Progress on the study of electromagnetic corrections to $K \to \pi\pi$ decay

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Including electromagnetism in $K \rightarrow \pi\pi$ decay calculations

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Motivation to study EM corrections to $K \to \pi\pi$

This morning's session is about the studies of $K \to \pi\pi$ decay and ϵ'

- Progresses reported by R. Mawhinney, T. Wang, C. Kelly, F. Romero-Lopez

Direct CP violation in $K \to \pi\pi$

$$\epsilon' = \frac{1}{3}(\eta_{+-} - \eta_{00}) = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right)$$

- Turn on EM interaction, $A_I \to A_I^\gamma$, $\delta_I \to \delta_I^\gamma$, $I = 0, 2$

Though $A_2^\gamma - A_2$ is an $O(\alpha_e)$ effect, its size could be enhanced by a factor of 22 due to the mixing with A_0 and $\Delta I = 1/2$ rule

- ChPT+Large-N_c: Cirigliano et al, hep-ph/0008290, hep-ph/0310351
  –"the isospin violating correction for ϵ' is below 15%"

Technical issues on including electromagnetism

- Lellouch-Lüscher's formalism relies on a short-range interaction
  $\Rightarrow$ long-range EM requires the change in the FV formalism
  Main topic of this talk

- EM interaction mixes $I = 0$ and $I = 2$ $\pi\pi$ scattering
  $\Rightarrow$ $K \to \pi\pi$ decay becomes a coupled-channel problem
  See Lat17 proceeding: EPJ Web Conf. 175 (2018) 13016

- Possible photon radiation
  $\Rightarrow$ coupled channels further mixed with 3-particle channel ($\pi\pi\gamma$)
  Under investigation

Strategy to include electromagnetism

Include EM interaction in the Coulomb gauge

$$\mathcal{L}_{\text{int}} = \underbrace{\sum_{q=u,d,s} e_q \vec{A}(x) \cdot \bar{q}\vec{\gamma}q(x)}_{\text{Transverse radiation}} - \underbrace{\sum_{q,q'=u,d,s} \int \frac{d^3\vec{x}'}{4\pi} \frac{\rho_q(\vec{x}',t)\rho_{q'}(\vec{x},t)}{|\vec{x}'-\vec{x}|}}_{\text{Coulomb potential}}$$

- Adding transverse photon to $\pi\pi \quad \Rightarrow \quad$ three-particle problem
- At current stage, focus on Coulomb potential only

Photon propagator in the Coulomb gauge

$$\underbrace{G_{00}(p) = \frac{1}{\vec{p}^2}}_{V(r)=\frac{1}{4\pi r}}, \quad G_{ij}(p) = \frac{1}{p^2}\left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2}\right), \quad G_{i0}(p) = G_{0i}(p) = 0$$

Coulomb potential in the finite volume

Encode long-range EM interaction in the finite box – QED$_L$

[helpful discussion with Luchang Jin]

- Coulomb potential in periodic box $V_L(\mathbf{r}) = \sum_n V(\mathbf{r} + \mathbf{n}L)$
  - $\forall \mathbf{n}$, $V(\mathbf{r} + \mathbf{n}L)$ has impact on $\mathbf{r} \approx \mathbf{0}$ region and $\sum_n$ causes divergence
- Modify $V_L(\mathbf{r}) \to \hat{V}_L(\mathbf{r}) = V_L(\mathbf{r}) - \frac{1}{L^3}\int d^3\mathbf{r}\, V(\mathbf{r})$ to remove the divergence
  - This is equivalent to remove zero mode: $\hat{V}_L(\mathbf{r}) = \frac{4\pi\alpha_e}{L^3}\sum_{\mathbf{p}\neq\mathbf{0}} \frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{p^2}$
- However, $\hat{V}_L$ introduces $O(1/L)$ FV effects

$$\delta V(\mathbf{r}) \equiv \hat{V}_L(\mathbf{r}) - V(\mathbf{r}) = \left(\frac{1}{L^3}\sum_{\mathbf{p}\neq\mathbf{0}} - \int \frac{d^3\mathbf{p}}{(2\pi)^3}\right)\frac{4\pi\alpha_e}{p^2}e^{i\mathbf{p}\cdot\mathbf{r}}, \quad \lim_{\mathbf{r}\to 0}\delta V(\mathbf{r}) = -\kappa\frac{\alpha_e}{L} \approx -2.8\frac{\alpha_e}{L}$$

