Light and strange quark masses from $N_f = 2 + 1$ simulations with Wilson fermions

ALPHA Collaboration

INFN
Istituto Nazionale di Fisica Nucleare

Jonna Koponen
INFN Roma Tor Vergata
with M. Bruno, I. Campos, P Fritzsch, T. Korzec, C. Pena, D. Preti, A. Ramos, S. Schaefer, A. Vladikas

Lattice 2018, East Lansing, MI, USA

Outline

- CLS lattice ensembles
- PCAC masses
- Renormalisation, RG-running
- Chiral & continuum extrapolation
- Preliminary results

ALPHA Collaboration

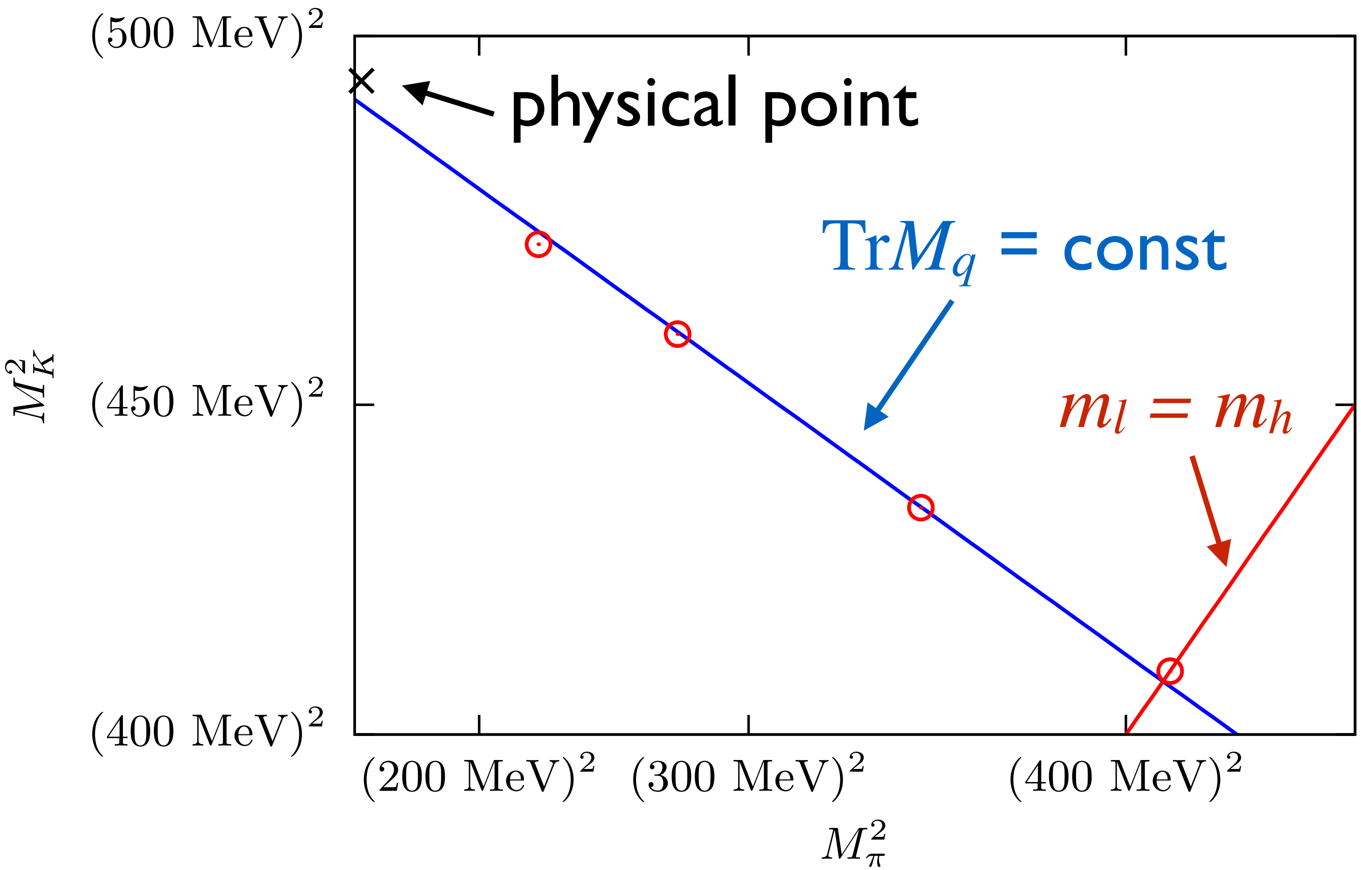
Lattice ensembles: CLS

- Lüscher-Weisz gauge action, tree level coefficients
- Symanzik-improved Wilson fermions, non-pert c_{SW}
- Open boundary conditions
- $N_f = 2+1$ (degenerate u/d, and s sea quarks)
- 4 lattice spacings in the range $0.05 \ldots 0.085$ fm (inverse coupling $\beta = 3.4 \ldots 3.7$)
- Multiple quark masses, $M_\pi = 200 \ldots 420$ MeV
- $M_\pi L \gtrsim 4$ for all ensembles

JHEP 02 (2015) 043, Phys. Rev. D 95 (2017) 074504

Tuning

- Subtracted quark masses $m_{q,r} = 1/(2\kappa_r) - 1/(2\kappa_{\text{crit}})$
- Choose masses so that $\mathrm{Tr}M_q$ = const ensuring that the improved bare coupling $\tilde{g}_0^2 = g_0^2[1 + b_g(a\mathrm{Tr}M_q)/3]$ stays constant to $O(a^2)$



- Chiral trajectory: choose starting point on the line $m_l = m_h$ so that you hit the physical point

Tuning (cont.)

- Note that $\mathrm{Tr}M_q$ = const means the trace of renormalised masses would not be constant (violated by $O(a)$ effects) → "constant physics" condition not $O(a)$ improved
- Solution: redefine the chiral trajectory using
$$\phi_4 \equiv 8t_0(M_{lh}^2 + M_{ll}^2/2)$$
$$\phi_2 \equiv 8t_0 M_{ll}^2$$
