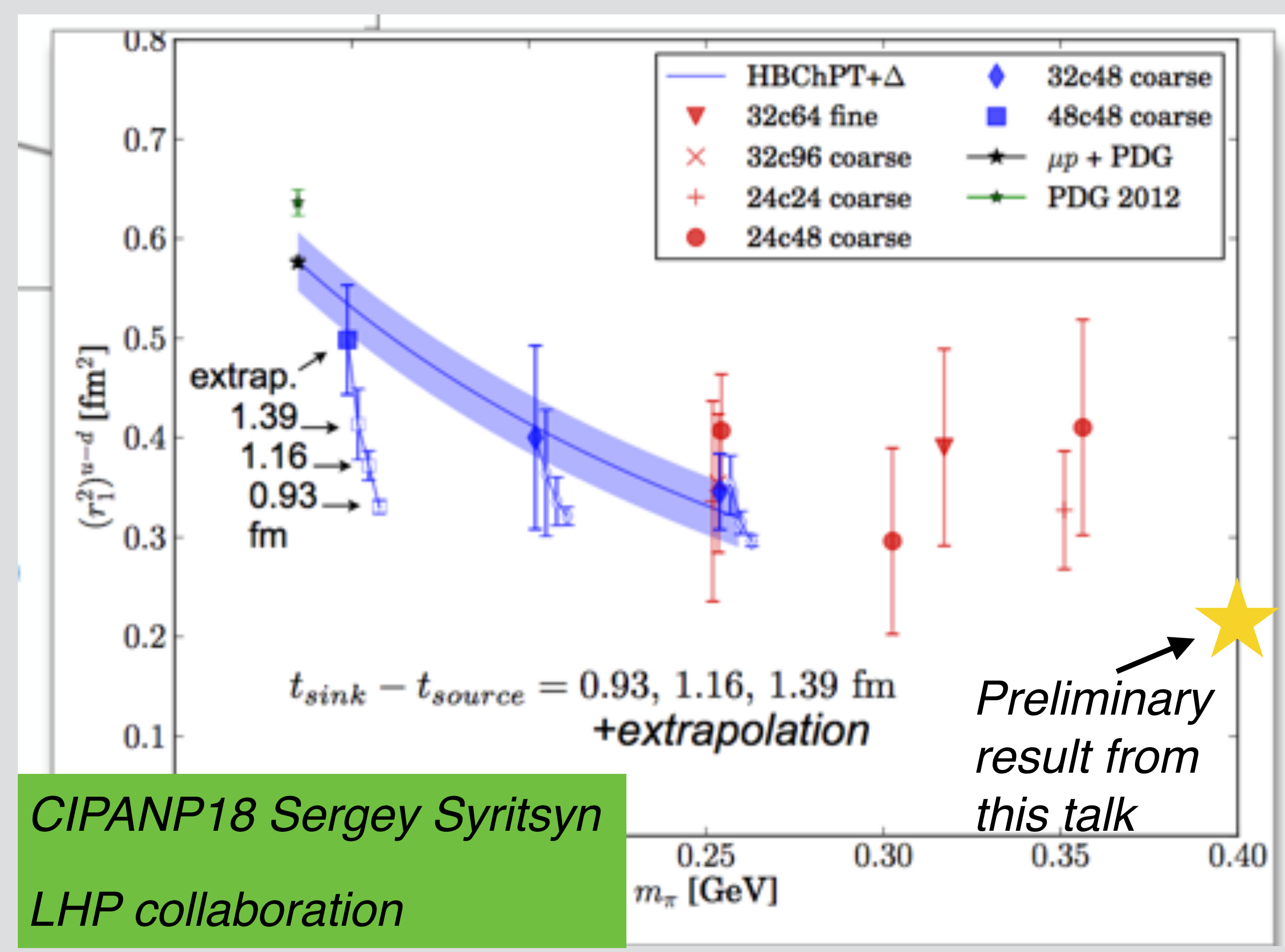
Check dispersion relation with Z

$$Z_s(k^2) \equiv \frac{\langle N(k^2)|0\rangle}{\sqrt{2E(k^2)}} = \frac{\langle N(0)|0\rangle}{\sqrt{2E(k^2)}}\left(1 \pm \mathrm{O}\left(ak\right)\right)$$

Light gray region marks $\mathrm{O}(ak)$ constraint
Dark gray region marks $\mathrm{O}(\alpha_s ak)$ constraint

Connected Dirac radius
Consistent with other results.





Summary and outlook

Requires little additional computation time.

Obtain slopes of matrix elements (radii).

Further constrains shape of form factors.

Possible path to precision radius calculation.

Thank you