THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

DOE CSGF

Strongly interacting rotating bosons via complex stochastic quantization

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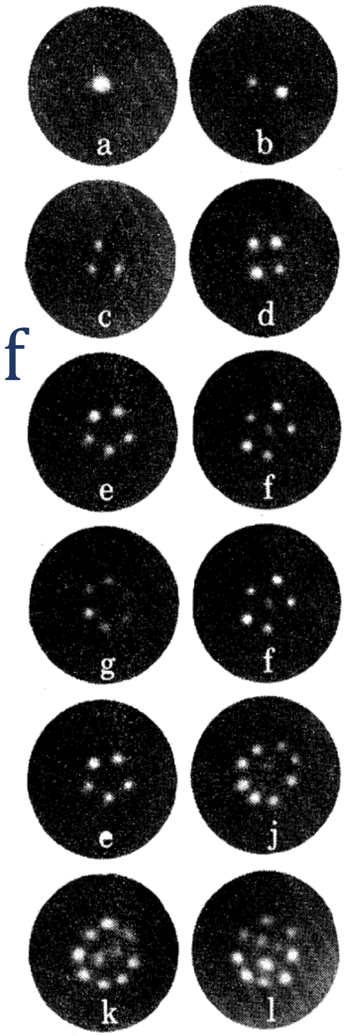
Rotating Bose-Einstein condensates

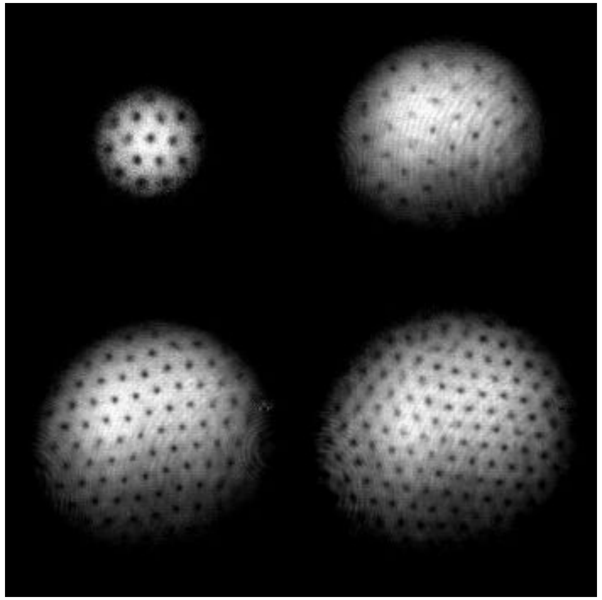
1949: Onsager predicts rotating superfluids will form vortices

1979: First observation of vortices in rotating ^{4}He



Phys. Rev. Lett. **4** 14 (1979)

1990s-2000s: rotating BECs in ^{4}He and dilute atomic gases



Science **292** 5516 (2001)

Advances in theory

Theoretical advancements in study of rotating superfluids since 1950s

?

Theoretical progress

- Why are we stuck?
- Many-body quantum systems → Quantum Monte Carlo

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-S[\phi]}$$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi \, e^{-S[\phi]} \mathcal{O}[\phi] = \int \mathcal{D}\phi \mathcal{P}[\phi] \mathcal{O}[\phi]$$

- Evaluate stochastically, with $\mathcal{P}[\phi] = \frac{e^{-S[\phi]}}{\mathcal{Z}}$

The sign problem

- Action for non-relativistic rotating bosons:

$$S = \int \phi^* \left(\mathcal{H} - \mu - \boxed{\omega_z L_z}\right) \phi + \lambda \int (\phi^* \phi)^2$$

$$\omega_z L_z = i\omega_z (x\partial_y - y\partial_x)$$

- Complex action
- Usual Quantum Monte Carlo methods do not work
- Proposed solution: Complex Langevin Method

The Complex Langevin method

- Generalization of stochastic quantization to complex dynamical variables

$$\begin{aligned} \phi &\to \phi^R + i\phi^I \\ S[\phi^R + i\phi^I] &= u[\phi^R + i\phi^I] + iv[\phi^R + i\phi^I] \end{aligned}$$

- Leads to two coupled SDEs:

$$\begin{aligned} d\phi^R &= \mathrm{Re}[K]dt + \eta\sqrt{dt} \\ d\phi^I &= \mathrm{Im}[K]dt \\ K &= -\nabla S[\phi^R + i\phi^I] \end{aligned}$$

CL: success stories

- Relativistic Bose gas at finite chemical potential

$$S = \int d^4x \left[|\partial_\nu \phi|^2 + (m^2 - \mu^2)|\phi|^2 + \mu(\phi^* \partial_t \phi - \partial_t \phi^* \phi) + \lambda \left(\phi^* \phi\right)^2\right]$$