where t_0 is used to set the scale
- Perform small shifts so that the condition is now $\phi_4 =$ const

PCAC masses

- Define pseudoscalar and axial correlation functions

$$f_P^{rs}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x}, \vec{y}} \left\langle P^{rs}(x_0, \vec{x}) P^{sr}(y_0, \vec{y}) \right\rangle$$

$$f_A^{rs}(x_0, y_0) = -\frac{a^6}{L^3} \sum_{\vec{x}, \vec{y}} \left\langle A_0^{rs}(x_0, \vec{x}) P^{sr}(y_0, \vec{y}) \right\rangle$$

where $P^{rs}(x) = \bar{\psi}^r(x)\gamma_5\psi^s(x)$ and

$$A_0^{rs}(x) = \bar{\psi}^r(x)\gamma_0\gamma_5\psi^s(x) + ac_A\partial_0 P^{rs}(x)$$

- The PCAC mass is defined as

$$m_{rs} = \frac{\tilde{\partial}_0 f_A^{rs}(x_0, y_0)}{2 f_P^{rs}(x_0, y_0)}$$

Notation

- Recall

$$\phi_4 \equiv 8t_0\left(M_{lh}^2 + \frac{M_{ll}^2}{2}\right) = \text{const}$$

$$\phi_2 \equiv 8t_0 M_{ll}^2$$

- Define dimensionless quantities

$$\phi_{rs} \equiv \sqrt{8t_0}\, m_{rs}$$

$$\phi_\eta \equiv 8t_0 \frac{4M_{lh}^2 - M_{ll}^2}{3} = \frac{4\phi_4 - 3\phi_2}{3}$$

Renormalisation

$$\phi_{rs}^{\mathrm{RGI}} = Z_M\big(1 + (\tilde{b}_A - \tilde{b}_P)am_{rs} + (\bar{b}_A - \bar{b}_P)a\mathrm{Tr}M_q\big)\phi_{rs} + \mathcal{O}(a^2)$$

$$Z_M(g_0) = \frac{M}{\bar{m}(\mu_{\mathrm{had}})}\frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\mathrm{had}})}$$

- The ratio of axial current and pseudoscalar density normalisation factors $Z_A(g_0^2)/Z_P(g_0^2, a\mu_{\mathrm{had}})$
- The ratio of the RGI quark mass M to the renormalised quark mass $\bar{m}(\mu_{\mathrm{had}})$
  - running done in the $N_f = 3$ massless scheme
  - $\bar{m}(\mu_{\mathrm{had}})$ in the same scheme and scale as Z_P

