Semi-leptonic form factors for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$

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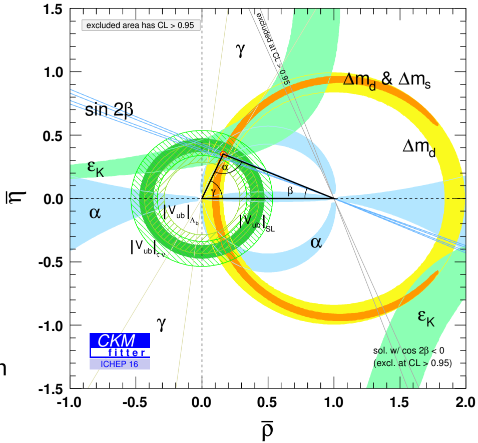
introduction

Why B_s meson decays?

- Alternative, tree-level determination of $|V_{cb}|$ and $|V_{ub}|$ from $B_s \to D_s\ell\nu$ and $B_s \to K\ell\nu$
  - → Commonly used $B \to \pi\ell\nu$ and $B \to D^{(*)}\ell\nu$
  - → Long standing $2-3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$)
  - → $B \to \tau\nu$ has larger error
  - → Alternative, exclusive ($\Lambda_b \to p\ell\nu$) determination [Detmold, Lehner, Meinel, PRD92 (2015) 034503]



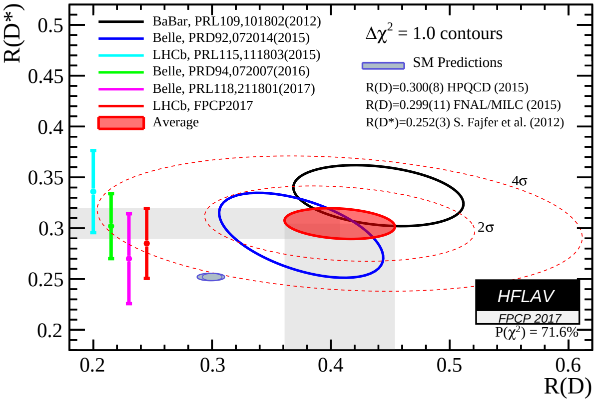
[http://ckmfitter.in2p3.fr]

Why B_s meson decays?

[HFLAV]

- Alternative tests of lepton flavor violations
  - → Determine e.g. $R_{D_s^{(*)}}$ from B_s decays to compare with $R_{D^{(*)}}$ from B decays

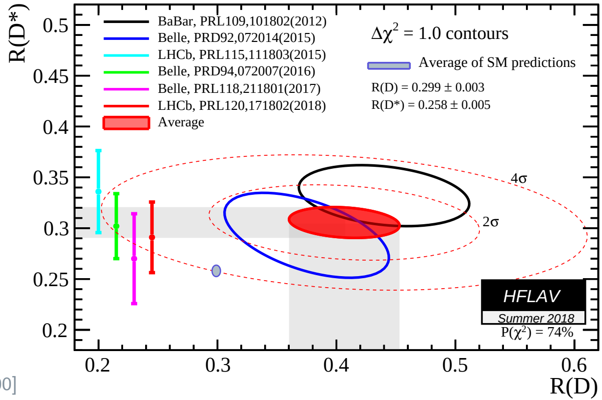
$$R^{\tau/\mu}_{D^{(*)}} \equiv \frac{BF(B \to D^{(*)}\tau\nu_\tau)}{BF(B \to D^{(*)}\mu\nu_\mu)}$$



- Nonperturbative lattice calculation favor B_s over B decays (higher precision)
- Only the spectator quark differs: $R_{D_s^{(*)}}$ may be a good proxy for $R_{D^{(*)}}$

Why B_s meson decays?

[HFLAV]



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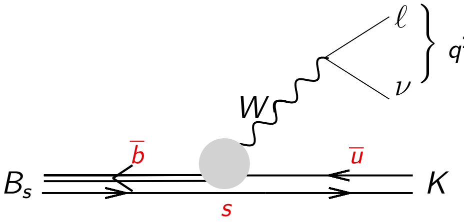
- HFLAV updated SM prediction, $R_{D^{(*)}}$: averaging [Bigi, Gambino PRD94(2016)094008] [Bernlochner, Ligeti, Papucci, Robinson PRD95(2017)11500] [Bigi, Gambino, Schacht JHEP11(2017)061][Jaiswal, Nandi, Patra JHEP12(2017)060]