$$S[\mu]^* = S[-\mu]$$

- Lattice action

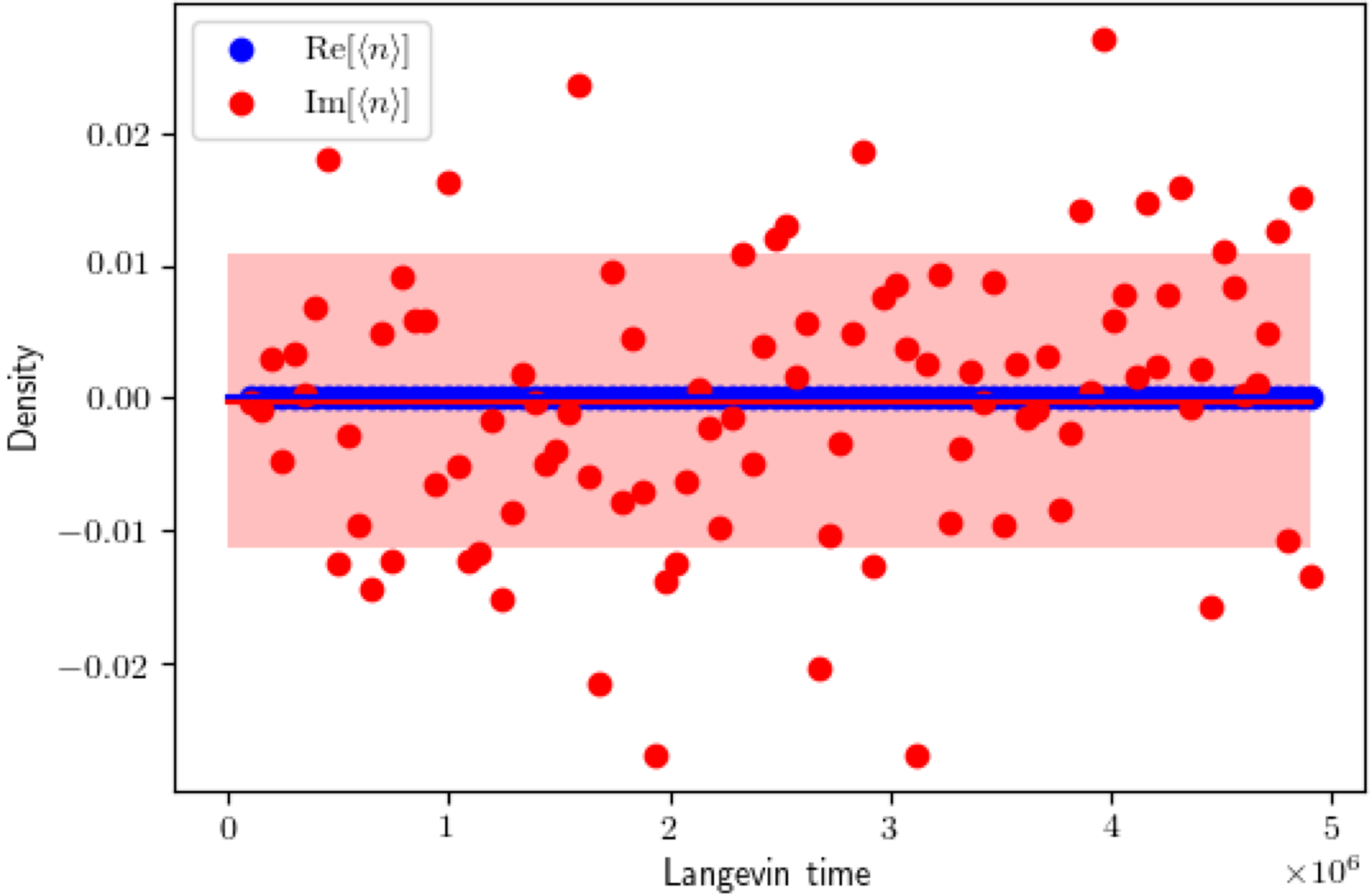
$$S = \sum_x \left[(2d + m^2)\phi_x^* \phi_x + \lambda(\phi_x^* \phi_x)^2 - \sum_{\nu=1}^{4} (\phi_x^* e^{-\mu\delta_{\nu,4}} \phi_{x+\hat{\nu}} + \phi_{x+\hat{\nu}}^* e^{\mu\delta_{\nu,4}} \phi_x)\right]$$

- Use CL to compute density, field modulus squared

Aarts, Phys. Rev. Lett (2009)