Similar situation happens for massive photon and C^* boundary condition

Adopt Lüscher's method

In the peridoic "exterior region" where strong interaction vanishes

- Without QED
  - $\psi(\mathbf{r})$ can be constructed by partial wave scattering amplitude
    $$\psi(\mathbf{r}) = \sum_{\ell m} b_{\ell m} Y_{\ell m}(\Omega_{\mathbf{r}}) \{\cos\delta_\ell \, j_\ell(kr) + \sin\delta_\ell \, n_\ell(kr)\}$$
    where $j_\ell(kr)$, $n_\ell(kr)$ are regular and irregular Bessel function
  - $\psi(\mathbf{r})$ is related to singular periodic solution of Helmholtz Eq.
    $$\psi(\mathbf{r}) = \sum_{\ell m} v_{\ell m} G^{(0)}_{\ell m}(\mathbf{r}, k^2)$$
  - This leads to quantization condition $\phi(k) + \delta(k) = n\pi$
- With QED
  - $j_\ell,\ n_\ell \quad \rightarrow \quad F_\ell,\ G_\ell$
    $$\psi_C(\mathbf{r}) = \sum_{\ell m} b_{\ell m} Y_{\ell m}(\Omega_{\mathbf{r}}) \{\cos\delta_\ell \, F_\ell(kr) + \sin\delta_\ell \, G_\ell(kr)\} + O(\frac{\alpha_e}{L})$$
    However, $V_L(r)$ is not of type $\frac{1}{r} \quad \rightarrow \quad O(\frac{\alpha_e}{L})$ effect
  - Solution of (Coulomb) Helmholtz Eq. can be perturbatively expanded
    $$\psi_C(\mathbf{r}) = \sum v_{\ell m} G_{C,\ell m}(\mathbf{r}, k^2), \quad G_{C,\ell m} = G^{(0)}_{\ell m} + G^{(1)}_{\ell m} + O(\alpha_e^2)$$

Lüscher's quantization condition

- Wave function can be written in two forms

$$\psi_C(\mathbf{r}) = \sum_{\ell m} b_{\ell m} Y_{\ell m}(\Omega_{\mathbf{r}}) \left\{ \cos\delta_\ell \, F_\ell(kr) + \sin\delta_\ell \, G_\ell(kr) \right\} + O\left(\frac{\alpha_e}{L}\right)$$

$$\psi_C(\mathbf{r}) = \sum_{\ell m} v_{\ell m} G_{C,\ell m}(\mathbf{r}, k^2), \qquad G_{C,\ell m} = G^{(0)}_{\ell m} + G^{(1)}_{\ell m} + O(\alpha_e^2)$$

- Equating two expressions yields quantization condition $\phi_c(k) + \delta(k) = n\pi$

$$\cot\phi_c(k) = (1+\pi\eta)\frac{1}{\pi}\frac{1}{kL}\sum_{\mathbf{n}} \frac{1}{-\mathbf{n}^2 + (\frac{kL}{2\pi})^2}$$
$$+ \lim_{r\to 0} 8\pi\eta \left\{ \sum_{\mathbf{n}\neq\mathbf{m}} \frac{e^{i\mathbf{n}\cdot\mathbf{r}\frac{2\pi}{L}}}{\pi(2\pi)^4} \frac{1}{\mathbf{n}^2 - (\frac{kL}{2\pi})^2} \frac{1}{(\mathbf{n}-\mathbf{m})^2} \frac{1}{\mathbf{m}^2 - (\frac{kL}{2\pi})^2} - \frac{1}{4\pi}\ln(1/kr) + \frac{1}{4\pi} \right\}$$

with $\eta = \frac{\alpha_e \mu}{k}$ the Sommerfeld parameter

(See also formula for scattering length [Bean & Savage, 1407.4846])

