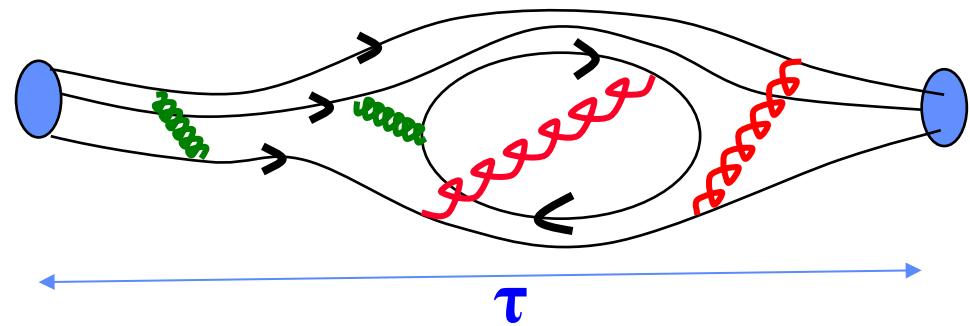


# Isovector and flavor diagonal charges of the Nucleon

Rajan Gupta  
Laboratory Fellow  
Theoretical Division  
Los Alamos National Laboratory, USA

$$\langle 0 | N(\tau) \bar{N}(0) | 0 \rangle = \sum_n A_n e^{-m_n \tau}$$



# Work done in collaboration with

## PNDME collaboration (Clover-on-HISQ)

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Yong-Chull Jang
- Huey-Wen Lin
- Boram Yoon

Bhattacharya et al, PRD85 (2012) 054512

Bhattacharya et al, PRD89 (2014) 094502

Bhattacharya et al, PRD92 (2015) 114026

Bhattacharya et al, PRL 115 (2015) 212002

Bhattacharya et al, PRD92 (2015) 094511

Bhattacharya et al, PRD94 (2016) 054508

Bhattacharya et al, PRD96 (2017) 114503

Gupta et al, arXiv:1806.09006

Huey-Wen Lin et al, arXiv:1806.10604

## NME collaboration (Clover-on-Clover)

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Yong-Chull Jang
- Bálint Joó
- Huey-Wen Lin
- Kostas Orginos
- David Richards
- Frank Winter
- Boram Yoon

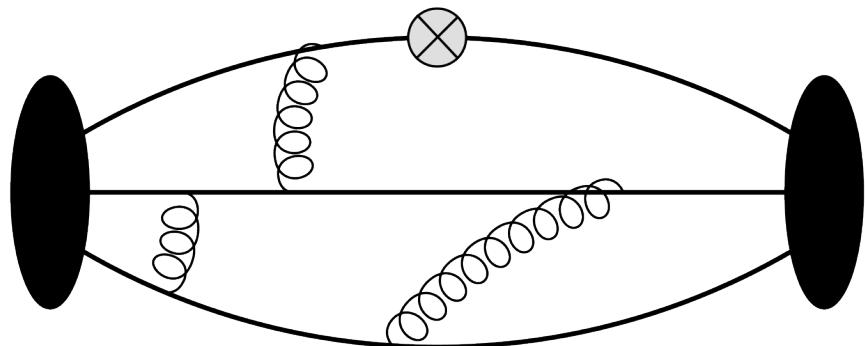
Yoon et al., PRD D93 (2016) 114506

Yoon et al., PRD D95 (2017) 074508

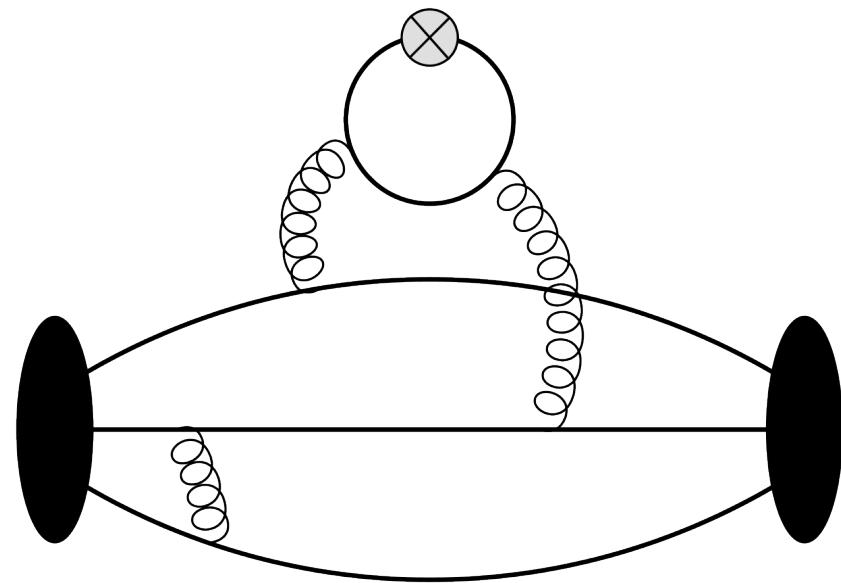
# Acknowledgement: Computing Resources

- **Clover-on-Clover:**
- Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725.
- Institutional Computing at Los Alamos National Laboratory.
- The USQCD Collaboration, funded by the Office of Science, U.S. DOE
- **Clover-on-HISQ**
- The National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. DOE under Contract No. DE-AC02-05CH11231
- Oak Ridge Leadership Computing Facility at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725.
- The USQCD Collaboration, funded by the Office of Science, U.S. DOE
- Institutional Computing at Los Alamos National Laboratory.
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**“Connected” and “disconnected” contributions to matrix elements of quark bilinear operators within the nucleon state are needed to address a number of physics questions**



**Connected**



**Disconnected**

# Examples of matrix elements within nucleon states being calculated

- Isovector charges  $g_A, g_S, g_T$   $\langle p | \bar{u} \Gamma d | n \rangle$
- Axial vector form factors  $\langle p(q) | \bar{u} \gamma_\mu \gamma_5 d(q) | n(0) \rangle$
- Vector form factors  $\langle p(q) | \bar{u} \gamma_\mu d(q) | n(0) \rangle$
- Flavor diagonal matrix elements  $\langle p | \bar{q} q | p \rangle$
- Quark EDM and quark chromo EDM, ...
- Generalized Parton Distribution Functions

