Renormalization and Matching of qPDF of Pion

Lattice 2018

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BNL

Work in progress with T. Izubuchi, L. Jin, K. Kallidonis, S. Mukherjee, P. Petreczky, C. Shugert, S. Syritsyn

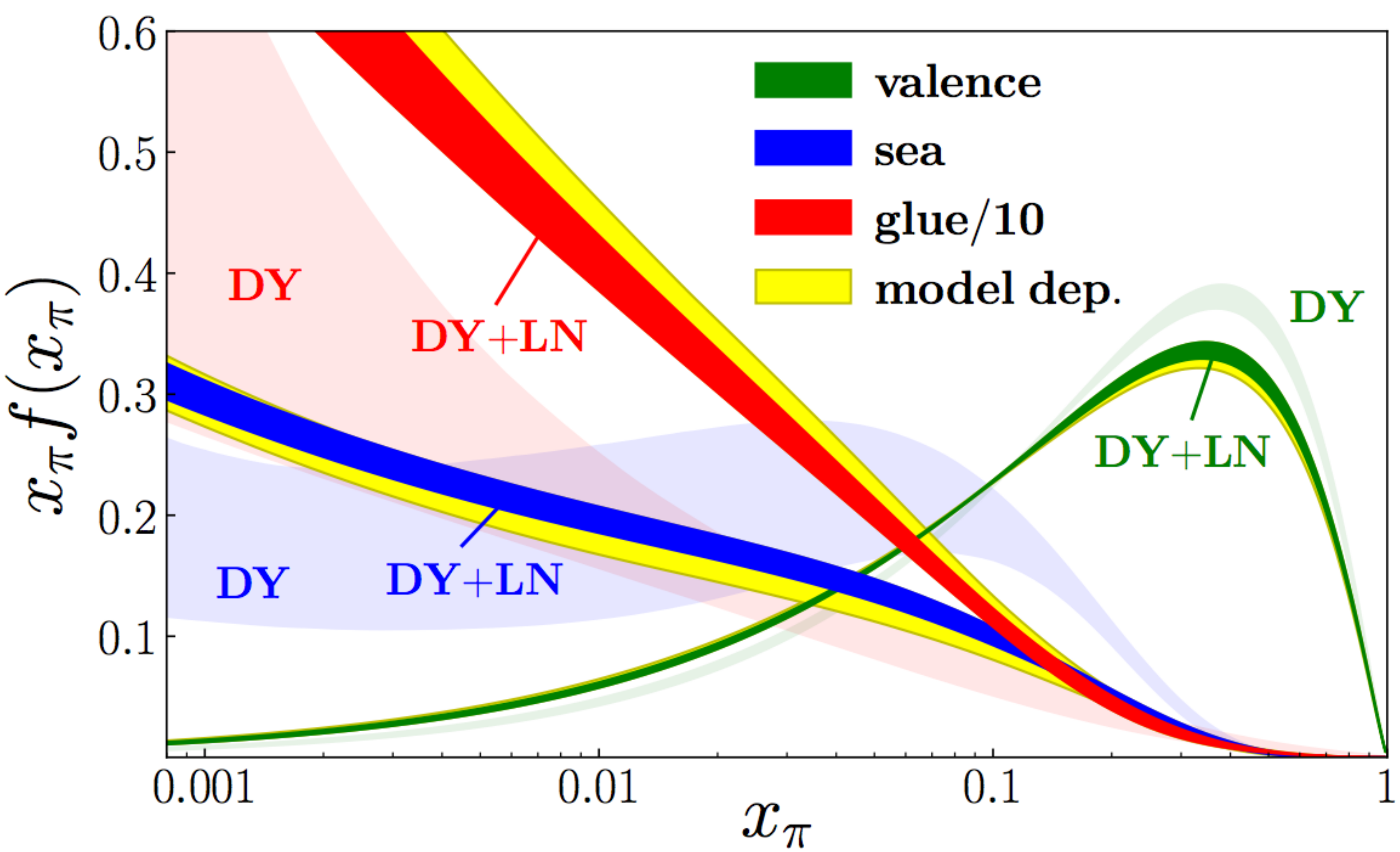
Valence PDF of $\pi^+(u\bar{d})$

We measure the valence PDF of charged pion:

$$q(x,\mu) = q_u(x,\mu) - q_d(x,\mu)$$

Flavor non-singlet ➡ No mixing with glue and no disconnected fermion diagrams

P. C. Barry et al, 2018



Work-flow

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Compute bare qPDF operators

$$\tilde{q}_b(z, \Gamma, P_z)$$

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$\Big\downarrow$ Renormalize at (P_z^R, μ^R)

$\tilde{q}_R(z, \gamma_z, P_z, P_z^R, \mu^R)$

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$\downarrow$ Fourier $z \to xP_z$

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Match qPDF to PDF

X. Ji '13,
(RIMOM-to-MSBAR:) Stewart and Zhao '17

$$q(y, \mu) = \int_{-1}^{+1} \frac{dx}{|x|} C\left(\frac{y}{x}, \gamma_z, \frac{P_z^R}{yP_z}, \frac{P_z^R}{\mu^R}\right) \tilde{q}\left(x, \cdots\right)$$

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$a \to p_R$

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$z \to xP_z$

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$a \to p_R$

$p_R \to \mu$

for $P_z >> \Lambda_{\mathrm{QCD}}, m_\pi$

$$q(y, \mu) = \int_{-1}^{+1} \frac{dx}{|x|} C\left(\frac{y}{x}, \gamma_z, \frac{P_z^R}{yP_z}, \frac{P_z^R}{\mu^R}\right) \tilde{q}\left(x, \cdots\right)$$

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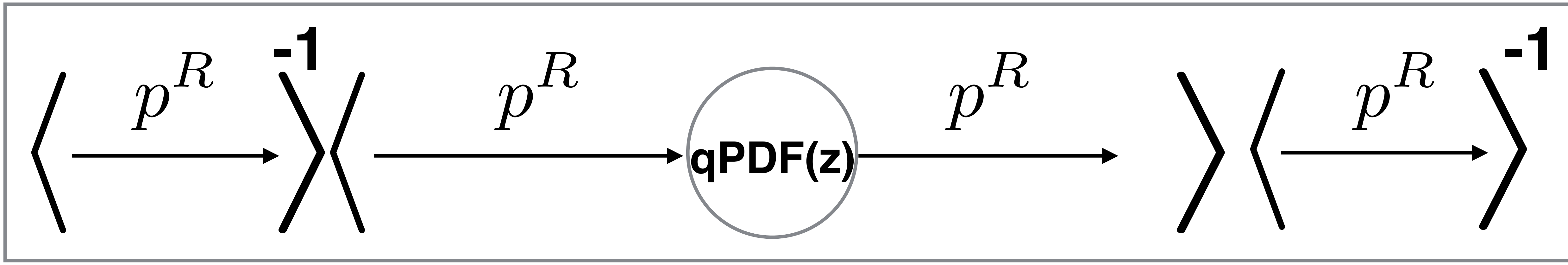
RI/MOM Renormalization

In Landau gauge: Stewart and Zhao ’17, J.W. Chen et.al ’18



γ_z mixes with scalar on the lattice

$$\mathrm{Tr}\left(\not{p}\Lambda^R_{\gamma_z}(p)\right)|_{p=p^R} = 12 p^R_z e^{ip^R_z z} \qquad \mathrm{Tr}\left(\Lambda^R_{\gamma_z}(p)\right)|_{p=p^R} = 0$$

$$\mathrm{Tr}\left(\not{p}\Lambda^R_{1}(p)\right)|_{p=p^R} = 0 \qquad \mathrm{Tr}\left(\Lambda^R_{1}(p)\right)|_{p=p^R} = 12 e^{ip^R_z z}$$