Eur. Phys. J C78 (2017) 5387

Quark mass RG-running

- Running done in the $N_f = 3$ massless scheme
- Use small lattices at fixed scale μ to obtain standard step-scaling function
- Two lattice setups used:
  - tree-level Symanzik improved (Lüscher-Weisz) gauge action at $\mu = 0.25 \dots 2$ GeV
  - plaquette Wilson gauge action at $\mu = 2 \dots 100$ GeV
  - Wilson clover fermion action
- Beyond $\mu = 100$ GeV RG-running is done perturbatively (at 2-loops for quark mass)

Renormalisation (cont.)

- Recall that

$$\phi_{rs}^{\mathrm{RGI}} = Z_M\Big(1 + (\tilde{b}_A - \tilde{b}_P)am_{rs} + (\bar{b}_A - \bar{b}_P)a\mathrm{Tr}M_q\Big)\phi_{rs} + \mathcal{O}(a^2)$$

$$Z_M(g_0) = \frac{M}{\bar{m}(\mu_{\mathrm{had}})}\frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\mathrm{had}})}$$

- Z_A and Z_P have been calculated in the Schrödinger functional scheme in the same β range as the PCAC masses - Z_P at scale μ_{had}

- Final result is summarised as
$$Z_M(g_0) = Z_M^{(0)} + Z_M^{(1)}(\beta - 3.79) + Z_M^{(2)}(\beta - 3.79)^2$$

- 1-loop PT for $(\tilde{b}_A - \tilde{b}_P) = -0.0012g_0^2$

- Ignore contribution $(\bar{b}_A - \bar{b}_P) \sim \mathcal{O}(g_0^4)$

Chiral & continuum fits

- The fit functions are

$$\phi_{ll}^R = \beta_0\phi_2\left[1-\beta_1\phi_4-\beta_2\phi_2-K\left(\bar{L}_\pi-\frac{1}{3}\bar{L}_\eta\right)\right]+c_{a1}\frac{a^2}{t_0}$$

$$\phi_{lh}^R = \beta_0\frac{1}{2}(2\phi_4-\phi_2)\left[1-\beta_1\phi_4-\beta_2\frac{1}{2}(2\phi_4-\phi_2)-\frac{1}{3}K\bar{L}_\eta\right]+c_{a2}\frac{a^2}{t_0}$$

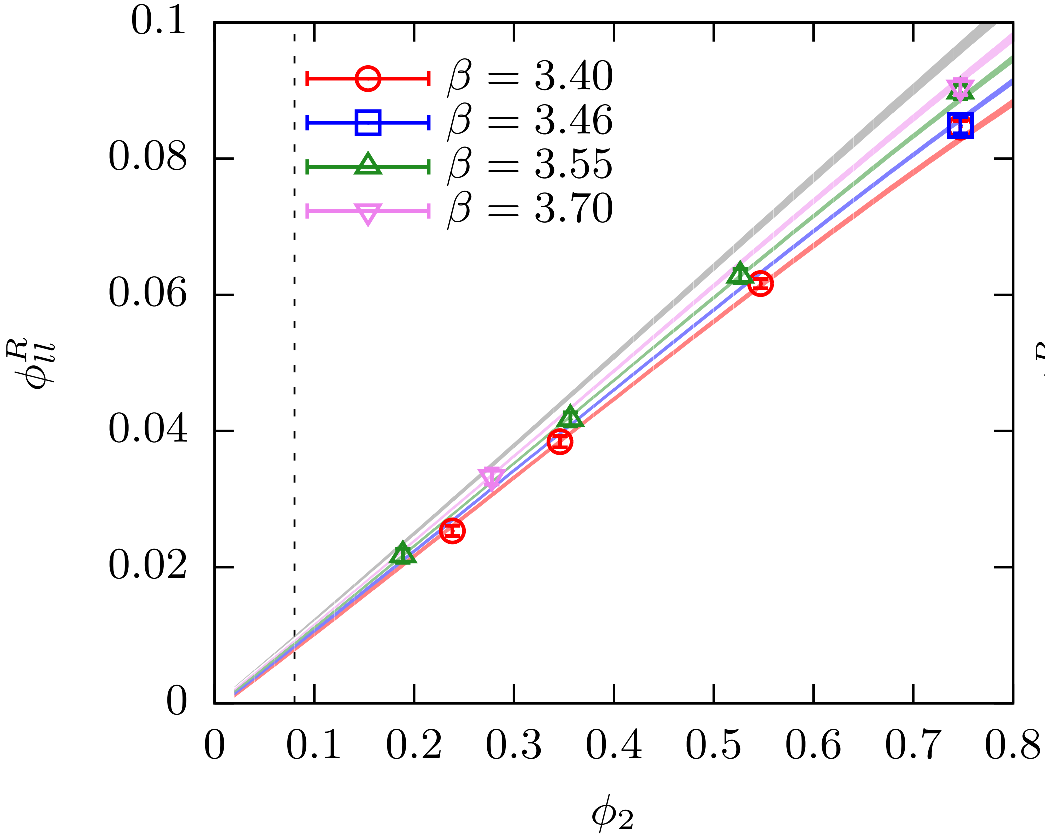
$$\frac{\phi_{ll}^R}{2\phi_{lh}^R} = \frac{\phi_2}{2\phi_4-\phi_2}\left[1-\frac{2}{3}\beta_2\phi_2+\beta_2\phi_4-K(\bar{L}_\pi-\bar{L}_\eta)\right]+c_{a3}\frac{a^2}{t_0}$$