# Challenges to obtaining high precision results for matrix elements within nucleon states

- High Statistics: O(1,000,000) measurements
- Demonstrating control over all Systematic Errors:
  - Non-perturbative renormalization of bilinear operators ( $\text{RI}_{\text{smom}}$  scheme)
  - Contamination from excited states
    - Finite volume effects
    - Chiral extrapolation to physical  $m_u$  and  $m_d$  (simulate at physical point)
    - Extrapolation to the continuum limit (lattice spacing  $a \rightarrow 0$ )

Perform simulations on ensembles with multiple values of

- Lattice size  $M_\pi L \rightarrow \infty$
- Light quark masses  $\rightarrow$  physical  $m_u$  and  $m_d$
- Lattice spacing  $a \rightarrow 0$

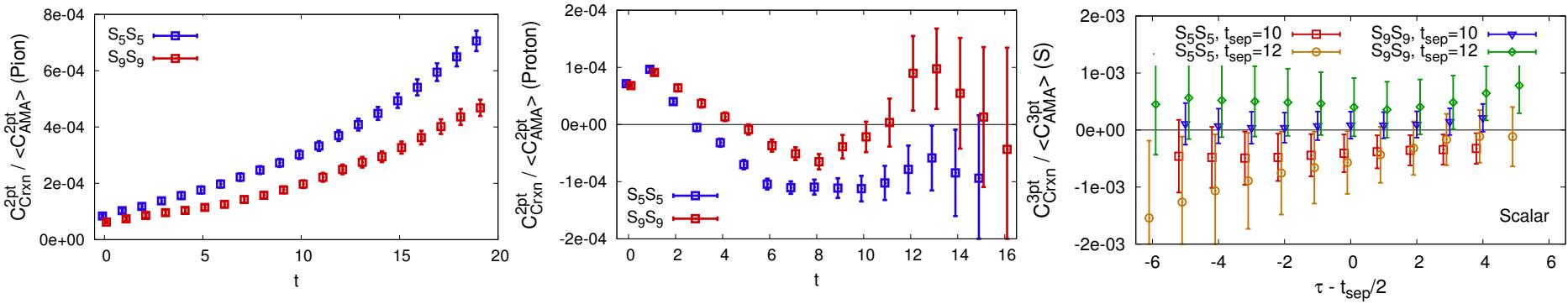
# Toolkit

- Multigrid Dirac invertor → propagator  $S_F = D^{-1}\eta$
- Truncated solver method with bias correction (TSM)
- Coherent source sequential propagator
- Deflation + hierarchical probing (for disconnected)
- 3-5 values of  $t_{\text{sep}}$  with smeared sources
- 2-state and 3-state fits to multiple values of  $t_{\text{sep}}$
- Non-perturbative methods for renormalization constants
- Combined extrapolation in  $a$ ,  $M_\pi$ ,  $M_\pi L$
- Variation of results with extrapolation Ansatz

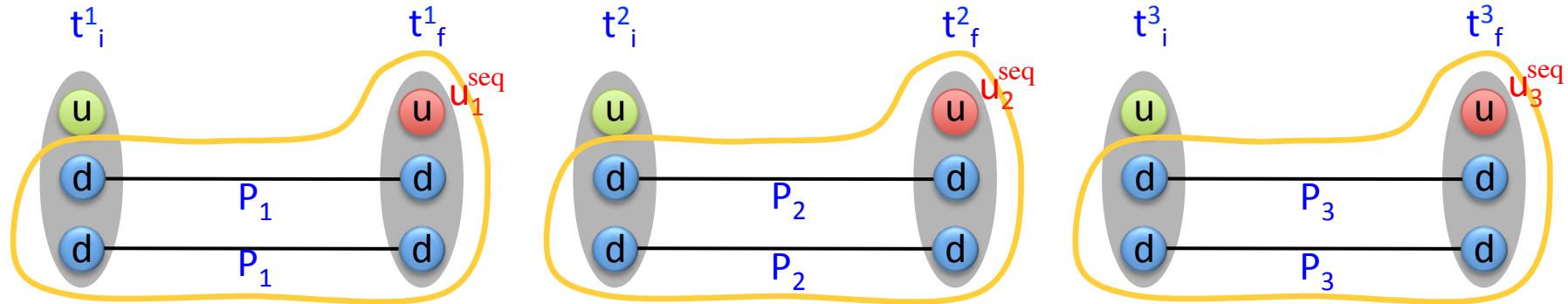
# Truncated solver + bias correction (TSM)

$$C^{AMA} = \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} C_{LP}(x_i^{LP}) + \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} \{C_{HP}(x_i^{HP}) - C_{LP}(x_i^{HP})\}$$

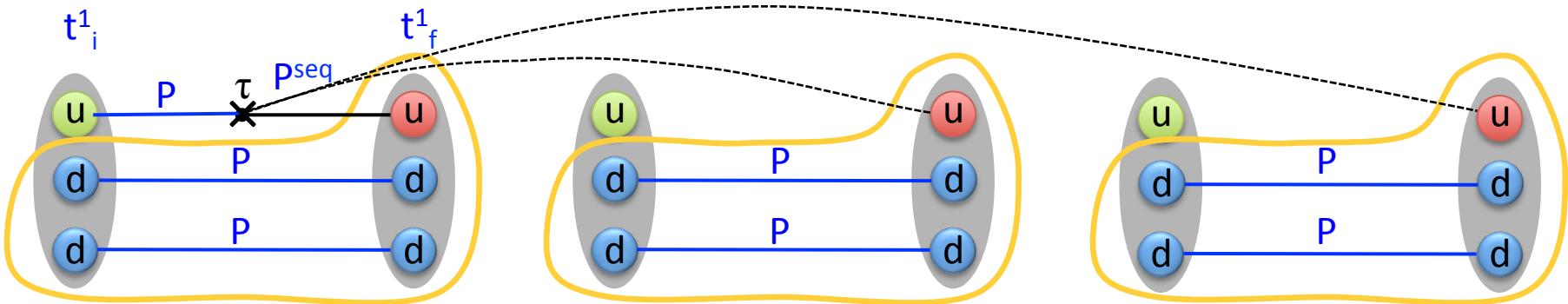
- Use multigrid inverter with
  - $r_{LP} = 10^{-3}$
  - $r_{HP} = 10^{-7} - 10^{-10}$
  - $N_{LP} = 64 - 160$ ,  $N_{HP} = 3 - 5$  per configuration
- The bias term is negligible ( $\sim 1\%$  of the error)
- The AMA error is  $< 15\%$  larger than LP