Renormalization factors $Z_{\gamma_z\gamma_z}(z), \quad Z_{\gamma_z 1}(z)$

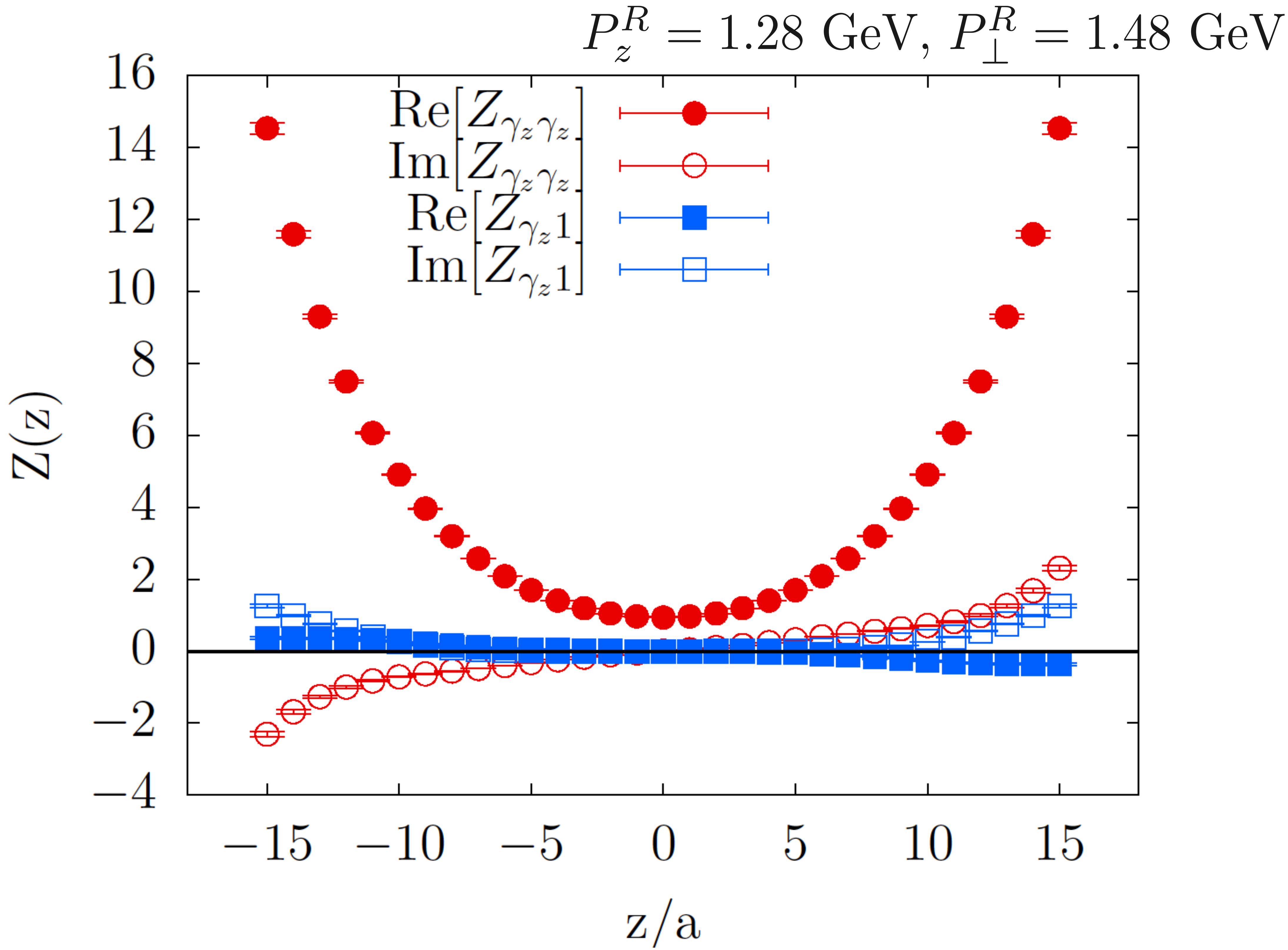
(Quark renormalization $Z_q \approx 1.03$ negligible)

Simulation details

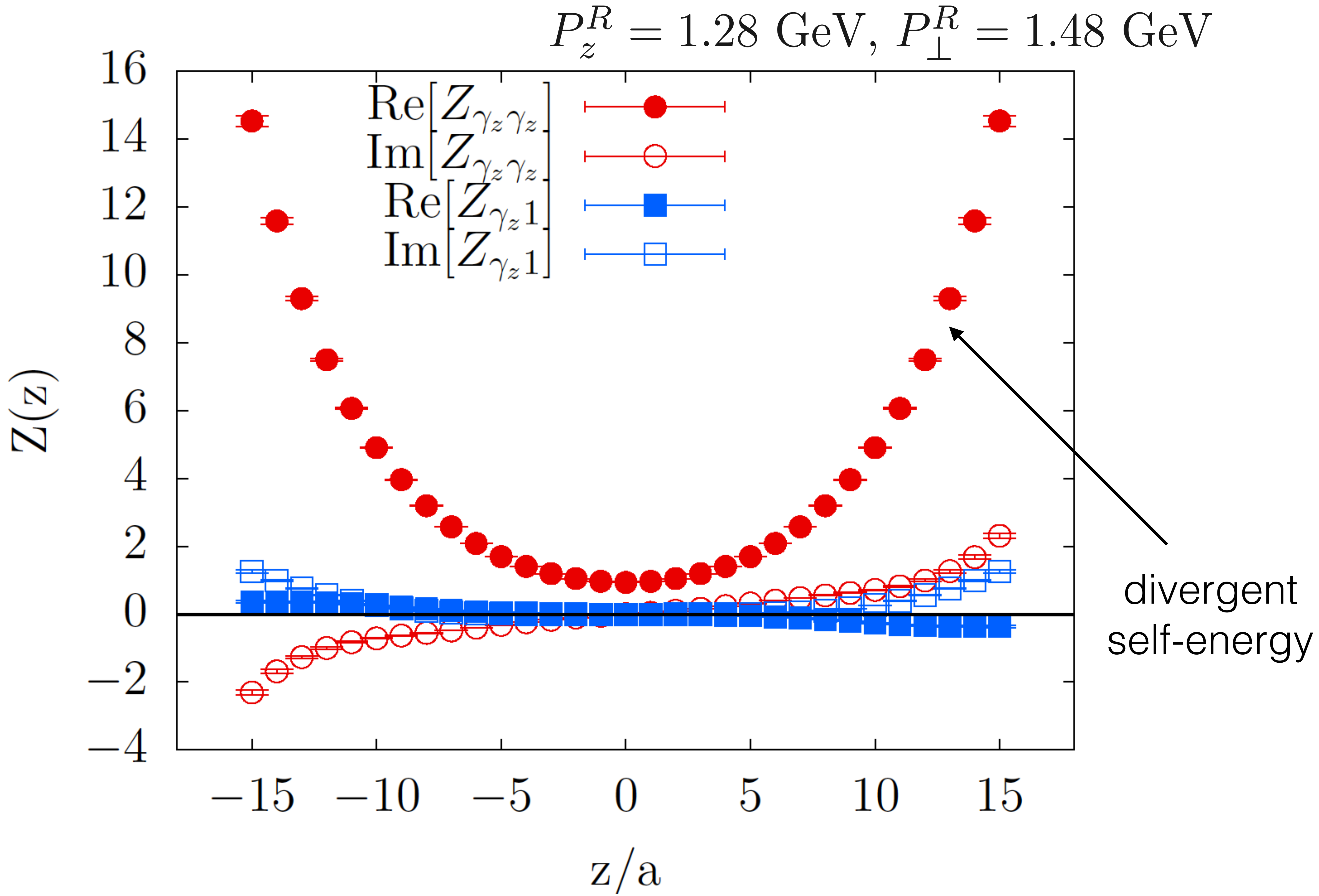
- HISQ sea quark from HotQCD ensemble
- 1-HYP smeared Wilson-Clover valence quark tuned to 300 MeV pion
- Lattice spacing a=0.06 fm
- 1-HYP smeared Wilson line
- Results presented here are at fixed source-sink separation of 10 lattice spacings

Comparison between 1-loop perturbative renormalization and non-perturbative renormalization of qPDF

