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Towards non-perturbative
renormalization of $\Delta S = 1$
four-quark operators with a
position-space procedure



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WMEs w/ 3-flavor LQCD

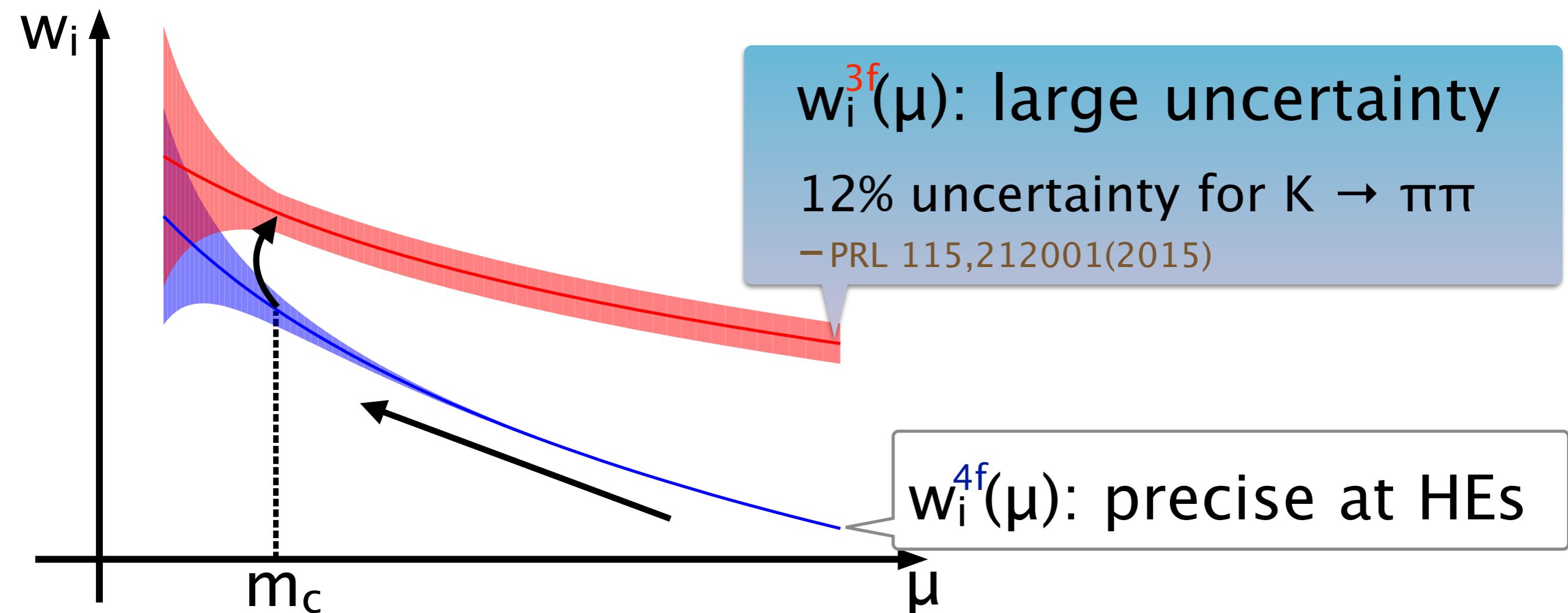
$$\langle f | H_w | i \rangle = \sum_i w_i(\mu) \frac{\langle f | O_i(\mu) | i \rangle}{\text{pQCD}} - \frac{\langle f | O_i(\mu) | i \rangle}{\text{LQCD}}$$

WMEs w/ 3-flavor LQCD

$$\langle f | H_w | i \rangle = \sum_i w_i^{3f}(\mu) \frac{\langle f | O_i^{3f}(\mu) | i \rangle}{\text{LQCD}}$$

WMEs w/ 3-flavor LQCD

$$\langle f | H_w | i \rangle = \sum_i w_i^{3f}(\mu) \frac{\langle f | O_i^{3f}(\mu) | i \rangle}{\text{LQCD}}$$

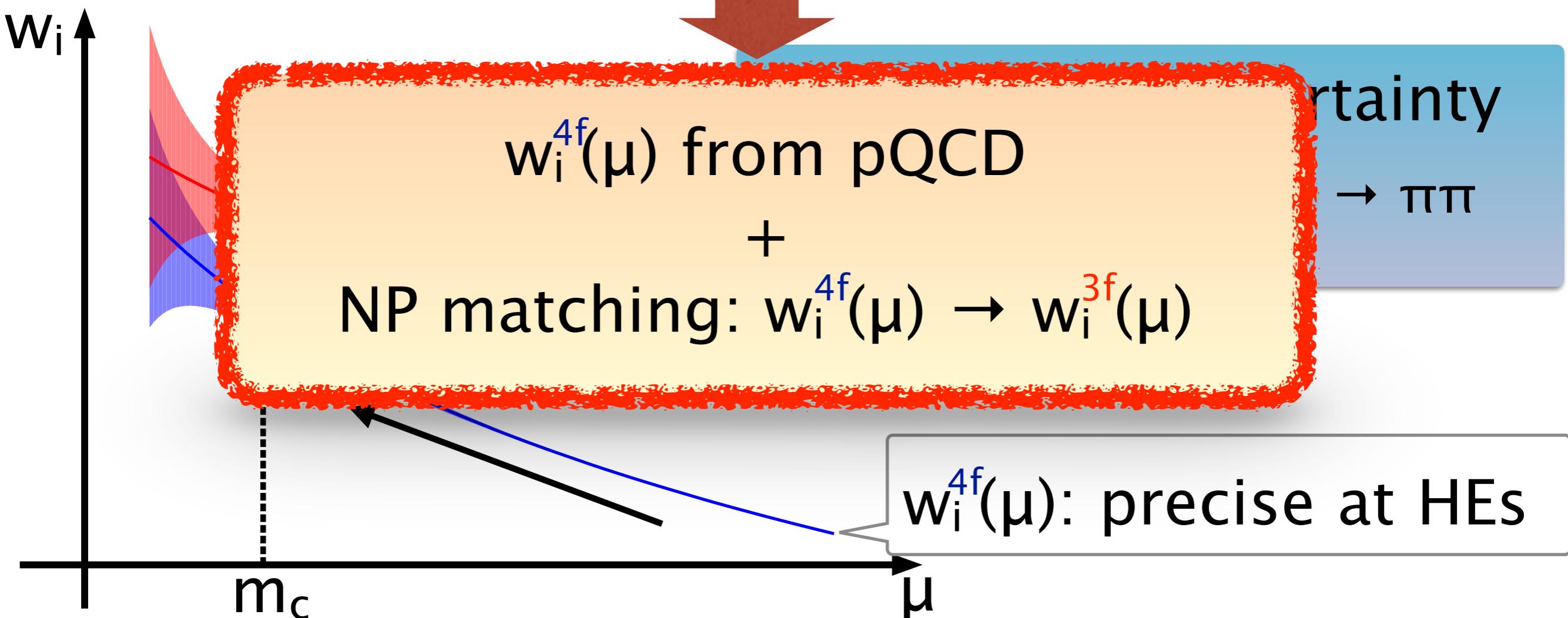


WMEs w/ 3-flavor LQCD

$$\langle f | H_w | i \rangle = \sum_i w_i^{3f}(\mu) \frac{\langle f | O_i^{3f}(\mu) | i \rangle}{LQCD}$$