# Coherent Source Sequential Propagator



3 measurements being done in a single computer job



Bias = 0 after gauge integral: Tiny increase in Variance

# HISQ Ensembles $N_f = 2 + 1 + 1$

$m_s$  tuned to the physical mass using  $M_{s\bar{s}}$

Ensemble ID	$a$ (fm)	$M_\pi^{\text{sea}}$ (MeV)	$M_\pi^{\text{val}}$ (MeV)	$L^3 \times T$	$M_\pi^{\text{val}} L$	$\tau/a$	$N_{\text{conf}}$	$N_{\text{meas}}^{\text{HP}}$	$N_{\text{meas}}^{\text{LP}}$
<b>a15m310</b>	0.1510(20)	306.9(5)	320(5)	$16^3 \times 48$	3.93	$\{5, 6, 7, 8, 9\}$	1917	7668	122,688
<i>a12m310</i>	0.1207(11)	305.3(4)	310.2(2.8)	$24^3 \times 64$	4.55	$\{8, 10, 12\}$	1013	8104	64,832
<b>a12m220S</b>	0.1202(12)	218.1(4)	225.0(2.3)	$24^3 \times 64$	3.29	$\{8, 10, 12\}$	946	3784	60,544
<b>a12m220</b>	0.1184(10)	216.9(2)	227.9(1.9)	$32^3 \times 64$	4.38	$\{8, 10, 12\}$	744	2976	47,616
<b>a12m220L</b>	0.1189(09)	217.0(2)	227.6(1.7)	$40^3 \times 64$	5.49	$\{8, 10, 12, 14\}$	1000	4000	128,000
<b>a09m310</b>	0.0888(08)	312.7(6)	313.0(2.8)	$32^3 \times 96$	4.51	$\{10, 12, 14, 16\}$	2263	9052	144,832
<b>a09m220</b>	0.0872(07)	220.3(2)	225.9(1.8)	$48^3 \times 96$	4.79	$\{10, 12, 14, 16\}$	964	7712	123,392
<i>a09m130</i>	0.0871(06)	128.2(1)	138.1(1.0)	$64^3 \times 96$	3.90	$\{10, 12, 14\}$	883	7064	84,768
<b>a09m130W</b>						$\{8, 10, 12, 14, 16\}$	1290	5160	165,120
<i>a06m310</i>	0.0582(04)	319.3(5)	319.6(2.2)	$48^3 \times 144$	4.52	$\{16, 20, 22, 24\}$	1000	8000	64,000
<b>a06m310W</b>						$\{18, 20, 22, 24\}$	500	2000	64,000
<i>a06m220</i>	0.0578(04)	229.2(4)	235.2(1.7)	$64^3 \times 144$	4.41	$\{16, 20, 22, 24\}$	650	2600	41,600
<b>a06m220W</b>						$\{18, 20, 22, 24\}$	649	2596	41,536
<b>a06m135</b>	0.0570(01)	135.5(2)	135.6(1.4)	$96^3 \times 192$	3.7	$\{16, 18, 20, 22\}$	675	2700	43,200

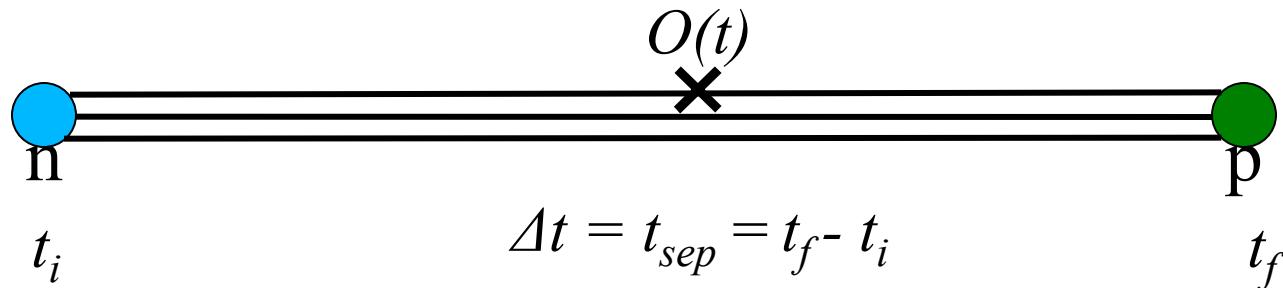
## Controlling excited-state contamination: n-state fit

$$\Gamma^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \dots$$

$$\begin{aligned}\Gamma^3(t, \Delta t) = & |A_0|^2 \langle 0 | O | 0 \rangle e^{-M_0 \Delta t} + |A_1|^2 \langle 1 | O | 1 \rangle e^{-M_1 \Delta t} + \\ & A_0 A_1^* \langle 0 | O | 1 \rangle e^{-M_0 \Delta t} e^{-\Delta M (\Delta t - t)} + A_0^* A_1 \langle 1 | O | 0 \rangle e^{-\Delta M t} e^{-M_0 \Delta t} + \dots\end{aligned}$$