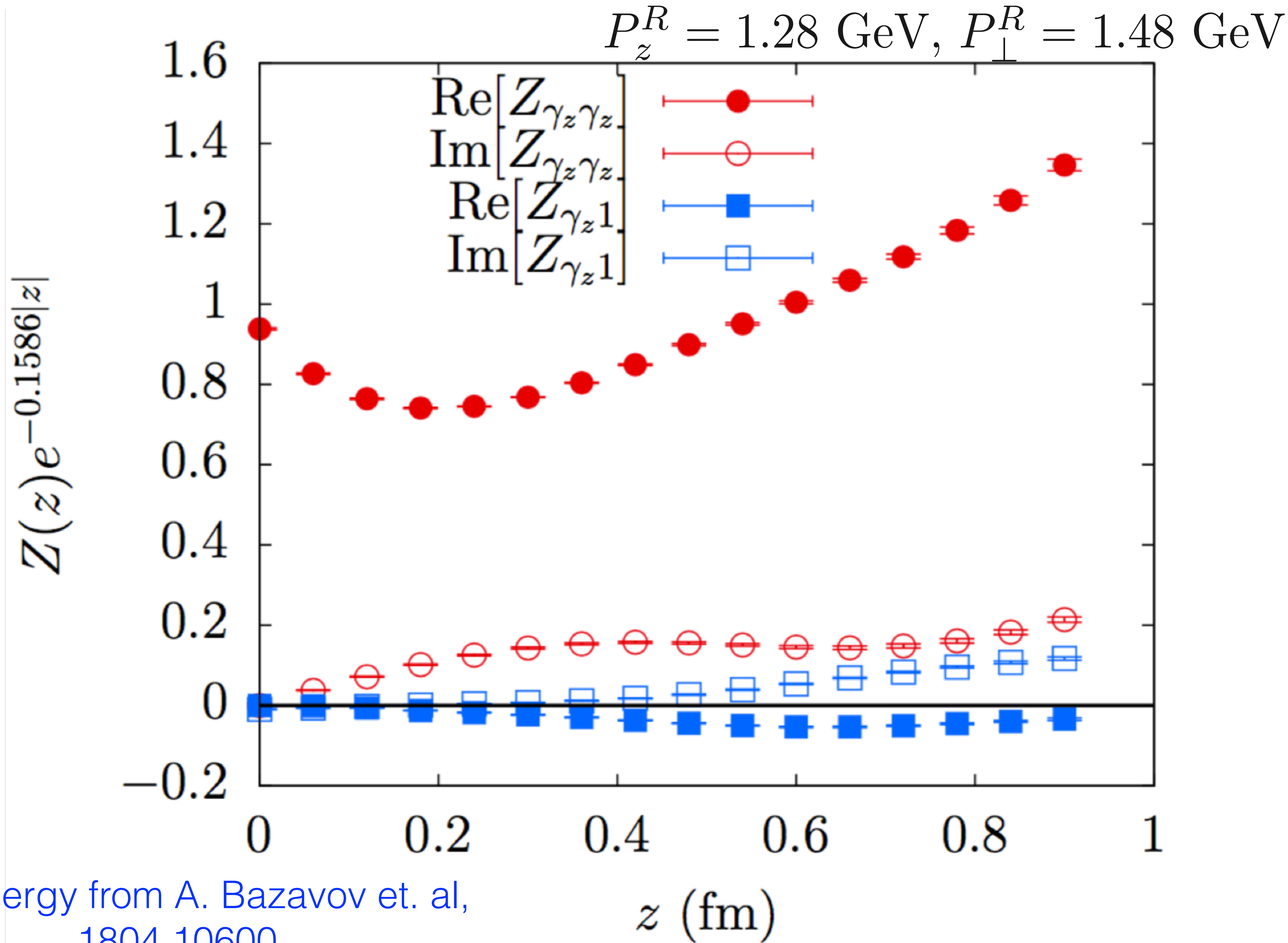
Renormalization Constants Including Self-Energy



Renormalization Constants Including Self-Energy



Renormalization constants excluding self-energy is O(1)



Self-energy from A. Bazavov et. al, 1804.10600

Comparison between lattice and perturbative quark qPDF

Define the RI-MOM quark PDF similar to 1-loop calculation:

$$\tilde{q}^{\mathrm{quark}}\left(z;p,p^R\right) \equiv Z_{\gamma_z\gamma_z}(z;p^R)\mathrm{Tr}\left(\not{p}\Lambda^b_{\gamma_z}(z;p)\right) + Z_{\gamma_z 1}(z;p^R)\mathrm{Tr}\left(\not{p}\Lambda^b_1(z;p)\right)$$

By renormalization condition, $\tilde{q}^{\mathrm{quark}}(z;p^R,p^R) = \tilde{q}^{\mathrm{quark}}_{\mathrm{free}}(z;p_R)$

We compare the lattice and 1-loop running of quark PDF away from renormalization point (Stewart, Zhao'17) :

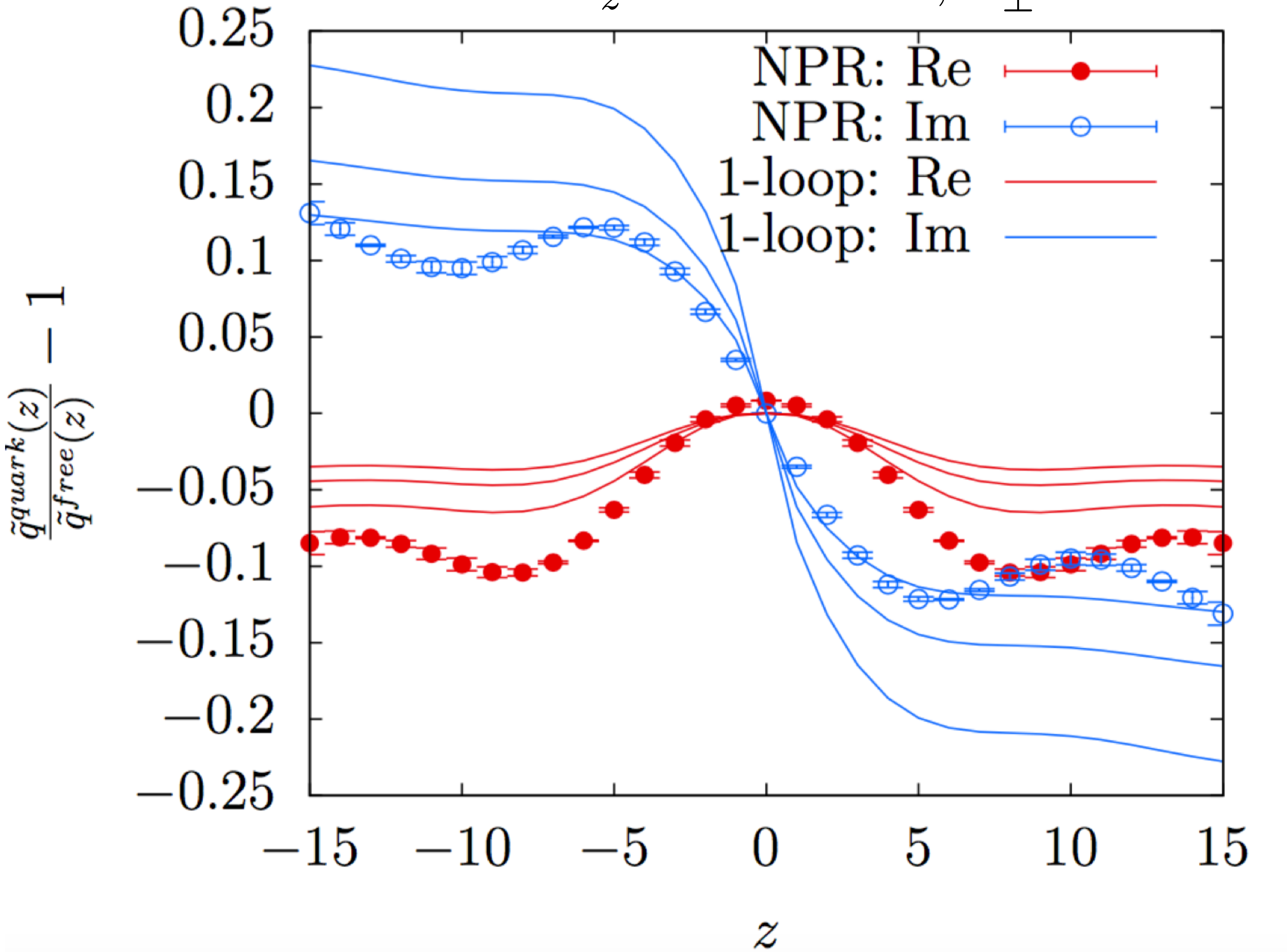
$$\frac{\tilde{q}^{\mathrm{quark}}(z;p,p^R)}{\tilde{q}^{\mathrm{quark}}_{\mathrm{free}}(z;p)} - 1 \stackrel{?}{=} \alpha_s f(p,p_R)$$ + negligible higher orders

1-loop results from $\alpha_s(p^R/2)$ $\alpha_s(p^R)$ and $\alpha_s(2p^R)$ are the three curves in next slide.

