

$K\pi$ scattering and the $K^*(892)$ resonance in 2+1 flavor QCD

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In collaboration with:

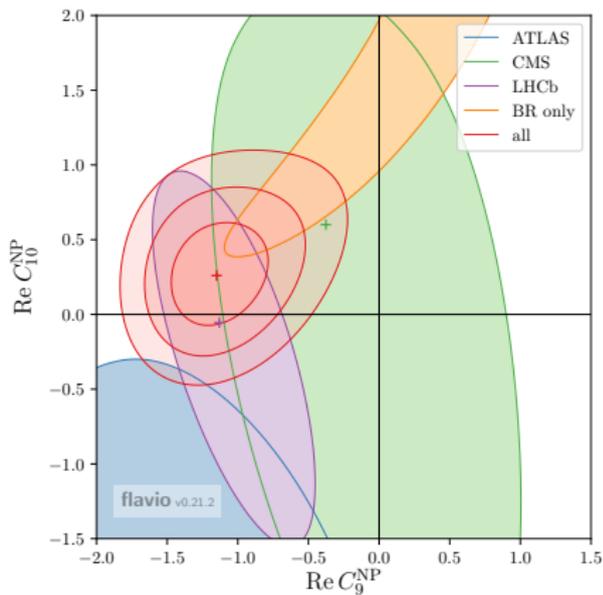
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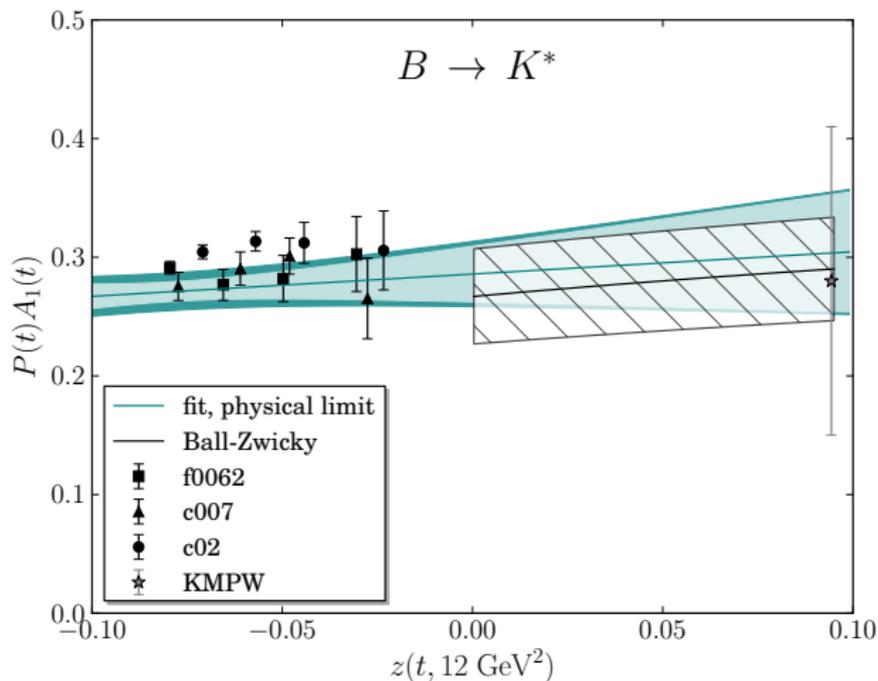
Fits of $C_i^{\text{NP}} = C_i - C_i^{\text{SM}}$
to experimental data for mesonic
 $b \rightarrow s \mu^+ \mu^-$ decays :

- $B_s \rightarrow \phi \mu^+ \mu^-$
- $B \rightarrow K \mu^+ \mu^-$
- $B \rightarrow X_s \mu^+ \mu^-$
- $B \rightarrow K^* (\rightarrow K \pi) \mu^+ \mu^-$