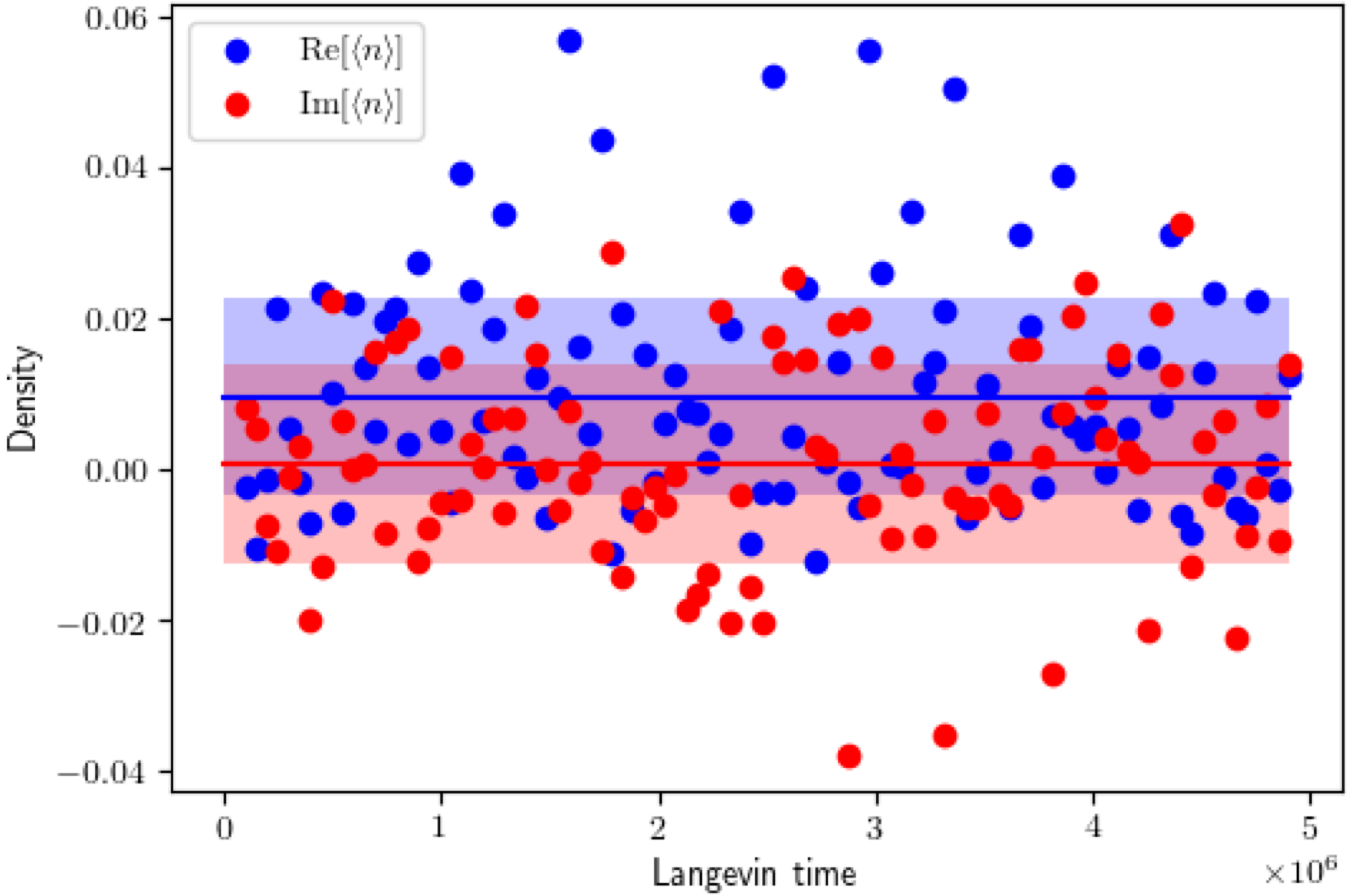
CL: success stories

$\mu = 0.0$



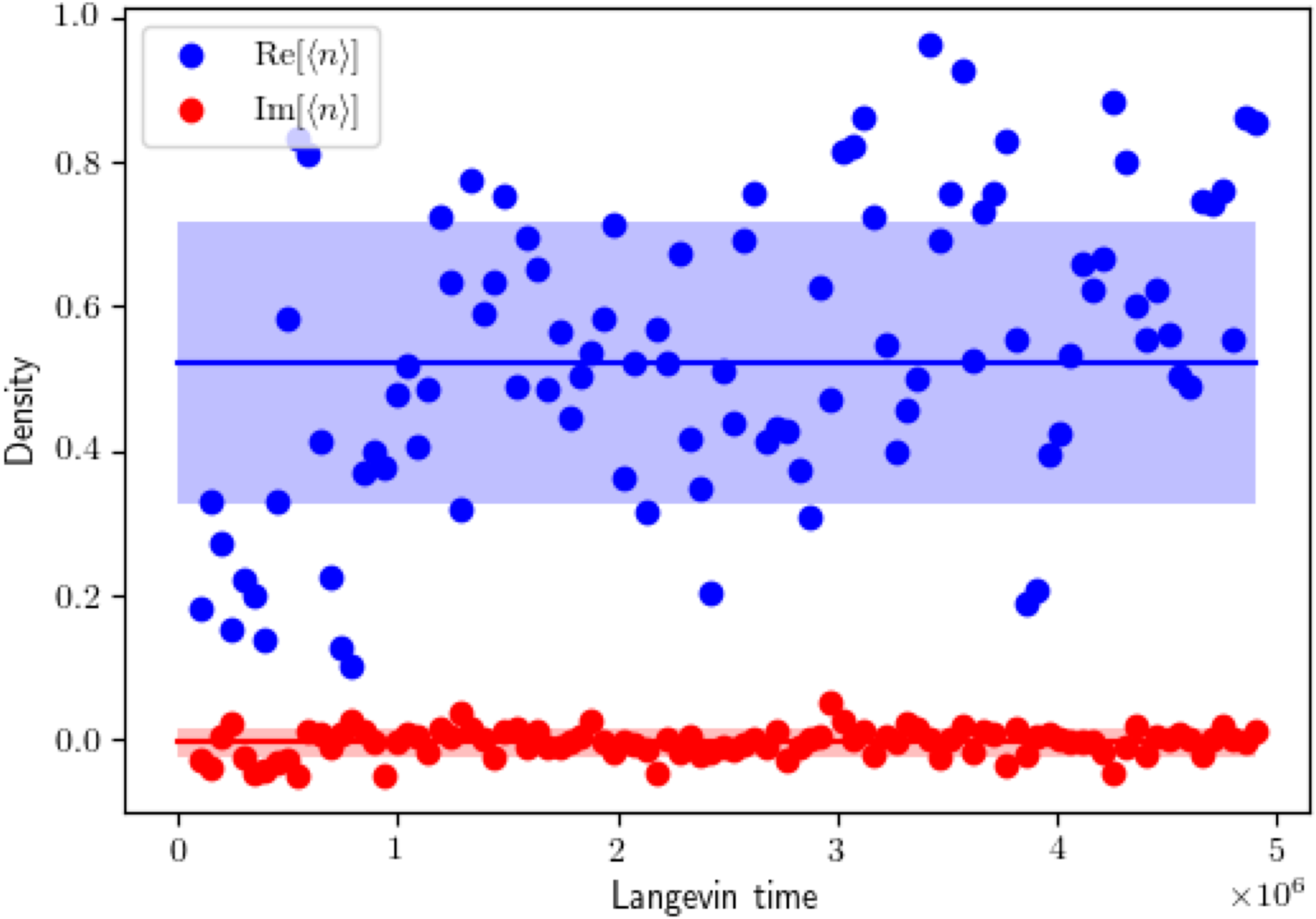
CL: success stories

$\mu = 0.7$