Comparison between lattice and perturbative quark qPDF

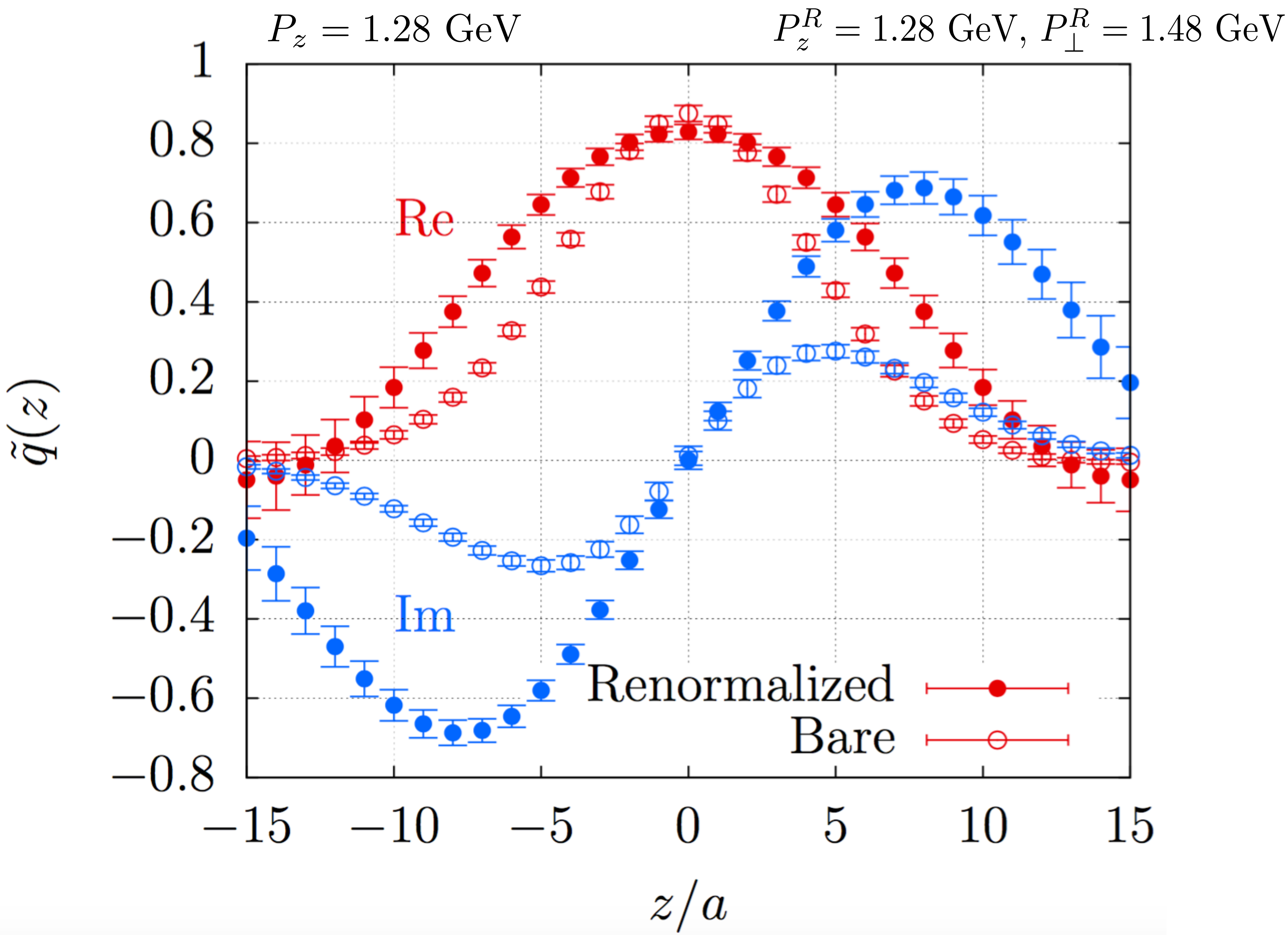
A generic case: $P_z = 1.28\ \mathrm{GeV},\ P_\perp = 1.48\ \mathrm{GeV}$