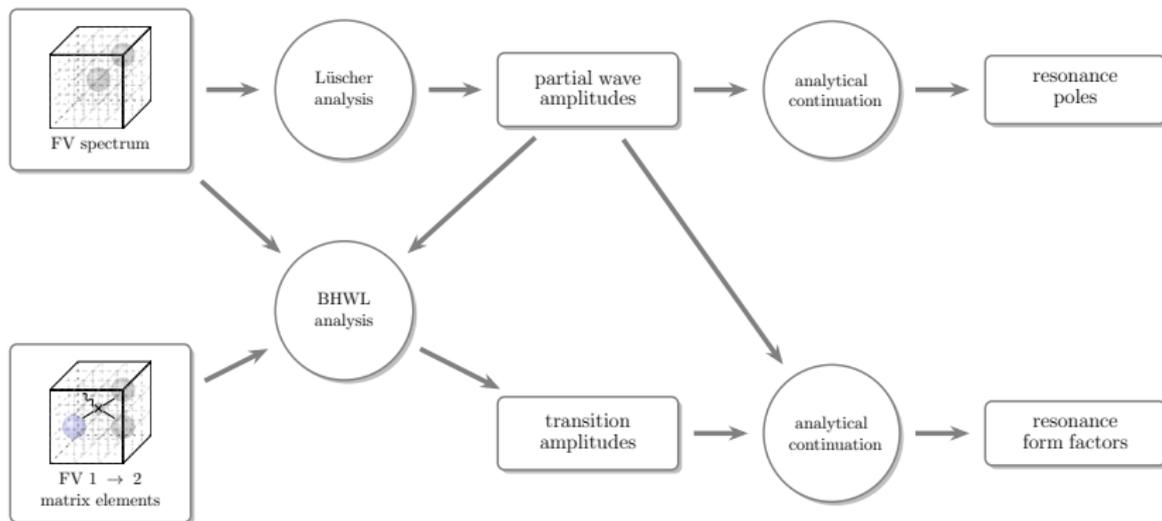
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1. Wolfgang ALTMANNSHOFER et al. "Status of the $B \rightarrow K^* \mu^+ \mu^-$ anomaly after Moriond 2017". In : **Eur. Phys. J. C** 77.6 (2017), p. 377. DOI : 10.1140/epjc/s10052-017-4952-0.



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2. Ronald R. HORGAN et al. "Lattice QCD calculation of form factors describing the rare decays $B \rightarrow K^* \ell^+ \ell^-$ and $B_s \rightarrow \phi \ell^+ \ell^-$ ". In: **Phys. Rev. D**89.9 (2014), p. 094501. DOI : 10.1103/PhysRevD.89.094501.

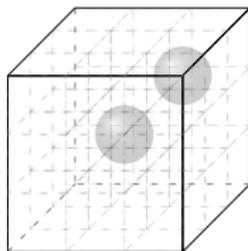


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[inspired by : Briceño@INT2016]

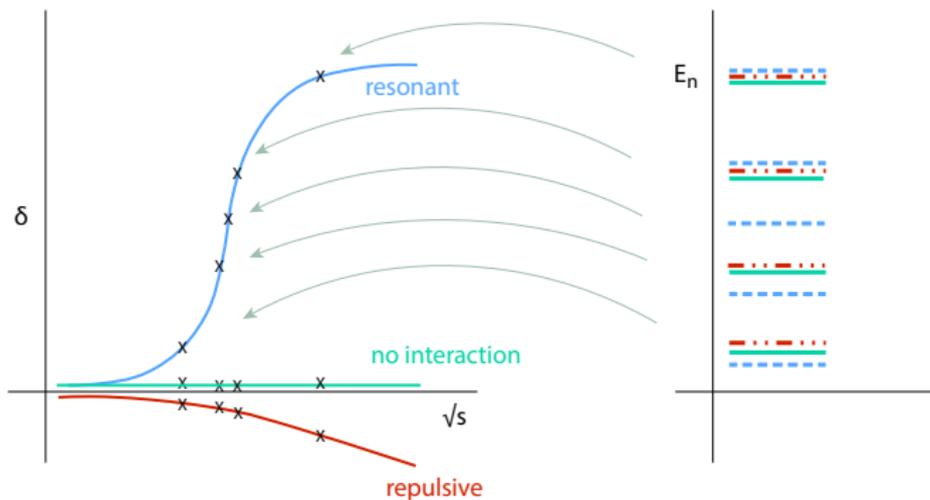
3. Raúl A. BRICEÑO, Maxwell T. HANSEN et André WALKER-LOUD. "Multichannel 1 → 2 transition amplitudes in a finite volume". In : **Phys. Rev. D**91.3 (2015), p. 034501. DOI : 10.1103/PhysRevD.91.034501.