$M_0, M_1, \dots$  masses of the ground & excited states

$A_0, A_1, \dots$  corresponding amplitudes



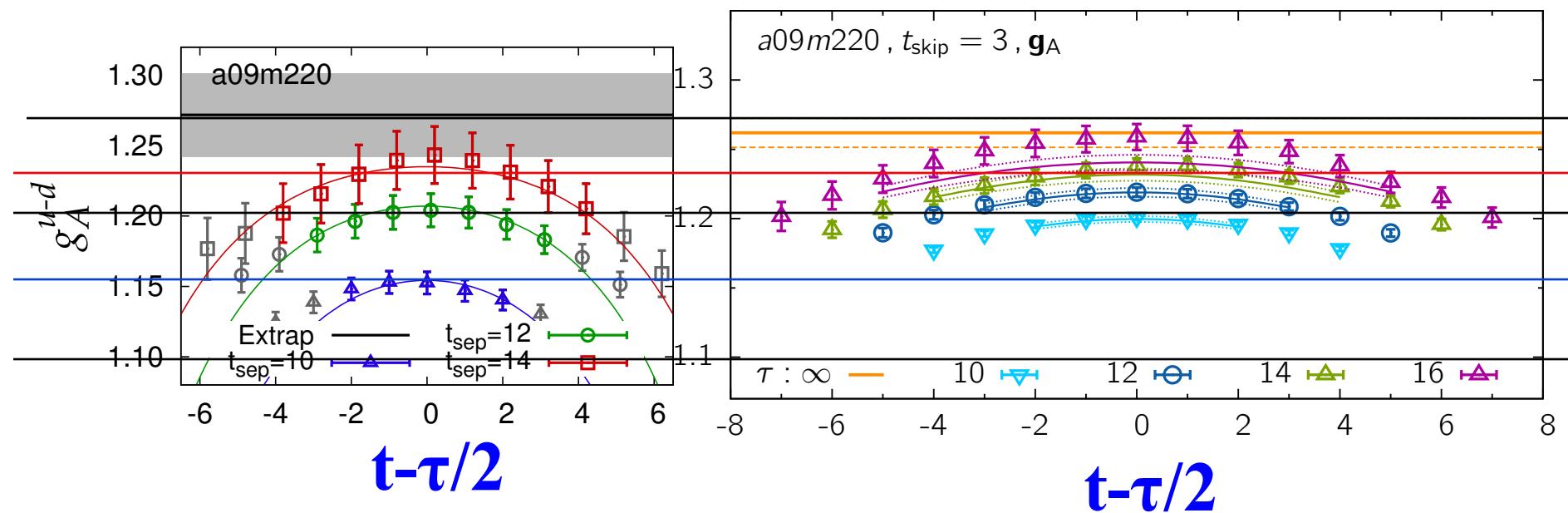
Make a simultaneous fit to data at multiple  $\Delta t = t_{sep} = t_f - t_i$

# Controlling excited-state contamination

- Reduce  $A_n/A_0$  in an n-state fit
  - Tune source smearing size  $\sigma$
  - Tune the interpolating operator
  - Variational method
- Generate data at multiple values of  $t_{sep}$
- $2 \rightarrow 3 \rightarrow n$ -state fits to data at multiple values of  $t_{sep}$

# Toolkit: Controlling Excited State Contamination

- Better smearing
- Higher statistics with TSM
- 4-5 values of source-sink separation  $\tau$
- 4-state fits to 2-point functions
- 3-state fits to 3-point functions
- Full covariance error matrix



# Analyzing lattice data $\Omega(a, M_\pi, M_\pi L)$ : Extrapolations in $a, M_\pi^2, M_\pi L$

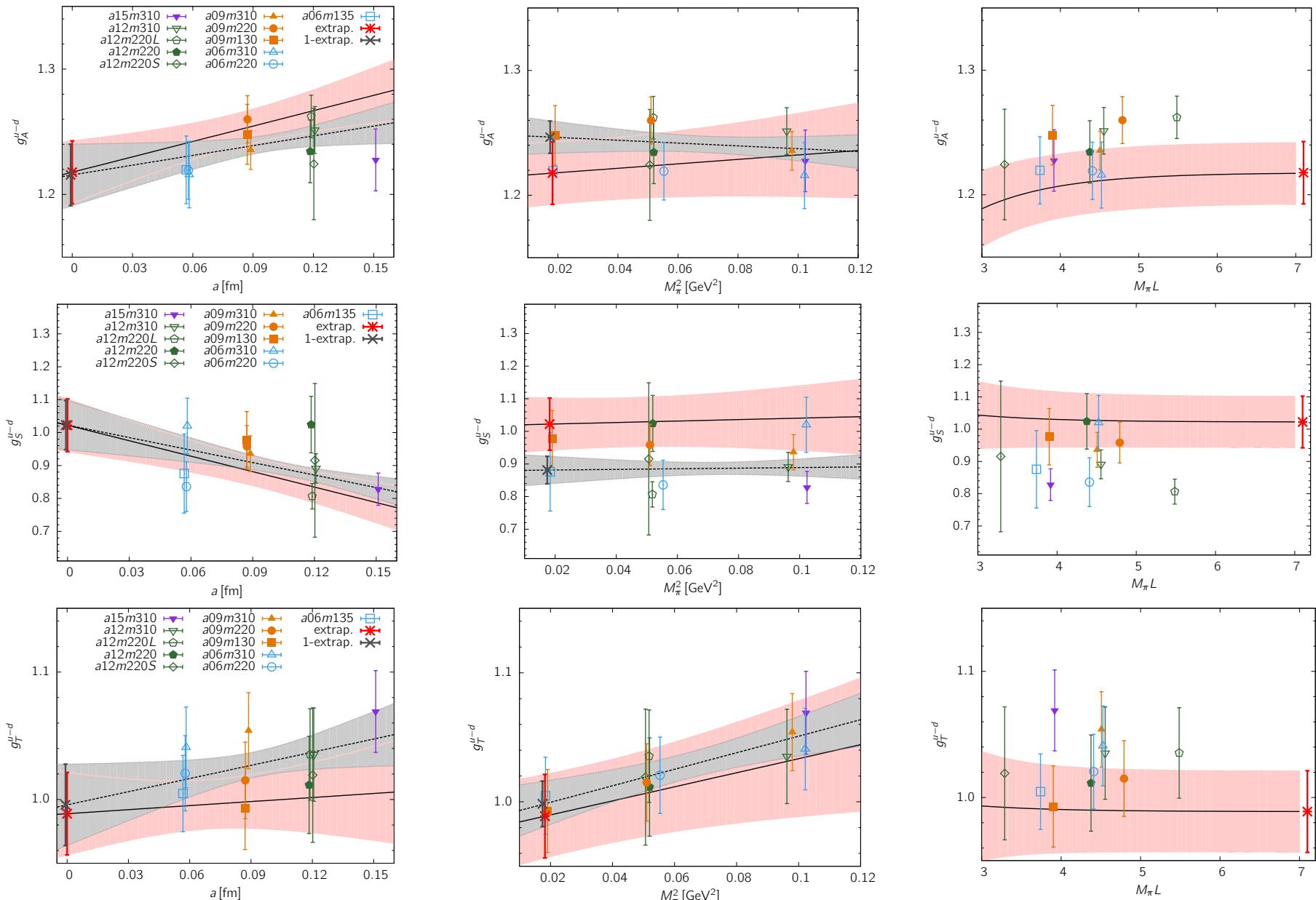
We use lowest order corrections when fitting lattice data w.r.t.