$P_z^R = 1.28\ \mathrm{GeV},\ P_\perp^R = 2.22\ \mathrm{GeV}$

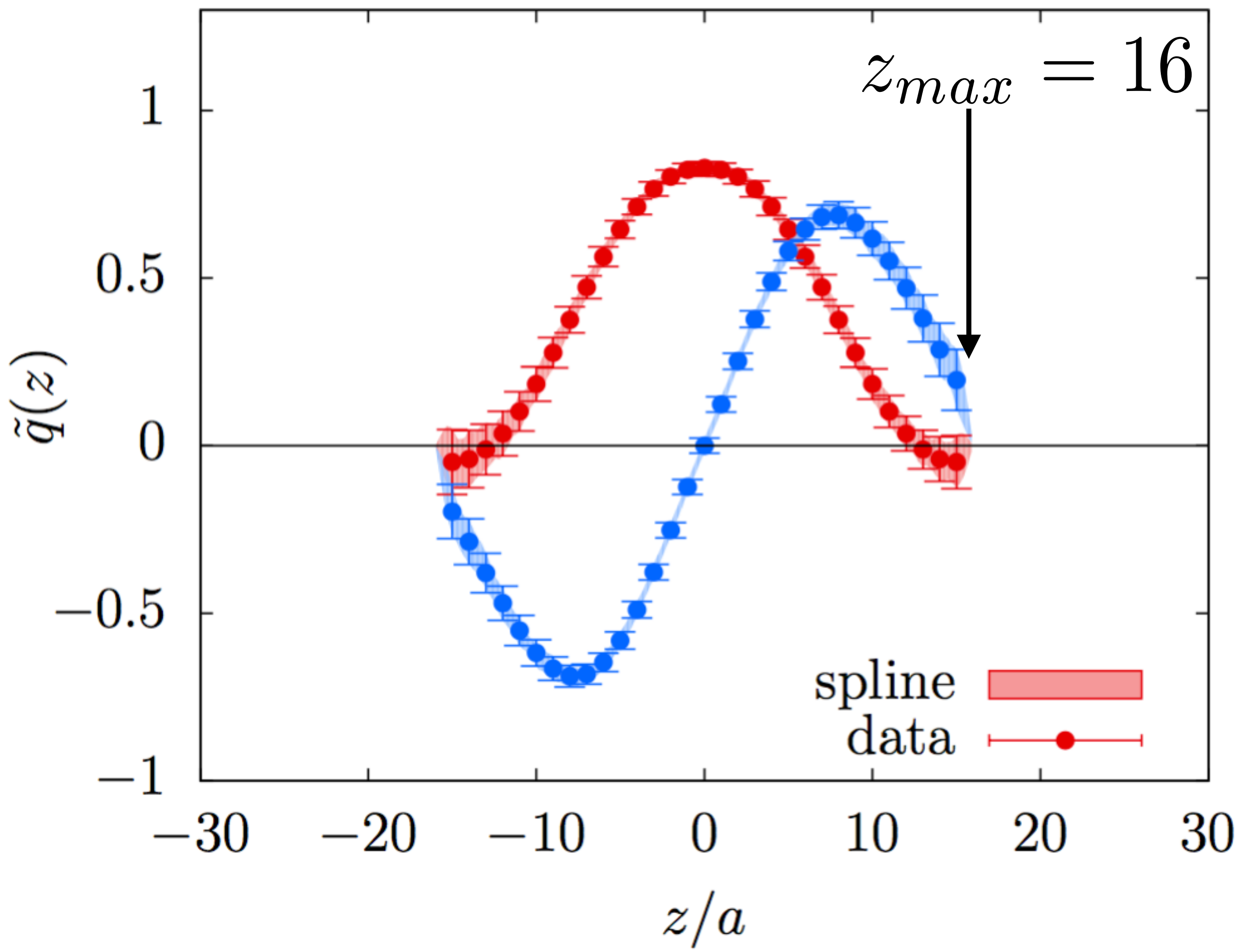


Renormalized quasi-PDF in real and Fourier space

Real-space pion qPDF



Pion qPDF from real-space to Fourier

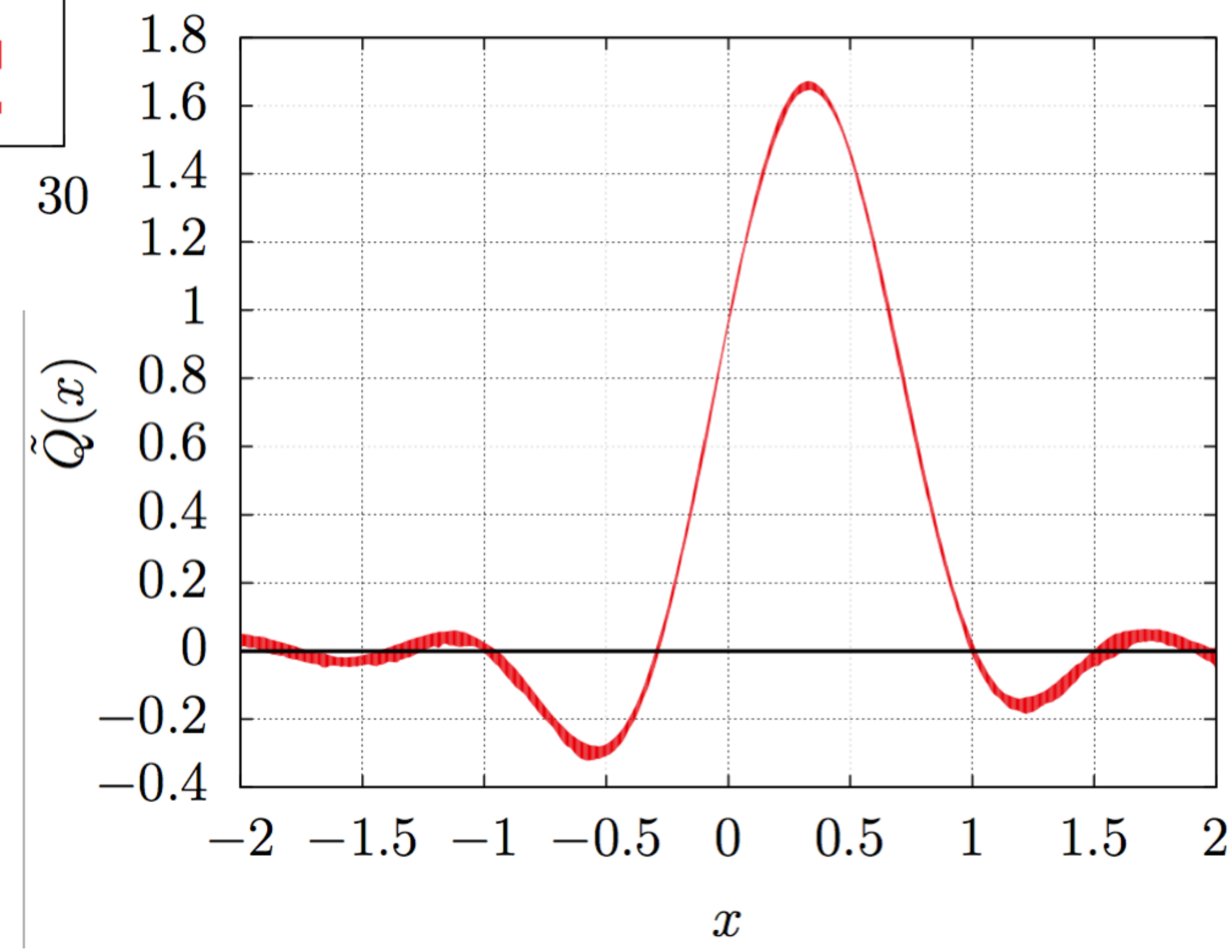


Quantify the unknown:

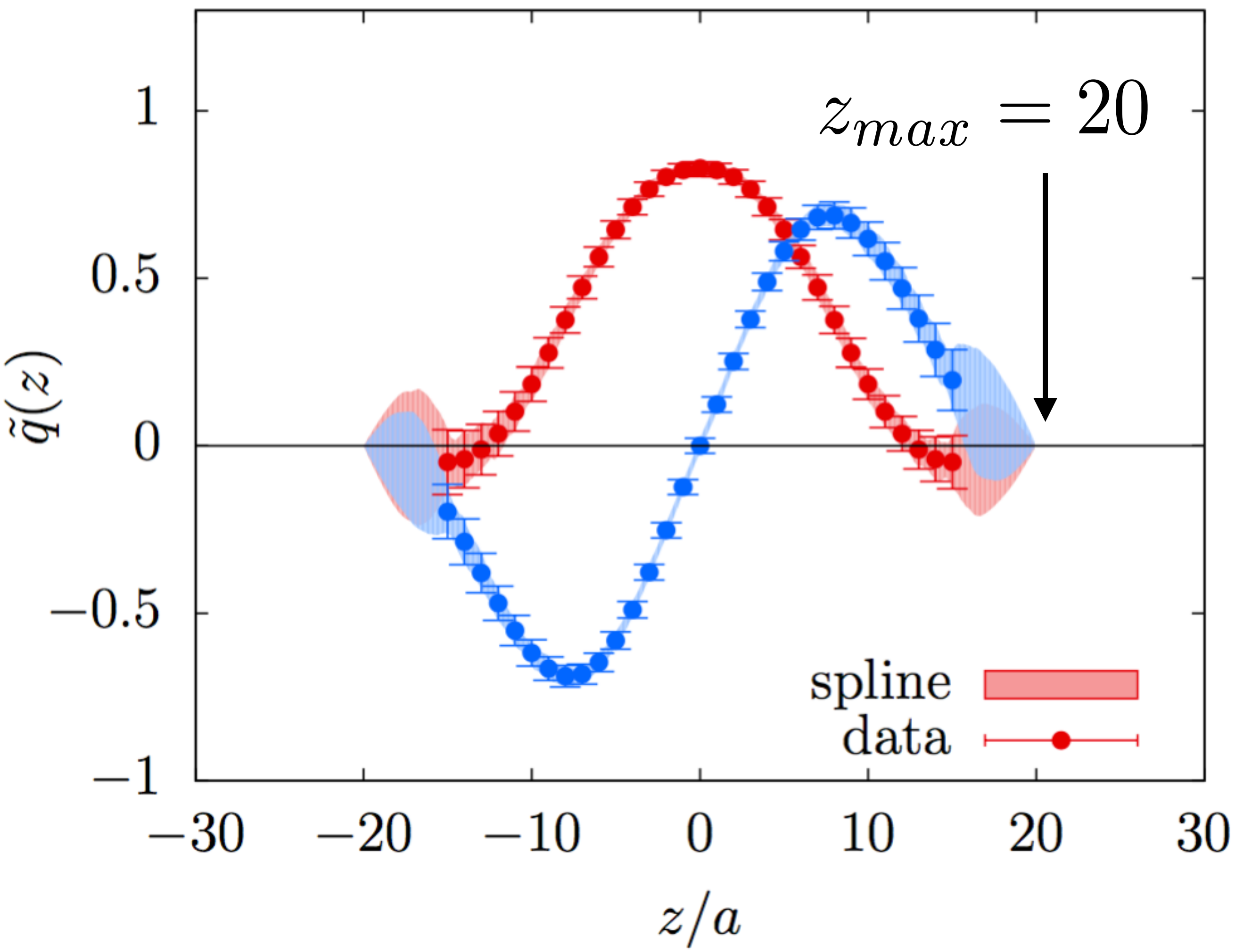
Cubic spline from z= $-z_{\mathrm{max}}$ to $+z_{\mathrm{max}}$