Elastic Scattering on the Lattice



- Calculation is done in box of length L
- $\phi(L) = \phi(x + L)$
- Discretized momentum space

We obtain discrete energy spectrum



Lüscher's formalism :

We obtain multi-hadron spectrum $E_n^{\vec{P},\Lambda}$ at different total momenta \vec{P}

COM Energy :

$$\sqrt{s_n} = \sqrt{(E_n^{\vec{P},\Lambda})^2 - (\vec{P})^2}$$

Solve for scattering momentum k :

$$\sqrt{s_n} = \sqrt{m_\pi^2 + (k_n^{\vec{P},\Lambda})^2} + \sqrt{m_K^2 + (k_n^{\vec{P},\Lambda})^2}$$

Quantization Condition [Lüscher NPB1991] :

$$\det \begin{bmatrix} \text{infinite} & \text{finite} \\ \text{volume} & \text{volume} \\ \cot \delta_J & -\mathcal{M}(k) \end{bmatrix} = 0$$

Determine $\delta(s)$ at discrete values of s_n .

Irreducible Representations

$\frac{L}{2\pi}\vec{P}$	Little Group ($LG^{\vec{P}}$)	irrep ($\Lambda^{\vec{P},r}$)	spin content	dimension
(0,0,0)	O_h	A_{1g}	J=0,4,...	1
(0,0,0)	O_h	T_{1u}	J=1,3,...	3
(0,0,1)	C_{4v}	$A_1(A_2)$	J=0,1,...	1
(0,0,1)	C_{4v}	E	J=1,2,...	2
(0,1,1)	C_{2v}	$A_1(B_3)$	J=0,1,...	1
(0,1,1)	C_{2v}	B_1	J=1,2,...	1
(0,1,1)	C_{2v}	B_2	J=1,2,...	1
(1,1,1)	C_{3v}	$A_1(A_2)$	J=0,1,...	1
(1,1,1)	C_{3v}	E	J=1,2,...	2

TABLE – All the highlighted irreps are the ones we consider in this analysis. ⁴

$$\vec{P} = \frac{2\pi}{L}(0,0,1), \quad \Lambda = E : \cot \delta_1(\sqrt{s_n^{\vec{P}}, \Lambda}) = w_{0,0}^{\vec{P}}(k_n^{\vec{P}}, \Lambda, L) - w_{2,0}^{\vec{P}}(k_n^{\vec{P}}, \Lambda, L)$$

4. Luka LESKOVEC et Sasa PRELOVSEK. "Scattering phase shifts for two particles of different mass and non-zero total momentum in lattice QCD". In : **Phys. Rev. D**85 (2012), p. 114507. DOI : 10.1103/PhysRevD.85.114507.

Lattices

- $N_f = 2 + 1$ Clover fermions
- Isotropic lattices (C13, D6) by Orginos et al.

Label	$N_L^3 \times N_t$	a (fm)	L (fm)	m_π (MeV)	m_K (MeV)	N_{config}
C13	$32^3 \times 96$	0.11403(77)	3.65	317(2)	527(4)	896/1048
D6	$48^3 \times 96$	0.08766(79)	4.21	178(2)	514(5)	60/690

$$\bar{q}q \text{ operators : } K_i^{*+}(p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x) \gamma_i u(x), \quad K_{ti}^{*+}(p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x) \gamma_t \gamma_i u(x),$$

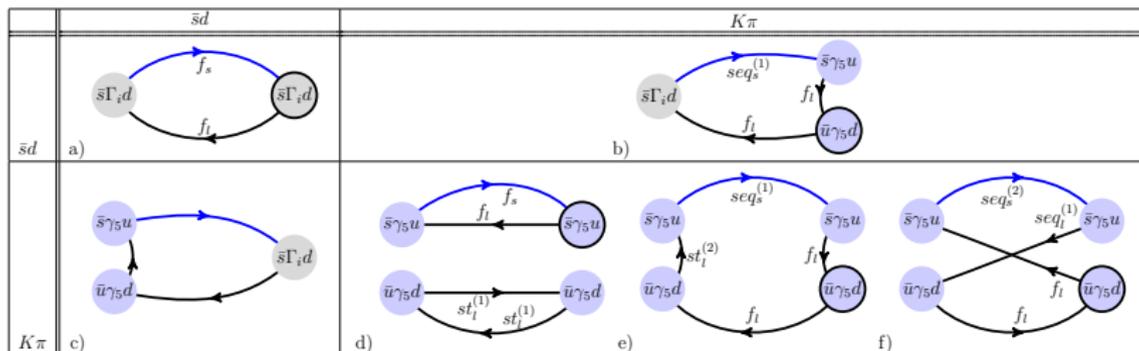
$$K\pi \text{ operators : } O_{K\pi}(p_1, p_2) = \sqrt{\frac{2}{3}} \pi^+(p_1) K^0(p_2) - \sqrt{\frac{1}{3}} \pi^0(p_2) K^+(p_1).$$