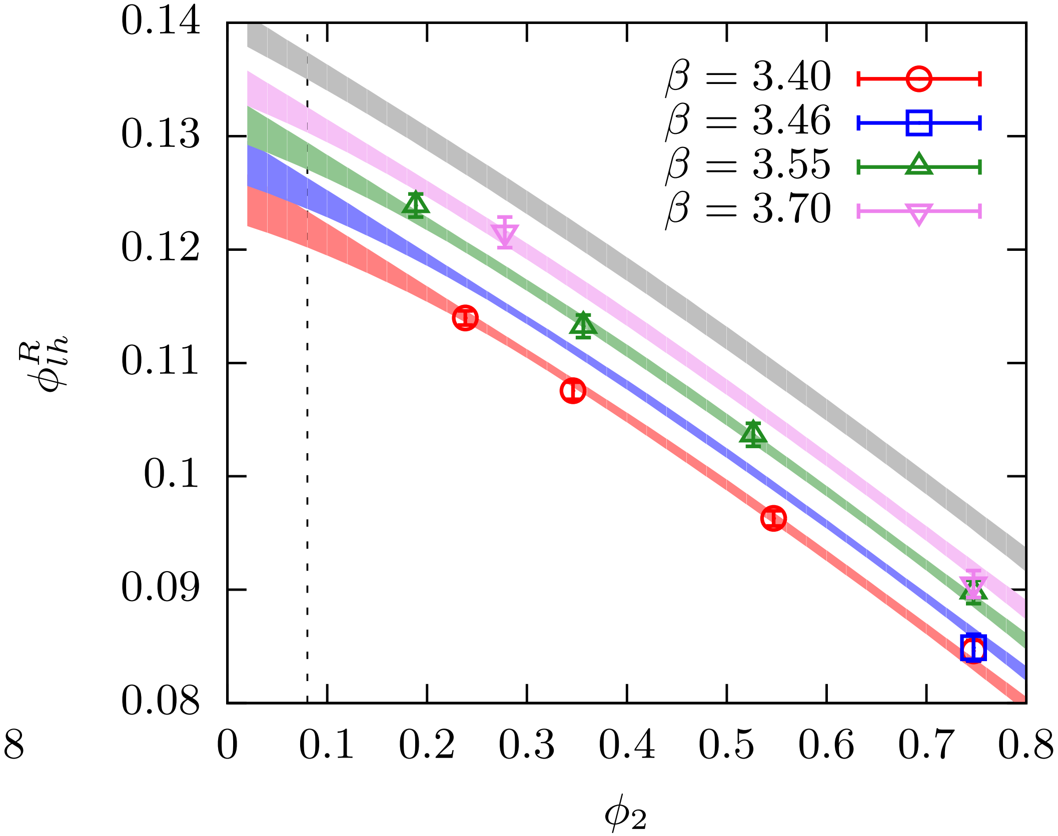
where

$$\beta_0=\frac{1}{2\sqrt{8t_0}B_0},\quad \beta_1=\frac{32(2L_6-L_4)}{8t_0f_0^2},\quad \beta_2=\frac{16(2L_8-L_5)}{8t_0f_0^2},\quad K=\frac{1}{8t_016\pi^2f_0^2}$$