Kim, Sachrajda and Sharpe's method

Finite volume effects arise from 2-particle propagators

$$\left(\int \frac{dp_0}{2\pi}\frac{1}{L^3}\sum_{\vec{p}} - \int \frac{d^4p}{(2\pi)^4}\right) f(p)\,\underbrace{\frac{1}{p^2-m^2+i\epsilon}\,\frac{1}{(P-p)^2-m^2+i\epsilon}}_{S_2(P,p)}\,g(p)$$

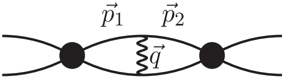
Integrating p_0 leaves two terms

$$\underbrace{\frac{1}{2\omega_p((E-\omega_p)^2-\omega_p^2)}}_{\text{power-law FV effects}}, \quad \underbrace{\frac{1}{2\omega_p((E+\omega_p)^2-\omega_p^2)}}_{\text{exponential FV effects}}, \quad \text{with } \omega_p = \sqrt{m^2+\vec{p}^2}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

on-shell amplitude — off-shell quantity

Include photon exchange



$(\vec{q} = \vec{p}_1 - \vec{p}_2)$

$$\left(\int \frac{dp_{10}}{2\pi}\int \frac{dp_{20}}{2\pi}\sum_{\vec{p}_1\neq\vec{p}_2} - \int \frac{d^4p_1}{(2\pi)^4}\int \frac{d^4p_2}{(2\pi)^4}\right) f(p_1)S_2(P,p_1)\frac{1}{\vec{q}^2}S_2(P,p_2)g(p_2)$$

$\vec{q} = \vec{p}_1 - \vec{p}_2 \neq \vec{0} \quad \Rightarrow \quad$ Off-shell quantity also contributes $O(1/L^n)$ FV effects

Coulomb potential with truncated range $R_T \leq L/2$

Truncate the Coulomb potential with a range R_T

$$V^{(T)}(\mathbf{r}) = \begin{cases} \alpha_e/r, & \text{for } r < R_T \\ 0, & \text{for } r > R_T \end{cases}$$

Build periodic potential

$$V_L^{(T)}(\mathbf{r}) = \sum_{\mathbf{n}} V^{(T)}(\mathbf{r} + \mathbf{n}L)$$

Lüscher's quantization condition holds for $V_s(\mathbf{r}) + V^{(T)}(\mathbf{r})$

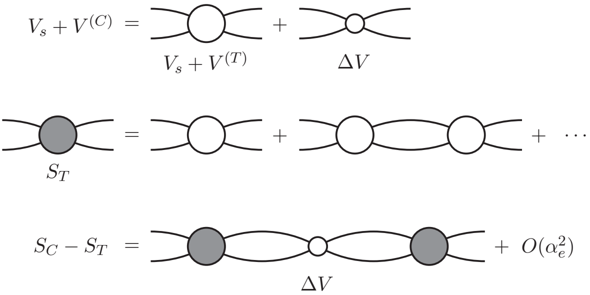
$$\phi(q) + \delta_T(k) = n\pi, \quad q = \frac{kL}{2\pi}$$

So does Lellouch-Lüscher formula

Both Lüscher's method in potential theory and KSS method in QFT work well

Remaining issue is to relate truncated δ_T and A_T to the physical ones

Truncation effects in scattering amplitude



The relation for scattering amplitude

$$S_C = S_T - i\,2\pi\delta(E - E')\,\langle E, -, T|\Delta V|E, +, T\rangle$$

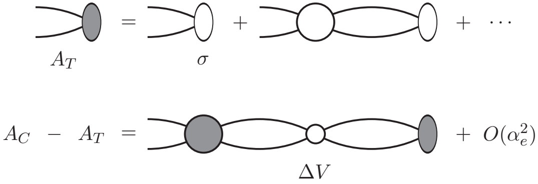
- $\Delta V(r)$ is non-zero only for $r > R_T$
- For $\psi_T^{(\pm)}(r) = \langle r|E, \pm, T\rangle$, the functional form is known for $r > R_T$

$$\psi_T^{(\pm)}(r) = \sqrt{\frac{\mu}{\pi k}}\frac{\sin(kr + \delta_T)}{r} e^{\pm i\delta_T}, \quad \text{for S-wave}$$