$$O_{K\pi, \Lambda}^{\vec{P}} = \frac{\dim(\Lambda)}{\text{order}(LG)} \sum_{R \in LG, \vec{m} \in Z^3} \chi(R) O_{K\pi}(\vec{d}/2 + R(\vec{d}/2 + \vec{m}), \vec{d}/2 - R(\vec{d}/2 + \vec{m}))$$

$$O_{K^*, \Lambda}^{\vec{P}} = \frac{\dim(\Lambda)}{\text{order}(LG)} \sum_{R \in LG} \chi(R) R_i K_i^{*+}(\vec{P}),$$

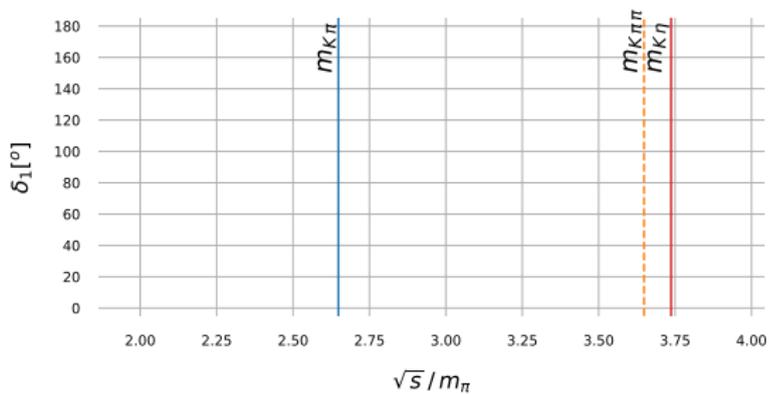
where $\vec{d} = \frac{L}{2\pi} \vec{P}$.

Wick Contractions

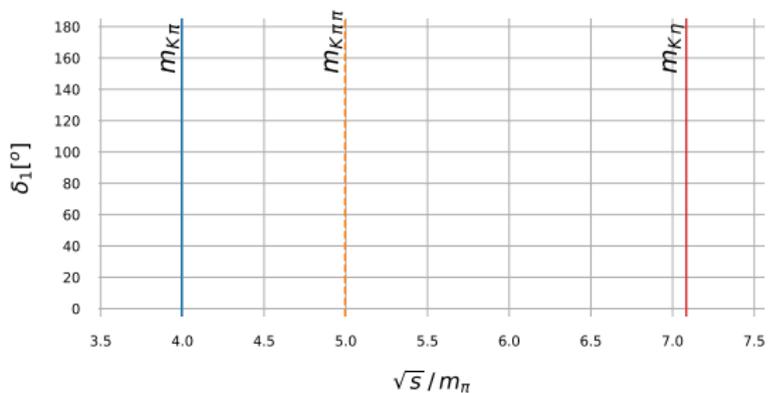


Thresholds

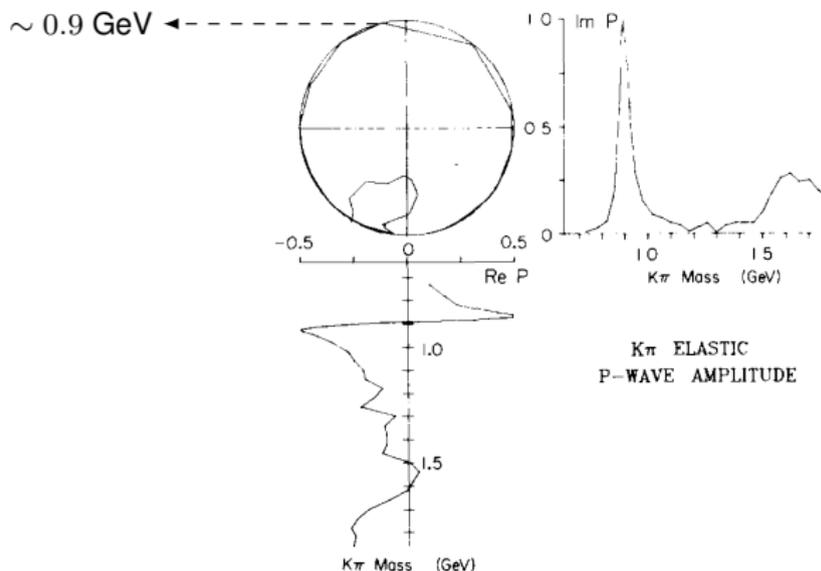
C13



D6



Elastic at $K\pi\pi$ threshold?