- Lattice spacing:  $a$
- Dependence on light quark mass:  $m_q \sim M_\pi^2$
- Finite volume:  $M_\pi L$

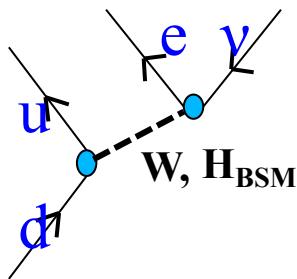
$$g_{A,T}(a, M_\pi, L) = g + A a + B M_\pi^2 + C M_\pi^2 e^{-M_\pi L} + \dots$$

$$g_S(a, M_\pi, L) = g + A a + B M_\pi + C M_\pi e^{-M_\pi L} + \dots$$

# Simultaneous extrapolation in $a$ , $M_\pi^2$ , $M_\pi L$



Fits using 11 clover-on-HISQ ensembles: Gupta et al, 1806.09006



# Isovector Charges

$$\langle p | \bar{u} \Gamma d | n \rangle$$

- Axial charge  $g_A^{u-d} = 1.218(25)(30)$
- Scalar charge  $g_S^{u-d} = 1.022(80)(60)$
- Tensor charge  $g_T^{u-d} = 0.989(32)(10)$

Competitive Results

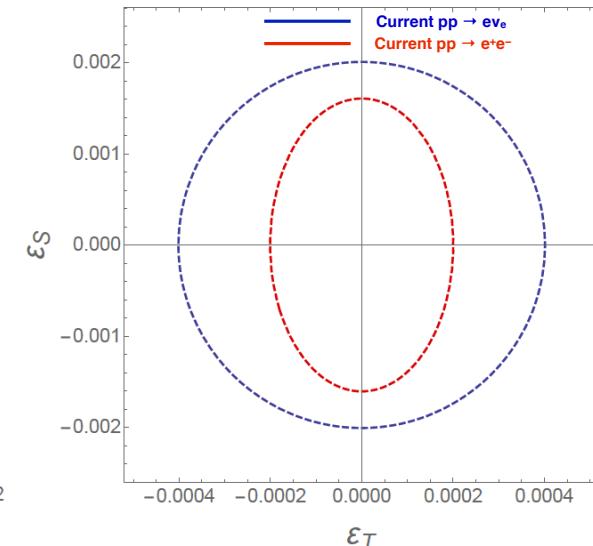
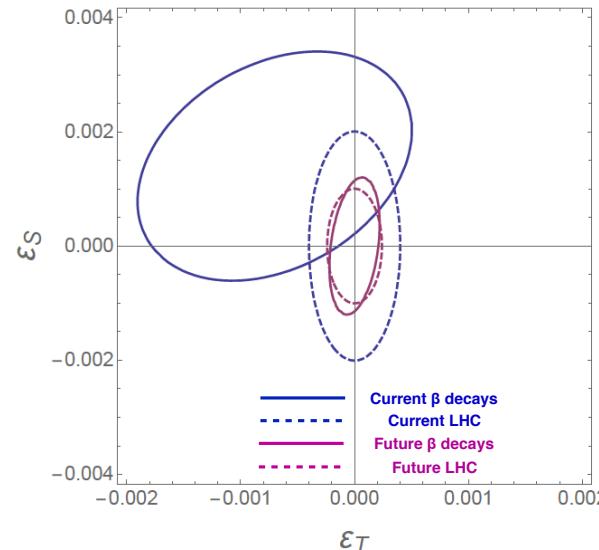
CalLat:  $g_A^{u-d} = 1.271(13)$

## Constraining novel scalar and tensor interactions

$$b \approx 0.34 g_S \epsilon_S - 5.22 g_T \epsilon_T$$

$$b_\nu \approx 0.44 g_S \epsilon_S - 4.85 g_T \epsilon_T$$

helicity suppressed parameters  $b$  and  $b_\nu$  extracted from neutron decay distribution



# $g_A, g_S$ : What is known

- Experiment: Neutron decay
  - $g_A = 1.2766(20)$

- CVC + Lattice QCD

Gonzalez-Alonso & Camalich  
Phy. Rev. Lett. 112 (2014) 042501

$$\frac{g_S}{g_V} = \frac{(M_N - M_P)^{QCD}}{(m_d - m_u)^{QCD}} = 1.02(8)(7)$$

In  $\overline{MS}$  at 2 GeV

$$[g_S^{\text{PNDME}} = 1.022(80)(60)] \odot [(m_d - m_u)^{\text{Lattice}}]$$

$(M_N - M_P)^{QCD} = 2.41(27)$  MeV for 2+1-flavors (BMW)

$(M_N - M_P)^{QCD} = 2.63(27)$  MeV for 2+1+1-flavors

(MILC/Fermilab/TUMQCD)

# $g_A^{u-d}$ : PNDME & CalLat agree within errors on 7 ensembles

CalLat: Nature: <https://doi.org/10.1038/s41586-018-0161-8>

PNDME: Gupta et al, 1806.09006

	PNDME	CalLat
a15m310	1.228(25)	1.215(12)
a12m310	1.251(19)	1.214(13)
a12m220S	1.224(44)	1.272(28)
a12m220	1.234(25)	1.259(15)
a12m220L	1.262(17)	1.252(21)
a09m310	1.235(15)	1.236(11)
a09m220	1.260(19)	1.253(09)

CalLat uses a variant of the summation method

Difference comes from the Chiral-Continuum fits:

- CalLat chiral fit anchored by heavier pion masses
- CalLat have not yet analyzed the  $a=0.06\text{fm}$  lattices