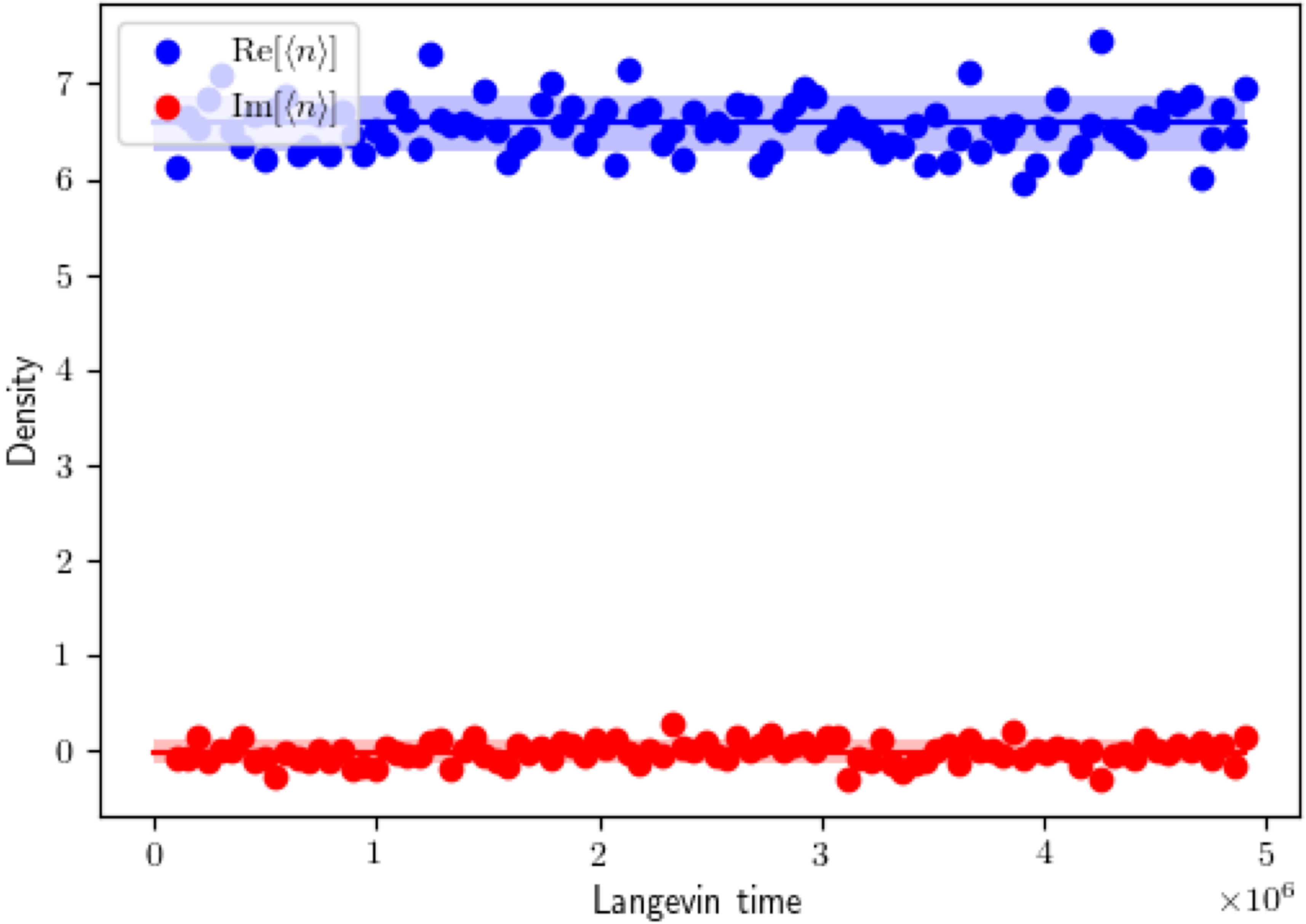
CL: success stories

$\mu = 1.125$

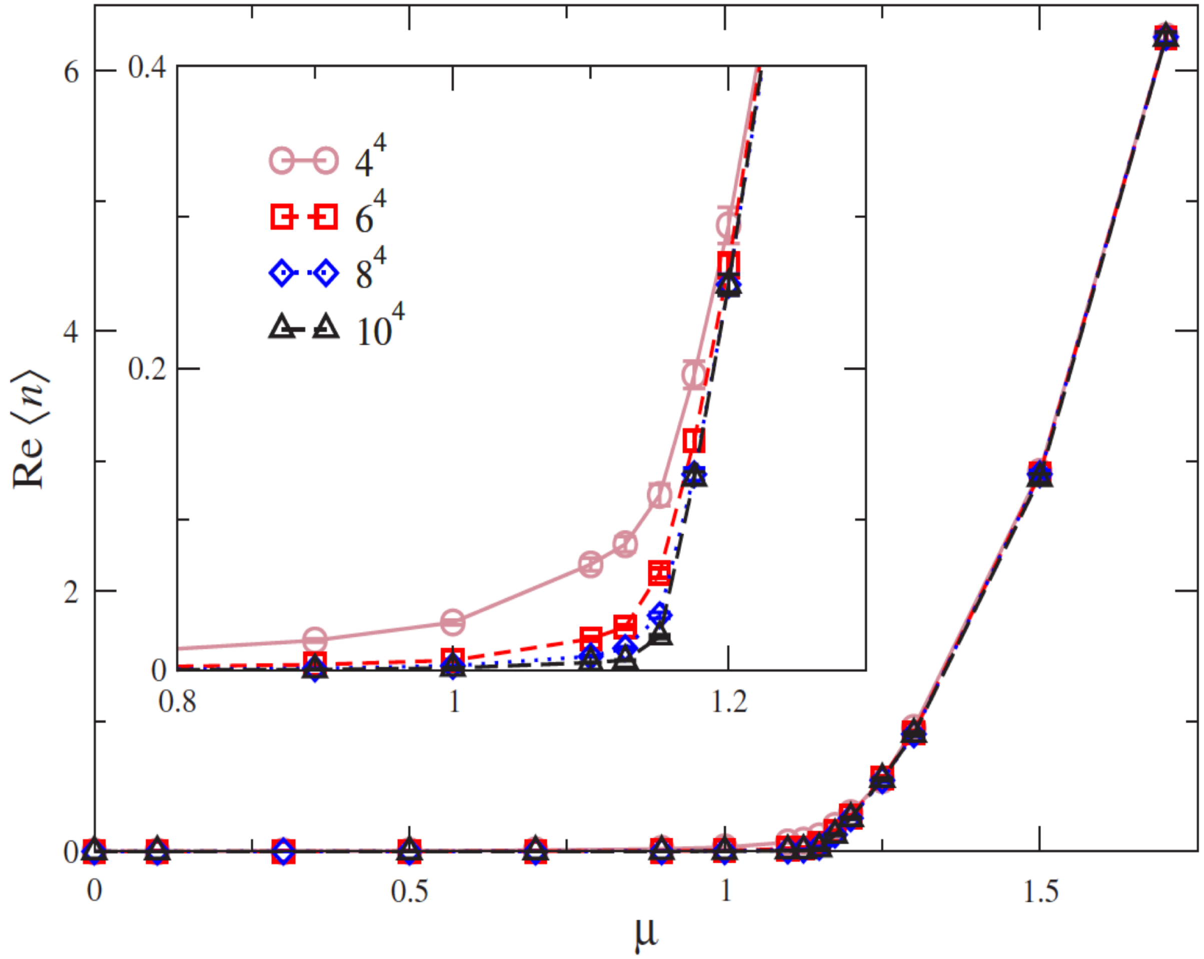


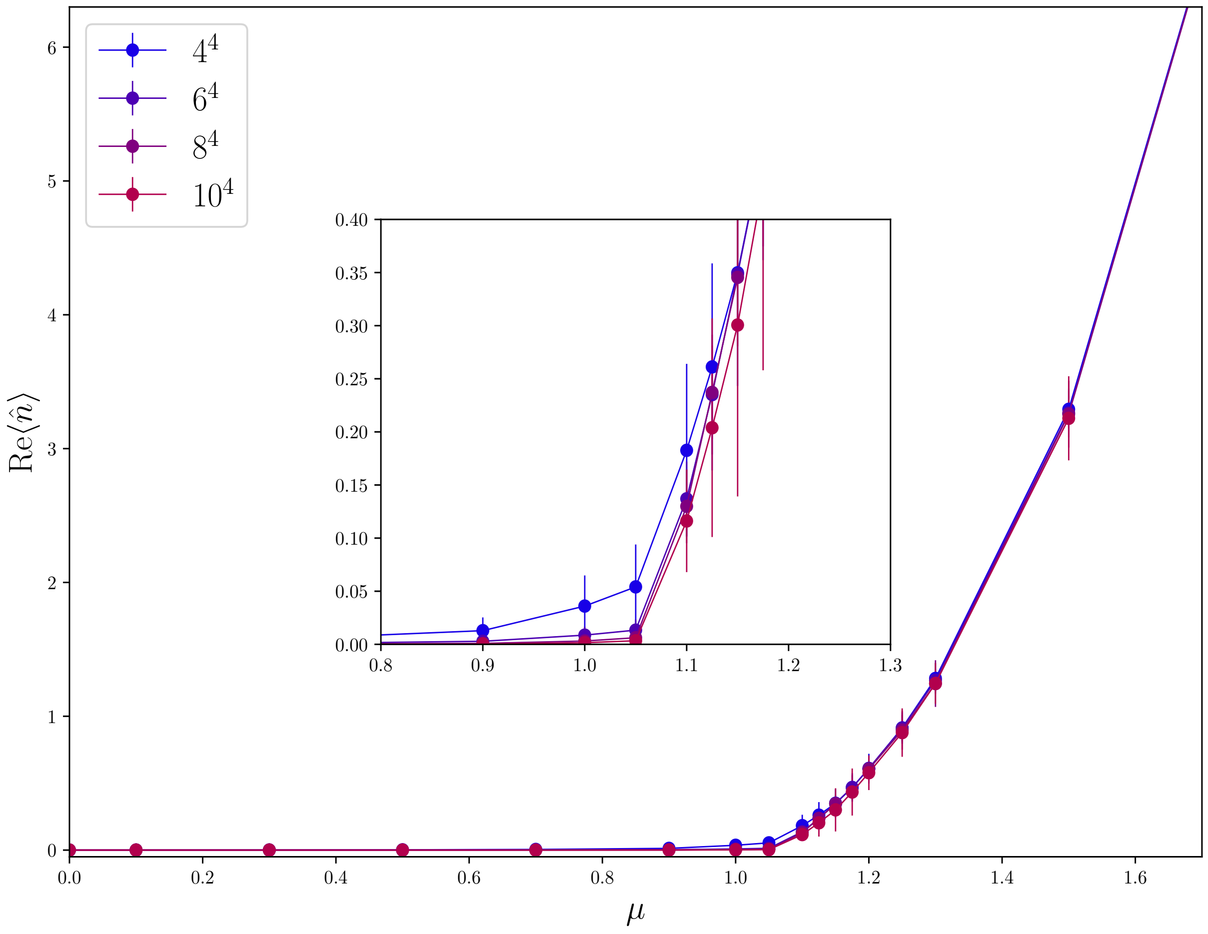