- Correction to scattering amplitude can be evaluated

$$\langle E, -, T|\Delta V|E, +, T\rangle = \int_{R_T}^{R_\infty} d^3\mathbf{r}\,\psi_T^{(-)*}(r)\frac{\alpha_e}{r}\psi_T^{(+)}(r)$$

$\sigma \to \pi\pi$ decay amplitude



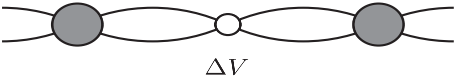
Truncation effects can be determined

$$\boxed{A_C - A_T = \int_{R_T}^{R_\infty} d^3\mathbf{r}\, \psi_T^{(-)*}(r) \frac{\alpha_e}{r} \psi_0(r) A_T}$$

ψ_0 is the free wave function: $\psi_0(r) = -\frac{1}{2}\sqrt{\frac{\mu}{\pi k}}\frac{e^{ikr}}{r}$

Examine in the quantum field theory

For scattering amplitude



$$\int \frac{d^4 p_1}{(2\pi)^4} \int \frac{d^4 p_2}{(2\pi)^4} f(p_1) S_2(P, p_1) \Delta V(\vec{q}) S_2(P, p_2) g(p_2), \quad \vec{q} = \vec{p}_1 - \vec{p}_2$$

- $\Delta V(\vec{q})$ can be written as

$$\Delta V(\vec{q}) = \int_{r > R_T} d^3 \vec{r}\, \frac{\alpha_e}{r} e^{-i\vec{q}\cdot\vec{r}}$$

- Integrating over p_{10} leaves two terms

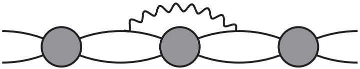
$$\underbrace{\int \frac{d^3 \vec{p}_1}{(2\pi)^3} f(p_1) \frac{e^{-i\vec{p}_1\cdot\vec{r}}}{2\omega_p((E-\omega_p)^2 - \omega_p^2)}}_{\text{on-shell scattering wave function}}, \quad \underbrace{\int \frac{d^3 \vec{p}_1}{(2\pi)^3} f(p_1) \frac{e^{-i\vec{p}_1\cdot\vec{r}}}{2\omega_p((E+\omega_p)^2 - \omega_p^2)}}_{\text{suppressed by } e^{-\Lambda_{\mathrm{QCD}} R_T}}$$

For decay amplitude

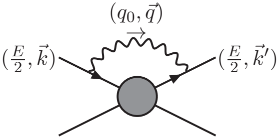
on-shell $\psi_T^{(-)*}$ $\qquad$ $e^{ikr} \times A_T$

One obtains the same structure in QFT as that in potential theory

When photon crosses the bubble



Check the singularity for the on-shell amplitude



$$(\tfrac{E}{2})^2 = \vec{k}^2 + m^2 = \vec{k}'^2 + m^2$$

$$\int \frac{dq_0}{2\pi} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{\left(\frac{E}{2} - q_0\right)^2 - (\vec{k} - \vec{q})^2 - m^2 + i\varepsilon} \frac{1}{\left(\frac{E}{2} - q_0\right)^2 - (\vec{k}' - \vec{q})^2 - m^2 + i\varepsilon} \frac{1}{\vec{q}^2}$$

Integrate over q_0

$$\int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{(\vec{k} - \vec{q})^2 - (\vec{k}' - \vec{q})^2} \frac{1}{\vec{q}^2} + \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{(\vec{k}' - \vec{q})^2 - (\vec{k} - \vec{q})^2} \frac{1}{\vec{q}^2}$$

Two residues cancels $\Rightarrow$ No worry about truncation effects here

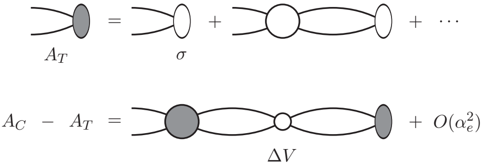
Situation changes when the transverse radiation part is included: $\frac{1}{\vec{q}^2} \to \frac{1}{q^2}$