~~pQCD~~

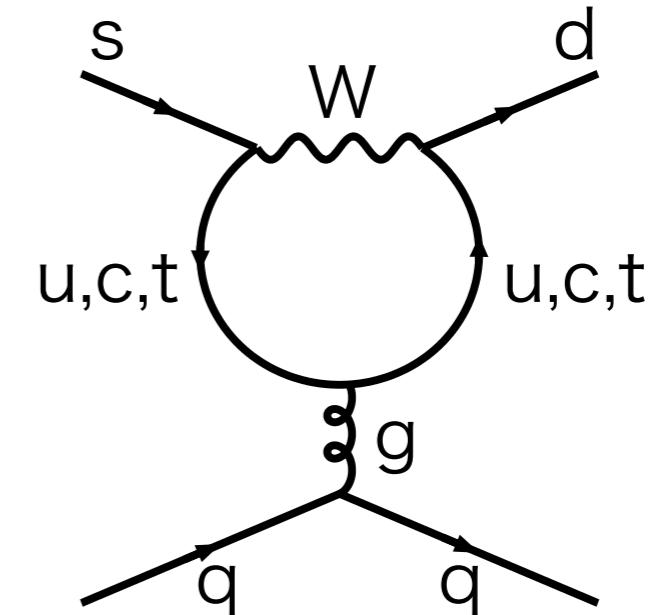
LQCD



Valence charm significant

- Operators in 4f theory w/ charm
 - Ex1) QCD penguin operators:

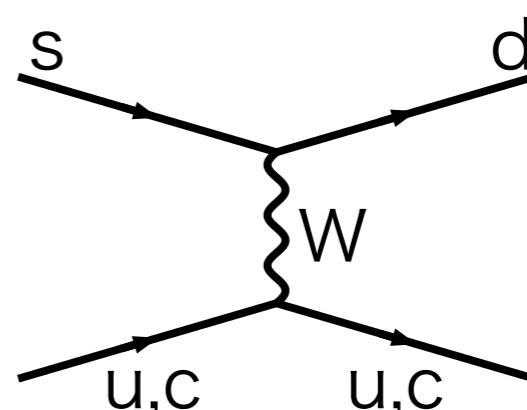
$$Q_i = (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\pm A}$$



- Ex2) current-current operators:

$$Q_i^u = (\bar{s}d)_{V-A} (\bar{u}u)_{V-A}$$

$$Q_i^c = (\bar{s}d)_{V-A} (\bar{c}c)_{V-A}$$



- Could affect significantly ($\sim 10\%$)
 - NP matching $w_i^{4f} \rightarrow w_i^{3f}$ is important for such cases

Why position-space scheme?

- 4f–3f matching should be done at LDs
 $x \gg 1/m_c$ or $p \ll m_c$
- RI schemes affected by mixing w/ irrelevant Ops.
 - Due to gauge non-invariant, off-shell
 - A lot of irrelevant Ops. significant at LDs
- Position-space scheme
 - Gauge Inv. & on-shell
 - No mixing w/ irrelevant Ops.

Contents

- Introduction
- Strategy for NP 4f–3f matching
 - Two-point functions in position space
- Technique for reducing discretization errors
 - Average over spheres
 - Quark mass renormalization in position space w/ scalar correlator as an example

NP 4f–3f matching

$$H_w = \sum_i w_i^{4f}(\mu) O_i^{4f}(\mu) = \sum_i w_i^{3f}(\mu) O_i^{3f}(\mu)$$

$$\langle H_w(x) O_j^{3f}(\mu; y)^\dagger \rangle$$

$$\sum_i w_i^{4f}(\mu) G_{ij}^{4f-3f}(\mu; x-y) = \sum_i w_i^{3f}(\mu) G_{ij}^{3f-3f}(\mu; x-y)$$

$$G_{ij}^{nf-n'f}(\mu; x-y) = \langle O_i^{nf}(\mu; x) O_j^{n'f}(\mu; y)^\dagger \rangle$$

$$w_i^{3f}(\mu) = \sum_{jk} w_j^{4f}(\mu) G_{jk}^{4f-3f}(\mu; x-y) (G^{3f-3f}(\mu; x-y))_{ki}^{-1}$$

※ Sea charm is neglected for simplicity but we have a certain procedure to take sea charm into account

Correlators on the lattice

- Needed for 4f-3f matching & NPR in position Sp.
 - Usually point-to-point correlators (no sum over t-slice)
- Discretization errors (DEs)
 - UV side of window problem
 - Significant even at LDs ($\sim \Lambda_{\text{QCD}}$)
 - Calculable only at lattice sites
 - › Hard to take continuum limit for renormalized quantities, step scaling functions, ...
- We propose a new treatment for DEs with an example of quark mass renormalization in position Sp.

Quark mass renormalization

- $Z_m = Z_S^{-1}$
- Position-space renormalization of scalar current

$$Z_S^{\overline{\text{MS}}/\text{lat}}(\mu, 1/a)^2 \Pi_S^{\text{lat}}(1/a; x) = \Pi_S^{\overline{\text{MS}}}(\mu; x)$$

$$Z_S^{\overline{\text{MS}}/\text{lat}}(\mu, 1/a) = \sqrt{\frac{\Pi_S^{\overline{\text{MS}}}(\mu; x)}{\Pi_S^{\text{lat}}(1/a; x)}}$$

- Π_S : two-point function of scalar currents
- $Z_S = Z_P$ if chirally symmetric lattice action (DWF in this work)
- We analyze

$$\tilde{m}_q^{\overline{\text{MS}}}(\mu; x; a) = m_q^{\text{bare, phys}}(a) \sqrt{\frac{\frac{1}{2}(\Pi_S^{\text{lat}}(1/a; x) + \Pi_P^{\text{lat}}(1/a; x))}{\Pi_S^{\overline{\text{MS}}}(\mu; x)}}$$