PNDME error estimate more realistic

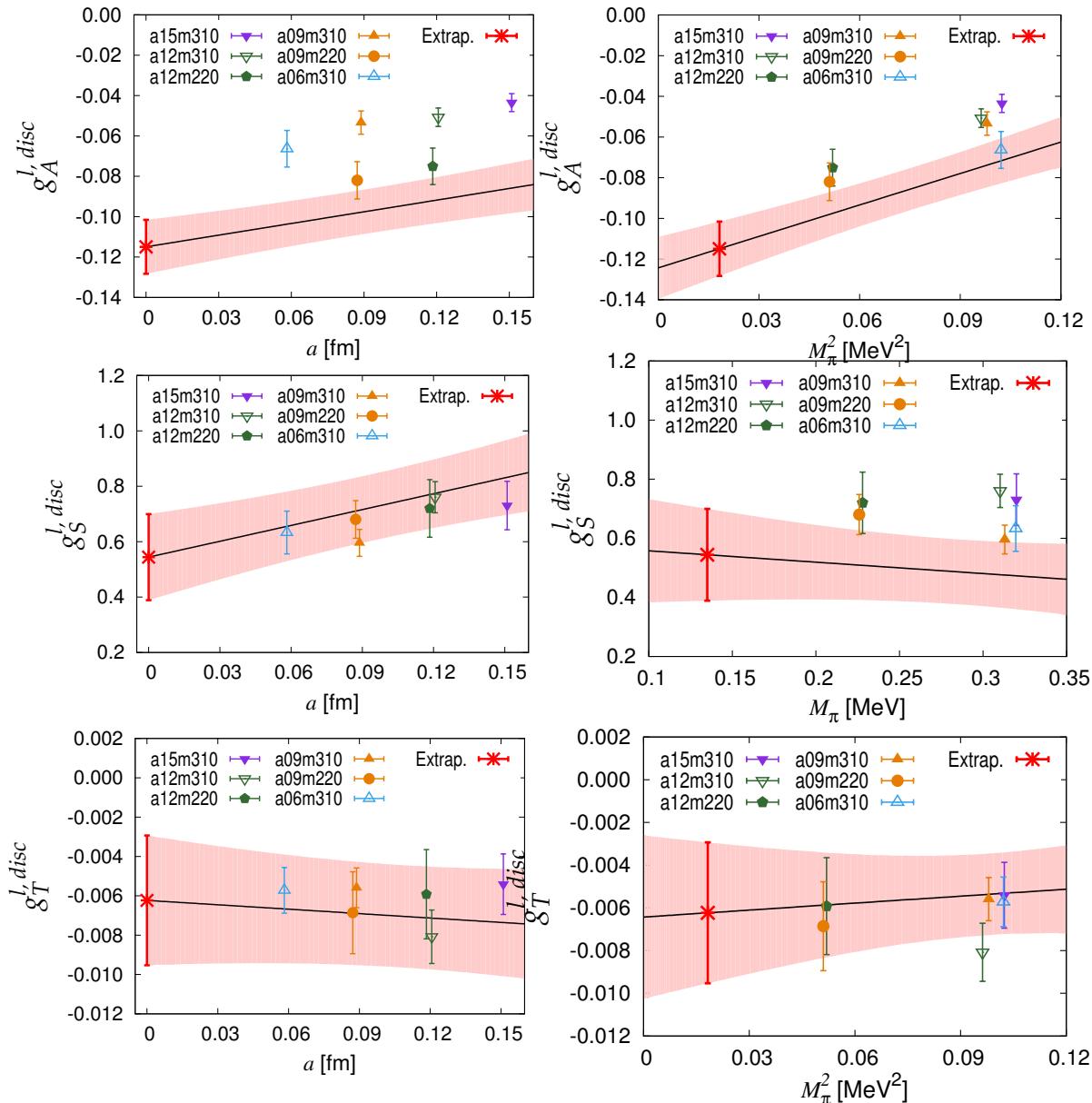
# Physics probed by flavor diagonal charges

$$g^q_\Gamma = \langle N | q \Gamma q | N \rangle$$

- Axial charges  $g_A$ 
  - Contribution of quarks to the nucleon spin ( $\Sigma = g_A^u + g_A^d + g_A^s$ )
  - Spin dependent interactions of dark matter
- Scalar charges  $g_S$ 
  - Dependence of nucleon mass on the quark masses ( $\sigma_{\pi N} = m_l \langle N | uu + dd | N \rangle; \sigma_s = m_s \langle N | ss | N \rangle$ )
  - Coupling of dark matter to nucleons (needed for direct detection of dark matter via scattering off nucleons)
  - Constraining BSM theories: bounds on effective scalar interactions (e.g. Higgs mediated lepton flavor violation)
- Tensor charges  $g_T$ 
  - Contributions of quarks to the neutron EDM
  - Constraining BSM theories: bounds on effective tensor interactions
  - Transversity
- Chromo EDM
  - Constraining BSM via novel CP violating interactions, e.g., Higgs CP odd couplings

# Disconnected light quark contribution

Extrapolation  
in  $a \rightarrow 0$  is  
significant for  
 $g_A^l$  and  $g_S^l$