Conclusion

- It can be foreseen that ϵ' will reach the precision of O(10%)
- Important to include the EM corrections, as enhanced by $\Delta I = 1/2$ rule
- To determine the EM correction, we try to solve three problems
  - Encode EM into Lüscher and Lellouch-Lüscher formalism
    $\Rightarrow$ Introduce truncated Coulomb potential
  - Solve the issue for the mixing between $I = 0$ and 2 channel
    $\Rightarrow$ Coupled channel problem simplified due to α_e-expansion
  - Remaining issue: Include the transverse radiation
- Pave the way for the realistic calculation of EM corrections $K \to \pi\pi$

Backup slides

$\sigma \to \pi\pi$ decay amplitude



The relation for decay amplitude

$$A_C - A_T = \langle E, -, T|\Delta V\, G_{TS}^{(+)}|\sigma\rangle = \langle E, -, T|\Delta V\, G_0^{(+)}\left(1 + V_{TS}G_{TS}^{(+)}\right)|\sigma\rangle$$

- ΔV is non-zero at $r > R_T$; $V_{TS} = V_s + V^{(T)}$ is non-zero at $r < R_T$
- The free Green function $\langle \mathbf{r}|G_0^{(+)}|\mathbf{r}'\rangle$ for $r > R_T$ and $r' < R_T$ is given by

$$\langle \mathbf{r}|G_0^{(+)}|\mathbf{r}'\rangle = \int \frac{dE'}{2\pi}\langle \mathbf{r}|E'\rangle \frac{1}{E - E' + i\varepsilon}\langle E'|\mathbf{r}'\rangle \quad \xrightarrow{r>r'} \quad -\frac{1}{2}\sqrt{\frac{\mu}{\pi k}}\frac{e^{ikr}}{r}\langle E|\mathbf{r}'\rangle$$

Truncation effects can be determined

$$\boxed{A_C - A_T = \int d^3\mathbf{r}\, \psi_T^{(-)*}(r)\frac{\alpha}{r}\left(-\frac{1}{2}\sqrt{\frac{\mu}{\pi k}}\frac{e^{ikr}}{r}\right)A_T}$$

Mixing of isospin states

Focus on Coulomb potential, no $\pi\pi\gamma$ state

However, $I=2$ and $I=0$ $\pi\pi$ states still mix with each other

- No EM: relation between charged $c=+-,00$ and isopsin $s=0,2$ $\pi\pi$ states

$$|(\pi\pi)_c\rangle^{\rm out} = \sum_{s=0,2} \Omega_{cs} |(\pi\pi)_s\rangle^{\rm out}, \quad \Omega_{cs} = \begin{pmatrix} \sqrt{2}/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{3} & \sqrt{2}/\sqrt{3} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- With EM:

$$|(\pi\pi)^\gamma_c\rangle^{\rm out} = \sum_{s=0,2} \Omega^\gamma_{cs} |(\pi\pi)^\gamma_s\rangle^{\rm out}, \quad \Omega^\gamma_{cs} = \begin{pmatrix} \cos\theta^\gamma & \sin\theta^\gamma \\ -\sin\theta^\gamma & \cos\theta^\gamma \end{pmatrix}$$

Define ${}^{\rm out}\langle(\pi\pi)^\gamma_s|H_W|K^0\rangle = e^{i\delta^\gamma_s} A^\gamma_s$

$$\epsilon' = \frac{1}{3}(\eta_{+-} - \eta_{00}) = \frac{\sin 2\theta}{\sin 2\theta^\gamma} \frac{i e^{i(\delta^\gamma_2 - \delta^\gamma_0)}}{\sqrt{2}} \frac{\mathrm{Re}\, A^\gamma_2}{\mathrm{Re}\, A^\gamma_0} \left(\frac{\mathrm{Im}\, A^\gamma_2}{\mathrm{Re}\, A^\gamma_2} - \frac{\mathrm{Im}\, A^\gamma_0}{\mathrm{Re}\, A^\gamma_0} \right)$$