Lattice calculation

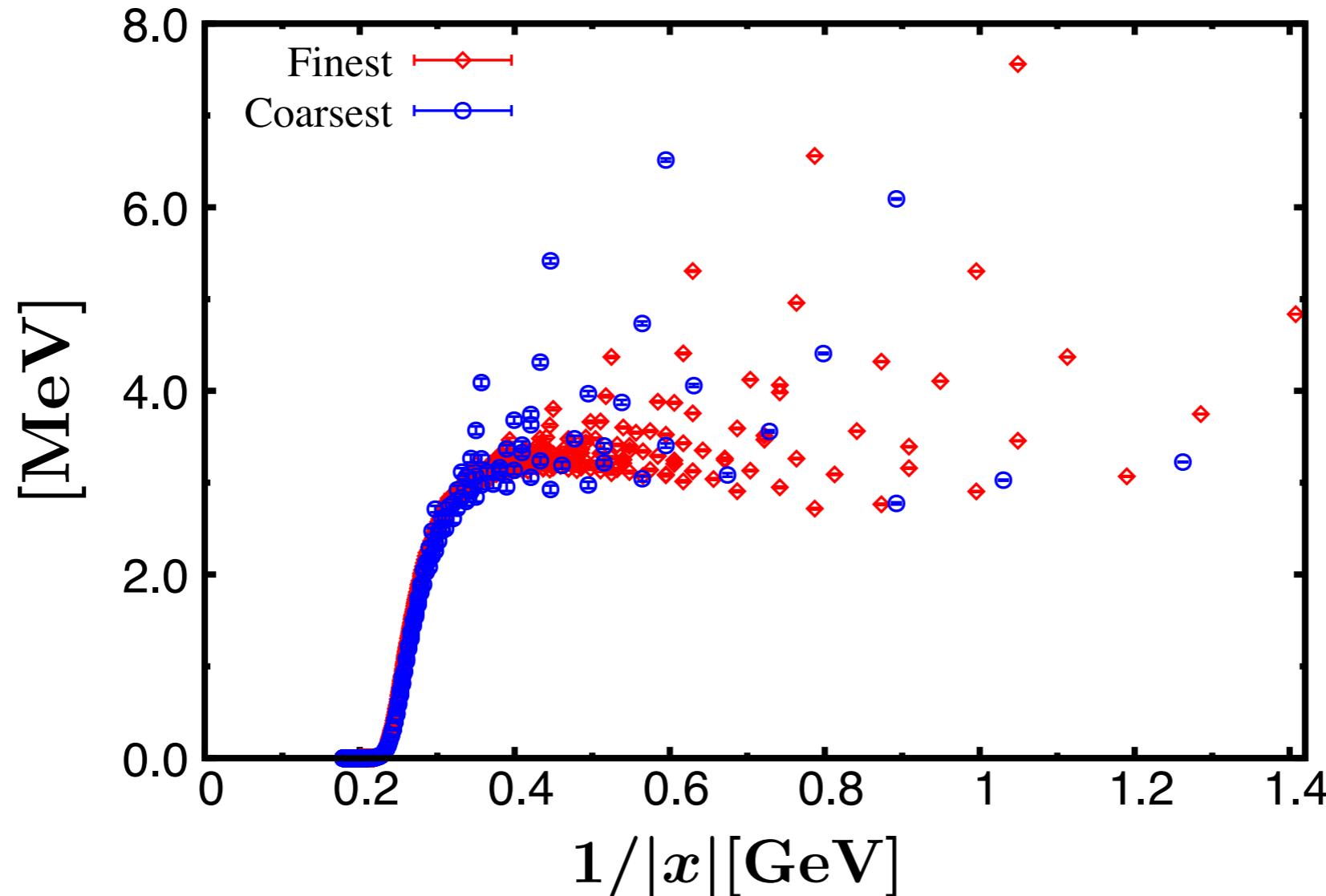
- Ensembles
 - 2+1 Domain-wall fermions
 - 3 lattice spacings: 1.7–3.1 GeV
 - Pion masses: 300–420 MeV
- For each ensemble, we analyze

$$\tilde{m}_q^{\overline{\text{MS}}}(\mu; x; a) = \frac{m_q^{\text{bare,phys}}(a)}{\sqrt{\frac{\frac{1}{2}(\Pi_S^{\text{lat}}(1/a; x) + \Pi_P^{\text{lat}}(1/a; x))}{\Pi_S^{\overline{\text{MS}}}(\mu; x)}}}$$

[RBC/UKQCD (2016)] 

Supposed to be a constant Z_m
if in renormalization window

$\tilde{m}_q^{\overline{\text{MS}}}(3 \text{ GeV}; x)$



- Different lattice points distinguished ($(1,1,1,1)$ vs $(0,0,0,2)$)
- Large discretization errors

Average over spheres

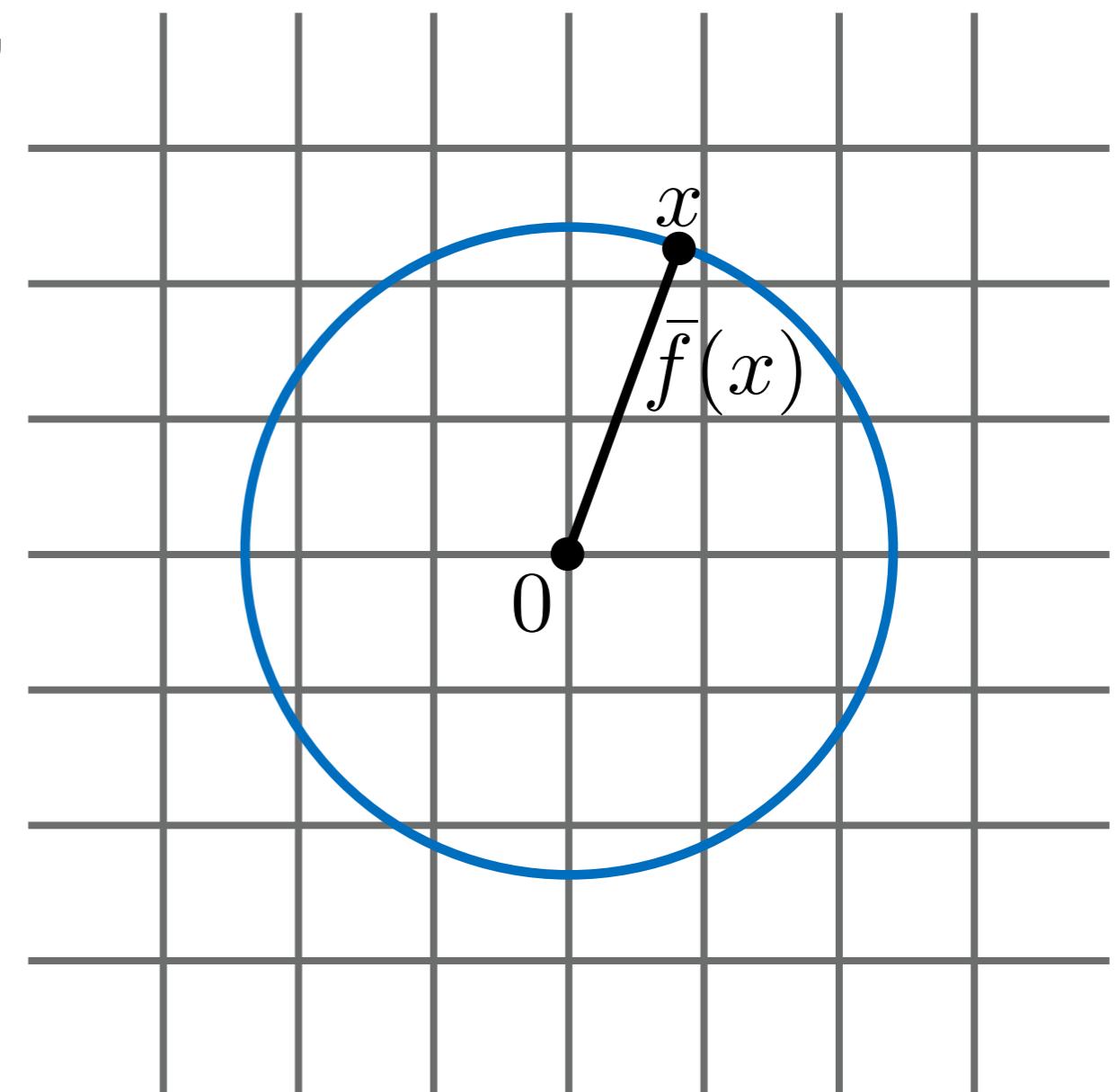
- Evaluate the value of a quantity at each 4d point from values at lattice points, with a guideline