$$\bar{L}_\pi=\phi_2\ln(\phi_2),\quad \bar{L}_\eta=\phi_\eta\ln(\phi_\eta)\quad \phi_\eta\equiv 8t_0\frac{4M_{lh}^2-M_{ll}^2}{3}=\frac{4\phi_4-3\phi_2}{3}$$

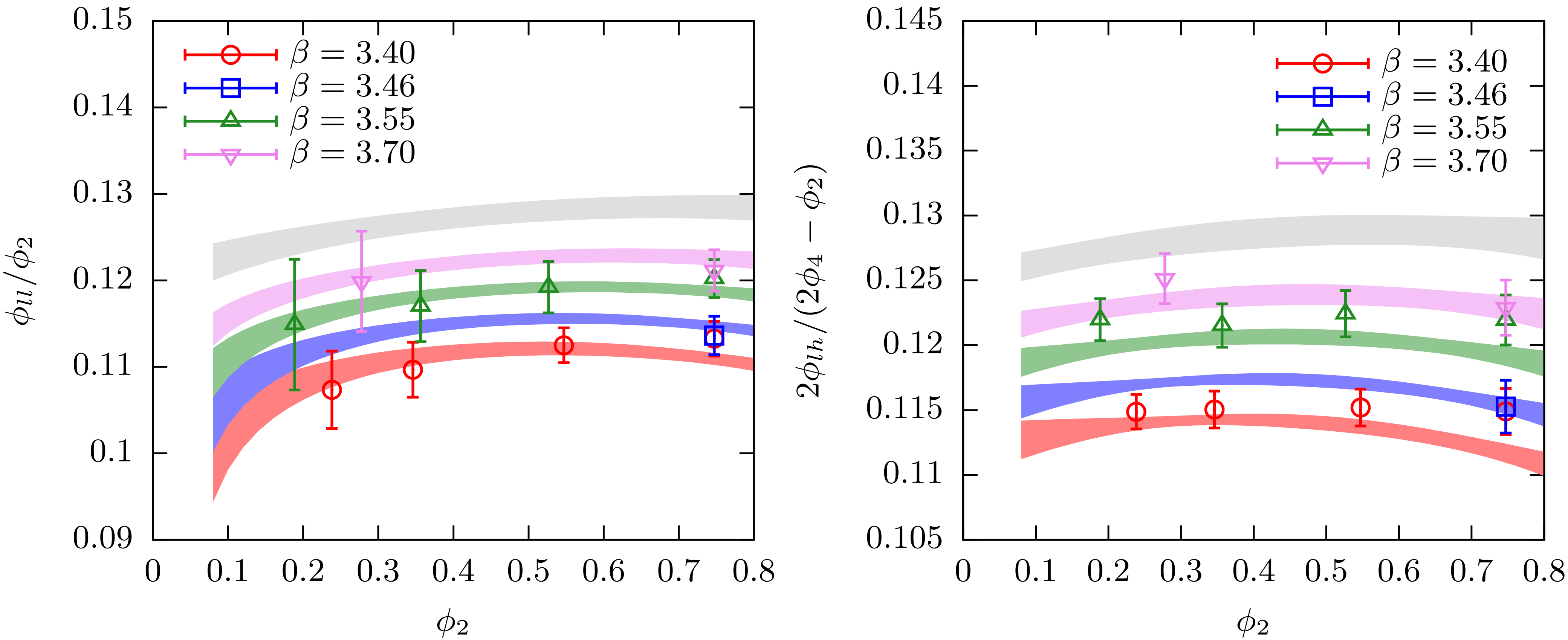
PCAC masses vs M_π^2





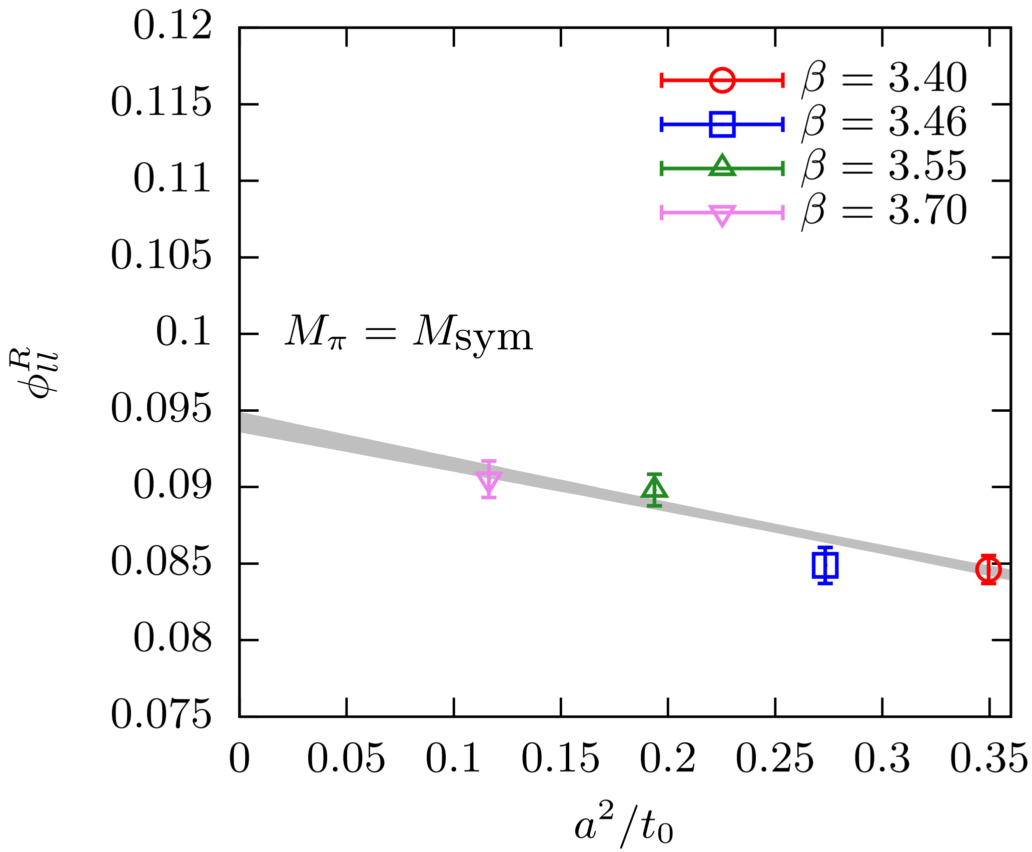
- Chiral and continuum extrapolation are done simultaneously

PCAC masses vs M_π^2



- ϕ_{ll}/ϕ_2 and $2\phi_{lh}/(2\phi_4 - \phi_2)$ are almost flat, which shows that higher order chiPT effects are small

PCAC masses vs a^2



- The data shows good a^2 scaling

Preliminary results

- After continuum and chiral extrapolation, we quote our preliminary results

$$m_{u/d}^{\text{RGI}} = 4.66 \pm 0.09 \text{ MeV}, \quad m_s^{\text{RGI}} = 125.2 \pm 1.6 \text{ MeV}$$

or in $\overline{MS}$ at 2 GeV and $N_f = 3$:

$$m_{u/d}^{\overline{\text{MS}}} = 3.50 \pm 0.08 \text{ MeV}, \quad m_s^{\overline{\text{MS}}} = 94.1 \pm 1.5 \text{ MeV}$$