Vary z_{max}

Fourier

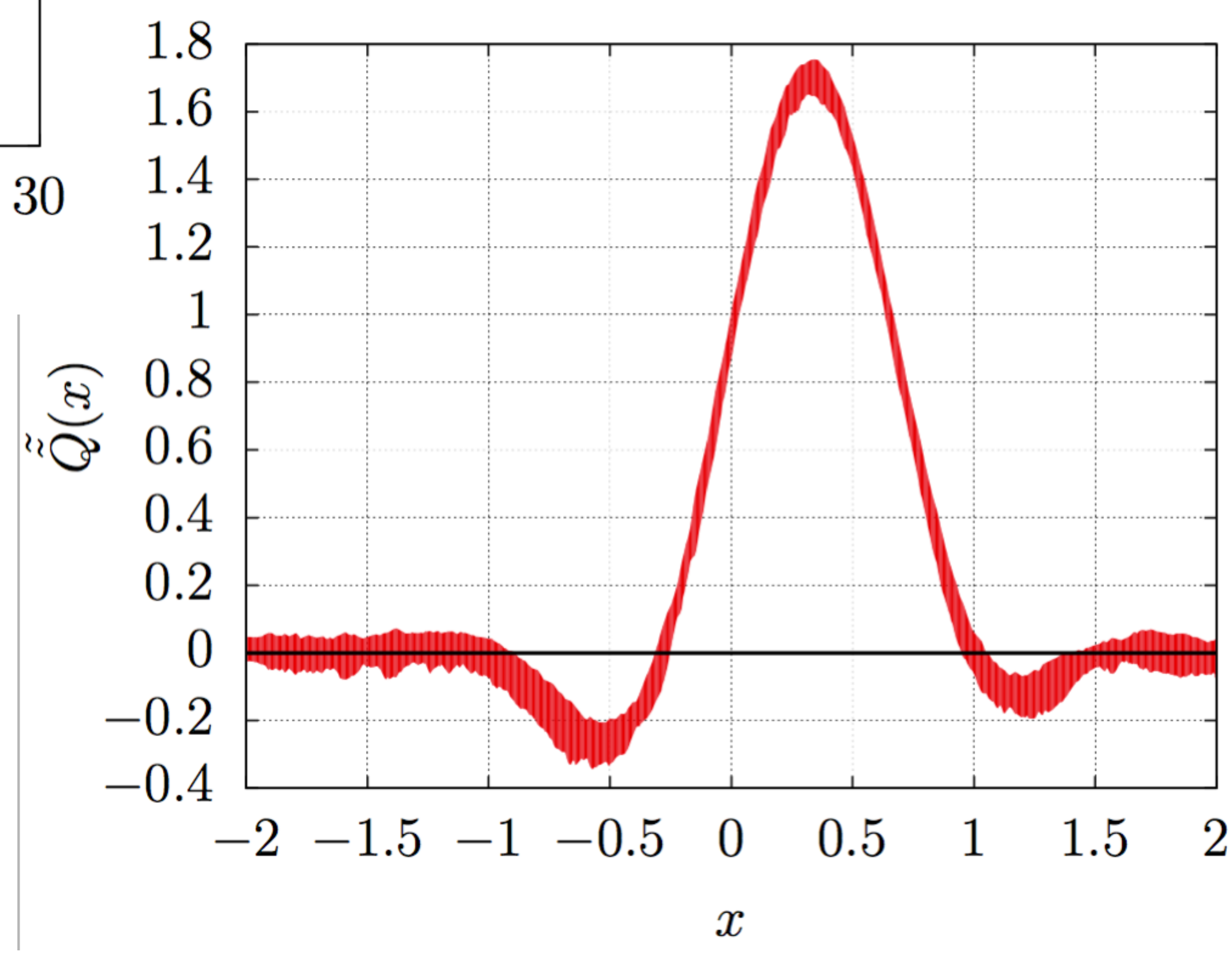


Pion qPDF from real-space to Fourier



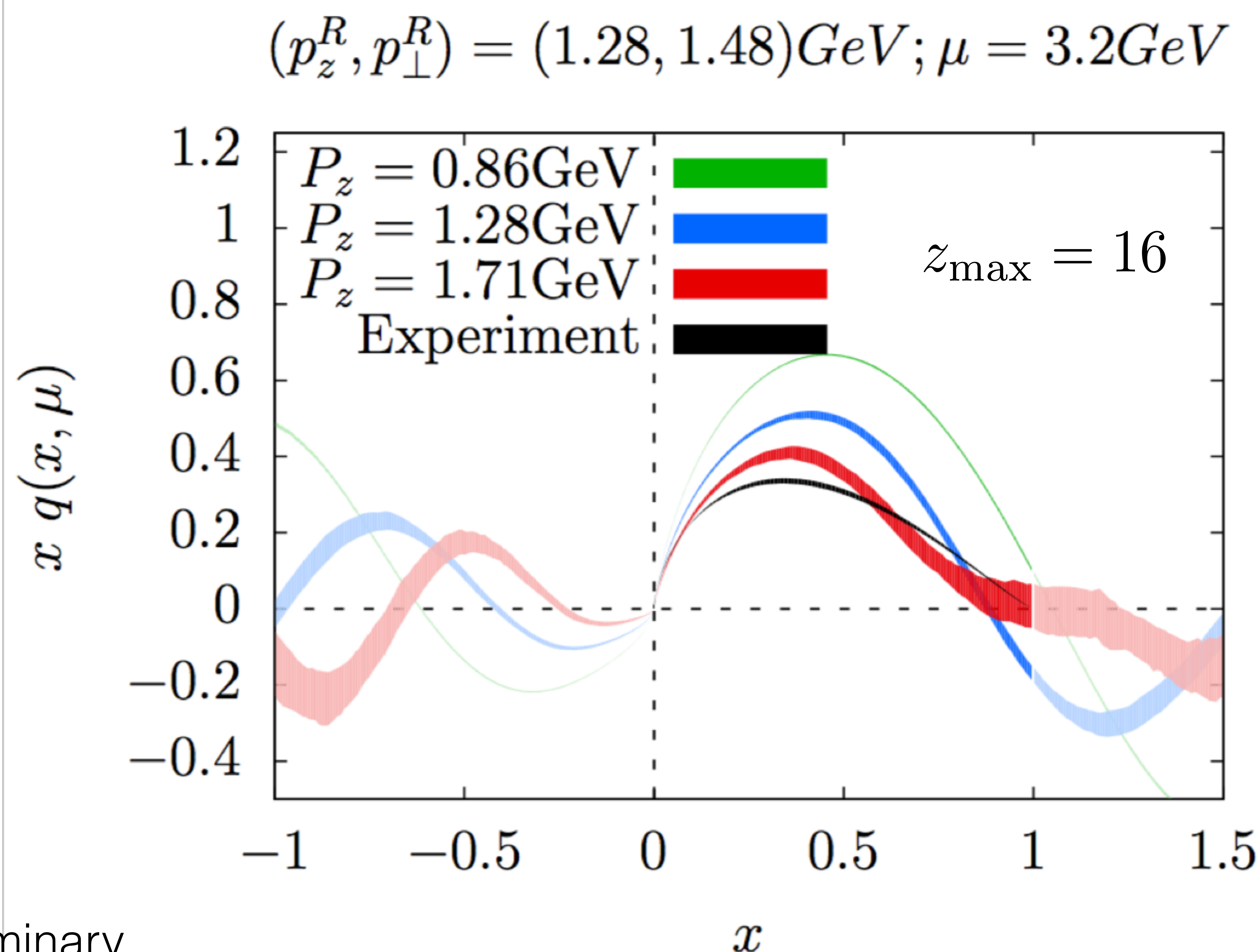
Shape is robust under $z_{\max}$ variations