CL: success stories

$\mu = 1.5$



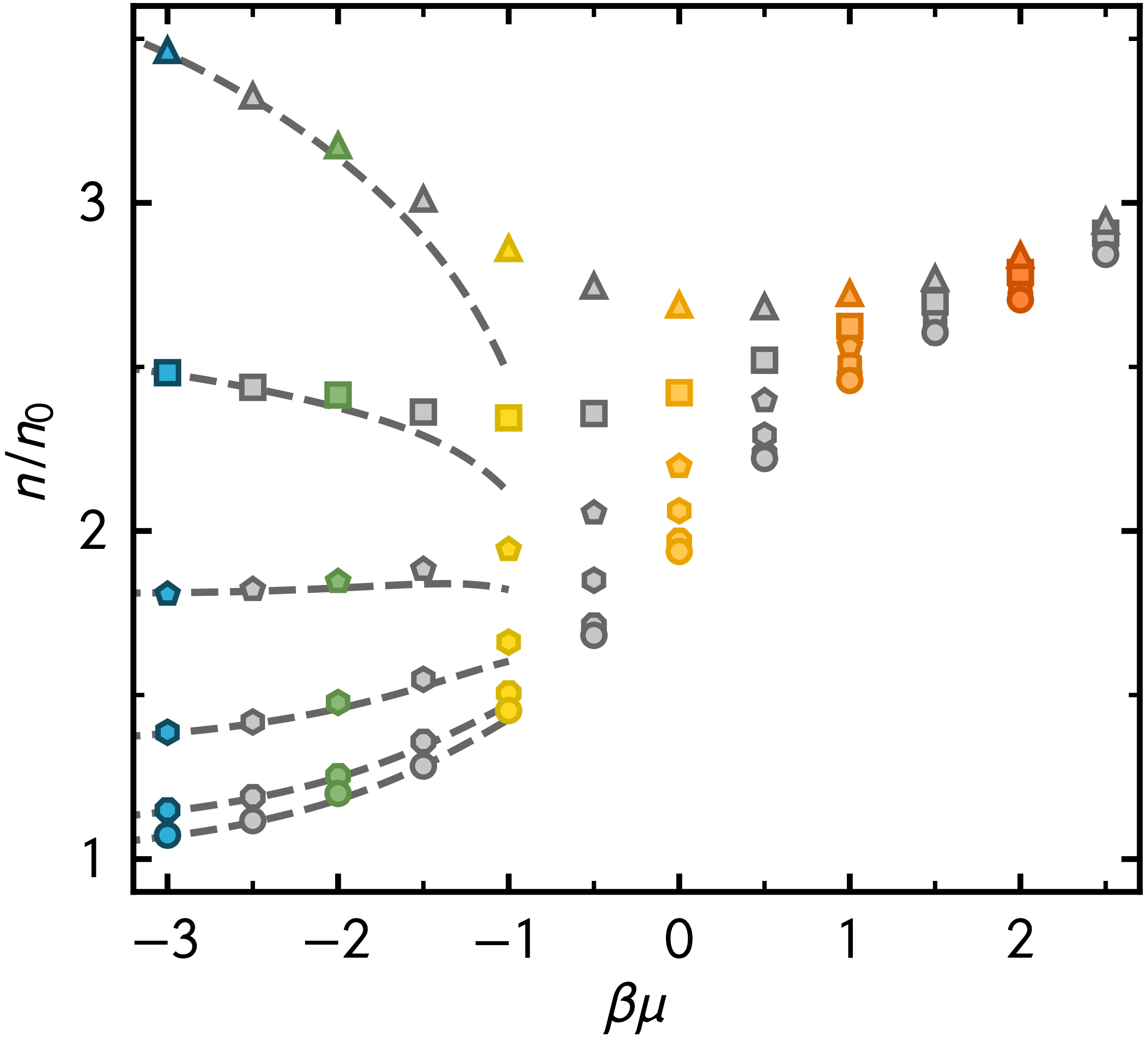
Relativistic Bose gas at finite chemical potential