$\frac{\sin 2\theta}{\sin 2\theta^\gamma}$ is a small correction $\Rightarrow$ focus on A^γ_s and δ^γ_s

Turn off EM and calculate correlators with $I = 0, 2$ operators

$$
\begin{aligned}
C_{II'}(t) &= \langle \phi_{\pi\pi,I}(t)\phi^\dagger_{\pi\pi,I'}(0)\rangle \\
&= \sum_{s=0,2} \langle 0|\phi_{\pi\pi,I}|(\pi\pi)_s\rangle e^{-E_s t}\langle(\pi\pi)_s|\phi^\dagger_{\pi\pi,I'}|0\rangle \delta_{s,I}\delta_{s,I'} \\
&= (UMU^\dagger)_{II'}
\end{aligned}
$$

where

$$
U = \begin{pmatrix} \langle 0|\phi_{\pi\pi,0}|(\pi\pi)_0\rangle & 0 \\ 0 & \langle 0|\phi_{\pi\pi,2}|(\pi\pi)_2\rangle \end{pmatrix}, \quad M = \begin{pmatrix} e^{-E_0 t} & \\ & e^{-E_2 t} \end{pmatrix}
$$

Turn on EM and calculate correlators with the same operators

$$
\begin{aligned}
C^\gamma_{II'}(t) &= \langle \phi_{\pi\pi,I}(t)\phi^\dagger_{\pi\pi,I'}(0)\rangle^\gamma \\
&= \sum_{s=0,2} {}^\gamma\langle 0|\phi_{\pi\pi,I}|(\pi\pi)^\gamma_s\rangle e^{-E^\gamma_s t}\langle(\pi\pi)^\gamma_s|\phi^\dagger_{\pi\pi,I'}|0\rangle^\gamma \\
&= (U^\gamma M^\gamma U^{\gamma\dagger})_{II'}
\end{aligned}
$$

where

$$
U^\gamma = \begin{pmatrix} {}^\gamma\langle 0|\phi_{\pi\pi,0}|(\pi\pi)^\gamma_0\rangle & {}^\gamma\langle 0|\phi_{\pi\pi,0}|(\pi\pi)^\gamma_2\rangle \\ {}^\gamma\langle 0|\phi_{\pi\pi,2}|(\pi\pi)^\gamma_0\rangle & {}^\gamma\langle 0|\phi_{\pi\pi,2}|(\pi\pi)_2\rangle \end{pmatrix}, \quad M^\gamma = \begin{pmatrix} e^{-E^\gamma_0 t} & \\ & e^{-E^\gamma_2 t} \end{pmatrix}
$$

Determination of A_s^γ and δ_s^γ from lattice QCD

- Use the coefficient matrix to construct a ratio $U^{-1}U^\gamma = 1 + \begin{pmatrix} N_{00}^{(1)} & N_{02}^{(1)} \\ N_{20}^{(1)} & N_{22}^{(1)} \end{pmatrix}$

- Build a ratio for the 2×2 correlation matrix: $R(t) = C^{-\frac{1}{2}}(t) C^\gamma(t) C^{-\frac{1}{2}}(t)$

- Time dependence of $R(t)$ yields

$$R(t) = \begin{pmatrix} 1 + 2N_{00}^{(1)} + E_0^{(1)} t & N_{20}^{(1)} e^{(E_2 - E_0)t/2} + N_{02}^{(1)} e^{(E_0 - E_2)t/2} \\ N_{20}^{(1)} e^{(E_2 - E_0)t/2} + N_{02}^{(1)} e^{(E_0 - E_2)t/2} & 1 + 2N_{22}^{(1)} + E_2^{(1)} t \end{pmatrix}$$

  - $E_s^{(1)} = E_s^\gamma - E_s$ can be used to determine δ_s^γ, $s = 0, 2$
  - $N_{II'}^{(1)}$ can be used to construct U^γ and compute $A_s^\gamma = \langle (\pi\pi)_s^\gamma | H_W | K^0 \rangle$

Need to modify Lüscher quantization condition and Lellouch-Lüscher relation to include EM effects