Fourier



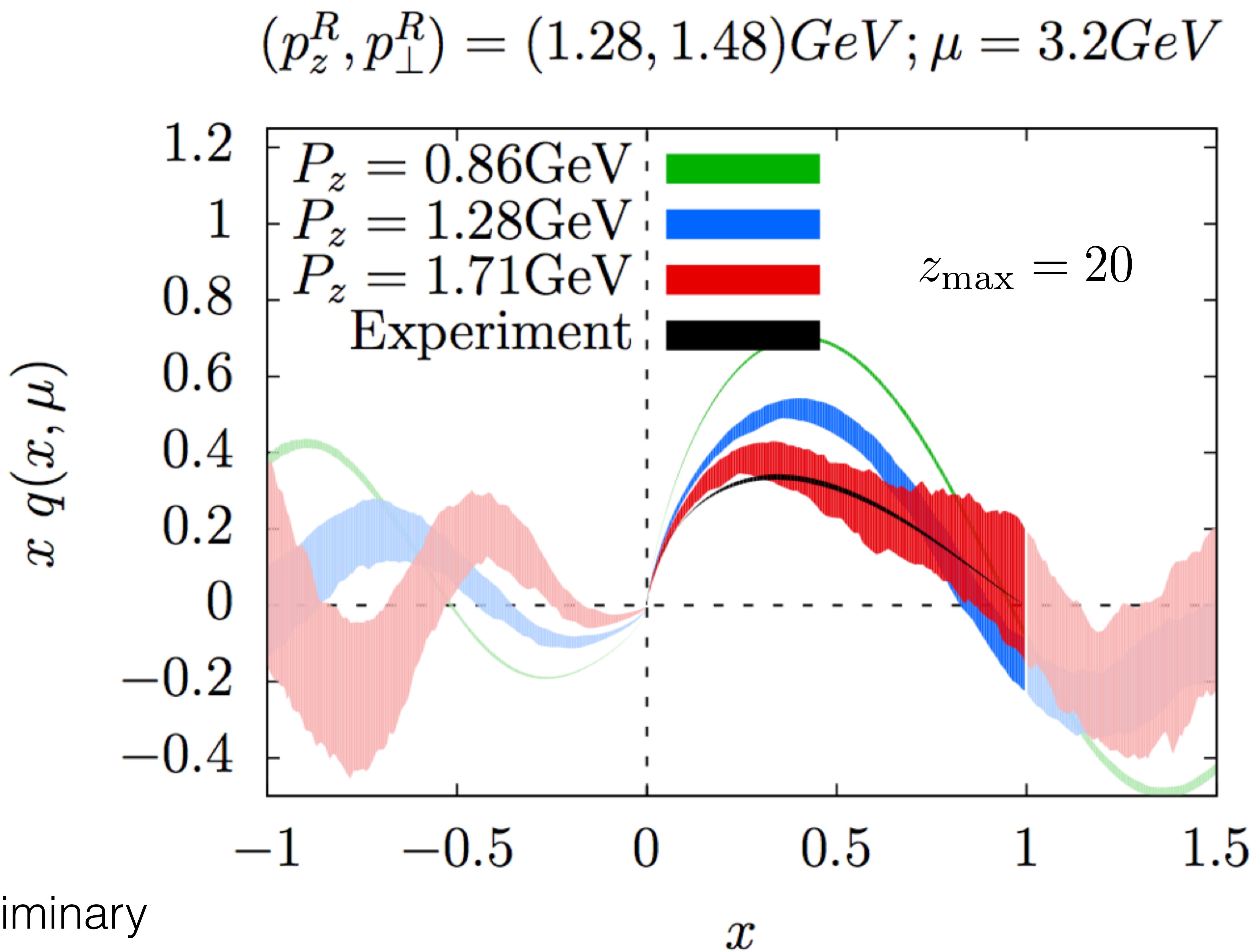
Matching quasi-PDF to PDF

Pion PDF at μ=3.2GeV from different P_z



Preliminary

Pion PDF at μ=3.2GeV from different P_z



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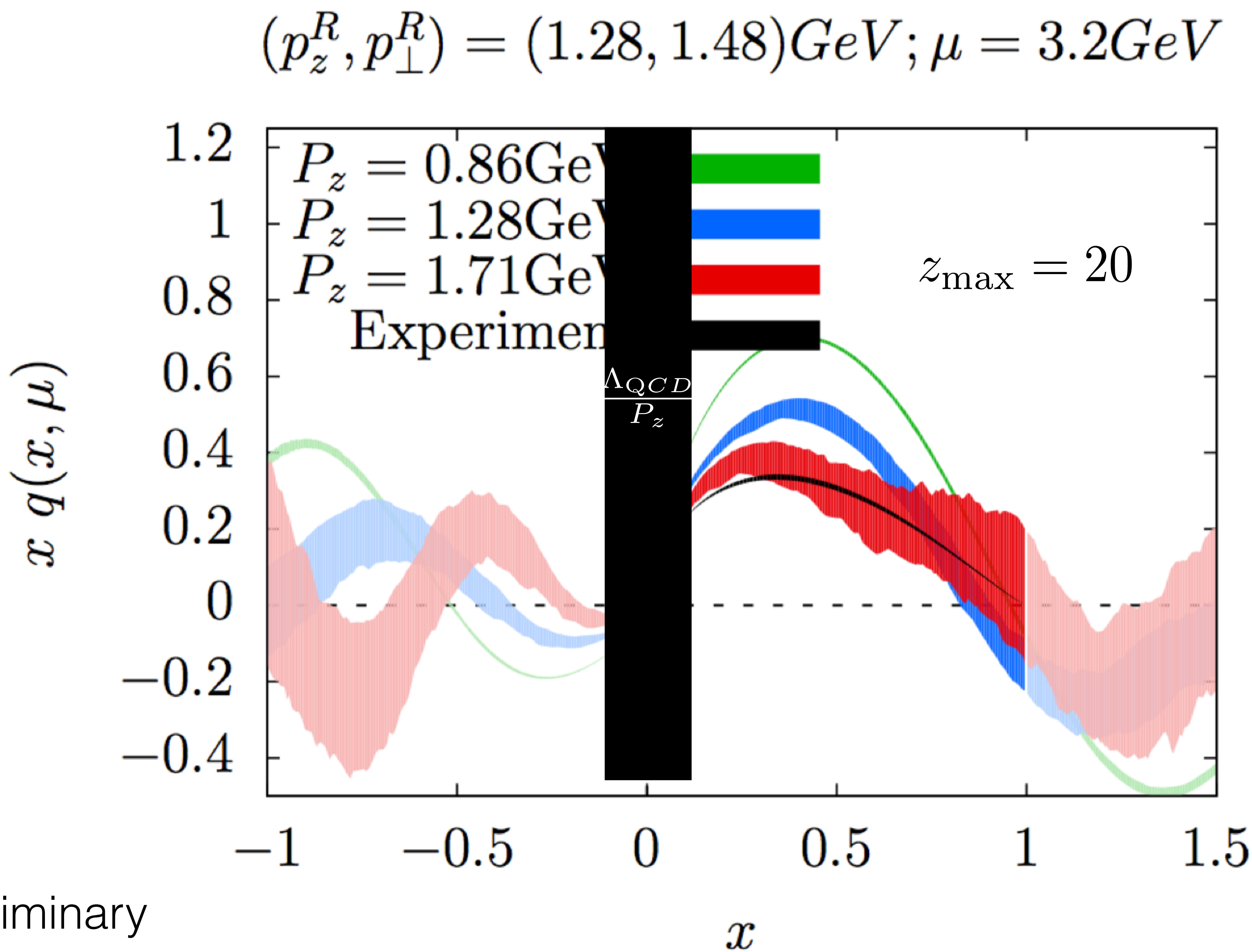
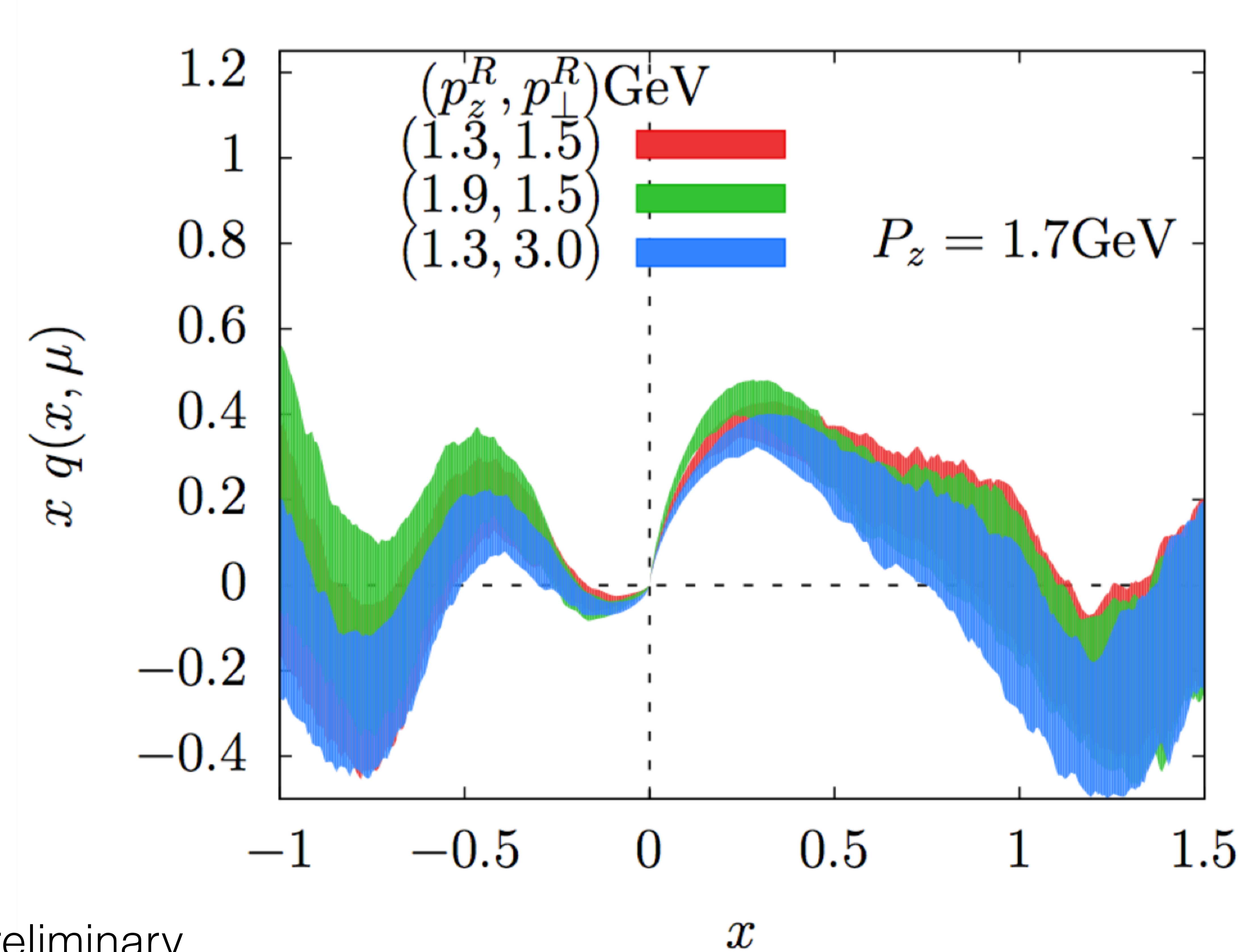