Aarts, Phys. Rev. Lett (2009)

Density EOS of spin polarized unitary Fermi gas



- βh from 0 to 2.0 (bottom to top)
- Dashed lines: 3rd order virial expansion
- 3+1 dimensional lattice
  - Nx = 11, Nt=160

CL results show good agreement with the virial expansion in the virial region

Rammelmüller et al, arxiv: 1807.04664 (2018)

CL: cautionary tales

- CL is not always successful
  - The Excursion Problem
    - The probability distribution is not suppressed enough for large values of the complexified variables
    - Causes the imaginary drift term to “run away”
  - The Singular Drift Problem
    - The probability distribution is not suppressed enough near singularities in the drift term
- We don’t yet know how to prove when CL will work
  - Important to have checks to ensure validity
  - Comparisons with existing theoretical benchmarks, experimental measurements

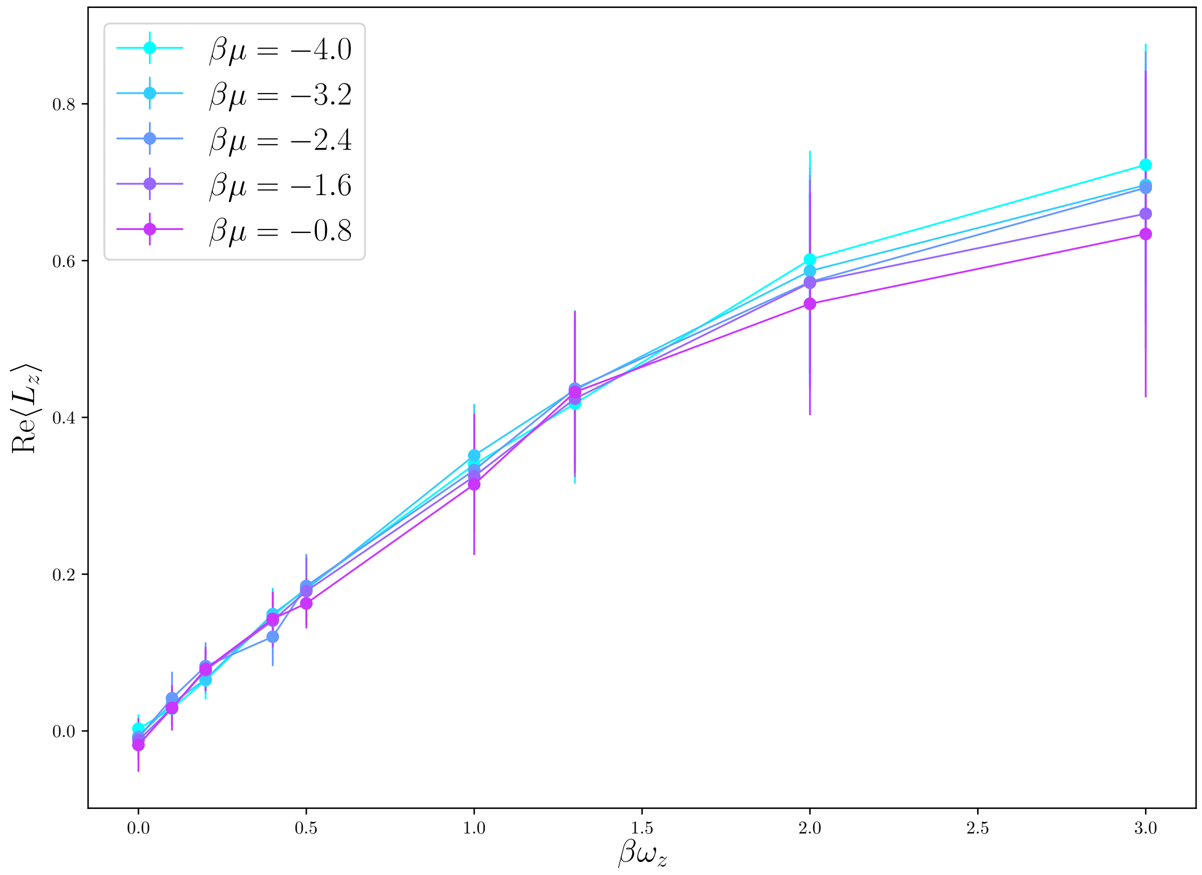
Action for our system

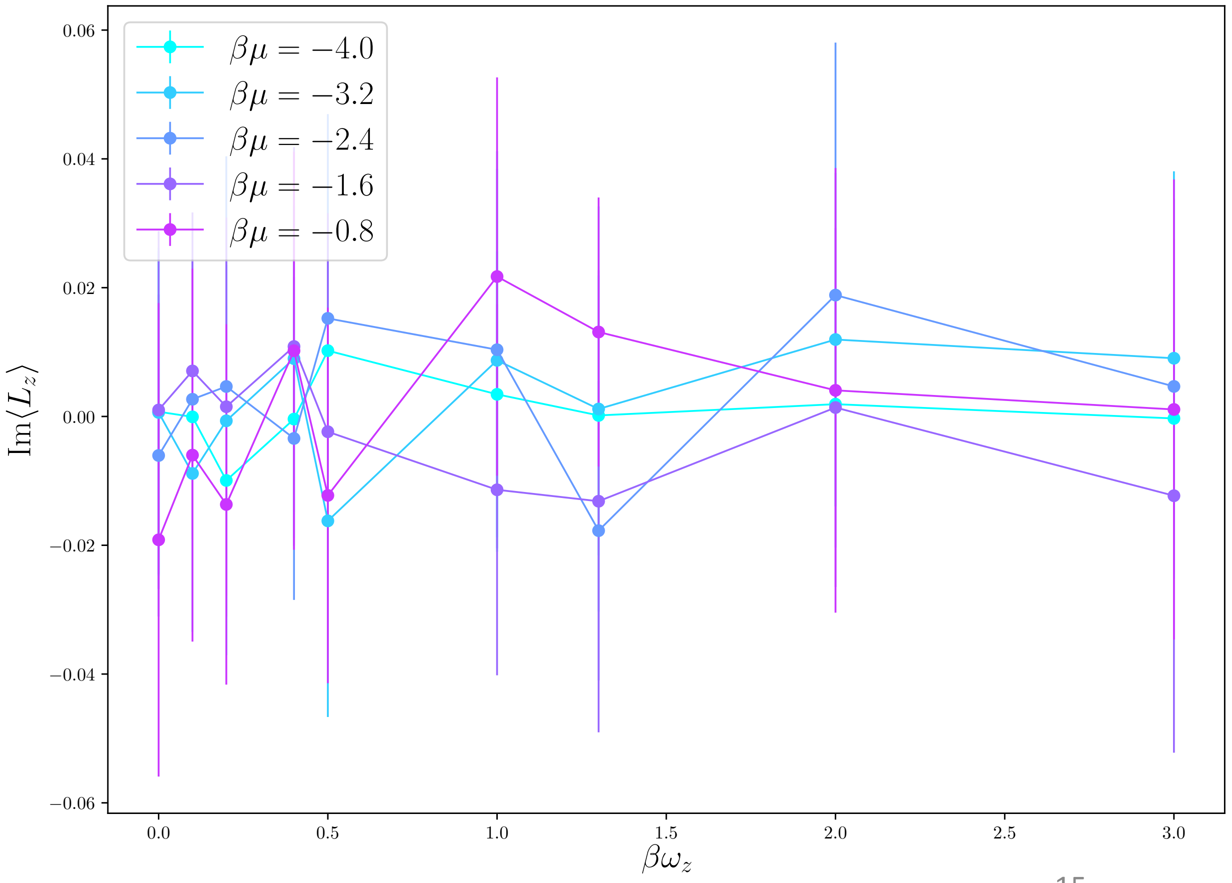
$$S = \int \phi^* \left(\mathcal{H} - \mu - \boxed{\omega_z L_z}\right) \phi + \lambda \int (\phi^* \phi)^2$$

$$\omega_z L_z = i\omega_z (x\partial_y - y\partial_x)$$

CL in non-relativistic rotating bosons

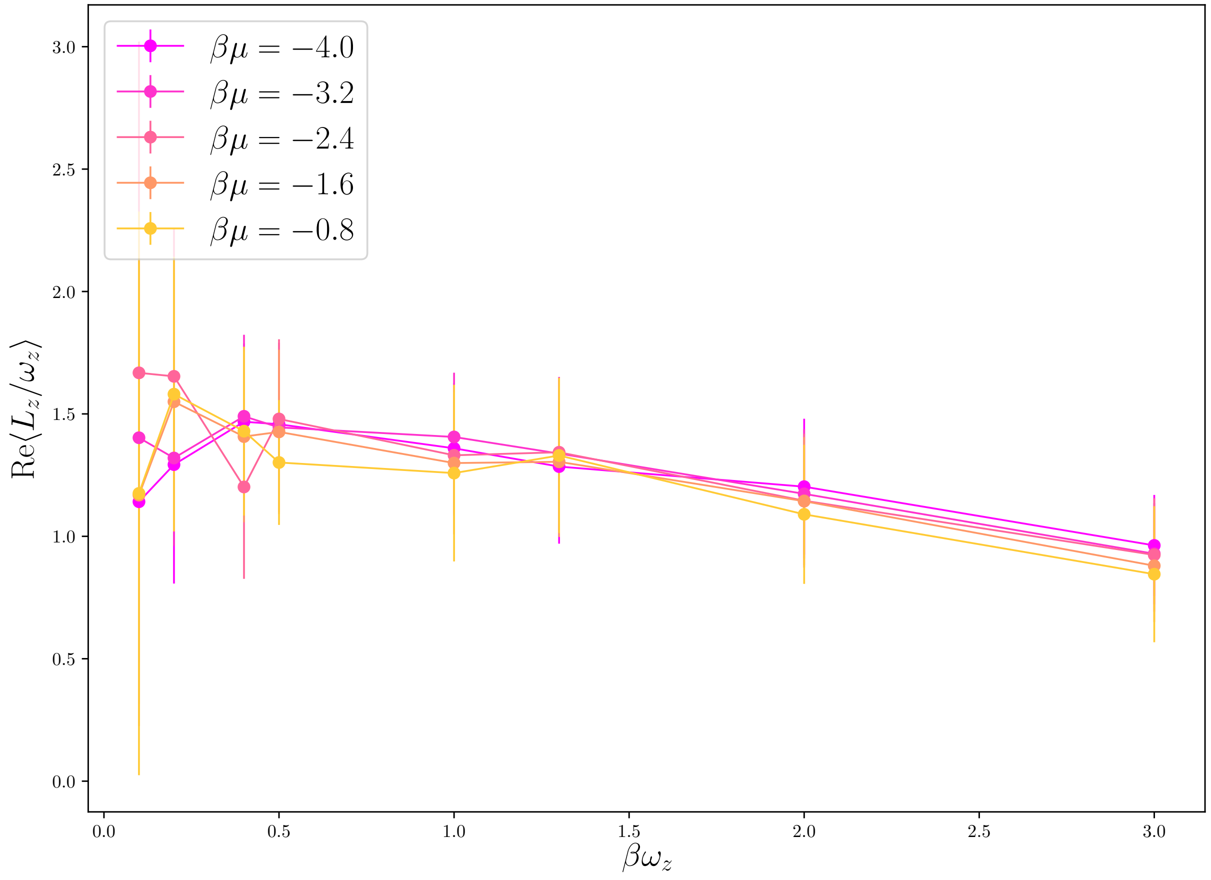
- Preliminary results for rotating, 2+1D system:
  - Average Angular Momentum dependence on rotation frequency
  - $Nx = 12$, $N\tau = 20$, $\tau = 0.2$