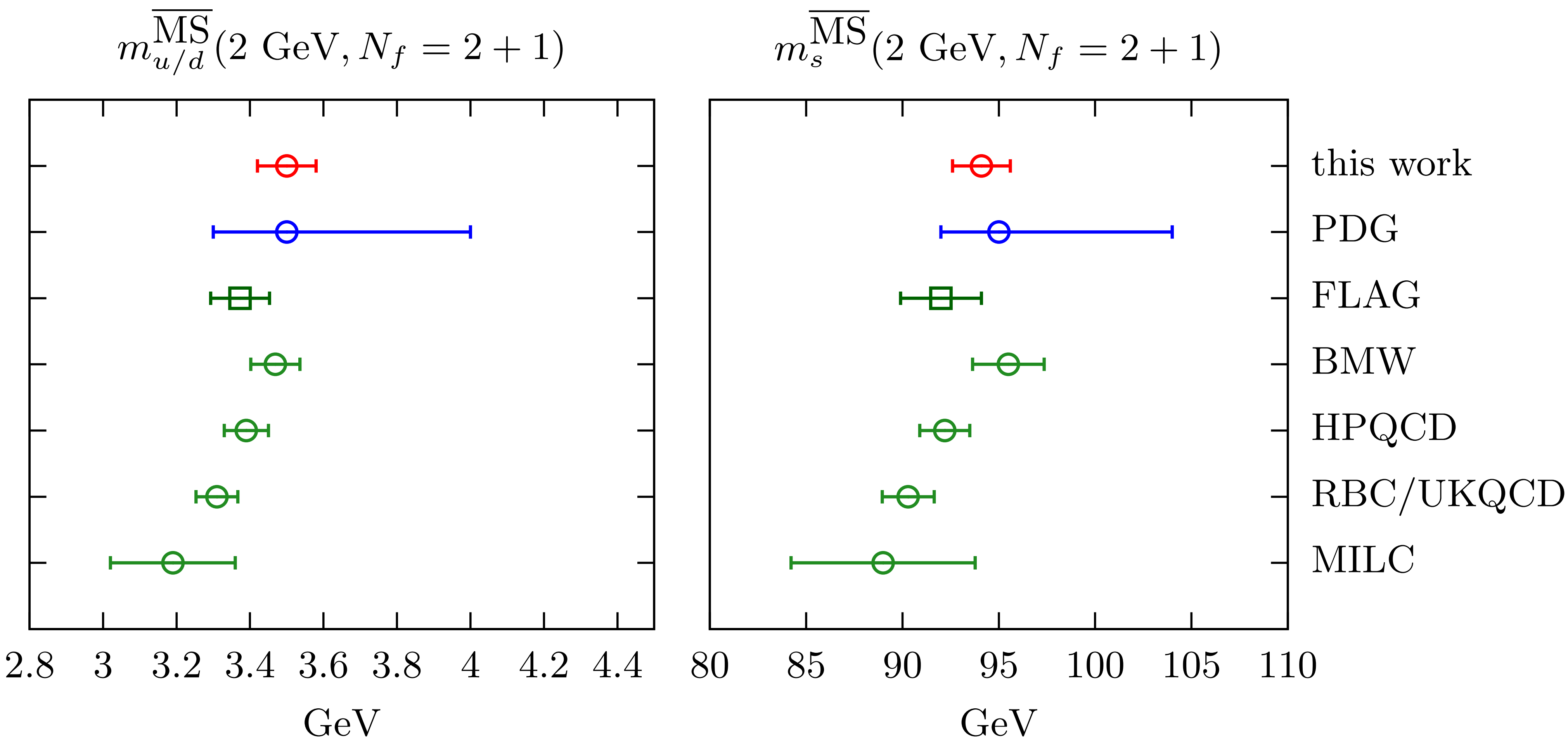
Errors include statistical + renormalisation

- Comparing to other results:

PDG $m_{u/d}^{\overline{\text{MS}}} = 3.5^{+0.5}_{-0.2}$ MeV, $m_s^{\overline{\text{MS}}} = 95^{+9}_{-3}$ MeV

FLAG $m_{u/d}^{\overline{\text{MS}}} = 3.373(80)$ MeV, $m_s^{\overline{\text{MS}}} = 92.0(2.1)$ MeV

Preliminary results



FLAG result is the average of BMW, HPQCD, RBC/UKQCD and MILC results

FLAG 2016: Eur. Phys. J C77 (2017) 112

Preliminary results

- For the ratio of the quark masses we quote

$$\frac{m_s}{m_{u/d}} = 26.9 \pm 0.4$$

Errors include statistical + systematic effects from renormalisation and running

- Comparing to other $N_f = 2{+}1$ results:

PDG $$\frac{m_s}{m_{u/d}} = 27.3(0.7)$$

FLAG $$\frac{m_s}{m_{u/d}} = 27.43(31)$$

Thank you!

INFN

Istituto Nazionale di Fisica Nucleare

Spare slides