$$\bar{f}(x) = \eta(f^{\text{lat}}; x)$$

※ details in next slides

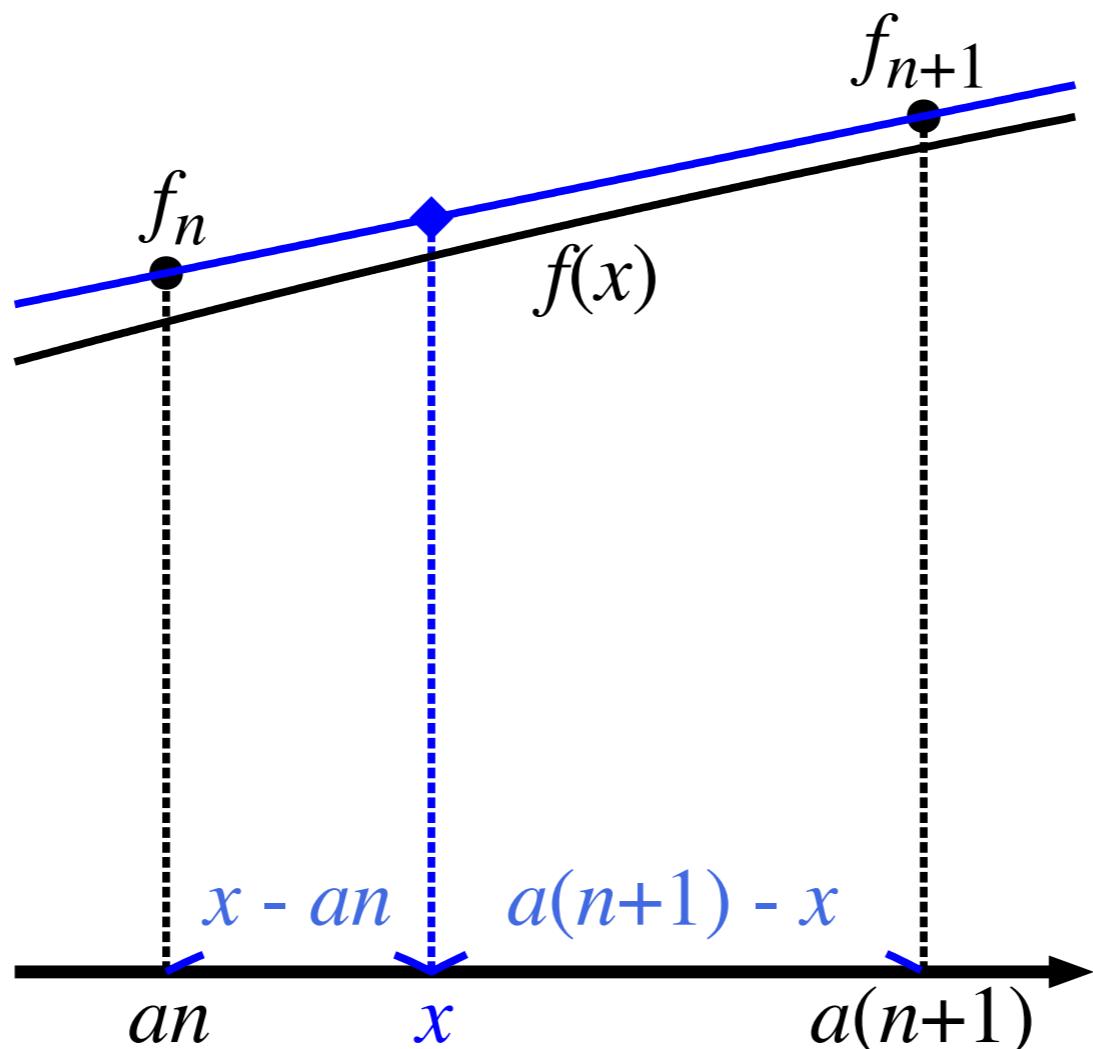
- Take the average over the sphere for each distance $|x|$

$$\hat{f}(|x|) = \frac{1}{2\pi^2} \oint_{S^3(|x|)} d\Omega \bar{f}(x)$$



Evaluation of $\bar{f}(x)$ (1-dim)

- Linear interpolation

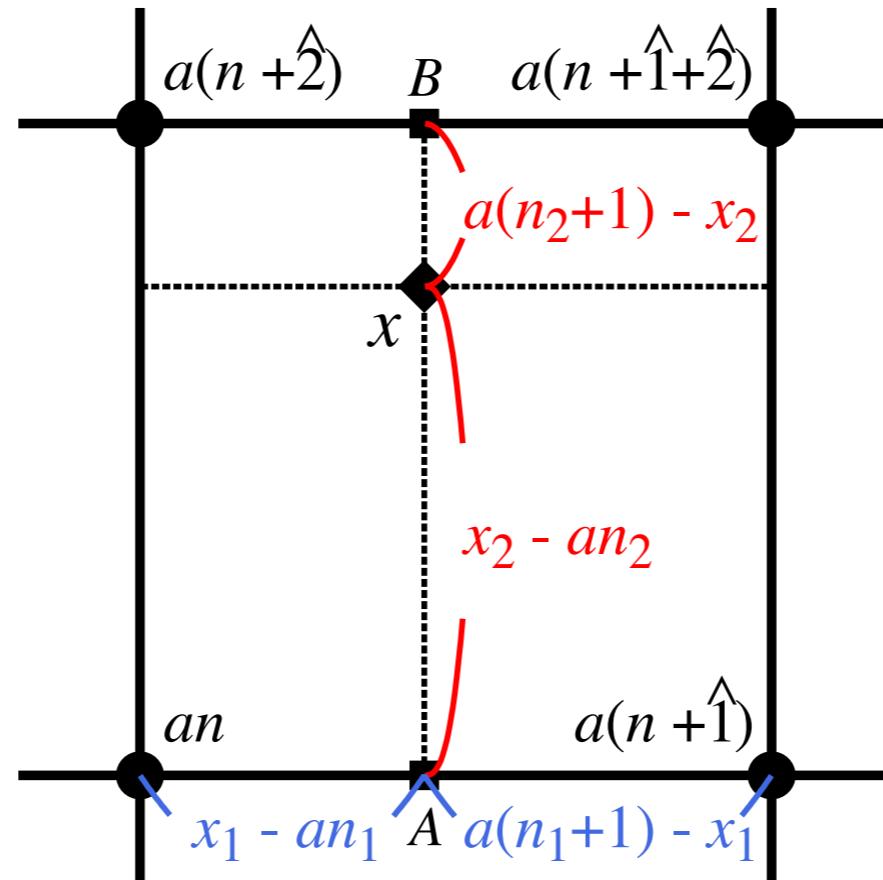


$$\bar{f}(x) = \frac{(a(n+1) - x)f_n + (x - an)f_{n+1}}{a} = f(x) + \underline{O(a^2)}$$