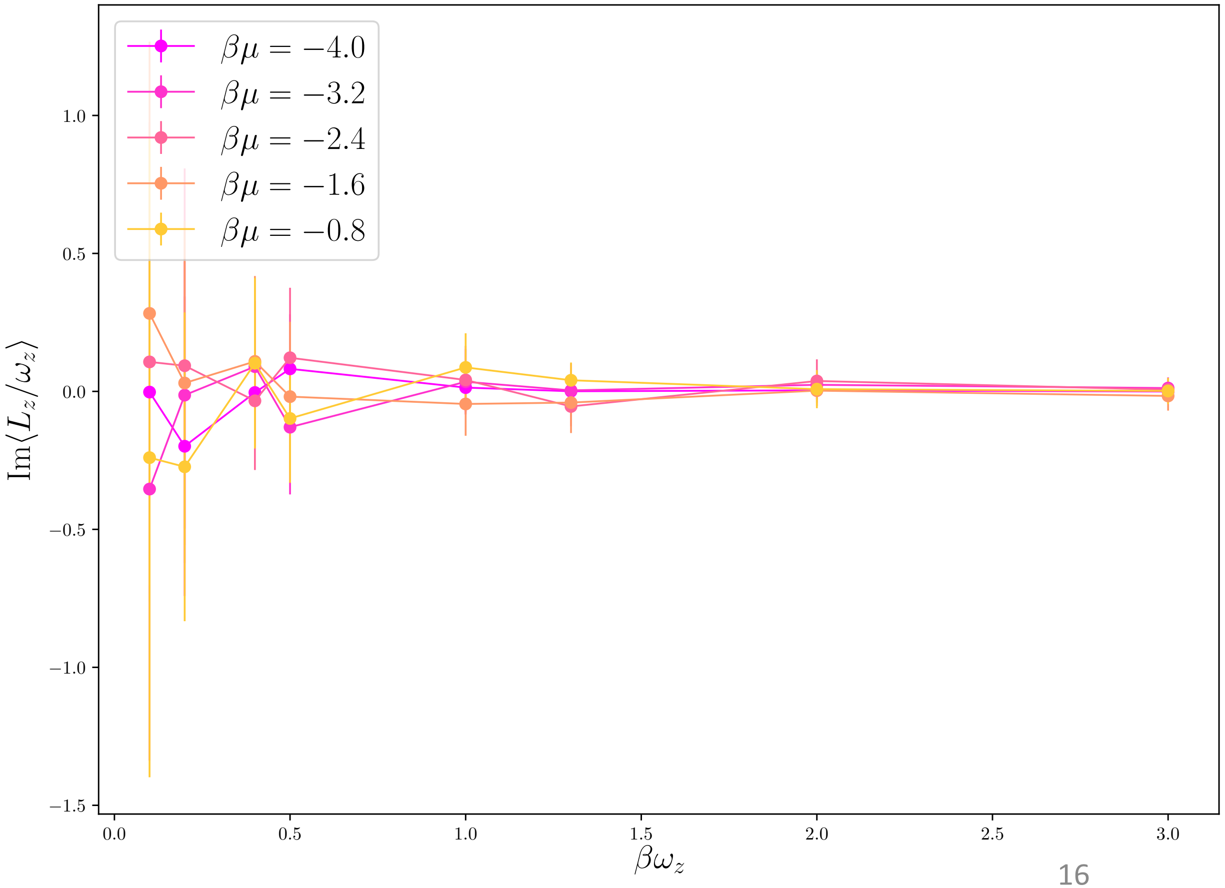




CL in non-relativistic rotating bosons

- Preliminary results for rotating, 2+1D system:
  - Moment of Inertia dependence on rotation frequency
  - Nx = 12, N$\boldsymbol{\tau}$ = 20, $\boldsymbol{\tau}$ = 0.2



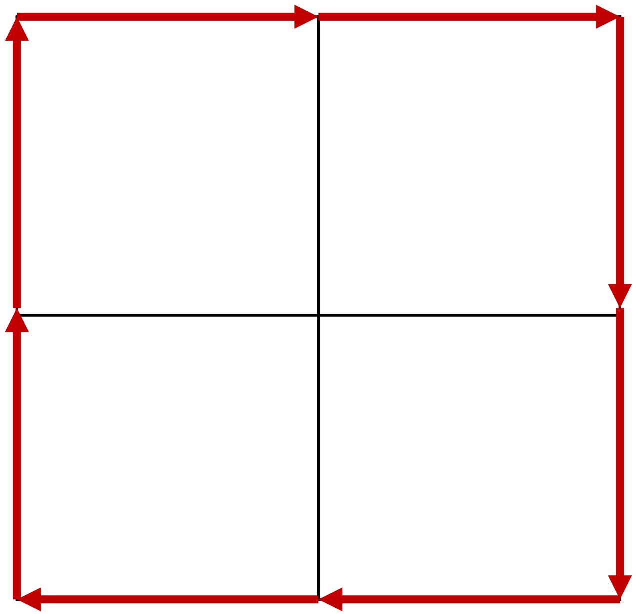


Future directions

- Decrease $|\beta\mu|$ to study superfluid regime
- Density should show triangular vortex lattice structure
- We expect to see discontinuities in the circulation observable

$$\Gamma[l] = \frac{1}{2\pi}\oint_{l\times l} dx\left(\theta_{t,x+\hat{j}} - \theta_{t,x}\right)$$

$$\theta_{t,x} = \tan^{-1}\left(\frac{\mathrm{Im}[\phi_{t,x}]}{\mathrm{Re}[\phi_{t,x}]}\right)$$



Summary and Conclusions

- Many systems of interest inaccessible to QMC due to sign problem
- CL allows us to circumvent the sign problem
- Under some circumstances, CL fails
- Preliminary results for rotating non-relativistic bosons are promising
- More work still to come

Thank you!

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