# Continuum and chiral extrapolations for $g^s$

## Ensembles

$a15m310$

$a12m310$

$a12m220$

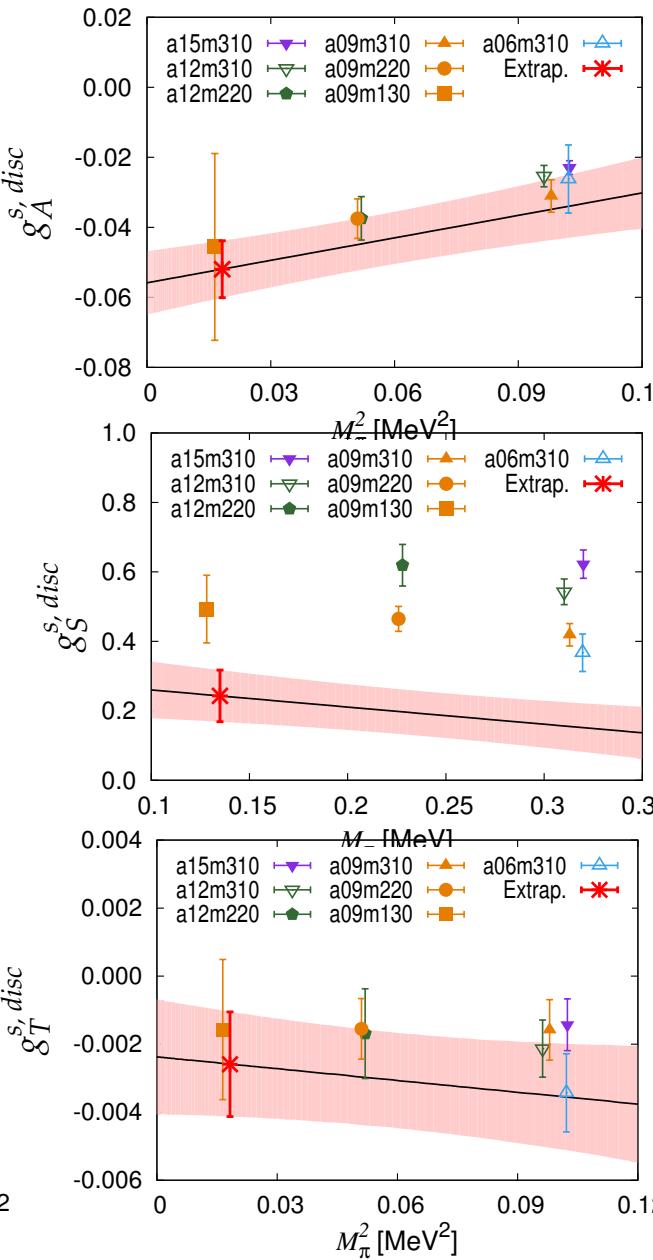
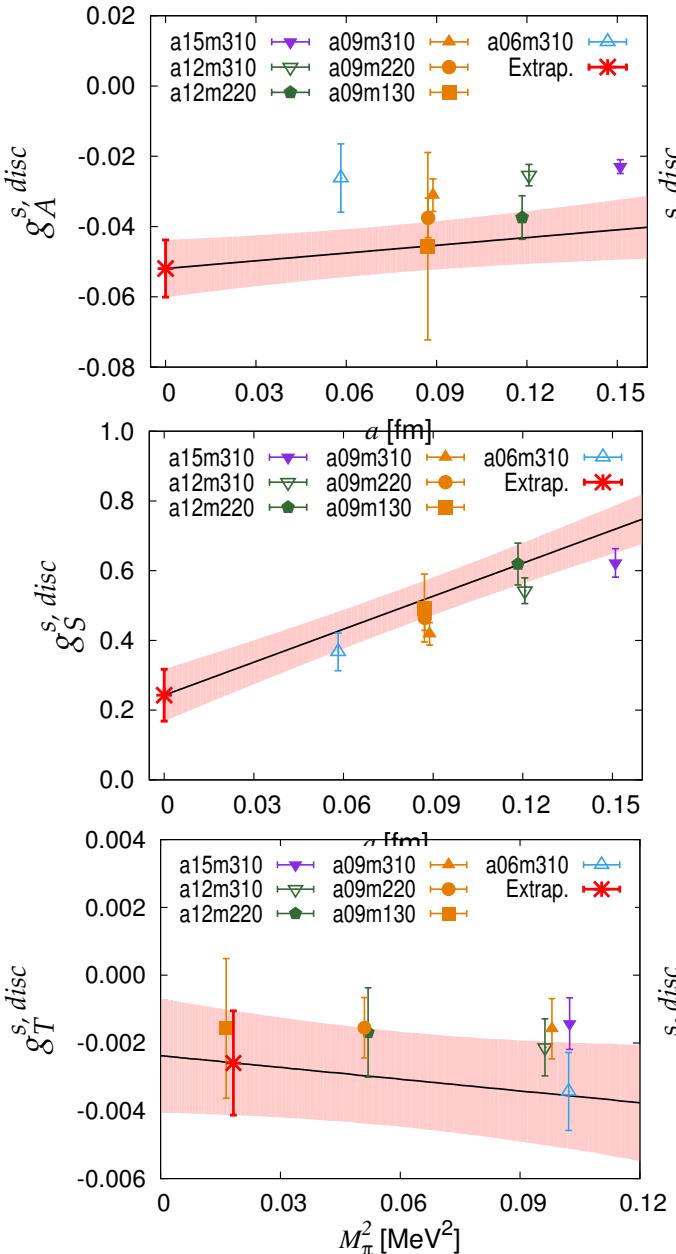
$a09m310$

$a09m220$

$a09m130$

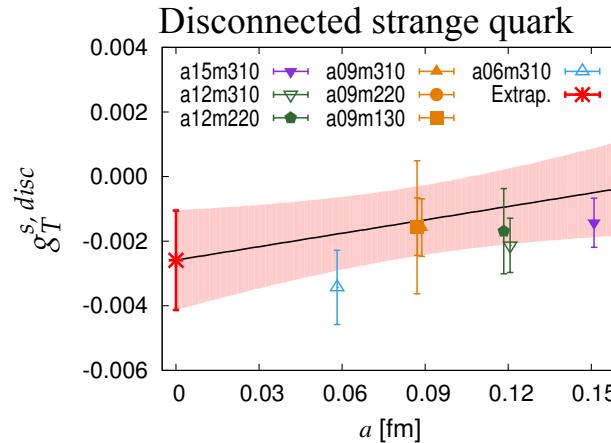
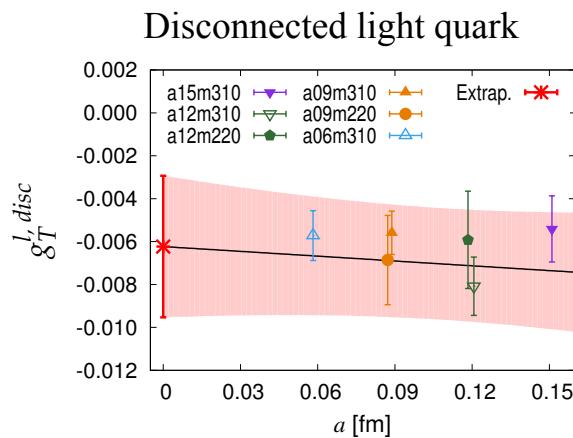
$a06m310$