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5. P. ESTABROOKS et al. "Study of K pi Scattering Using the Reactions $K^+ p \rightarrow K^+ \pi^+ n$ and $K^+ p \rightarrow K^+ \pi^- \Delta^{++}$ at 13-GeV/c". In : **Nucl. Phys.** B133 (1978), p. 490–524. DOI : 10.1016/0550-3213(78)90238-9.

Operator bases for each irrep

$\frac{L}{2\pi} \vec{P}$	Little Group ($LG^{\vec{P}}$)	irrep ($\Lambda^{\vec{P},r}$)	# of Operators	dimension
(0,0,0)	O_h	T_{1u}	$2 \times O_{qq} + 3 \times O_{K\pi}$	3
(0,0,1)	C_{4v}	E	$2 \times O_{qq} + 4 \times O_{K\pi}$	2
(0,1,1)	C_{2v}	B_1	$2 \times O_{qq} + 3 \times O_{K\pi}$	1
(0,1,1)	C_{2v}	B_2	$2 \times O_{qq} + 2 \times O_{K\pi}$	1
(1,1,1)	C_{3v}	E	$2 \times O_{qq} + 2 \times O_{K\pi}$	2

TABLE – Number of operators used for each irrep.

Operator bases for each irrep

Example

$$\vec{P} = \frac{2\pi}{L}(0, 0, 1), \quad \Lambda : E, \quad \text{polarization} : x$$

$$O_{qq}^1 = K_x^{*+}(\vec{P})$$

$$O_{qq}^2 = K_{tx}^{*+}(\vec{P})$$

$$O_{K\pi}^3 = O_{K\pi} \left(\frac{2\pi}{L}(-1, 0, 0), \frac{2\pi}{L}(1, 0, 1) \right) - O_{K\pi} \left(\frac{2\pi}{L}(1, 0, 0), \frac{2\pi}{L}(-1, 0, 1) \right)$$

$$O_{K\pi}^4 = O_{K\pi} \left(\frac{2\pi}{L}(-1, 0, 1), \frac{2\pi}{L}(1, 0, 0) \right) - O_{K\pi} \left(\frac{2\pi}{L}(1, 0, 1), \frac{2\pi}{L}(-1, 0, 0) \right)$$

$$O_{K\pi}^5 = -O_{K\pi} \left(\frac{2\pi}{L}(1, 1, 0), \frac{2\pi}{L}(-1, -1, 1) \right) - O_{K\pi} \left(\frac{2\pi}{L}(1, -1, 0), \frac{2\pi}{L}(-1, 1, 1) \right)$$

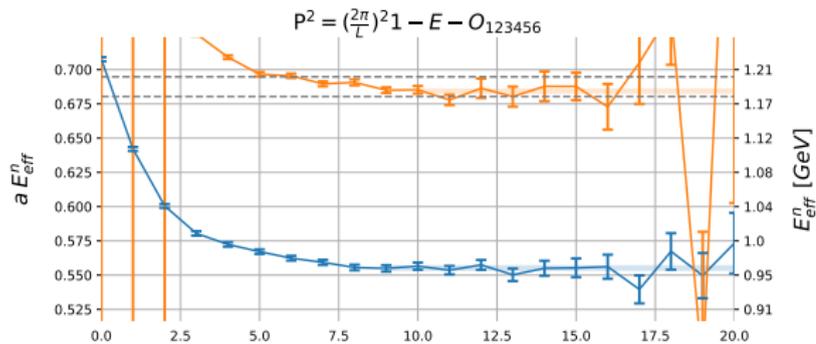
$$+ O_{K\pi} \left(\frac{2\pi}{L}(-1, 1, 0), \frac{2\pi}{L}(1, -1, 1) \right) + O_{K\pi} \left(\frac{2\pi}{L}(-1, -1, 0), \frac{2\pi}{L}(1, 1, 1) \right)$$

$$O_{K\pi}^6 = -O_{K\pi} \left(\frac{2\pi}{L}(1, 1, 1), \frac{2\pi}{L}(-1, -1, 0) \right) - O_{K\pi} \left(\frac{2\pi}{L}(1, -1, 1), \frac{2\pi}{L}(-1, 1, 0) \right)$$