- Nonperturbative lattice calculation favor B_s over B decays (higher precision)
- Only the spectator quark differs: $R_{D_s^{(*)}}$ may be a good proxy for $R_{D^{(*)}}$

$|V_{ub}|$ from exclusive semileptonic $B_s \to K\ell\nu$ decay



$q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K$

- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

$$\frac{d\Gamma(B_s\to K\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3}\left[\left(M_{B_s}^2 + M_K^2 - q^2\right)^2 - 4M_{B_s}^2 M_K^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{ub}|^2$$

experiment known nonperturbative input CKM

Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD

$\Rightarrow$ Nonperturbative calculation: lattice QCD

$\rightarrow$ Additional challenge $m_b = 4.18\text{GeV} \sim 1000 \times m_d$

$m_c = 1.28\text{GeV} \sim 270 \times m_d$

Set-up

- RBC-UKQCD's 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - → Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions
    [PRD 78 (2008) 114509][PRD 83 (2011) 074508][PRD 93 (2016) 074505][JHEP 1712 (2017) 008]

- Unitary and partially quenched domain-wall up/down quarks
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Domain-wall strange quarks at/near the physical value
- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  - → Simulate 3 or 2 charm-like masses then extrapolate/interpolate

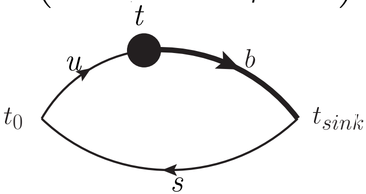
- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  - → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  - → Allows to tune the three parameters $(m_0 a,\ c_P,\ \zeta)$ nonperturbatively [PRD 86 (2012) 116003]
  - → Smooth continuum limit; heavy quark treated to all orders in $(m_b a)^n$

$$B_s \to K \ell \nu$$

$B_s \to K\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$



- Calculate 3-point function by
  - → Inserting a quark source for a "light" propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Use another "light" quark propagating from t_0 and contract both at t

Determining $B_s \to K\ell\nu$ form factors f_+ and f_0 on the lattice

- Updating calculation [PRD 91 (2015) 074510] with new values for a^{-1} and RHQ parameters
- New analysis directly fitting form factors and accounting for excited state contributions
- On the lattice we prefer using the B_s-meson rest frame and compute

$$f_{\parallel}(E_K) = \langle K|V^0|B_s\rangle/\sqrt{2M_{B_s}} \quad \text{and} \quad f_{\perp}(E_K)p_K^i = \langle K|V^i|B_s\rangle/\sqrt{2M_{B_s}}$$

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2}\left[(M_{B_s} - E_K)f_{\parallel}(E_K) + (E_K^2 - M_K^2)f_{\perp}(E_K)\right]$$

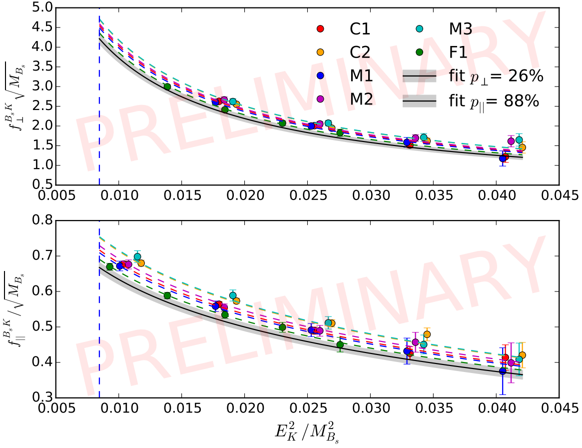
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_{\parallel}(E_K) + (M_{B_s} - E_K)f_{\perp}(E_K)\right]$$

Chiral-continuum extrapolation using SU(2) hard-kaon χPT

$$f_\perp(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_\perp^{(1)} \times \left[1 + \frac{\delta f_\perp}{(4\pi f)^2} + c_\perp^{(2)} \frac{M_K^2}{\Lambda^2} + c_\perp^{(3)} \frac{E_K}{\Lambda} + c_\perp^{(4)} \frac{E_K^2}{\Lambda^2} + c_\perp^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right]$$

$$f_\parallel(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c_\parallel^{(1)} \times \left[1 + \frac{\delta f_\parallel}{(4\pi f)^2} + c_\parallel^{(2)} \frac{M_K^2}{\Lambda^2} + c_\parallel^{(3)} \frac{E_K}{\Lambda} + c_\parallel^{(4)} \frac{E_K^2}{\Lambda^2} + c_\parallel^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4}\right]$$

with δf non-analytic logs of the kaon mass
and hard-kaon limit is taken by $M_K/E_K \to 0$