Add more  
points and  
reduce errors



# Contribution of quark EDM to neutron EDM

$$g_T^q = \langle n(0) | \bar{q} \sigma_{\mu\nu} q | n(0) \rangle$$



Disconnected

$$g_T^l = -0.0064(32); \quad g_T^s = -0.0027(16)$$

Connected + Disconnected

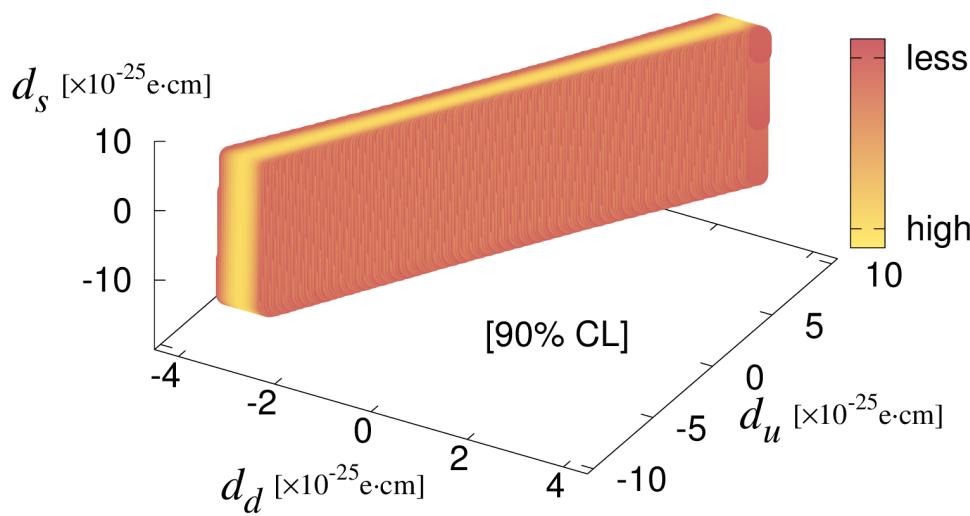
$$g_T^u = 0.784(28); \quad g_T^d = -0.204(11); \quad g_T^s = -0.0027(16)$$

# Contribution of quark EDM to neutron EDM

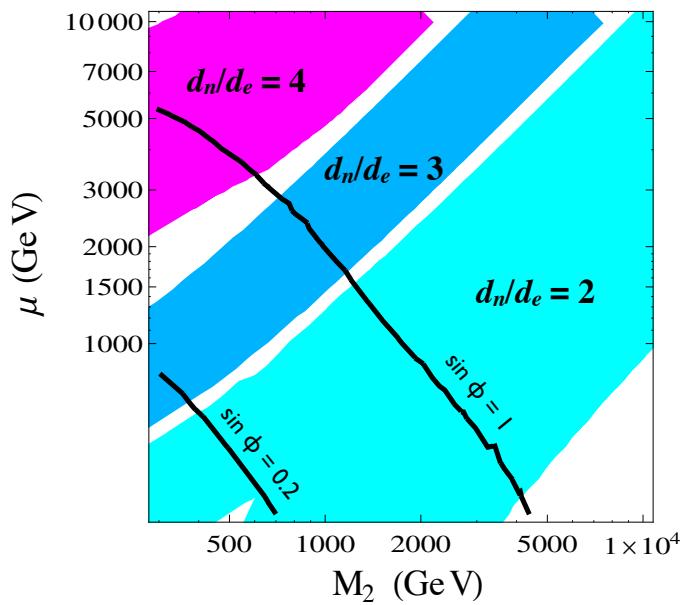
$$g_T^u = 0.784(28); \quad g_T^d = -0.204(11); \quad g_T^s = -0.0027(16)$$

Relation between charges  $g_T^q$ , couplings  $d_q^\gamma$ , and the neutron EDM  $d_n$

$$d_n = d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s + \dots$$



Constraint on  $d_n$  in Split SUSY



This is the only lattice QCD result on nEDM

# Contribution of quark spin to proton spin

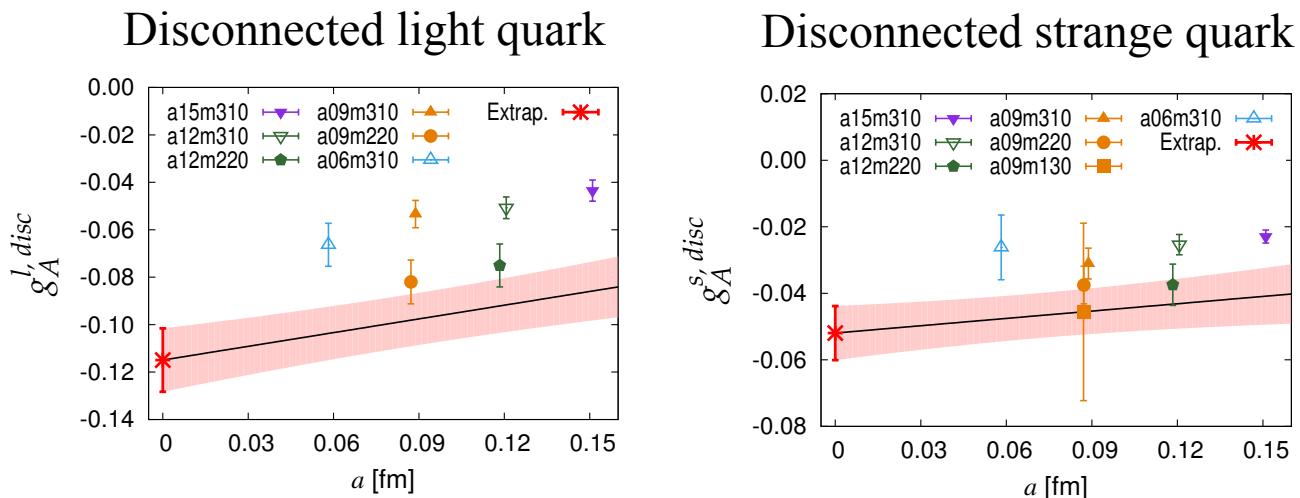
arXiv:1806:10604

$$g_A^q = \langle n(0) | \bar{q} \gamma_\mu \gamma_5 q | n(0) \rangle$$

- $g_A^u = 0.777(25)$
- $g_A^d = -0.438(18)$
- $g_A^s = -0.053(8)$

$$\frac{1}{2} \Delta\Sigma = g_A^u + g_A^d + g_A^s + \dots = 0.143(31)(29)$$

COMPASS Analysis (2016):  $0.13 < \Delta\Sigma/2 < 0.18$



Only lattice QCD result with continuum and chiral extrapolation