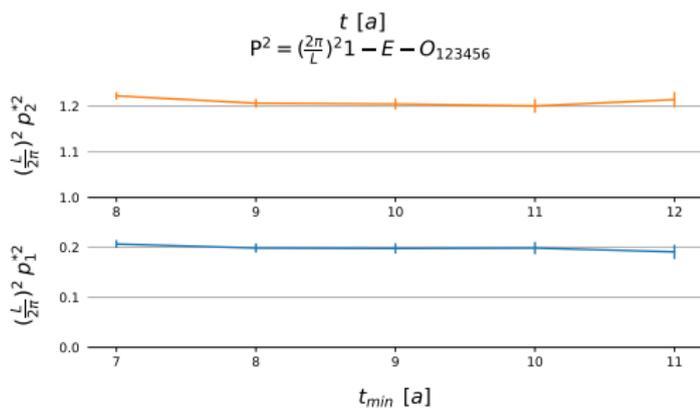
$$+ O_{K\pi} \left(\frac{2\pi}{L}(-1, 1, 1), \frac{2\pi}{L}(1, -1, 0) \right) + O_{K\pi} \left(\frac{2\pi}{L}(-1, -1, 1), \frac{2\pi}{L}(1, 1, 0) \right)$$

Preliminary Results

E_n



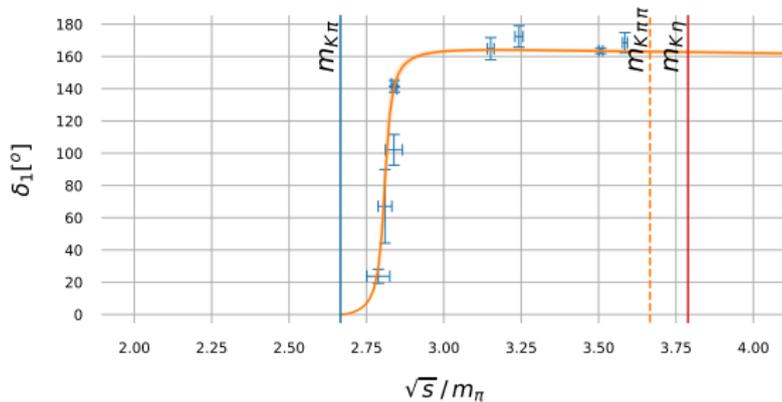
C13



Preliminary Results

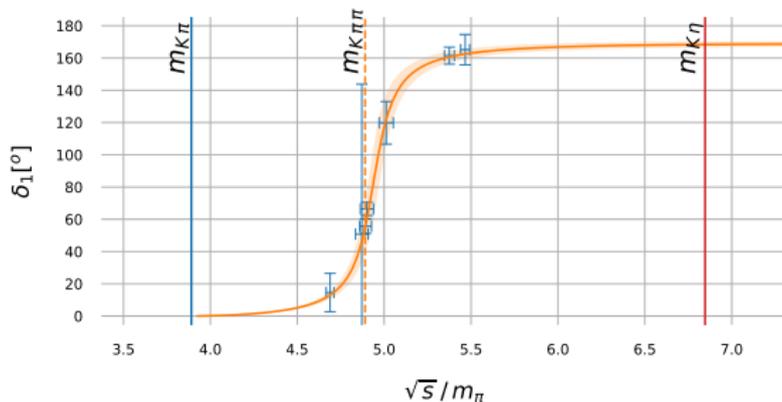
$$\delta^{J=1}(s)$$

C13



$$g_K = 7.87(38)$$
$$m_{K^*} = 890(7) \text{ MeV}$$

D6



$$g_K = 5.46(45)$$
$$m_{K^*} = 879(13) \text{ MeV}$$

- Add irreps with mixed S&P waves.
- Increase statistics.
- Determine $B \rightarrow K^*(\rightarrow K \pi)\ell^+\ell^-$ form factors using BHWL formalism.