Accurate up to $O(a^2)$

Evaluation of $\bar{f}(x)$ (2-dim)

- Bilinear interpolation



$$\bar{f}(x) = \frac{(a(n_2 + 1) - x_2)\bar{f}(A) + (x_2 - an_2)\bar{f}(B)}{a}$$

$$= a^{-2} \begin{pmatrix} a(n_1 + 1) - x_1 & x_1 - an_1 \end{pmatrix} \begin{pmatrix} f_n & f_{n+2} \\ f_{n+1} & f_{n+1+2} \end{pmatrix} \begin{pmatrix} a(n_2 + 1) - x_2 \\ x_2 - an_2 \end{pmatrix}$$

$$= f(x) \underline{+ O(a^2)}$$

Evaluation of $\bar{f}(x)$ (4-dim)

- Quadrilinear interpolation

$$\bar{f}(x) = a^{-4} \sum_{i,j,k,l=0}^1 \Delta_{1,i} \Delta_{2,j} \Delta_{3,k} \Delta_{4,l} f_{n+i\hat{1}+j\hat{2}+k\hat{3}+l\hat{4}}$$

$$\Delta_{\mu,i} = |a(n_\mu + 1 - i) - x_\mu|$$

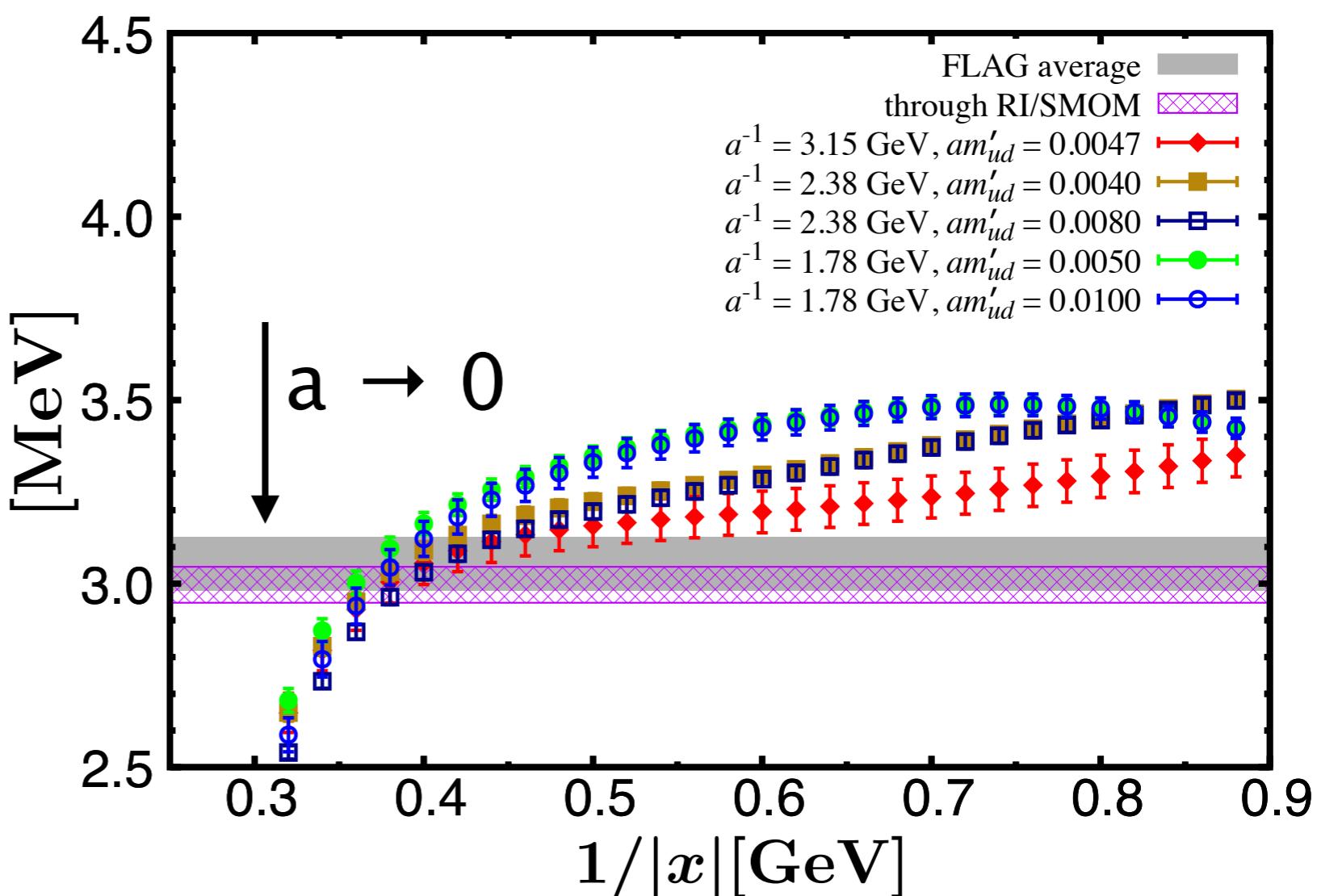