Preliminary

Mild P_z^R and $P_\perp^R$ dependence



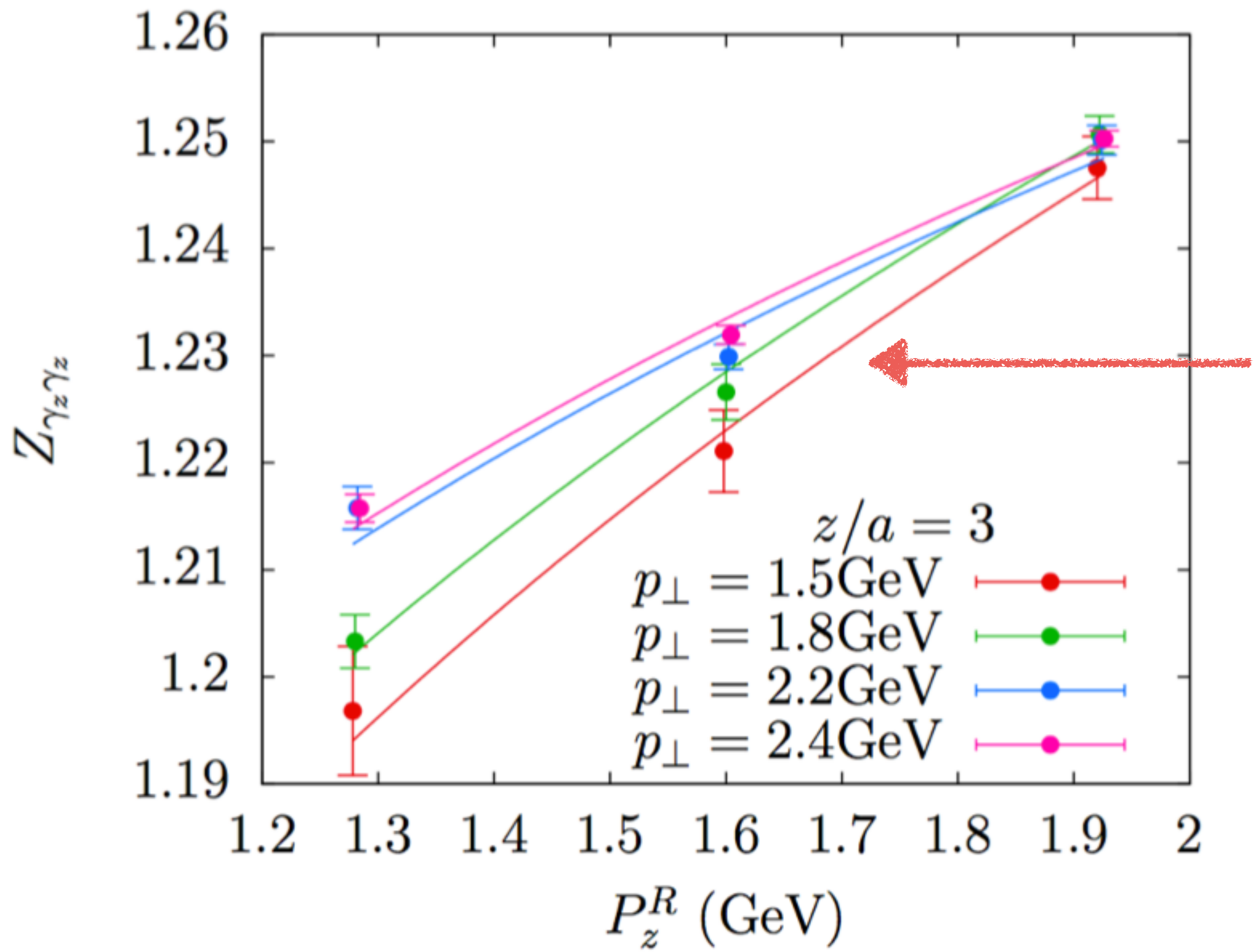
Preliminary

Conclusions

* We studied valence pion quasi-PDF using HISQ sea quarks and Wilson-Clover valence quarks.
* We investigated the validity of 1-loop renormalization in describing NPR.
* The NPR Z-factor, even at z=1 fm, close to 1 after removing self-energy contribution. We found qualitative agreement in NPR and 1-loop running of quark qPDF.
* We matched the pion qPDF to the PDF at mu=3.2GeV. We found the agreement with the experimental data to get better as Pz is changed from 0.86 GeV to 1.71 GeV.
* To be investigated: removing the effect of source-sink separation, taming contribution from large-z, towards continuum including a=0.04 fm ensemble.

P_z^R dependence of $Z(z = 3a)$

Weak P_z^R dependence. We see some evidence for the $\log\left(P_z^R\right)$ dependence on the magnified scale.



$\sim \log\left(P_z^R\right)$

Oscillatory short-distance to damped long-distance

$P_z = 1.28$ GeV

$P_z^R = 1.28$ GeV, $P_\perp^R = 1.48$ GeV

$x_{\text{eff}} \approx 0.35,\ M_{\text{eff}} \approx 300 - 600$ GeV