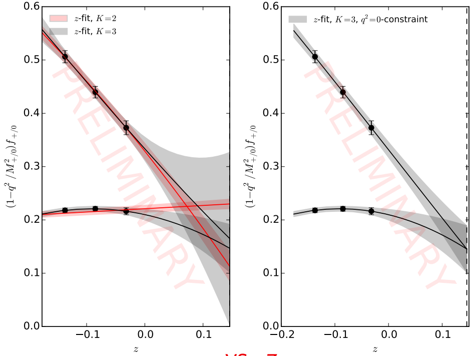
▶ Error budget not **yet** released for presentation

Kinematical extrapolation (z-expansion)

- Map q^2 to z with minimized magnitude in the semileptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}} \quad \text{with} \quad \begin{aligned} t_\pm &= (M_B \pm M_\pi)^2 \\ t_0 \equiv t_{\text{opt}} &= (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2 \end{aligned}$$

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603]
[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]



vs. z

- Express f_+ as convergent power series
- f_0 is analytic, except for B^* pole
- BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV
- Exploit kinematic constraint $f_+ = f_0$ at $q^2 = 0$

→ Include HQ power counting to constrain size of f_+ coefficients

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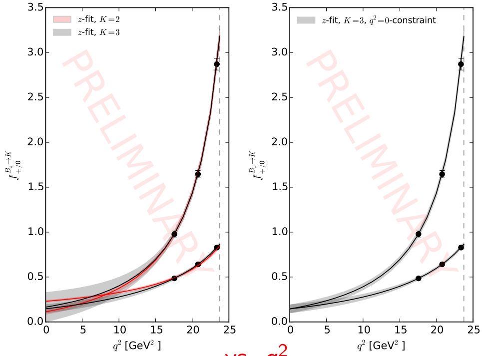


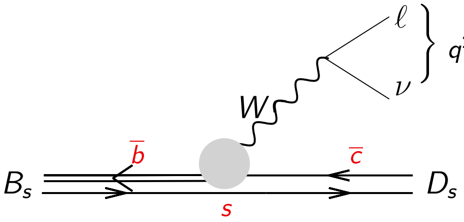
vs. q^2

- Allows to compare shape of form factors
  - → Obtained by other lattice calculations
  - → Predicted by QCD sum rules and alike

- Combination with experiment leads to the overall normalization: $|V_{ub}|$

$$B_s \to D_s \ell \nu$$

$|V_{cb}|$ from exclusive semileptonic $B_s \to D_s\ell\nu$ decay



$$q^2 = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}E_{D_s}$$

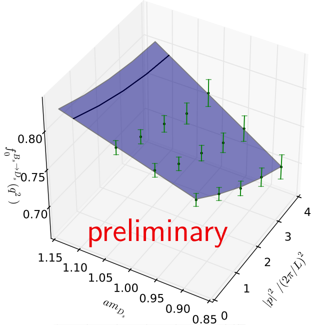
- Conventionally parametrized by (neglecting term $\propto m_\ell^2 f_0^2$)

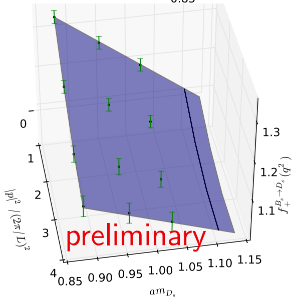
$$\frac{d\Gamma(B_s \to D_s\ell\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 M_{B_s}^3}\left[\left(M_{B_s}^2 + M_{D_s}^2 - q^2\right)^2 - 4M_{B_s}^2 M_{D_s}^2\right]^{3/2} \times |f_+(q^2)|^2 \times |V_{cb}|^2$$

experiment known nonperturbative input CKM

Charm extra-/interpolation for $B_s \to D_s\ell\nu$

- Simulate charm quarks using MDWF
  - → Similar action as for u, d, s quarks
  - → "Fully" relativistic setup simplifies renormalization
  - → Established by calculating $f_{D_{(s)}}$
    [Boyle et al. JHEP 1712 (2017) 008]
- Coarse ensembles
  - → Extrapolate three charm-like masses
- Medium and fine ensembles
  - → Interpolate between two charm-like masses
- Analysis of data at third, finer lattice spacing will help to better estimate uncertainty