- Accurate up to $O(a^2)$

Result for spherical average

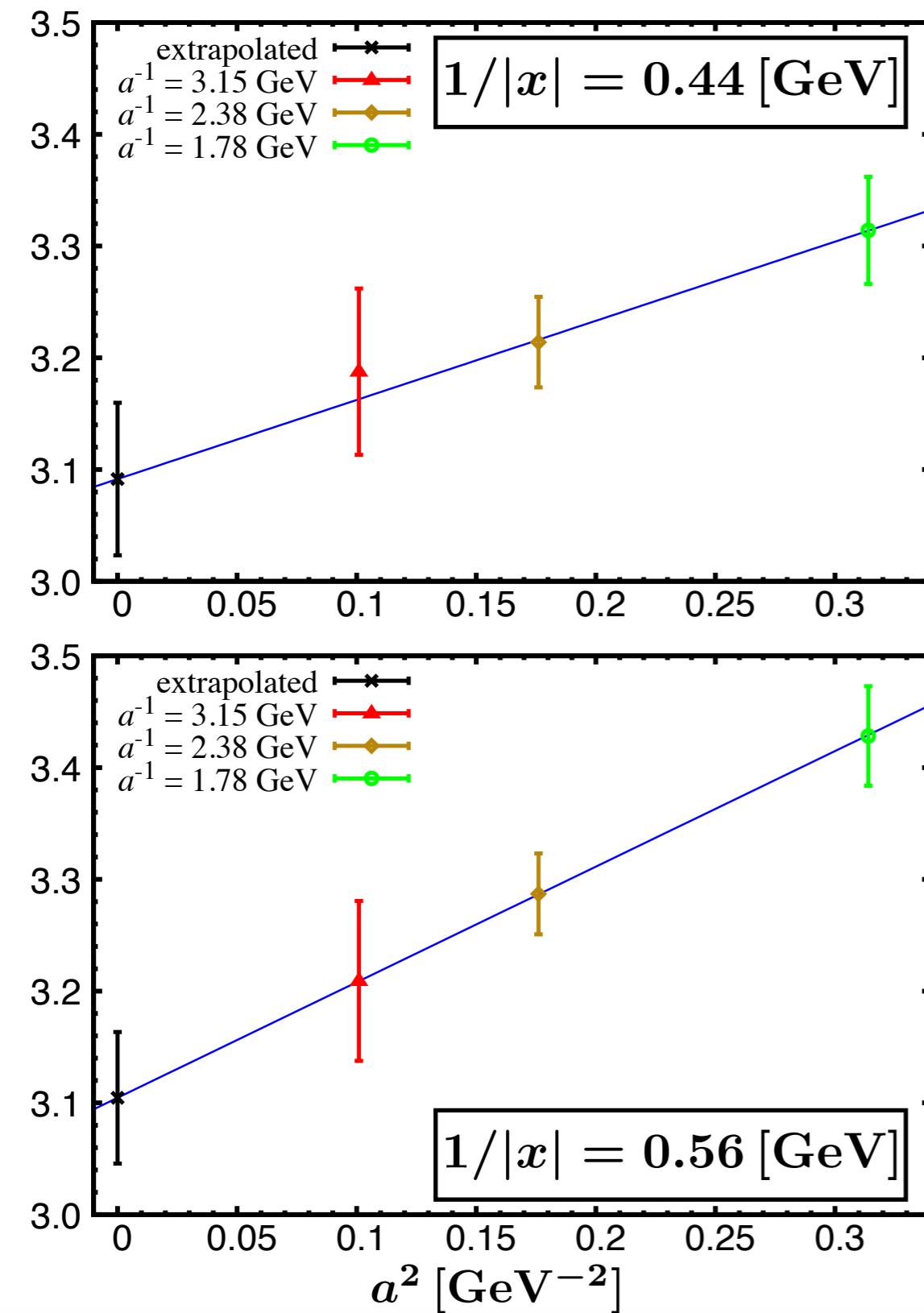
- Sphere average of

$$\tilde{m}_q^{\overline{\text{MS}}}(\mu; x; a) = m_q^{\text{bare,phys}}(a) \sqrt{\frac{\frac{1}{2}(\Pi_S^{\text{lat}}(1/a; x) + \Pi_P^{\text{lat}}(1/a; x))}{\Pi_S^{\overline{\text{MS}}}(\mu; x)}}$$

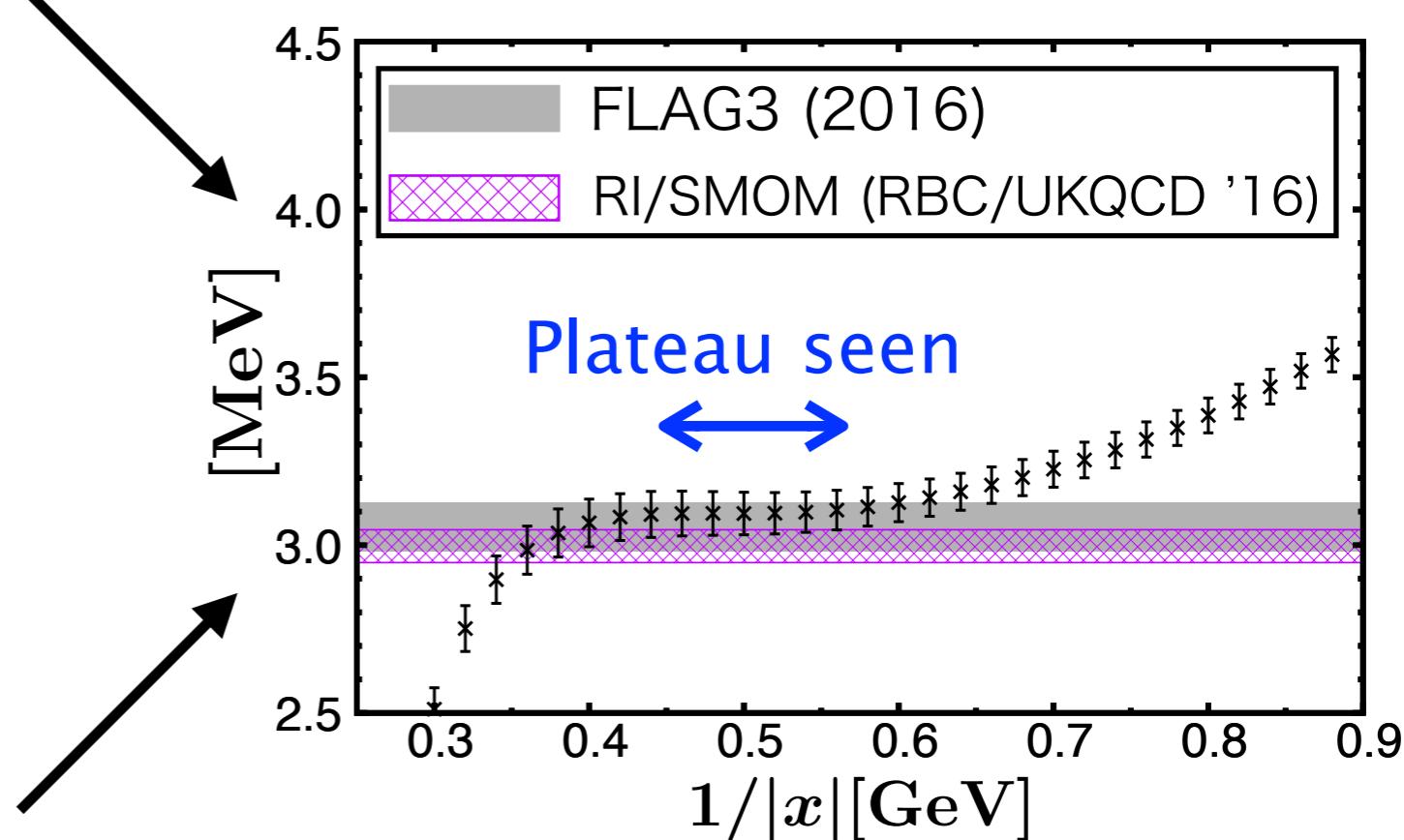
- Able to calculate at any distance
- Plateau seen better for finer lattices



Continuum limit



Chiral limit of Z_m is
simultaneously taken



$$m_{ud}^{\overline{MS}}(3 \text{ GeV})|_{2+1f} = 3.09 (6)_{\text{stat}} (6)_{\text{sys}}$$

$$m_s^{\overline{MS}}(3 \text{ GeV})|_{2+1f} = 85.3 (1.6)_{\text{stat}} (1.7)_{\text{sys}}$$

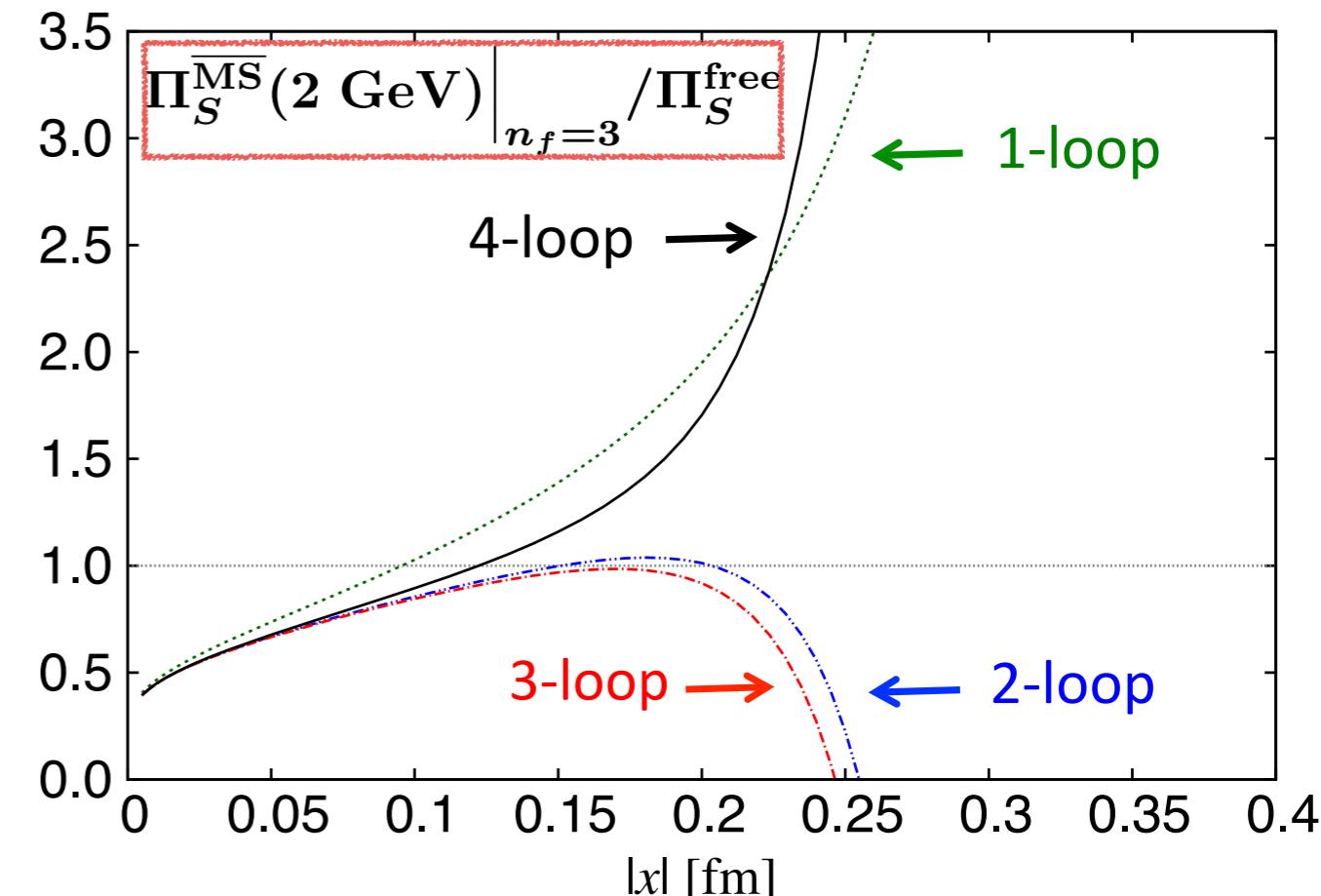
$\overline{\text{MS}}$ correlator

- Available up to $O(\alpha_s^4)$ [Chetyrkin, Maier (2011)]
- Bad convergence of original perturbative series

$$\begin{aligned} & \Pi_S^{\overline{\text{MS}}}(\mu_x = 1/x; x) \Big|_{n_f=3} \\ &= \frac{3}{\pi^4 x^6} \left(1 + 0.20 a_s - 19 a_s^2 - 11 a_s^3 + \color{red}{579 a_s^4} \right) \\ & \quad (a_s = \alpha_s(\mu_x = 1/|x|)/\pi) \end{aligned}$$

- large coef. at $O(a_s^4)$
- a_s blows up at ~ 0.3 fm

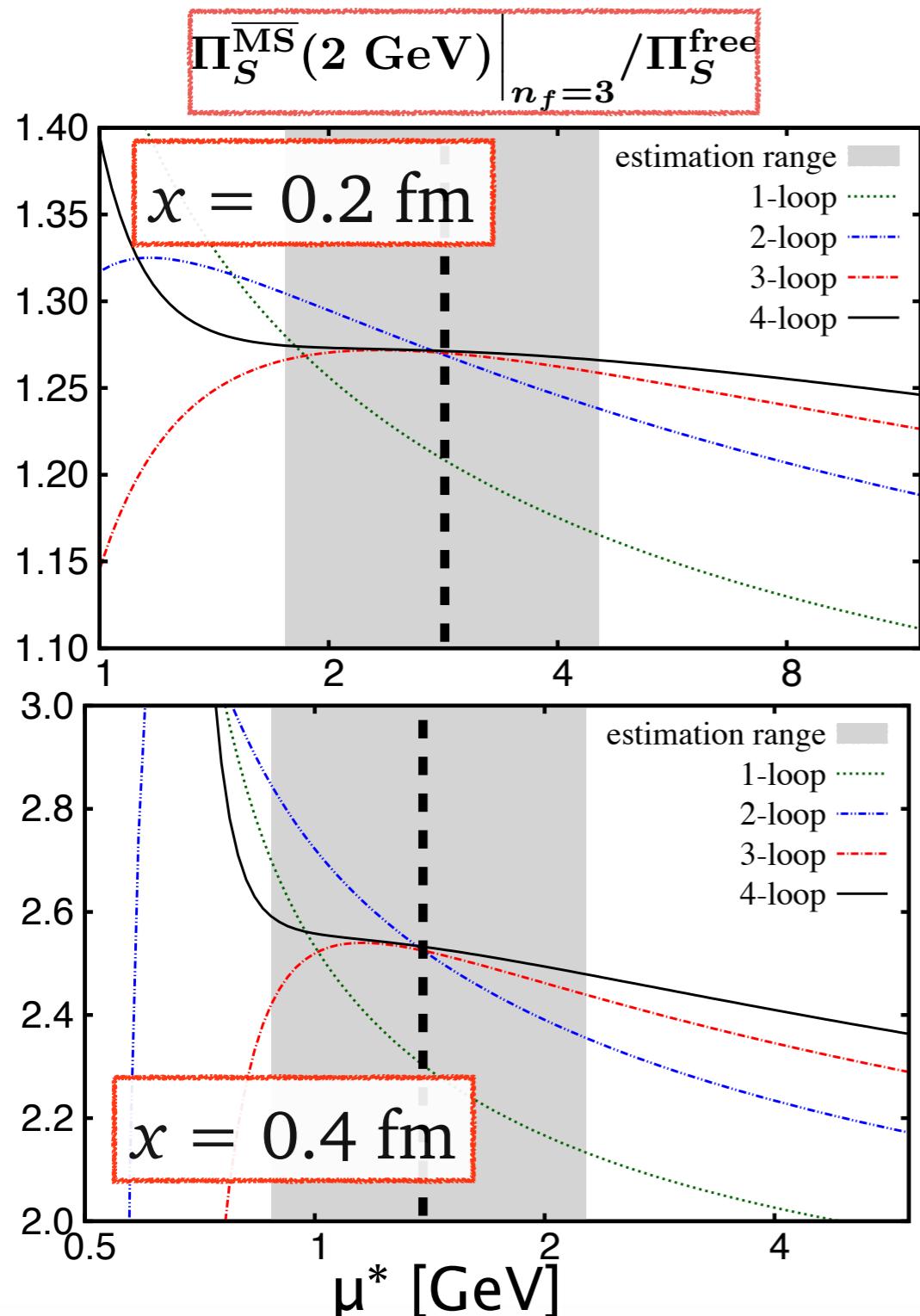
- No window for recently available lattice spacings



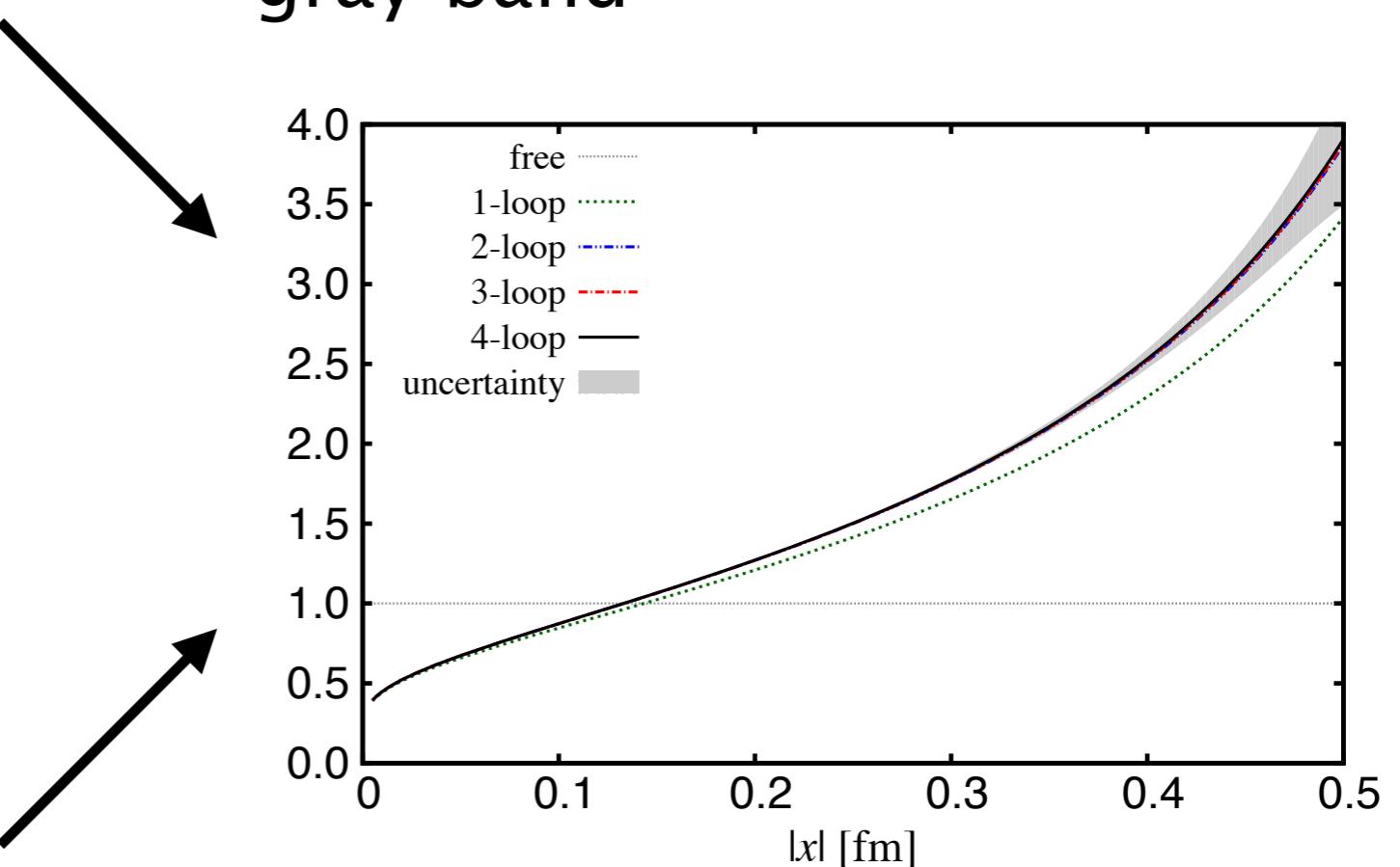
Scale of α_s

[MT et al (2015)]

- Dependence on the scale μ^* of α_s



- We use $\mu_x^* = 2.9/x$
- Truncation error estimated by gray band



Summary

- Position-space NPR is convenient for 4f-3f matching
 - Gauge invariant & on-shell
 - Free from mixing w/ irrelevant operators
- New treatment for discretization errors is proposed
 - Interpolation + Spherical average
 - Likely working well for quark mass renormalization
(agrees with FLAG & our previous result through RI/SMOM)
- Renormalization condition in position-space scheme for operators that mix is being investigated (will allow 4f-3f matching including sea effects)

NP 4f–3f matching

Guideline:

$$H_w = \sum_i w_i^{4f}(\mu) O_i^{4f}(\mu) = \sum_i w_i^{3f}(\mu) O_i^{3f}(\mu)$$

- $w_i^{3f}(\mu)$ can be obtained by comparing
 - $w_i^{4f}(\mu)$ from pQCD
 - $\langle \bullet' | O_i^{4,3f}(\mu) | \bullet \rangle$ and/or $\langle O_i^{4,3f}(\mu; x) O'(0) \rangle$ etc... from lattice
- Guideline invalid for SD physics ($x < 1/m_c$ or $p > m_c$)
 - Has to be done at LDs ($x \gg 1/m_c$)
 - RI schemes induce huge mixing w/ irrelevant Ops.
 - Gauge Inv. & on-shell schemes desired

Treatments for DEs in previous

- Free-field improvement

$$\Pi^{\text{lat}}(x) \rightarrow \Pi^{\text{lat}}(x) - (\Pi^{\text{lat,free}}(x) - \Pi^{\text{cont,free}}(x))$$

or

$$\Pi^{\text{lat}}(x) \rightarrow \Pi^{\text{lat}}(x) \frac{\Pi^{\text{cont,free}}(x)}{\Pi^{\text{lat,free}}(x)}$$

- May work well at SDs
- Might not work at LDs, where 4f-3f matching will be done

- Preferring (1,1,1,1)-direction, Hating (1,0,0,0)-direction
 - Disables or inconsistent w/ sphere average

Scale of α_s

- Tuning the scale of α_s
 - $a_s(\mu) = a_s(\mu^*)(1 + a_s(\mu^*) \beta_0 \ln(\mu^{*2}/\mu^2) + \dots)$
 - $c_0 + c_1 a_s(\mu) + c_2 a_s(\mu)^2 + \dots$
 $= c_0 + c_1 a_s(\mu^*) + c_2^* a_s(\mu^*)^2 + \dots$
- Equivalent in all order calculation