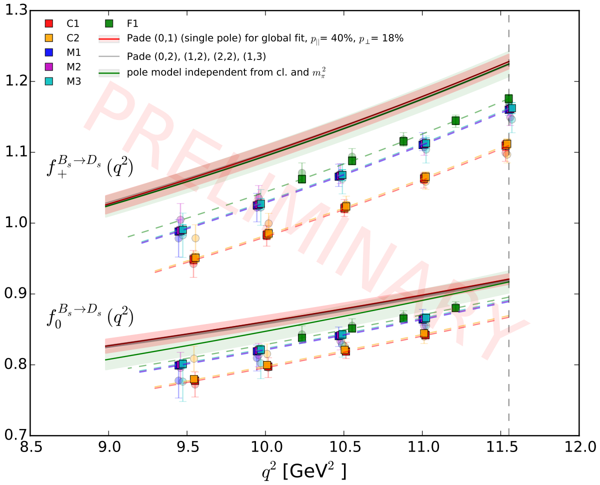




Chiral-continuum extrapolation

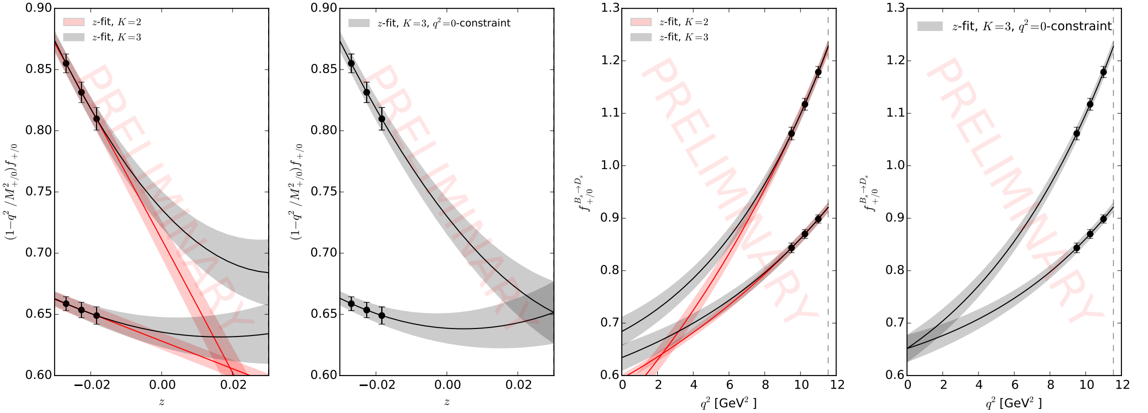
- No light valence quarks, no need for χPT
- Account for dependence on
  - $\to$ charm quark mass
  - $\to$ lattice spacing
  - $\to$ light sea-quark mass

$$f(q,a) = \frac{\alpha_0 + \alpha_1 M_{D_s} + \alpha_2 a^2 + \alpha_3 M_\pi^2}{1 + \alpha_4 q^2/M_{B_s}^2}$$



z-expansion

- BCL with poles $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV



conclusion

Conclusion

- In the final stages to complete $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ form factor calculation
  - → As usual, carefully estimating all systematic uncertainties is tedious
  - → Can even require additional simulations

- Our lattice calculation also includes
  - → $B \to \pi\ell\nu$, $B \to \pi\ell^+\ell^-$
  - → $B \to K^*\ell^+\ell^-$
  - → $B \to D^{(*)}\ell\nu$
  - → $B_s \to K^*\ell^+\ell^-$
  - → $B_s \to D_s^*\ell\nu$
  - → $B_s \to \phi\ell^+\ell^-$
  - → ...

Resources and Acknowledgments

USQCD: Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab)
RBC qcdcl (RIKEN) and cuth (Columbia U)
UK: ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)

appendix

2+1 Flavor Domain-Wall Iwasaki ensembles

	L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources	
C1	24	1.784	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
C2	24	1.784	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
C0	*48*	*1.730*	*0.00078*	*0.0362*	*139*	*40*	*81/1**	[PRD 93 (2016) 074505]
M0	64	2.359	0.000678	0.02661	139	—	—	[PRD 93 (2016) 074505]
F1	48	2.774	0.002144	0.02144	234	70 + 28	24	[JHEP 1712 (2017) 008]

* All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. PRD 88 (2012) 094503]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm