

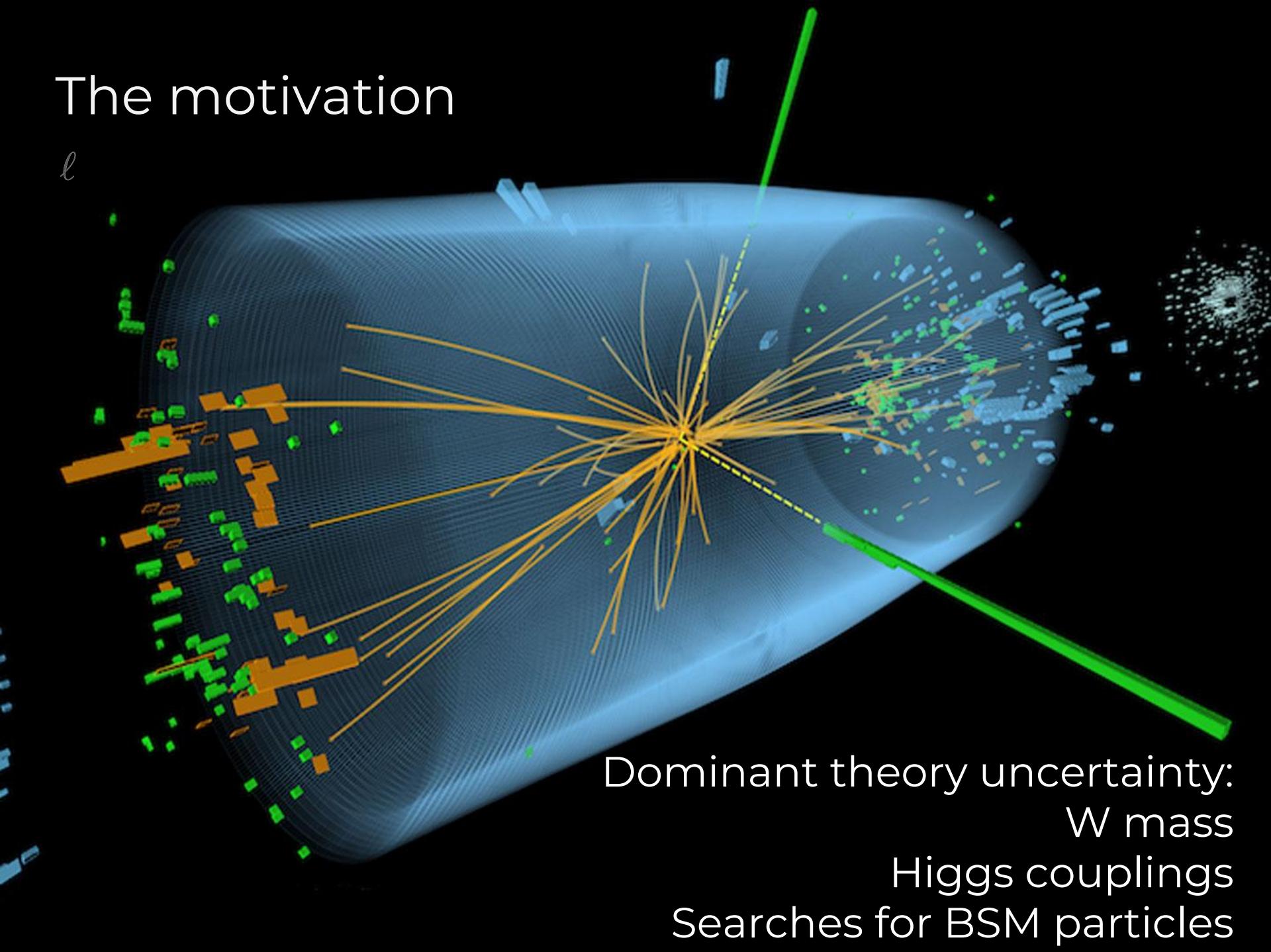
Recent developments in x-dependent structure calculations

Chris Monahan

*Institute for Nuclear Theory
University of Washington*



The motivation

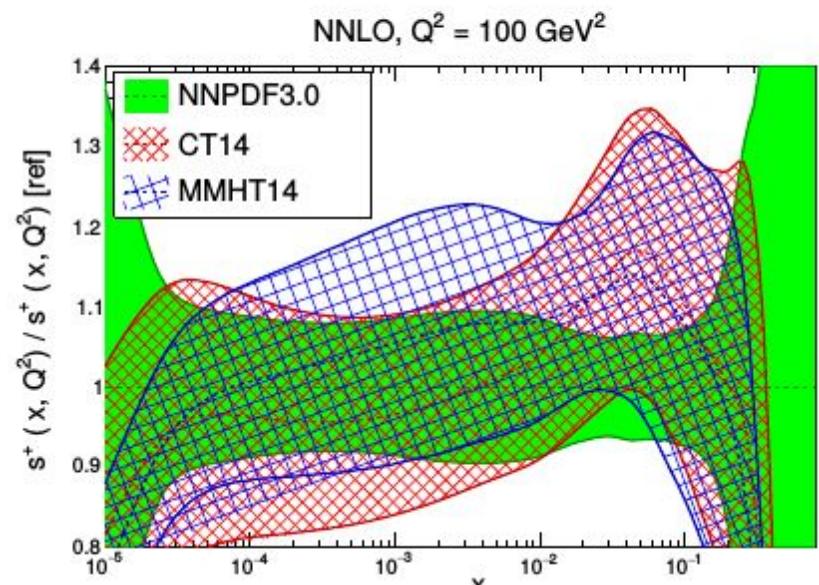
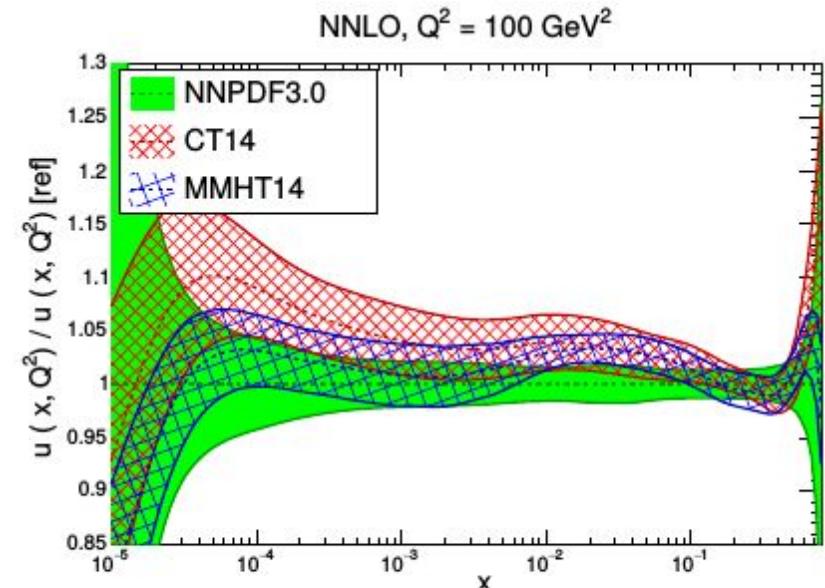
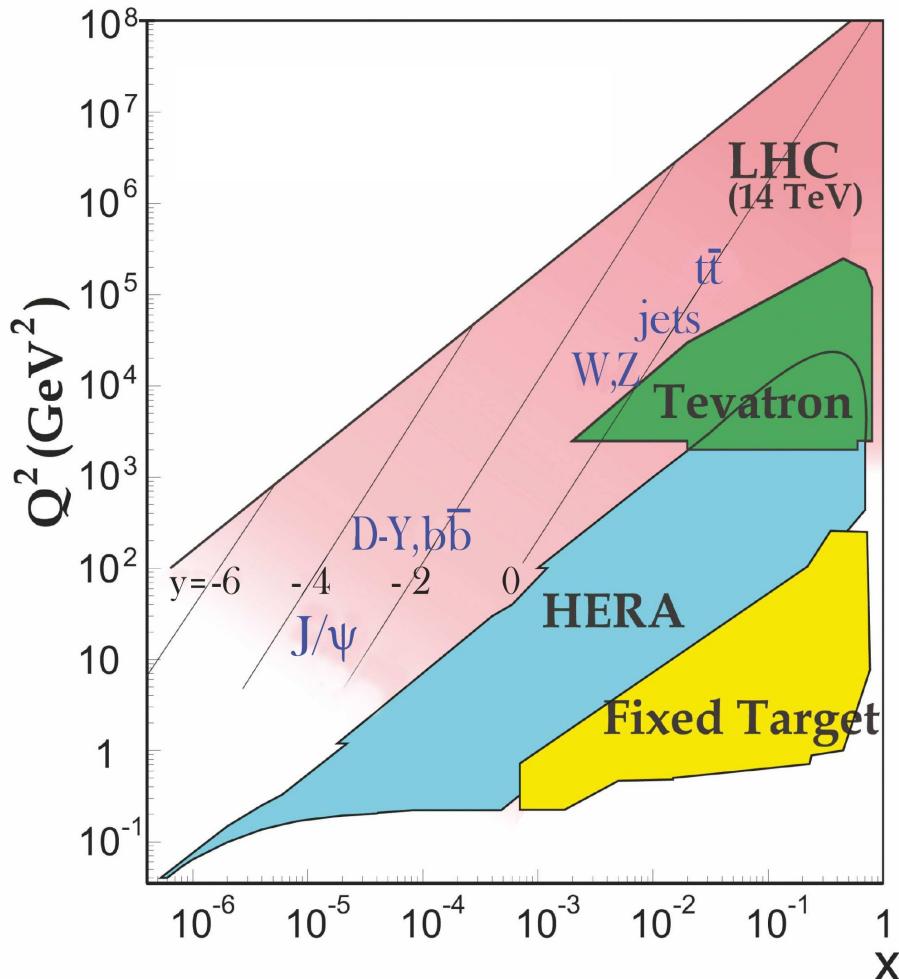


Dominant theory uncertainty:
 W mass
Higgs couplings
Searches for BSM particles

Parton distribution functions (PDFs)

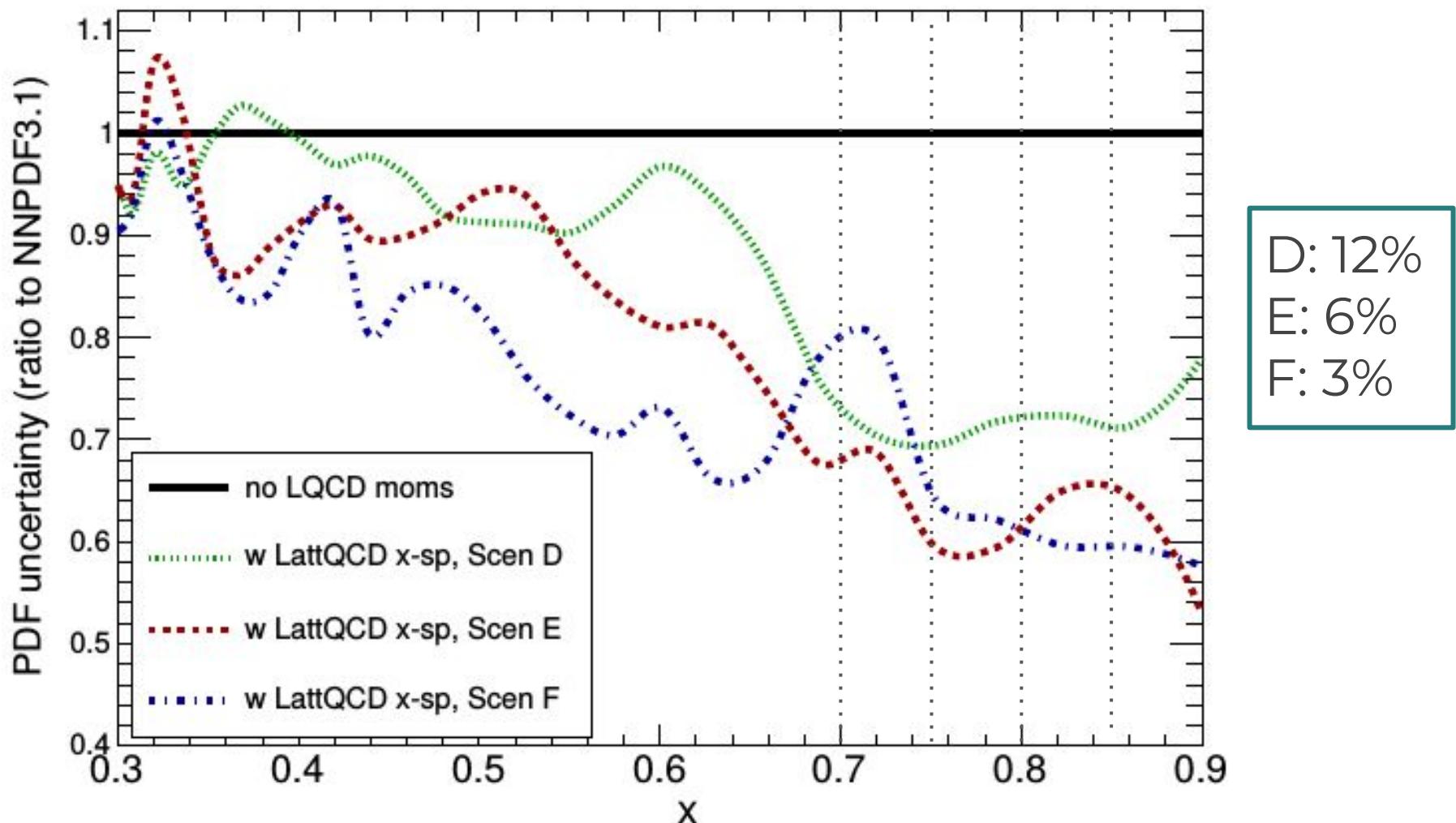
$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2q \cdot P}$$

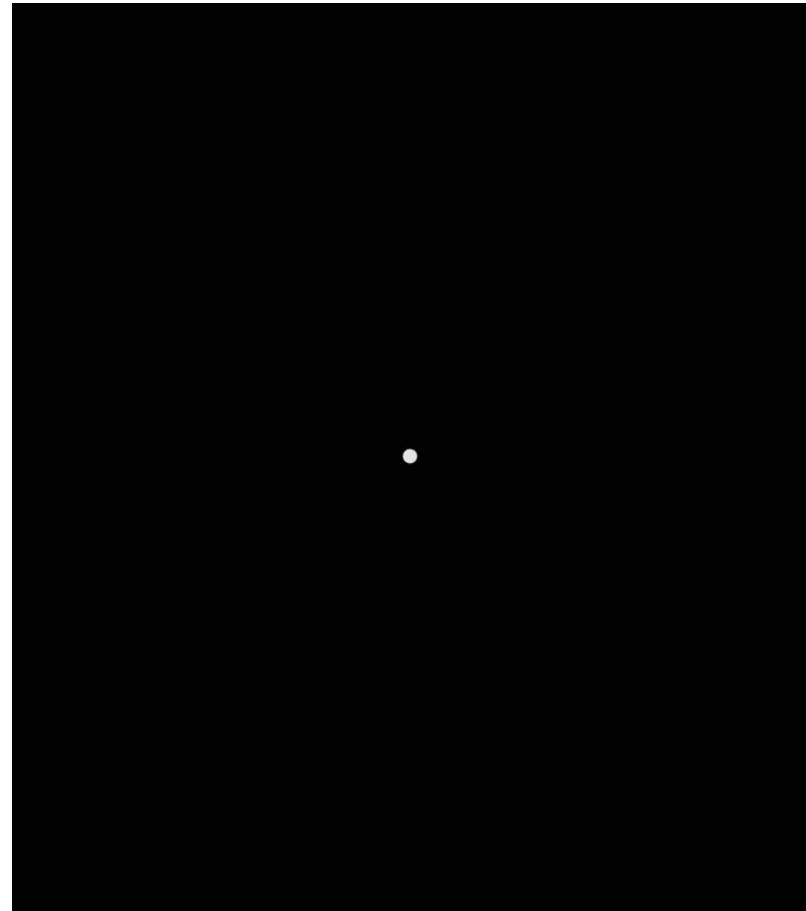
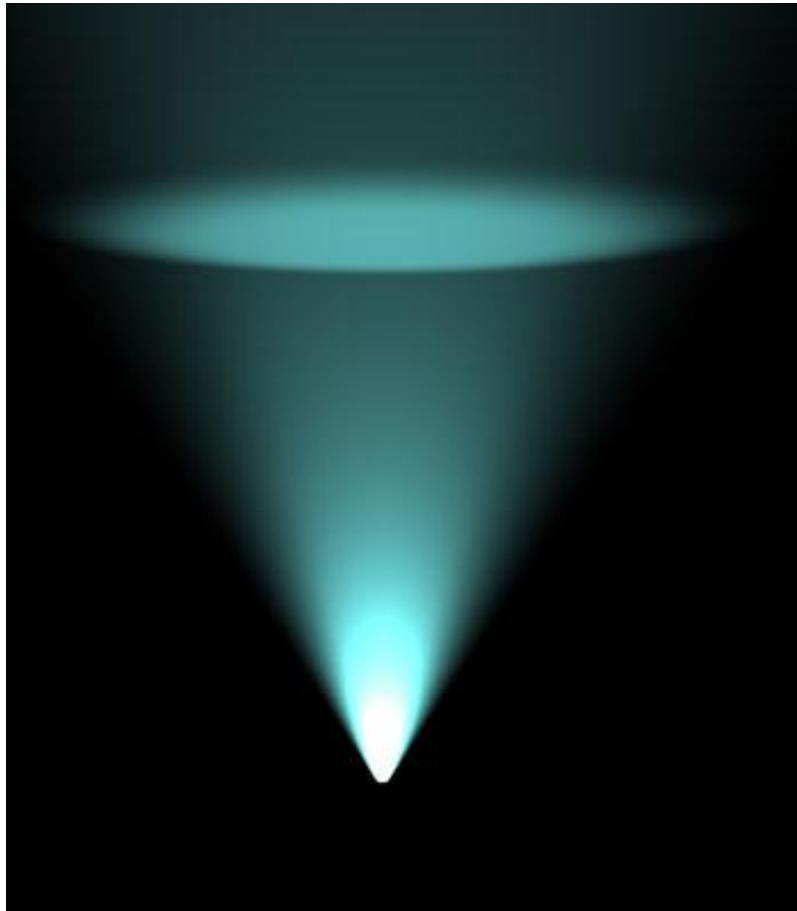


PDFs: the potential impact of lattice QCD

$\delta(\bar{d}) @ Q^2=4 \text{ GeV}^2, \text{NNPDF3.1}$



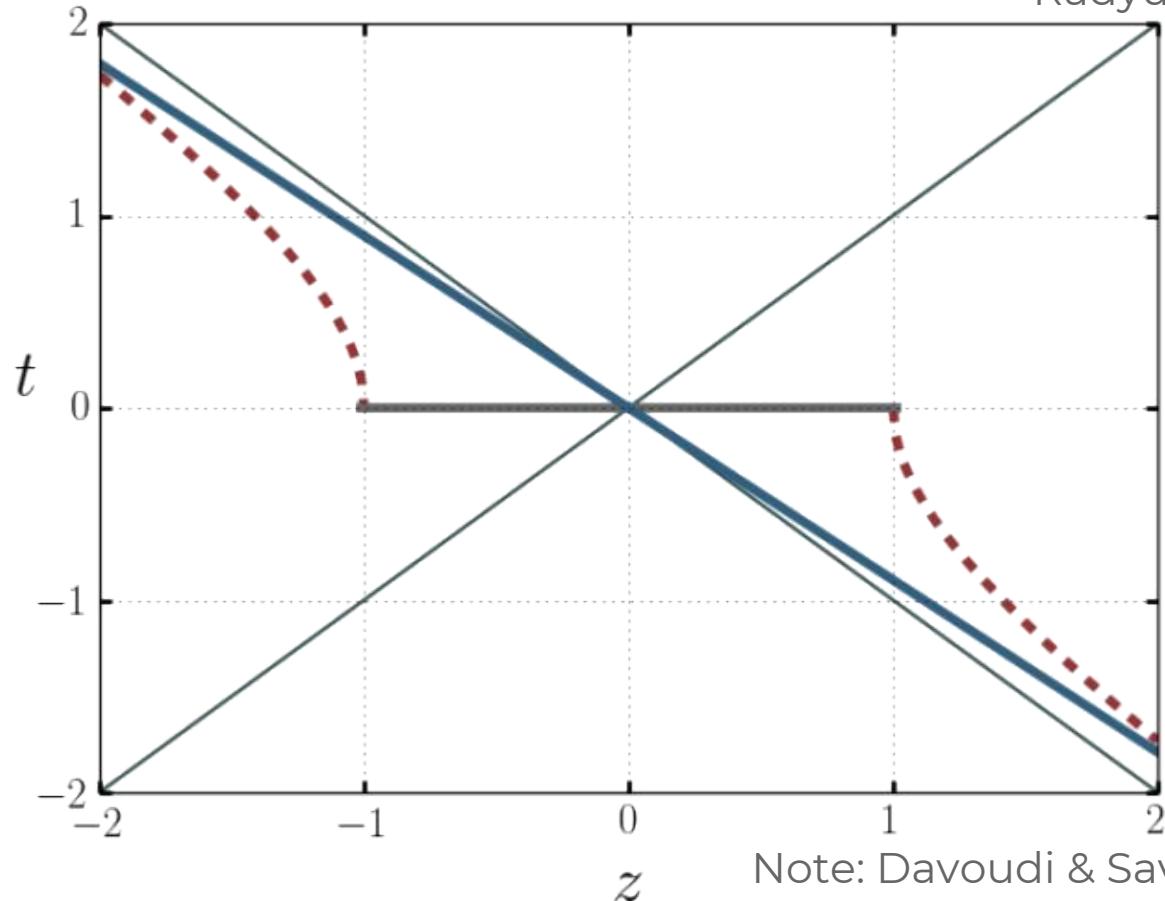
The problem



The solution(s) ... a recent renaissance

Ji, PRL 110 (2013) 262002

Radyushkin, PRD 96 (2017) 034025



Note: Davoudi & Savage, PRD 86 (2012) 054505

Musch et al., PRD 83 (2011) 094507

Braun & Müller, EPJ C55 (2008) 349

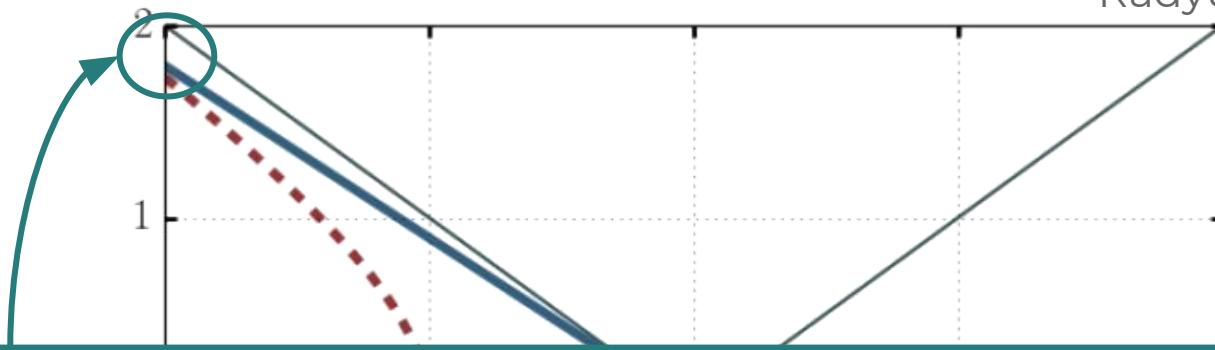
Detmold & Lin, PRD 73 (2006) 014501

Liu & Dong, PRL 72 (1994) 1790

The solution(s) ... a recent renaissance

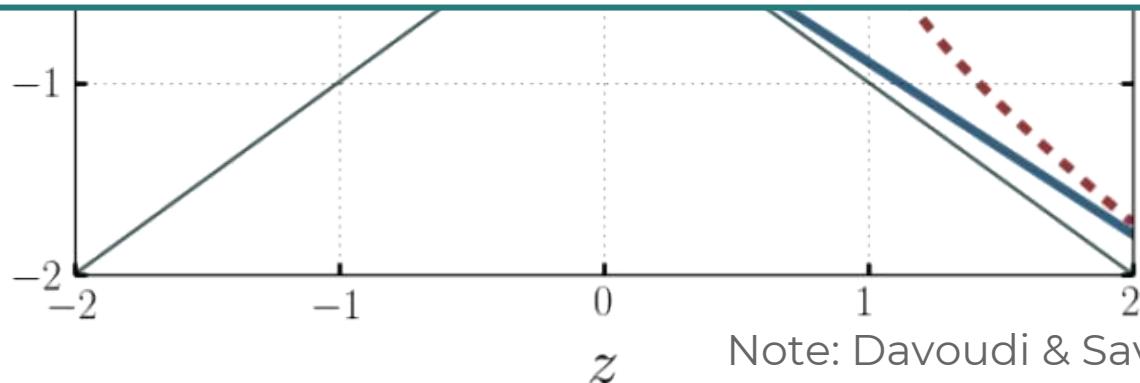
Ji, PRL 110 (2013) 262002

Radyushkin, PRD 96 (2017) 034025



Large Momentum Effective Theory (LaMET)

Ji, Sci.Ch. Phys.Mech.Ast. 57 (2014) 1407



Note: Davoudi & Savage, PRD 86 (2012) 054505

Musch et al., PRD 83 (2011) 094507

Braun & Müller, EPJ C55 (2008) 349

Detmold & Lin, PRD 73 (2006) 014501

Liu & Dong, PRL 72 (1994) 1790

The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Ioffe-time distributions

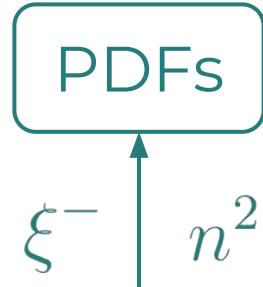
$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle P | \bar{\psi}(n) W(n, 0) \Gamma_\mu \psi(0) | P \rangle$$

$$W(n(u), 0) = \mathcal{P} \exp \left[-ig_0 \int_0^u dv \frac{dy^\mu}{dv} A_\mu^a(y(v)) T^a \right]$$

Radyushkin, PRD 96 (2017) 034025
Musch et al., PRD 83 (2011) 094507

A panoply of distributions: PDFs

$$f_{j/H}^{(0)}(x) = \int_{-\infty}^{\infty} \frac{d\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \langle P | \bar{\psi}(0, \omega^-, \mathbf{0}_T) W(\omega^-, 0) \Gamma \psi(0) | P \rangle$$

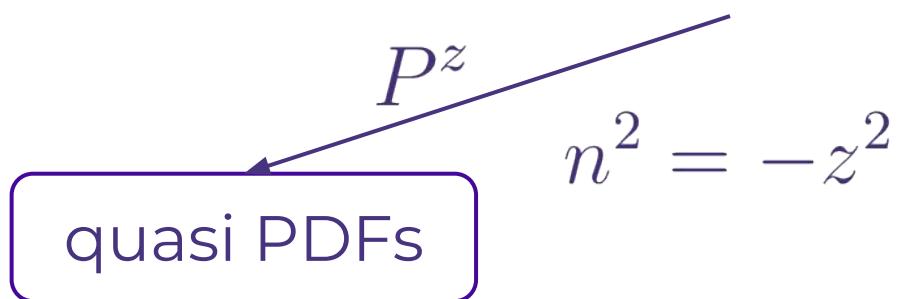


$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle P | \bar{\psi}(n) W(n, 0) \Gamma_\mu \psi(0) | P \rangle$$

A panoply of distributions: quasi PDFs



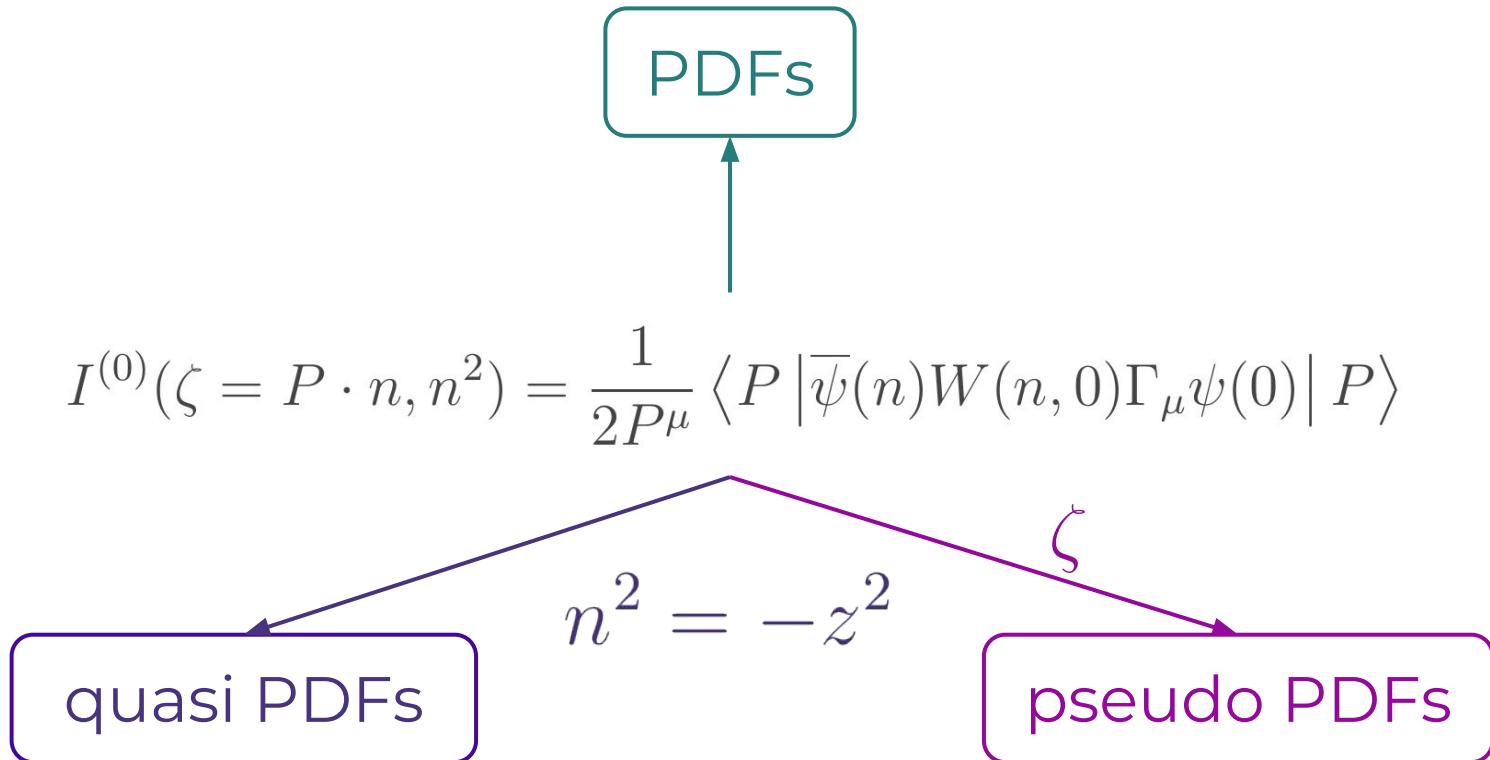
$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle P | \bar{\psi}(n) W(n, 0) \Gamma_\mu \psi(0) | P \rangle$$



$$\tilde{f}_{j/H}^{(0)}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{i\xi P_z z} \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle$$

A panoply of distributions: pseudo PDFs

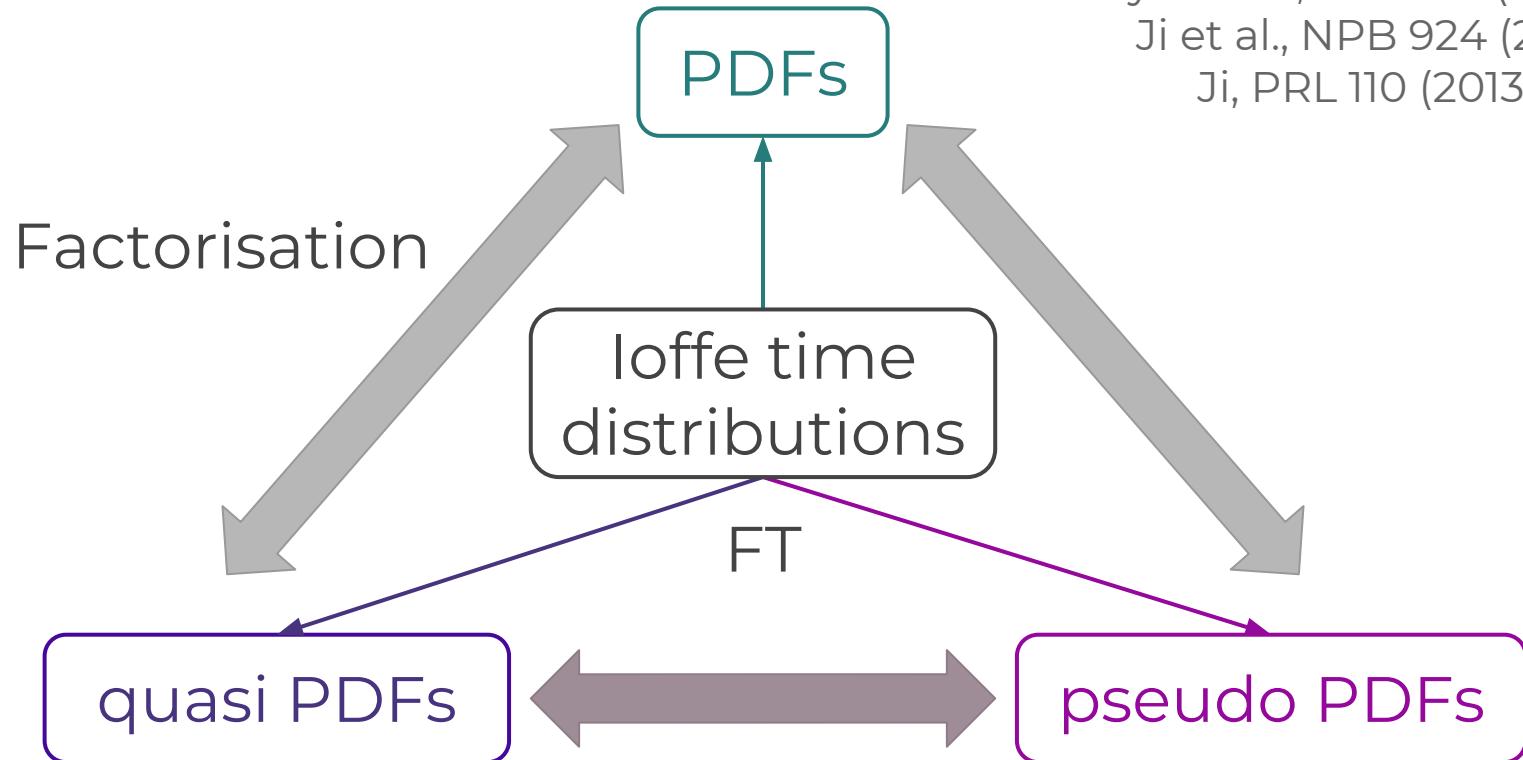
Radyushkin, PRD 96 (2017) 034025



$$\tilde{p}_{j/H}^{(0)}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{d\zeta}{4\pi} e^{i\xi\zeta} \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle$$

A panoply of distributions: factorisation

Izubuchi et al., 1801.03917
Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002



Factorisation

Izubuchi et al., 1801.03917
Zhang, Chen & CJM, PRD 97 (2018) 074508
Radyushkin, PLB 781 (2018) 433
Ji et al., NPB 924 (2017) 326
Ji, PRL 110 (2013) 262002

Factorisation theorems

$$\tilde{f}_{j/H}(\xi, P^z, \mu_R) = \int_{-1}^1 \frac{dy}{|y|} C^{(\tilde{f})} \left(\frac{\xi}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{yP^z} \right) f_{j/H}(y, \mu) + \mathcal{O} \left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(P^z)^2} \right)$$

$$\tilde{p}_{j/H}(\xi, z^2 \mu_R^2) = \int_{-1}^1 \frac{dy}{|y|} C^{(\tilde{p})} \left(\frac{\xi}{y}, \frac{\mu_R^2}{\mu^2}, \mu^2 z^2 \right) f_{j/H}(y, \mu) + \mathcal{O} (M^2 z^2, \Lambda_{\text{QCD}}^2 z^2)$$

Coefficients related by

Izubuchi et al., 1801.03917

$$C^{(\tilde{f})} \left(\xi, \frac{\mu}{yP^z} \right) = \int \frac{d\xi}{2\pi} e^{i\xi\xi} \int_{-1}^1 d\alpha e^{-i\alpha\xi} C^{(\tilde{p})} \left(\alpha, \frac{\mu^2 \xi^2}{(yP^z)^2} \right)$$

Example of “lattice ~~cross sections~~” factorisable matrix elements

$$\sigma_n(\xi \cdot P, \xi^2, P^2) = \int_{-1}^1 \frac{dx}{x} K_n(x \xi \cdot P, \xi^2, x^2 P^2, \mu^2) f(x, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\text{QCD}}^2)$$

Ma & Qiu, PRL 120 (2018) 022003
Ma & Qiu, 1404.6860

Renormalisation & systematics

bare lattice
matrix element

remove power
divergence/
renormalise

renormalised
matrix element

continuum,
infinite volume
limits

continuum
matrix element

Fourier
transform

continuum
quasi/pseudo PDF

factorise

PDF

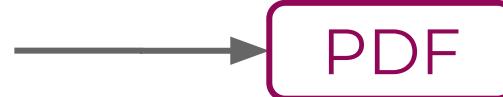
bare lattice
matrix element



Renormalisation & systematics

Matrix element extracted from long-time behaviour of Euclidean correlators is identical to that obtained from an LSZ reduction in Minkowski spacetime.

Carlson & Freid, PRD 095 (2017) 094504
Briceno, Hansen & CJM, PRD 96 (2017) 014502



bare lattice
matrix element

Renormalisation & systematics

remove power
divergence/
renormalise

renormalised
matrix element

Operator multiplicatively renormalisable in coordinate space

Ji et al., PRL 120 (2018) 112001
Ishikawa et al., PRD 96 (2017) 094019

Original conjectured convolution relation for
renormalisation now understood to be incomplete

e.g. Ji & Zhang, PRD 92 (2015) 034006

$$\tilde{f}_{j/H}(\xi, P^z, \mu_R) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{\xi}{y}\right) \tilde{f}_{j/H}^{(0)}(y, P^z)$$

Testa & Rossi, PRD 96 (2017) 014507
Testa & Rossi, 1806.00808

A. Radyushkin, 1807.07509
Karpie, Orginos & Zafeiropoulos, online soon

bare lattice
matrix element

Renormalisation & systematics

Y. Zhao Mon. 14:40

remove power
divergence/
renormalise

renormalised
matrix element

Operator multiplicatively renormalisable in coordinate space

1. Exponential mass counterterm

Ishikawa et al., 1609.02018
Chen et al., NPB 12 (2016) 004

2. RI/MOM, RI', and RI-xMOM schemes

Spanoudes & Panagopoulos, 1805.01164
Stewart & Zhao, PRD 97 (2018) 054512
Green et al., 1707.07152
Alexandrou et al., NPB 923 (2017) 394

3. “Reduced” pseudo PDFs

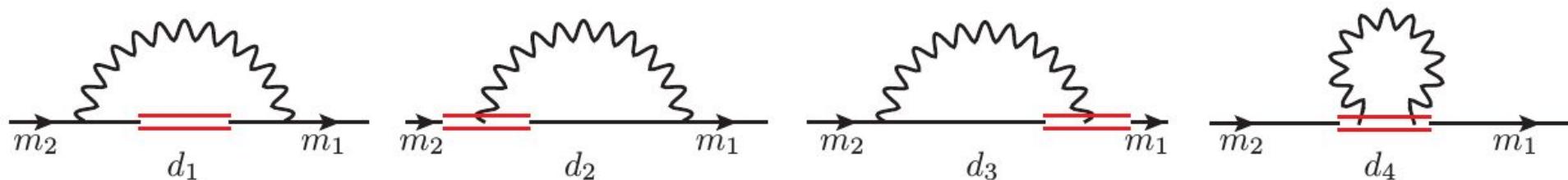
Radyushkin, PRD 96 (2017) 034025

4. Gradient flow

CJM, PRD 97 (2018) 054507
CJM & Orginos, JHEP 03 (2017) 116

Renormalisation

Effects of quark masses recently studied perturbatively



Additional flavour nonsinglet operator mixing induced by flavour symmetry breaking with different mass quarks

Requires an extended RI' scheme

Effects significant even for strange quarks

Renormalisation & systematics

bare lattice
matrix element

remove power
divergence/
renormalise

renormalised
matrix element

J. Guerrero Wed. 14:00

continuum,
infinite volume
limits

P. Wein Wed. 14:40

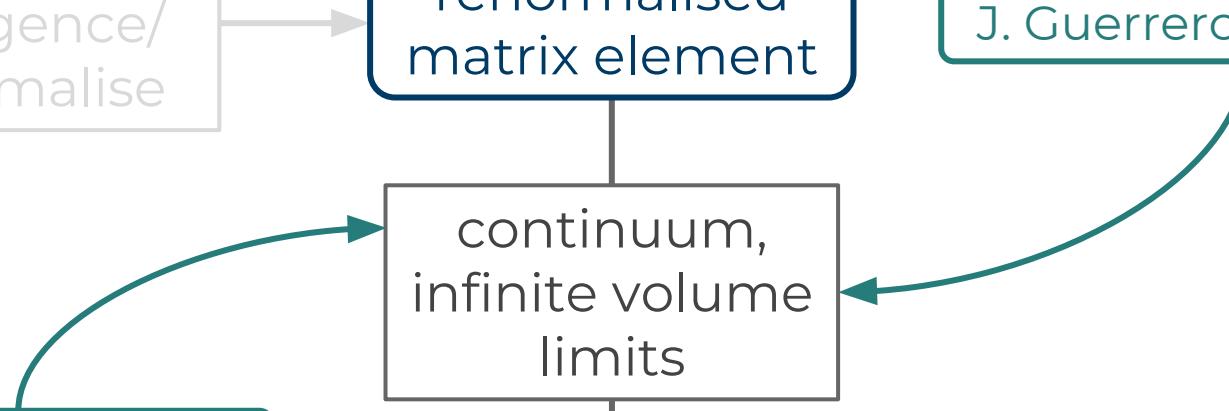
continuum
matrix element

Fourier
transform

continuum
quasi/pseudo PDF

factorise

PDF



Finite volume effects

Analytic study in a toy effective theory

- two species of scalar field, “pion” and “nucleon”
- spatially-extended current operators

Pion external states

$$\delta \mathcal{M}_L^{\text{LO}} \sim \frac{e^{-m_\pi|L-\xi|}}{(L-\xi)^{3/2}}$$

C. Lauer poster

Nucleon external states

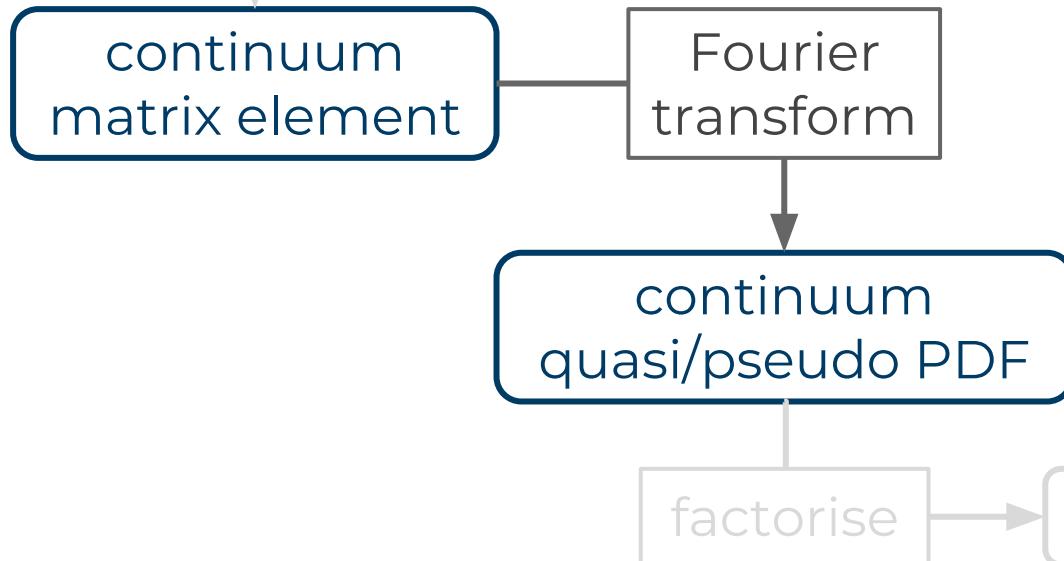
$$\delta \mathcal{M}_L \sim \sqrt{\frac{L-\xi}{\xi^3}} e^{-m_\pi L}$$

Renormalisation & systematics

Finite number of data points leads to spurious oscillations in quasi/pseudo PDFs, arising from the Fourier transform of the lattice matrix element.

Chen et al., PRD 97 (2018) 014505
Chen et al., 1711.07858
Lin et al., 1708.05301

S. Zafeiropoulos Mon. 16:10

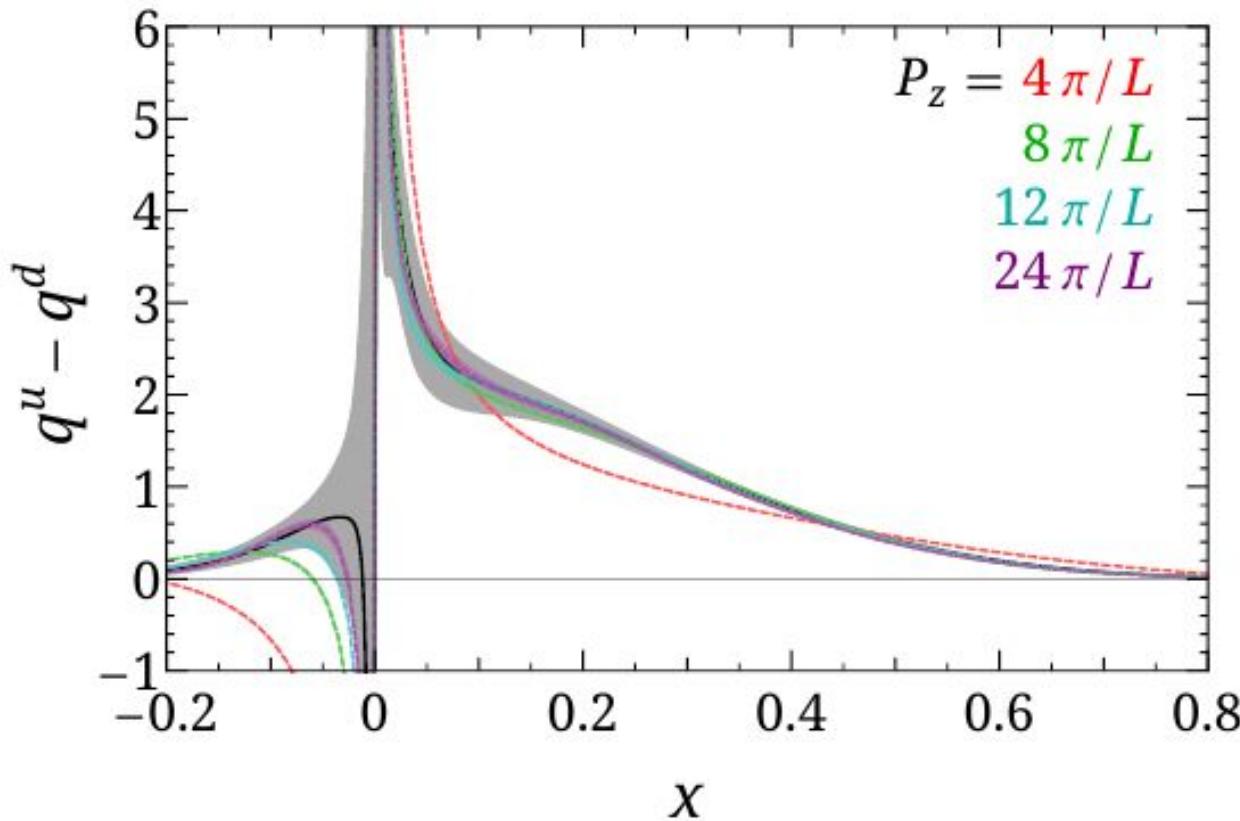


Spurious oscillations

Multiple methods studied, e.g. “derivative method”

$$\tilde{f}_{j/H}^{\text{deriv}}(\xi, P^z, \mu_R) = \frac{i}{2P^z\xi} \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} e^{ixP^zz} \left[h(z+1, P^z, \mu_R) - h(z-1, P^z, \mu_R) \right]$$

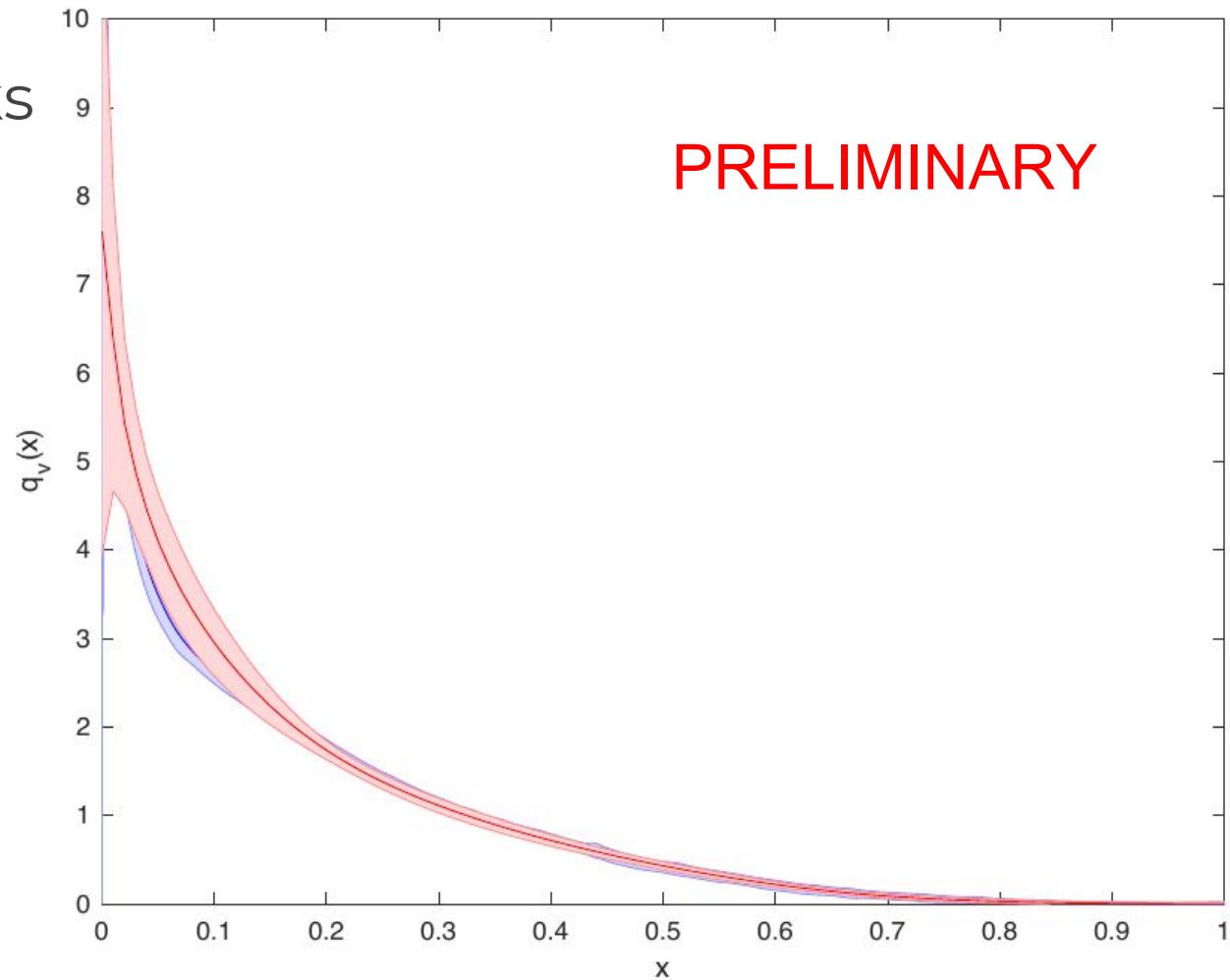
$$h(z, P^z, \mu_R) = \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle$$



Spurious oscillations

Multiple methods studied for pseudo PDFs:

- Bayesian inference
- Backus-Gilbert
- Neural networks



bare lattice
matrix element

remove power
divergence/
renormalise

Renormalisation & systematics

Target-mass/higher-twist corrections

Radyushkin, PLB 770 (2017) 514

Chen et al., 911 NPB (2016) 246

Bali et al., PRD 93 (2016) 094515

Alexandrou et al., PRD 92 (2015) 01502

Y. Zhao Mon. 14:40

Scheme matching

Y.-S. Liu et al., 1807.06566

Stewart & Zhao, PRD 97 (2018) 054512

Chen et al., PRD 97 (2018) 014505

Alexandrou et al., NPB 923 (2017) 394

Chen et al., NPB 915 (2017) 1

Xiong et al., 1705.00246

Alexandrou et al., PRD 92 (2015) 014502

Xiong et al., PRD 90 (2014) 014051

Factorisation

Izubuchi et al., 1801.03917

Zhang, Chen & CJM, PRD 97 (2018) 074508

Radyushkin, PLB 781 (2018) 433

Ji et al., NPB 924 (2017) 326

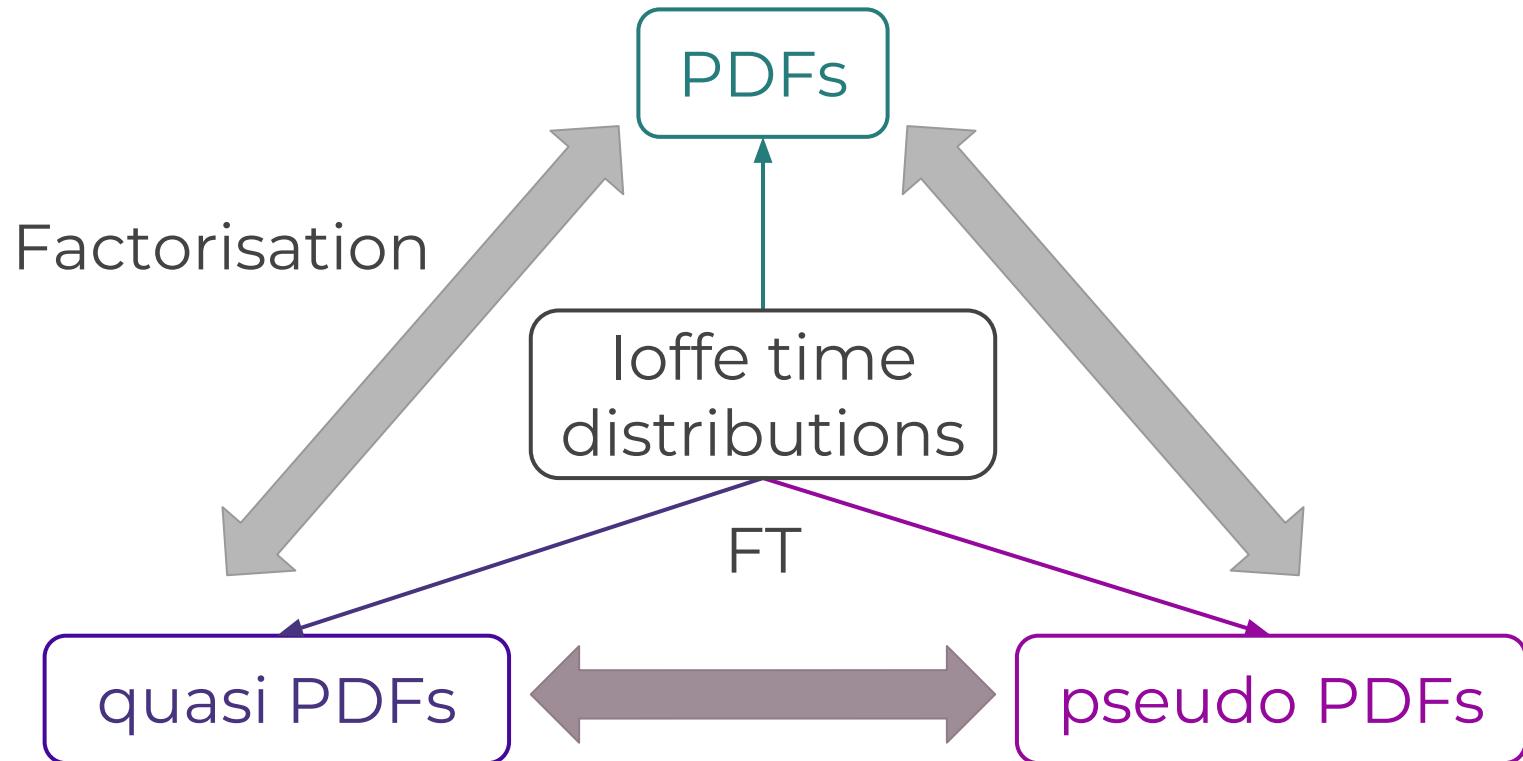
Ji, PRL 110 (2013) 262002

continuum
quasi/pseudo PDF

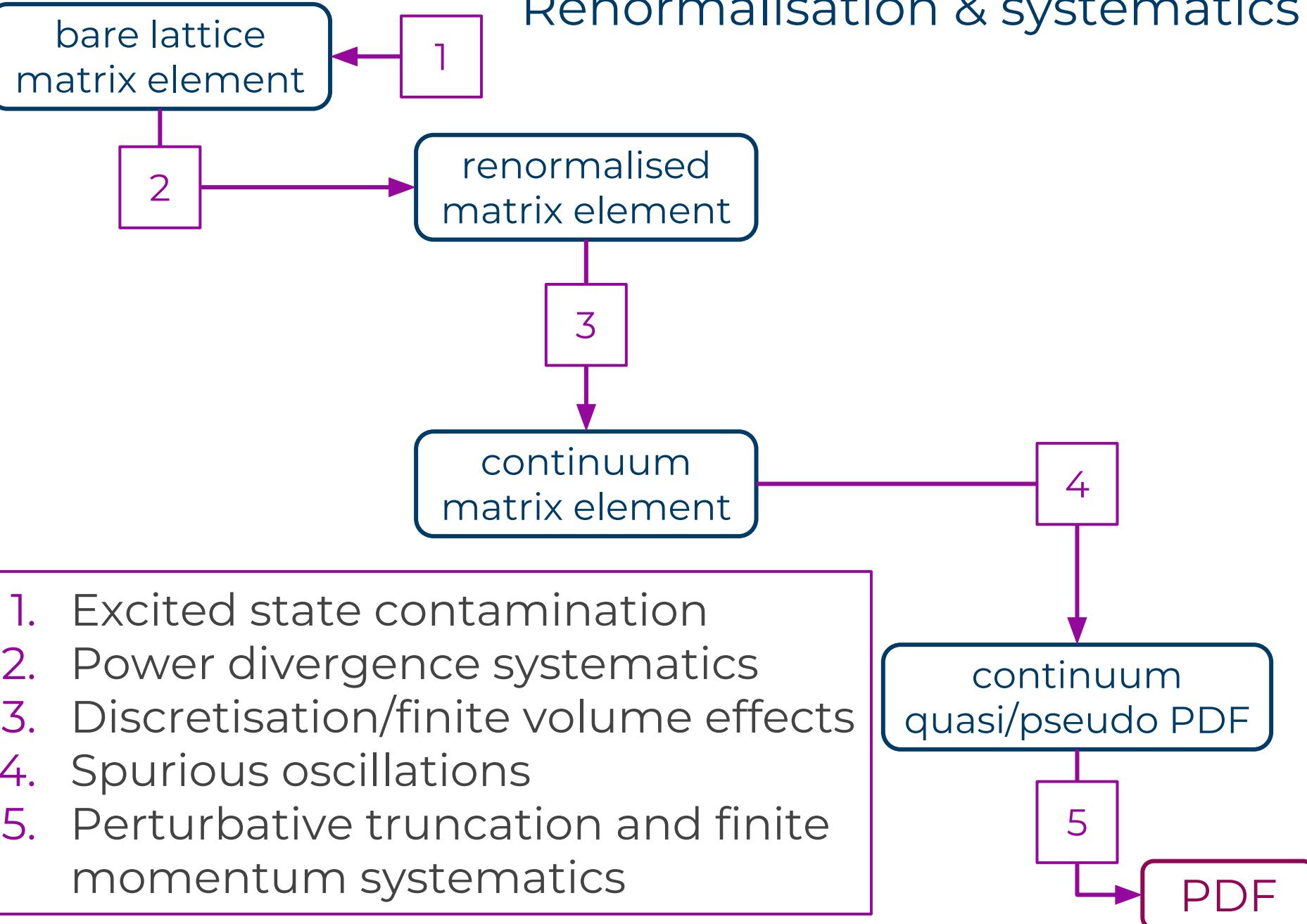
factorise

PDF

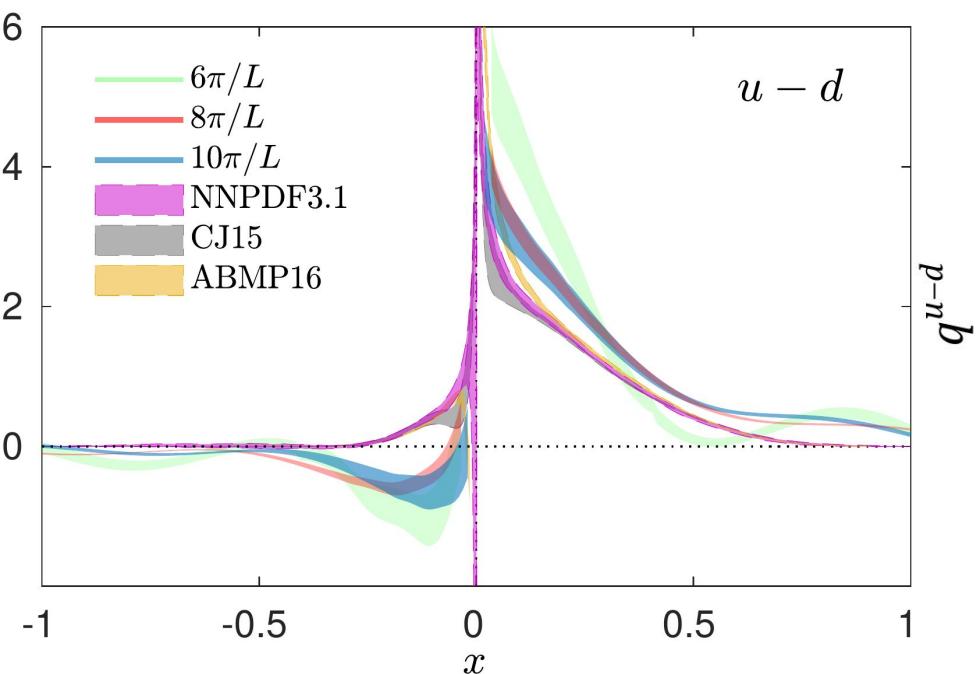
A panoply of distributions: reminder



Renormalisation & systematics



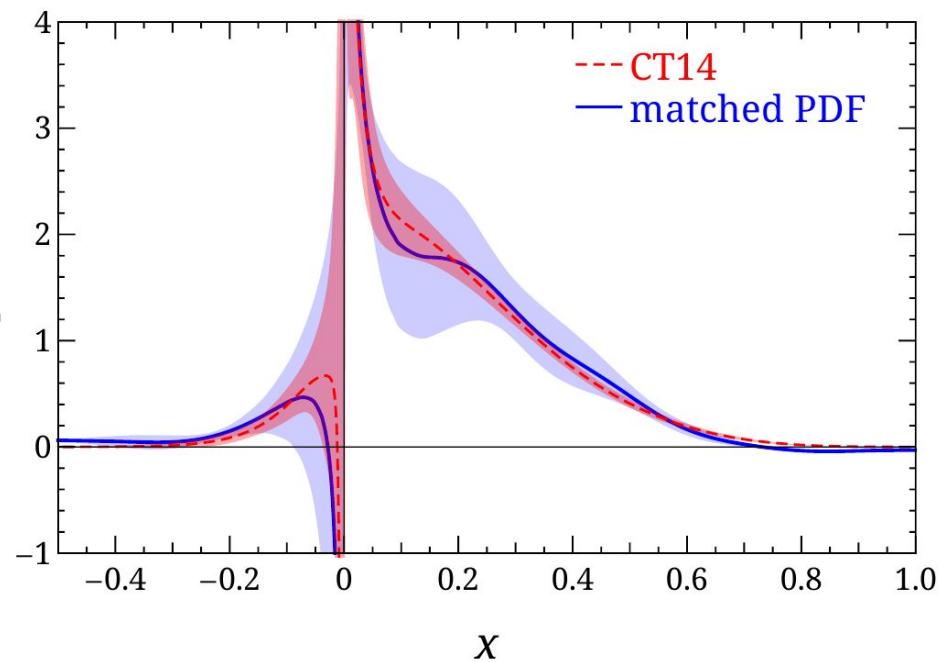
Unpolarised nucleon PDF



Alexandrou et al., 1803.02685

K. Cichy Mon. 14:00

A. Scapellato Mon. 14:20

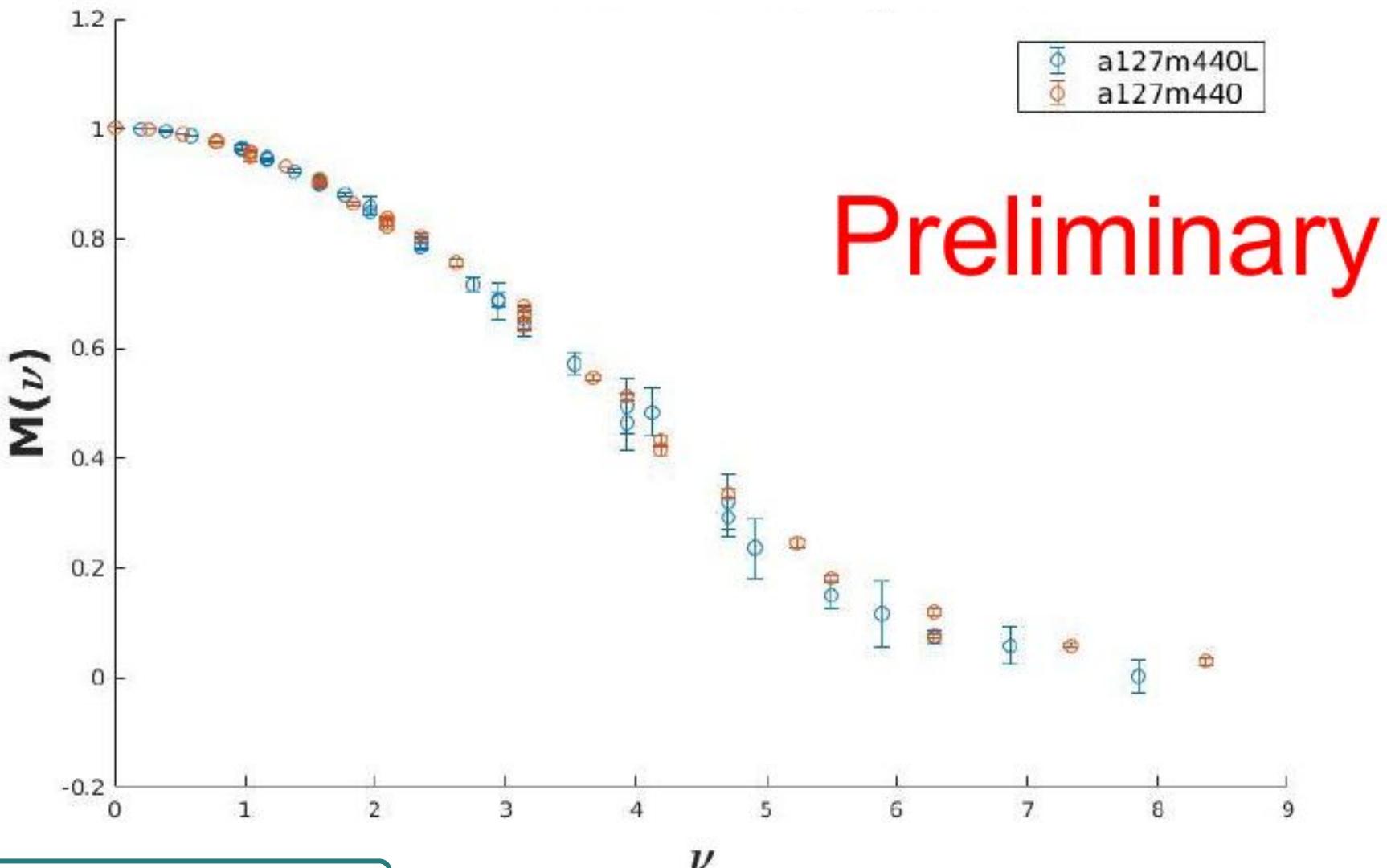


Chen et al., 1803.04393

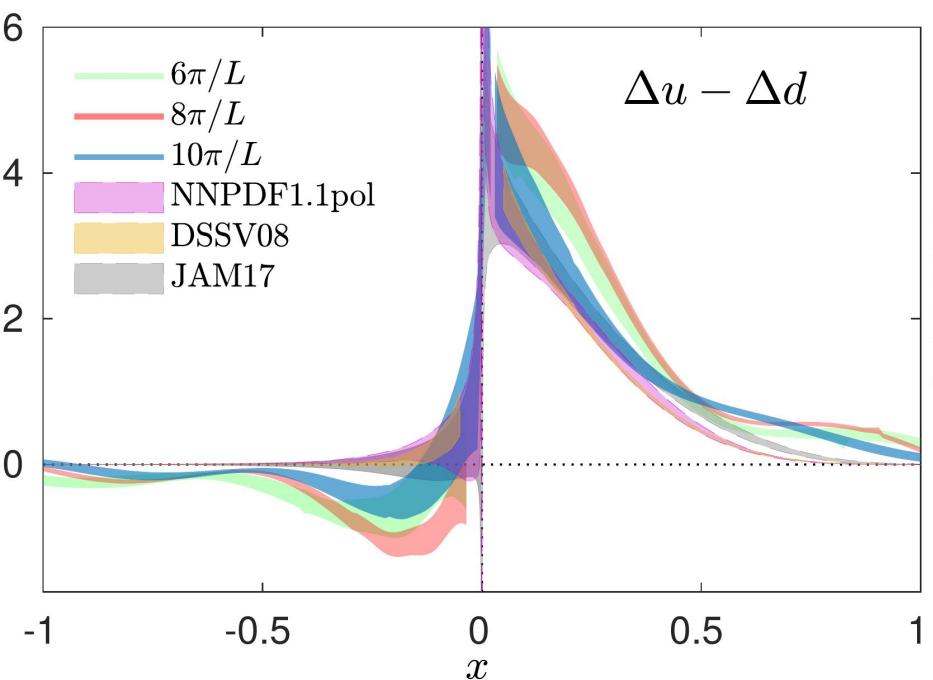
Y. Yang Mon. 15:00

Y.-S. Liu Mon. 15:20

JLab results: unpolarised nucleon PDF



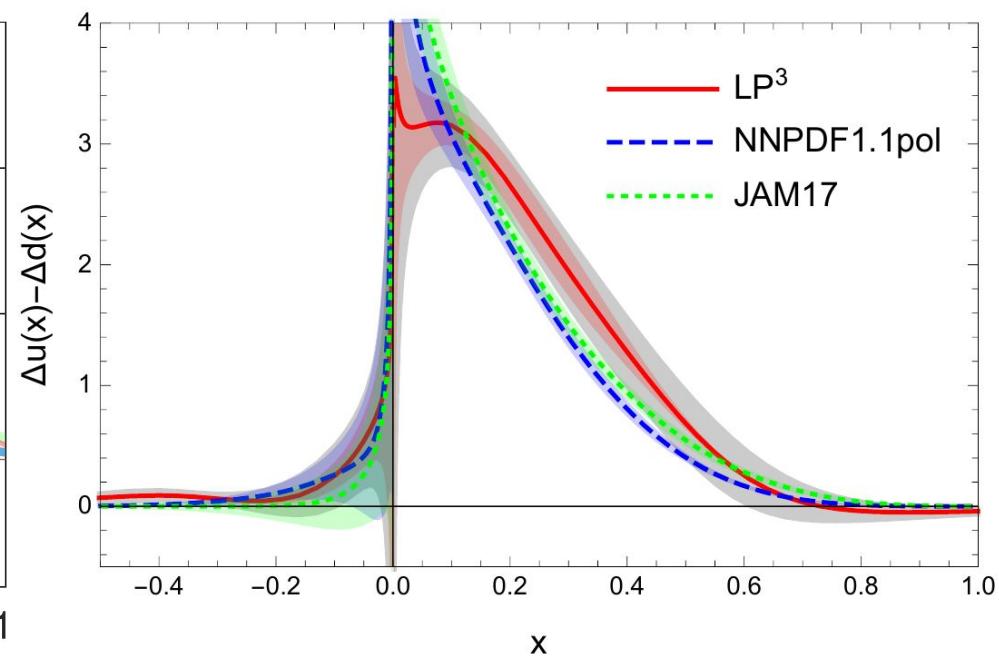
Polarised nucleon PDF



Alexandrou et al., 1803.02685

K. Cichy Mon. 14:00

A. Scapellato Mon. 14:20



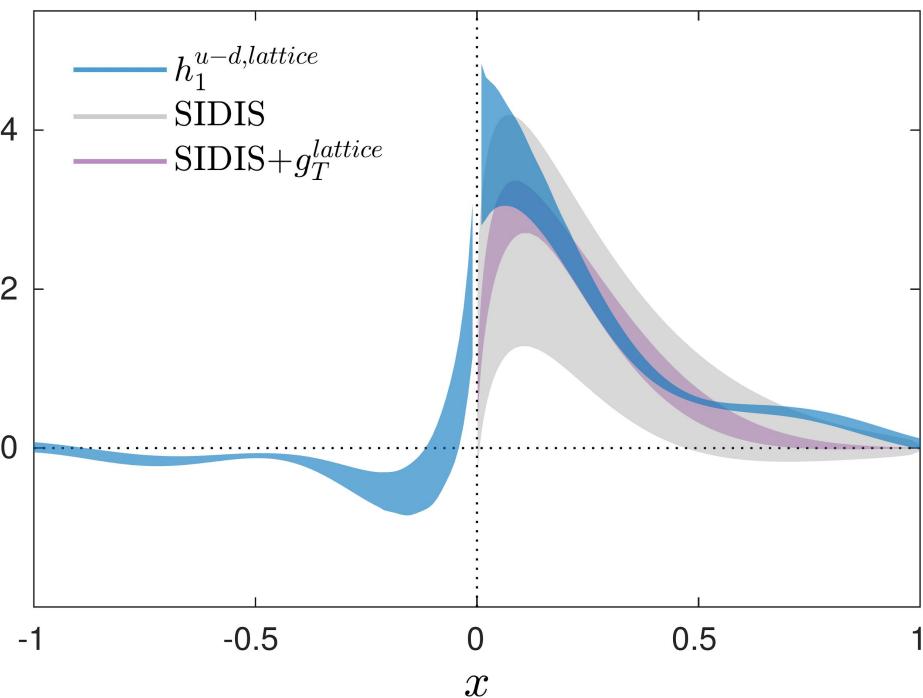
Chen et al., 1807.07431

Y. Yang Mon. 15:00

Y.-S. Liu Mon. 15:20

Nucleon transversity

First nucleon
transversity results!



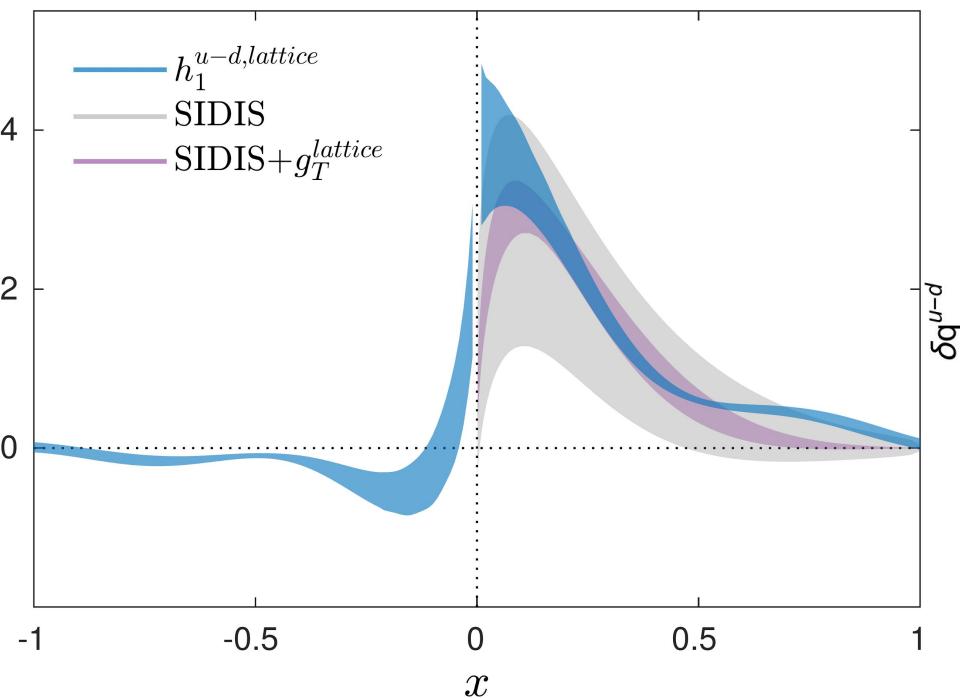
Alexandrou et al., 1807.00232

K. Cichy Mon. 14:00

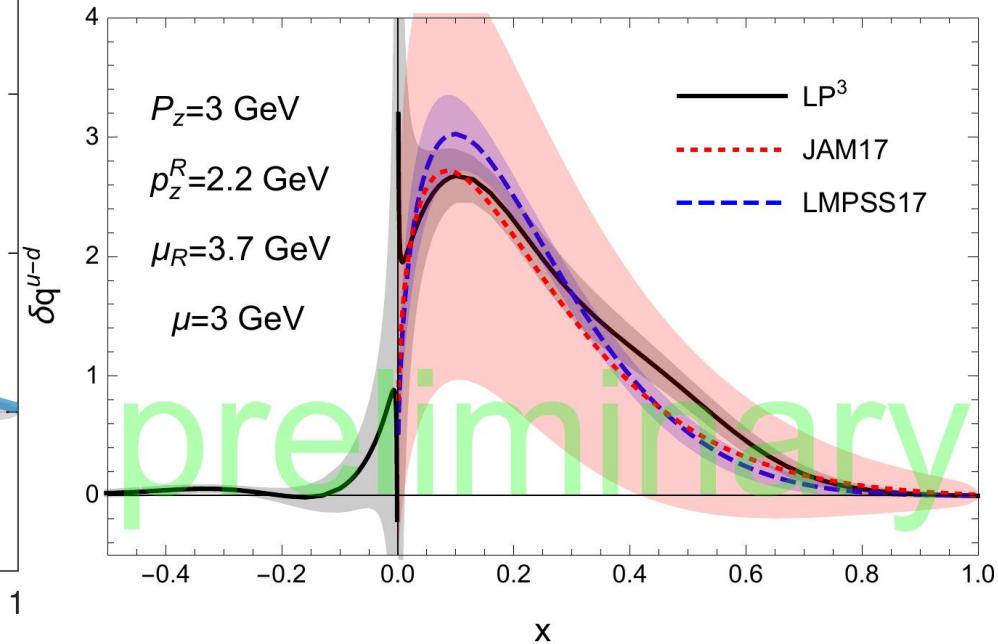
A. Scapellato Mon. 14:20

Nucleon transversity

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transversity results!



Alexandrou et al., 1807.00232



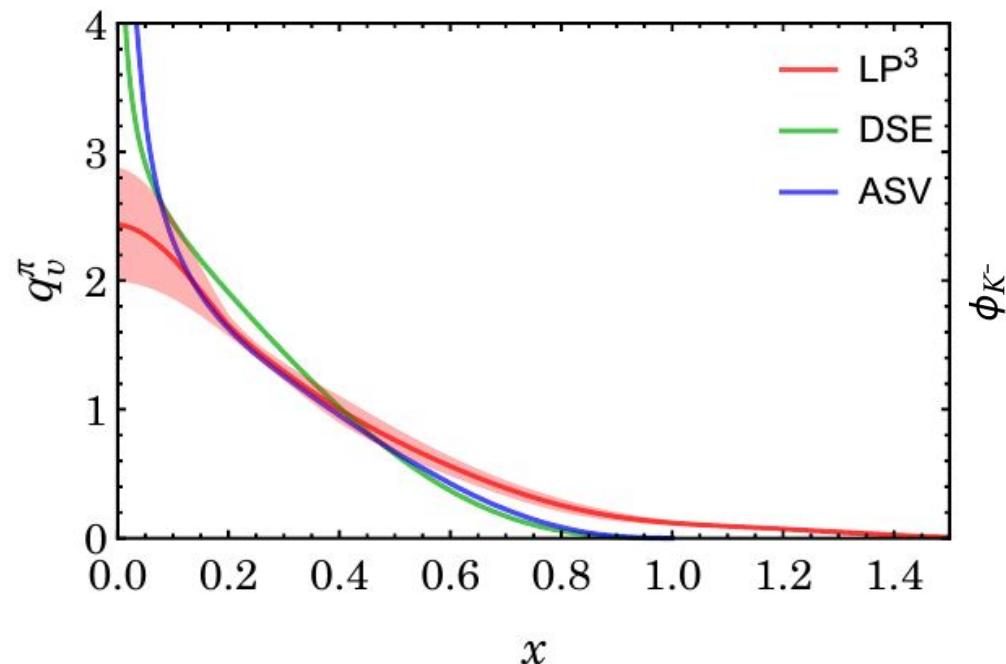
K. Cichy Mon. 14:00

A. Scapellato Mon. 14:20

Y.-S. Liu Mon. 15:20

Meson PDFs and DAs

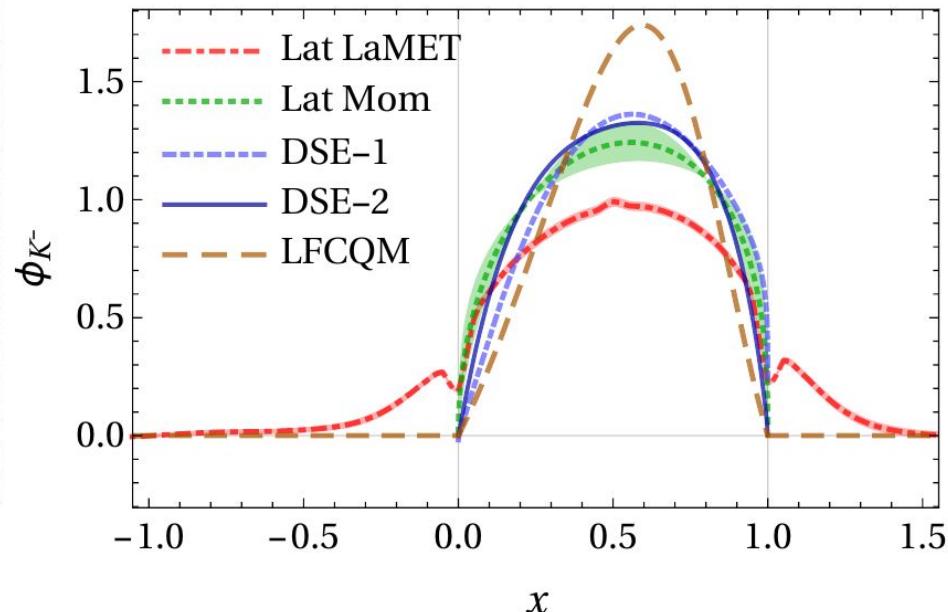
First pion
PDF results!



Chen et al., 1804.01483

J. Zhang Tue. 16:10

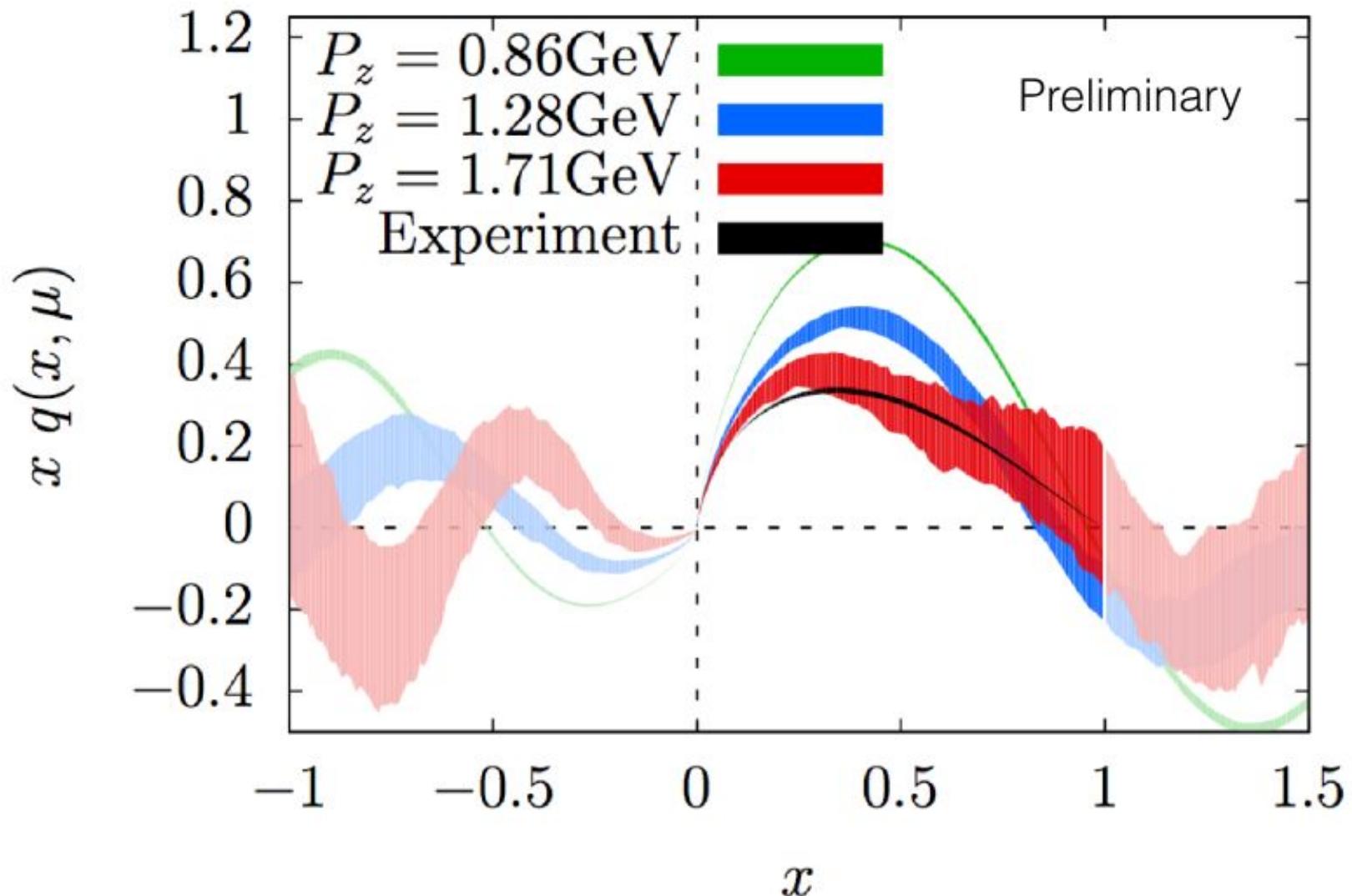
First kaon
DA results!



Chen et al., 1712.10025

R. Zhang Tue. 14:40

Unpolarised pion PDF



The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Factorisable matrix elements

Position-space correlation functions

- encode all necessary information
- avoid challenges of nonlocal renormalisation

Match lattice matrix element to perturbative QCD

$$\Phi_\pi(p \cdot z) = \frac{2(\pi z^2)^2}{f_\pi p \cdot z} \langle \pi^0(\mathbf{p}) | [\bar{u}q](z/2)[\bar{q}\gamma_5 u](-z/2) | 0 \rangle$$

$$\Phi_\pi(p \cdot z) = \int_0^1 dx e^{i(x-1/2)p \cdot z} \phi_\pi(x)$$

Require

- small z^2 for factorisation
- large $p.z$ only to distinguish pion DA shapes

Current combinations access higher twist DAs

Factorisable matrix elements

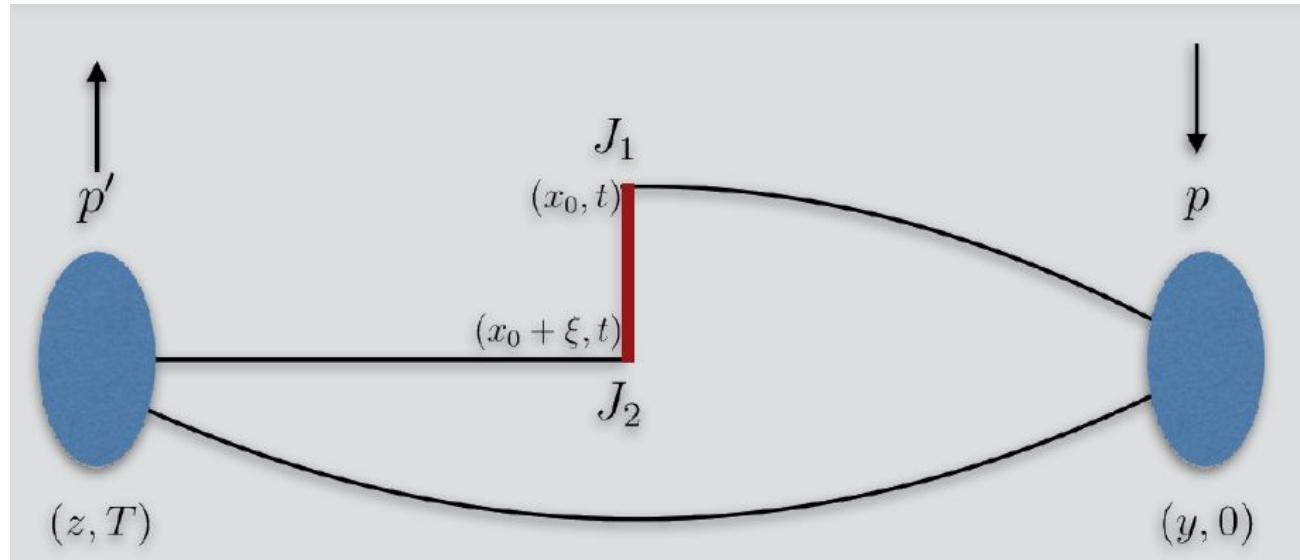
Single hadron matrix elements

- calculable on the lattice
- UV finite
- share perturbative collinear divergences with PDFs
- factorisable, with IR-safe coefficients

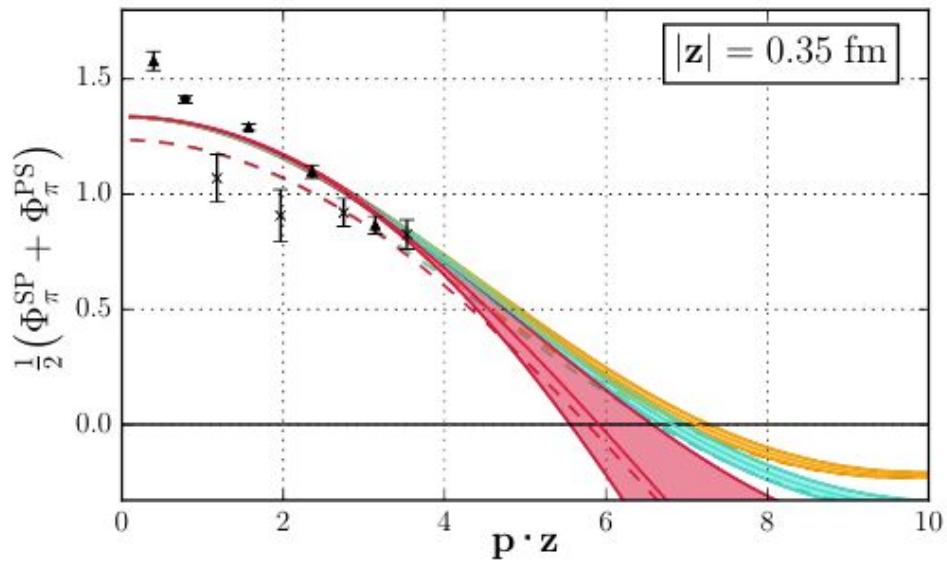
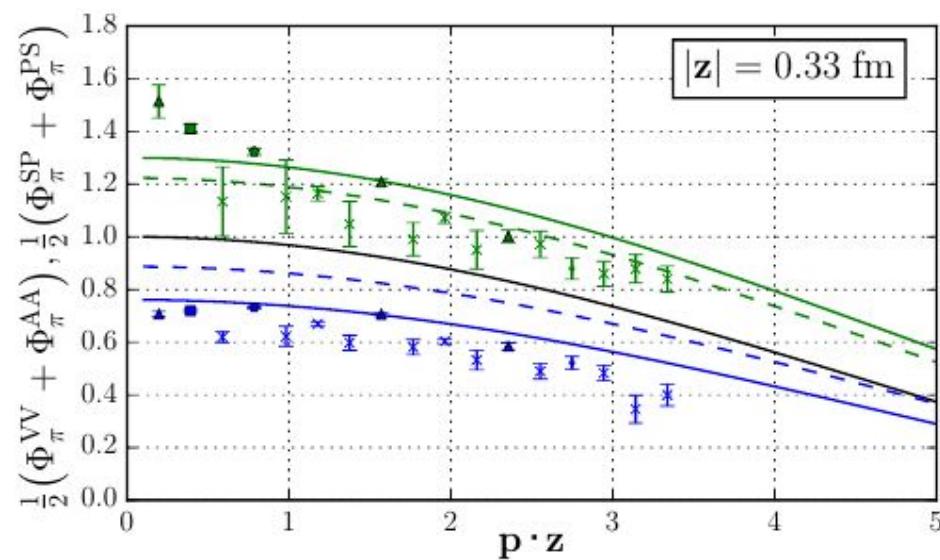
R. Sufian Tues. 14:00

B. Chakraborty Tues. 14:20

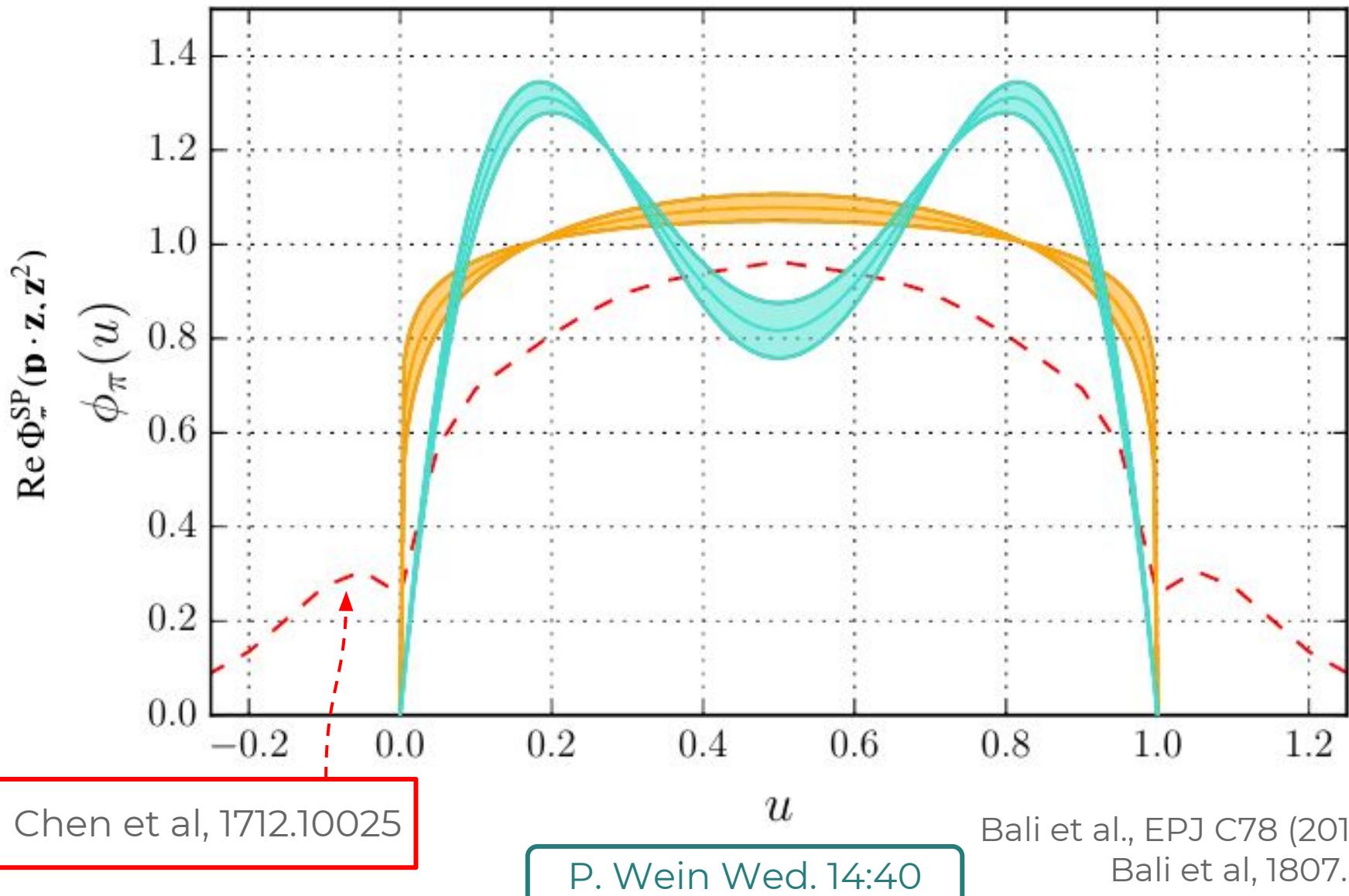
Fit data from various current combinations



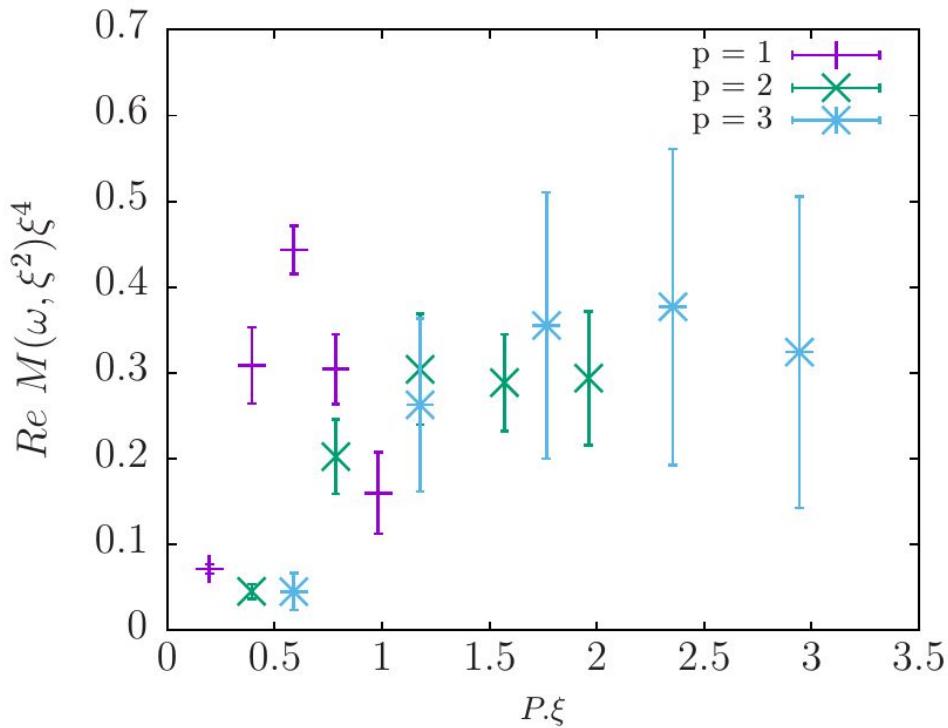
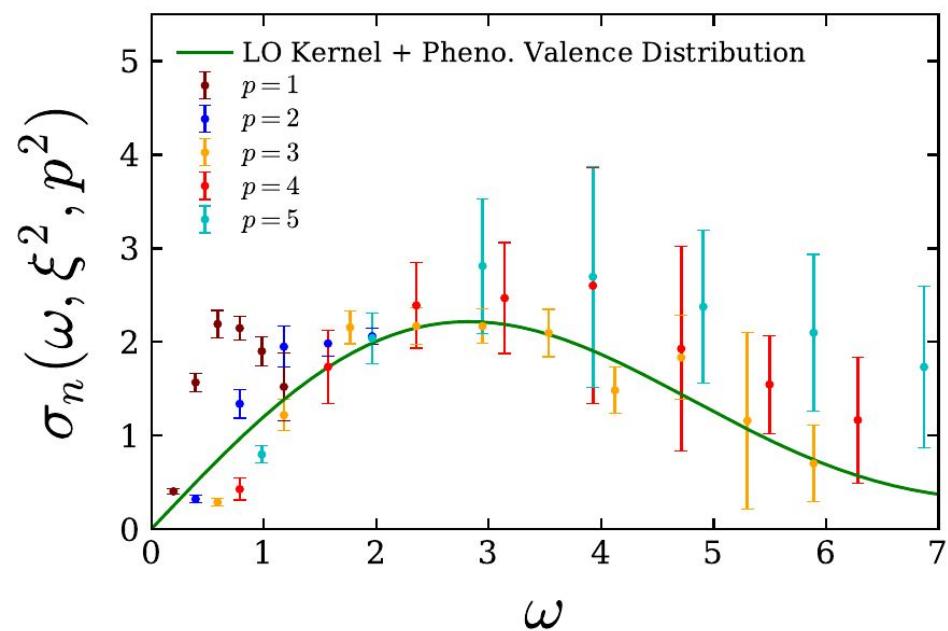
Factorisable matrix elements: pion DA results



Factorisable matrix elements: pion DA results



Factorisable matrix elements: meson DA



R. Sufian Tues. 14:00

$$\omega = P \cdot \xi$$

B. Chakraborty Tues. 14:20

The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Euclidean hadronic tensor

Liu & Dong, PRL 72 (1994) 1790
Liu, PRD 62 (2000) 074501
Liu, PoS(LATTICE 2015) 115
Hansen, Meyer & Robaina, PRD 96 (2017) 094513

Formulate in Euclidean path-integral formalism

- renormalisation straightforward
- frame invariant

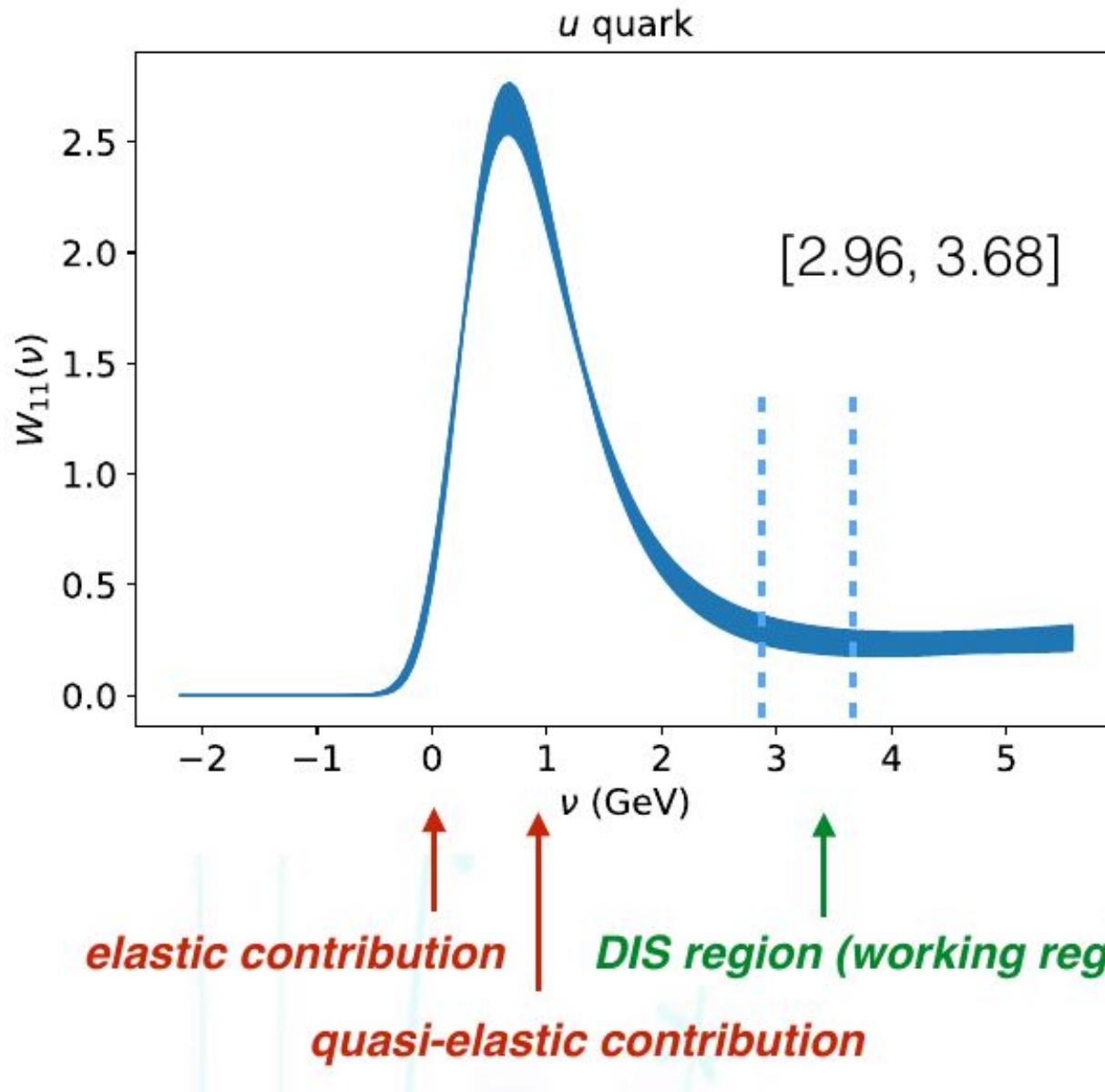
$$W_{\mu\nu}(\mathbf{q}, \tau) = \frac{1}{4\pi} \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}} \langle P | J_\mu(\mathbf{x}, \tau) J_\nu(0, 0) | P \rangle$$

$$W_{\mu\nu}(q^2, q \cdot P) = \frac{1}{i} \int_{c-i\infty}^{c+i\infty} d\tau e^{q \cdot P \tau} W_{\mu\nu}(\mathbf{q}, \tau)$$

Challenges:

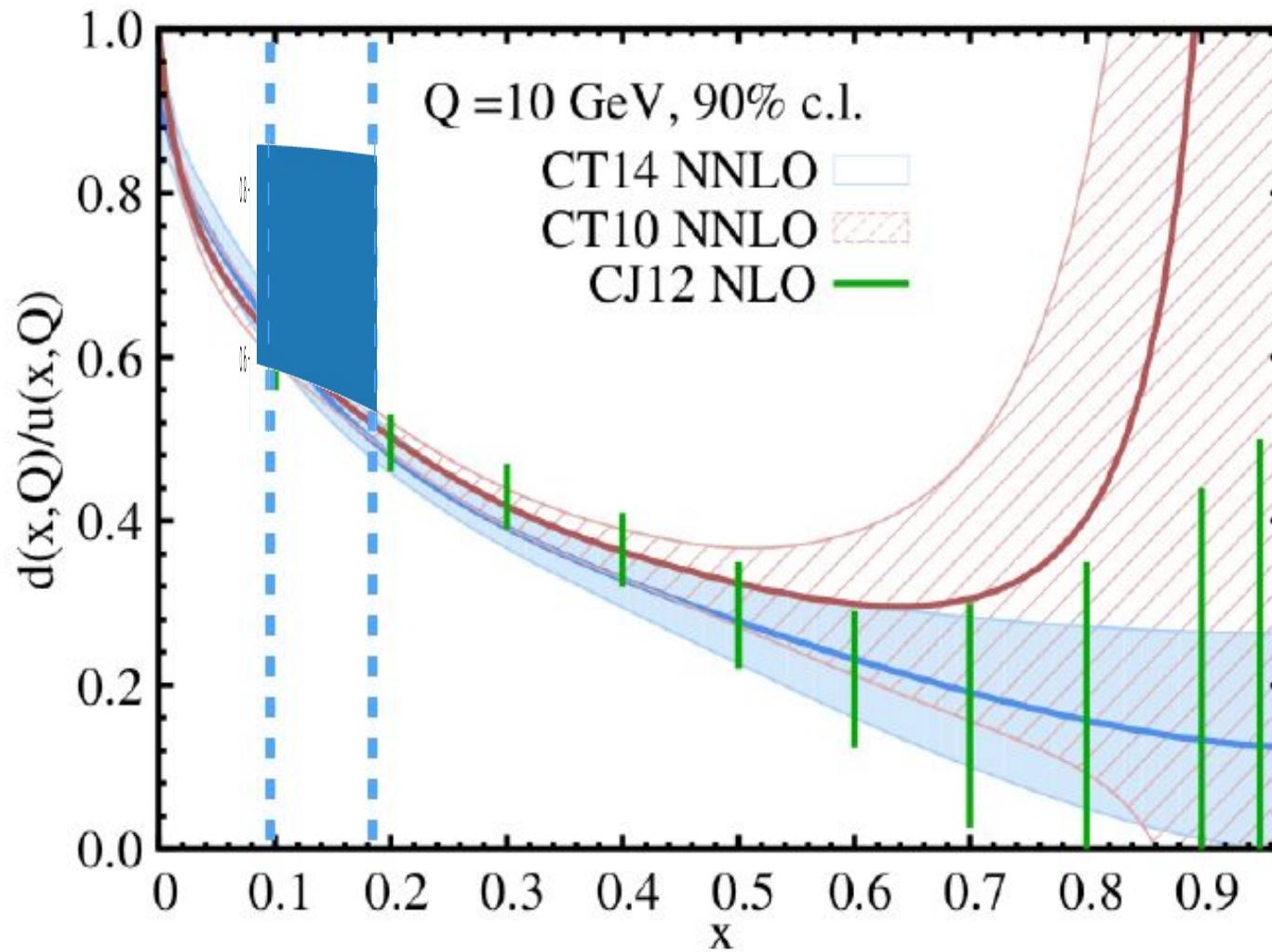
- requires calculation of four-point function
- difficult inverse problem to convert signature

Euclidean hadronic tensor: results



$$\nu = E_n - E_p$$

Euclidean hadronic tensor: results



The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Compton amplitude

Calculate via a Feynman-Hellman method

- avoids renormalisation and mixing issues
- can disentangle higher-twist contributions

$$T_{\mu\nu}(P, q) = \int d^4x e^{iq \cdot x} \langle P | \mathcal{T} J_\mu(x) J_\nu(0) | P \rangle$$

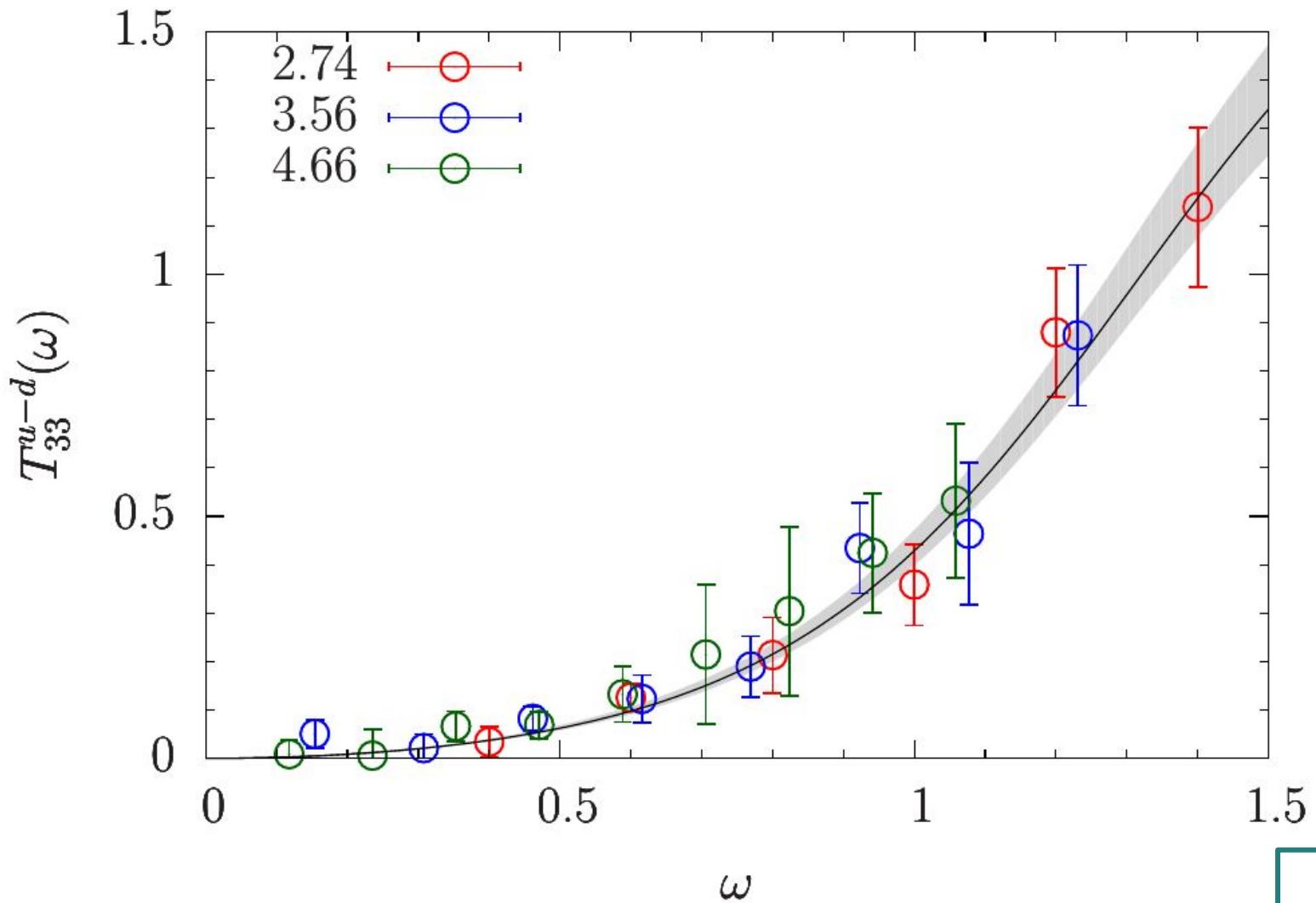
$$T_{33}(P, q) = \sum_{n=2,4,\dots}^{\infty} 4\omega^n \int_0^1 dx x^{n-1} F_1(x, q^2) \quad \omega = \frac{2P \cdot q}{q^2}$$

$$T_{33}(P, q) = 4\omega \int_0^1 dx \frac{\omega x}{1 - (\omega x)^2} F_1(x, q^2)$$

Challenges

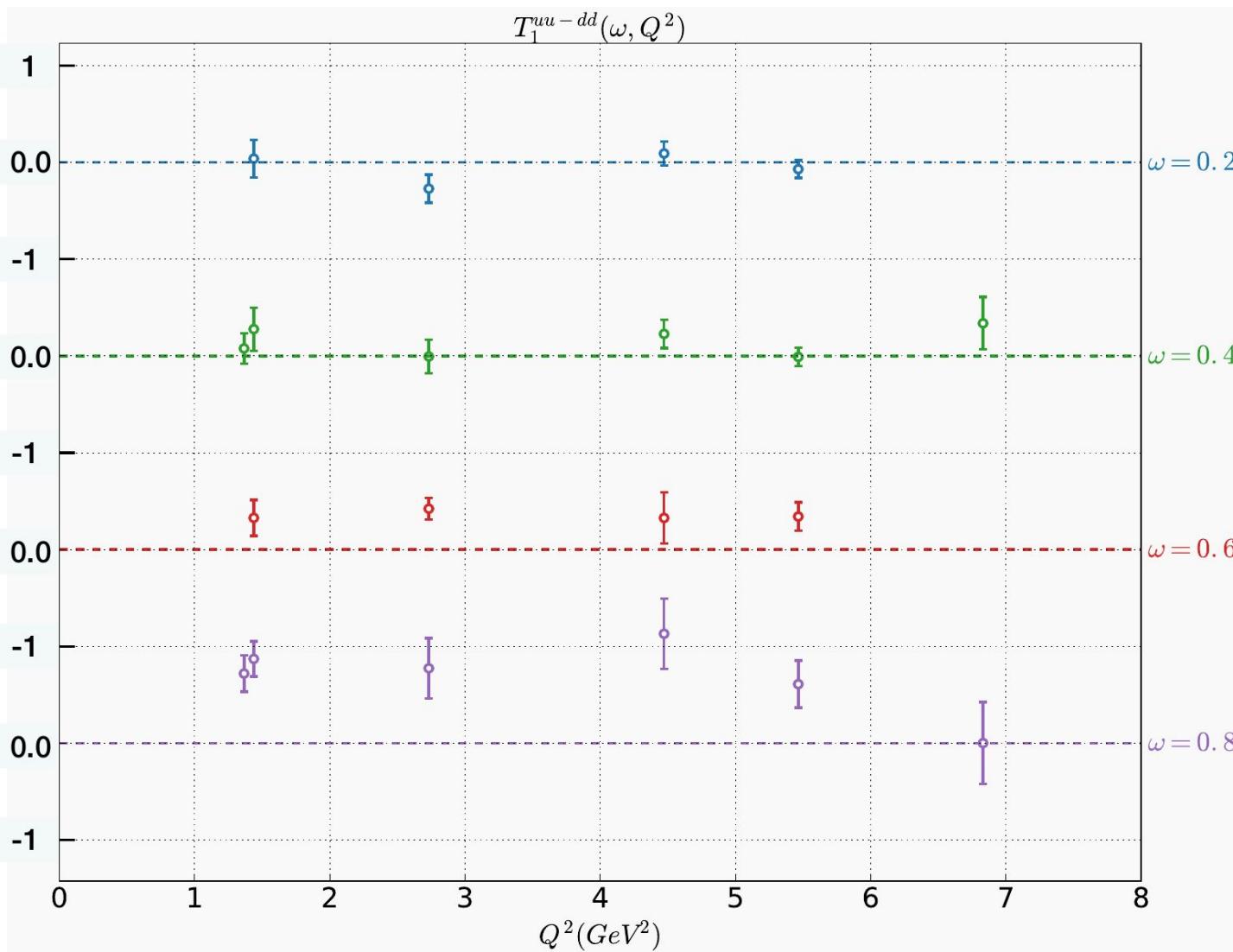
- must reconstruct inverse Mellin transform

Compton amplitude: results



$$\omega = \frac{2p \cdot Q}{Q^2}$$

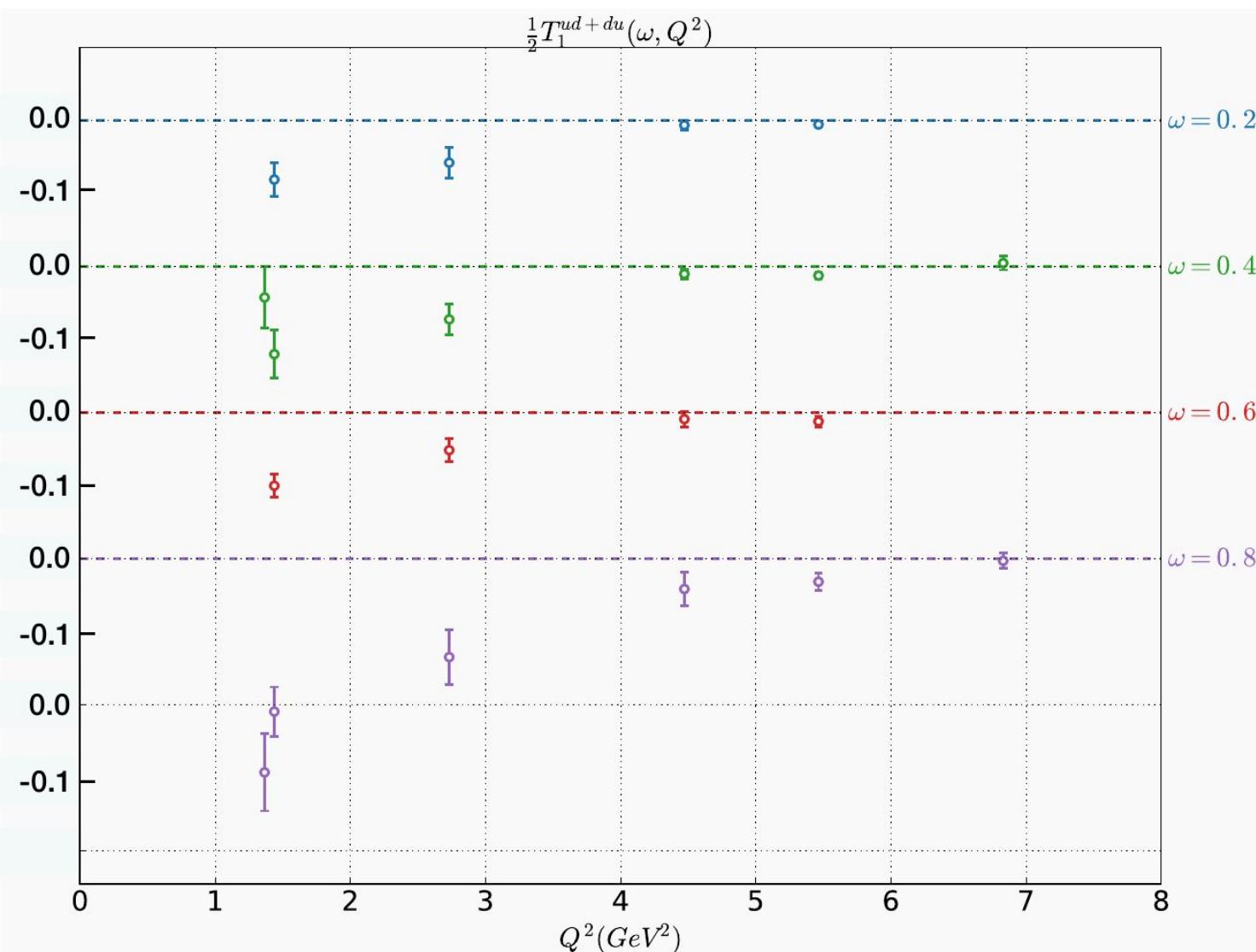
Compton amplitude: results



Scaling study for isovector currents

$$\omega = \frac{2p \cdot Q}{Q^2}$$

Compton amplitude: results



Scaling study of higher twist effects

$$\omega = \frac{2p \cdot Q}{Q^2}$$

The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Fictitious heavy quarks

Calculate Mellin moments of pion DA from

$$U_{[\mu\nu]}^A(q, p) = \int d^4x \langle O | T\{ A_{[\mu}^{(\Psi,\psi)}(x) A_{\nu]}^{(\Psi,\psi)}(0) \} | \pi^+(p) \rangle$$

via flavour-changing axial current

$$A_\mu^{(\Psi,\psi)}(x) = \bar{\Psi} \gamma_\mu \gamma_5 \psi + \bar{q} \gamma_\mu \gamma_5 \Psi$$

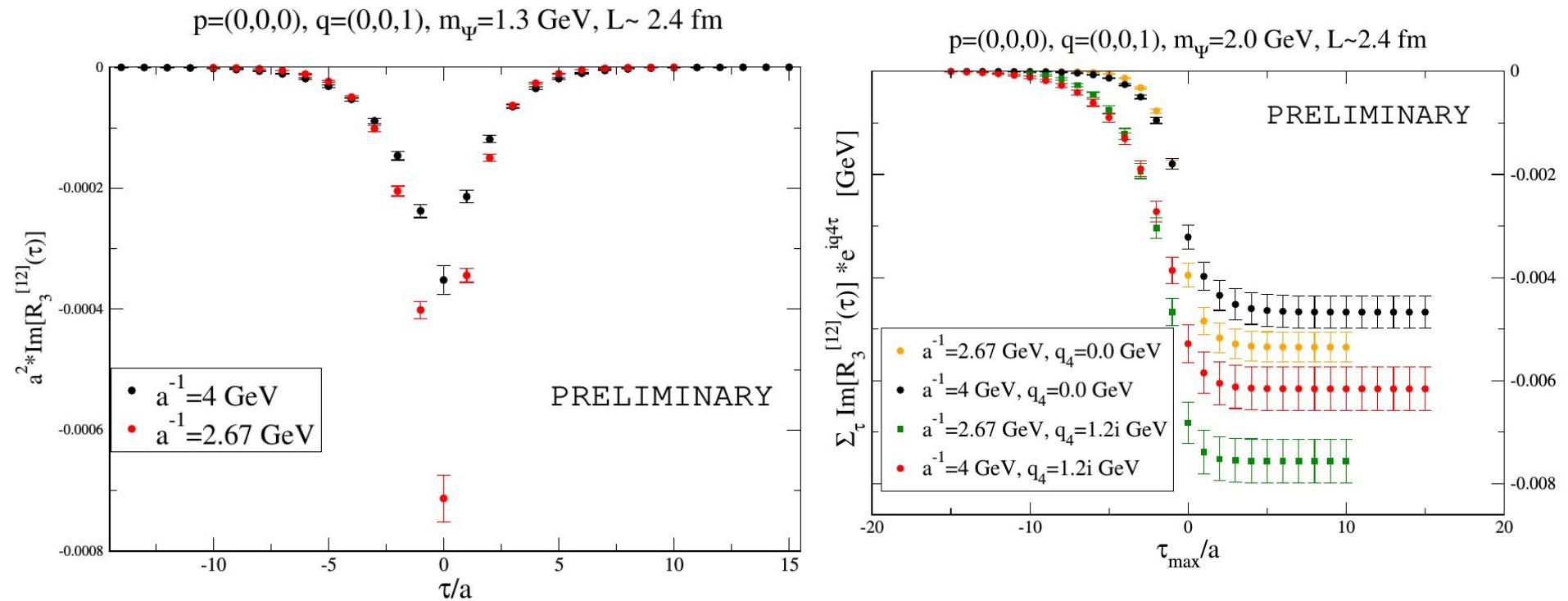
in “unphysical” region

$$(p_M + q_M)^2 < (m_\Psi + \Lambda_{\text{QCD}})^2$$

Requires Fourier transform of lattice matrix element

$$U_{[\mu\nu]}^A(q, p) = \int_{\tau_{\min}}^{\tau_{\max}} d\tau e^{iq_4\tau} \int d^3\mathbf{x} \langle O | T\{ A_\mu^{(\Psi,\psi)}(\mathbf{x}, \tau) A_\mu^{(\Psi,\psi)}(0) \} | \pi(p) \rangle$$

Fictitious heavy quarks: pion DA results



The solutions ...

Quasi and pseudo PDFs

Factorisable matrix elements

Euclidean hadronic tensor

Compton amplitude

Fictitious heavy quarks

TMDs

Higher moment method

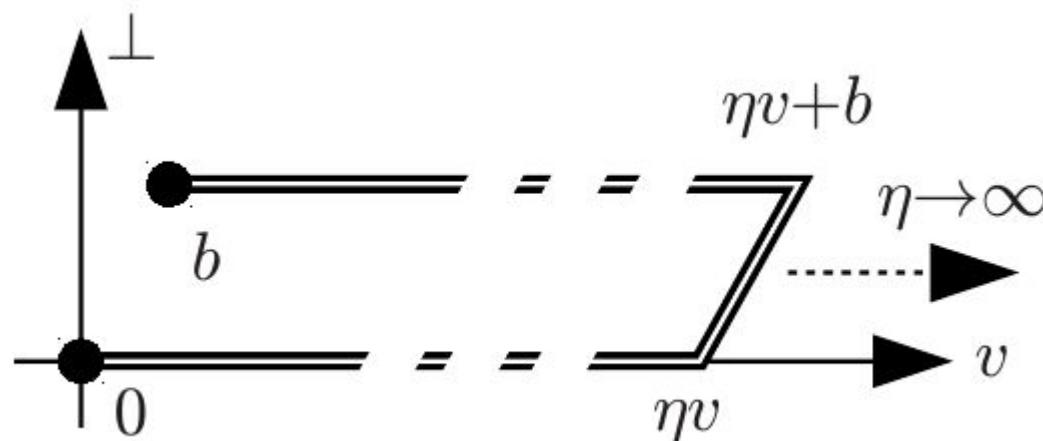
Davoudi & Savage, PRD 86 (2012) 054505

Z. Davoudi Wed. 14:20

TMDs

Calculate matrix element of staple-link Wilson operator

$$\Phi^{(\Gamma)}(x, \mathbf{k}_T) = \int \frac{d^2 \mathbf{b}_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{4\pi P^+} e^{i\xi b \cdot P - i\mathbf{b}_T \cdot \mathbf{k}_T} \langle P | \bar{\psi}(b) W(b, \eta\nu + b, \eta\nu, 0) \Gamma \psi(0) | P \rangle$$

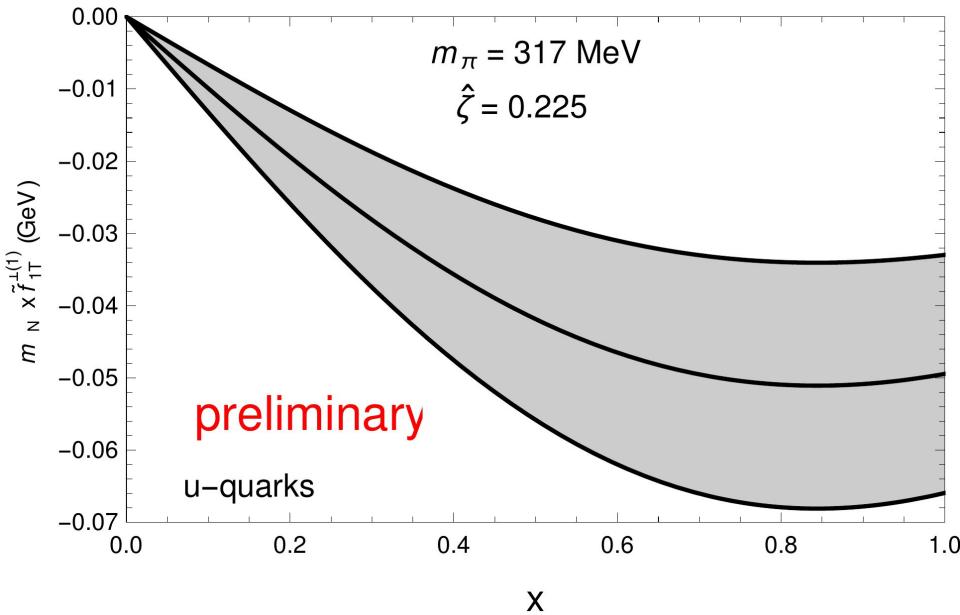
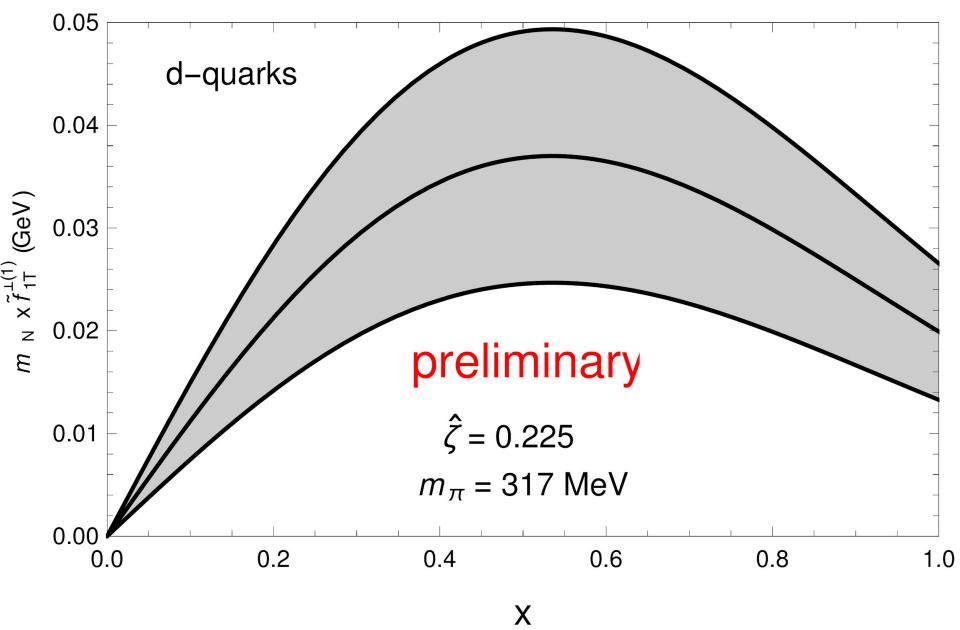


Semi-inclusive deep inelastic scattering limit: infinite η

Spatial staple direction introduces extra dependence on

$$\hat{\zeta} = \frac{\nu \cdot P}{|\nu||P|}$$

TMDs: Sivers function results



First x-dependent
TMD results!

A photograph of a clear blue sky. In the upper left quadrant, a bright sun is visible, surrounded by a starburst effect and a small lens flare. The sky is dotted with several wispy white clouds and larger, more billowy clouds in the lower half.

Outlook

The background of the slide features a clear blue sky with scattered white clouds. A bright sun is positioned in the upper left quadrant, emitting radial rays of light. A small lens flare or rainbow-like effect is visible near the bottom center.

Outlook

Much has been understood:
factorisation theorems
nonperturbative renormalisation
most early issues resolved

Preliminary results encouraging:
multiple complementary approaches
lattice inputs to global PDF fits

Outlook

Much has been understood:
factorisation theorems
nonperturbative renormalisation
most early issues resolved

Preliminary results encouraging:
multiple complementary approaches
lattice inputs to global PDF fits

Any unknown (and unresolved) theoretical issues?

Systematic uncertainties unexplored

excited state effects?

enhanced discretisation or finite volume effects?

need larger momenta

spurious oscillations

Thank you

cjm373@uw.edu

Particular thanks to:

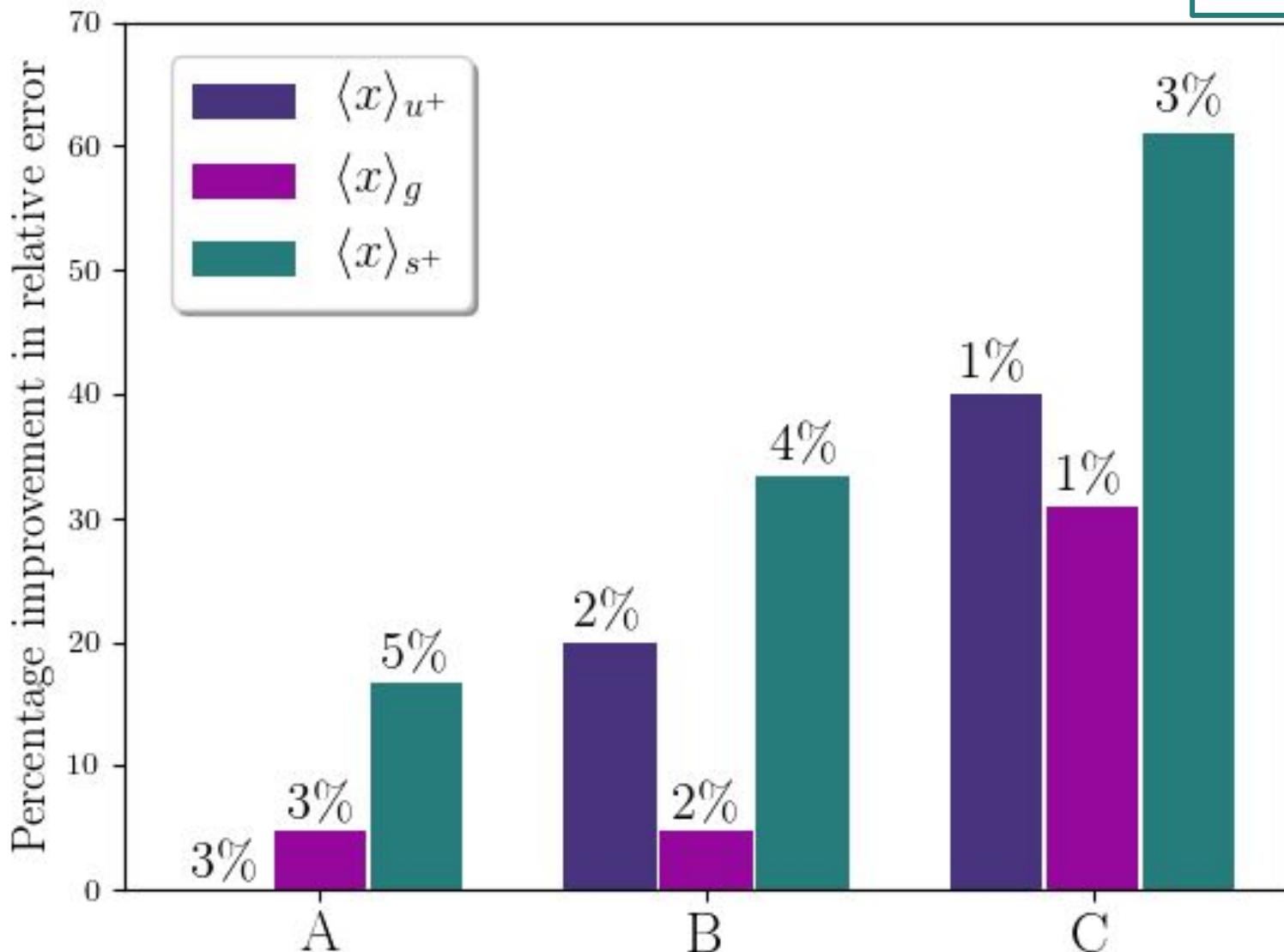
Bipasha Chakraborty, Martha Constantinou,
Michael Engelhardt, Nikhil Karthik, Joe Karpie,

David Lin, Yu-Sheng Liu, Giancarlo Rossi,
Gerrit Schierholz, Gregoris Spanoudes, Raza Sufian,
Savvas Zafeiropoulos, James Zanotti, Yong Zhao



PDFs: the impact of lattice QCD

A: (3,3,5)%
B: (2,2,4)%
C: (1,1,3)%



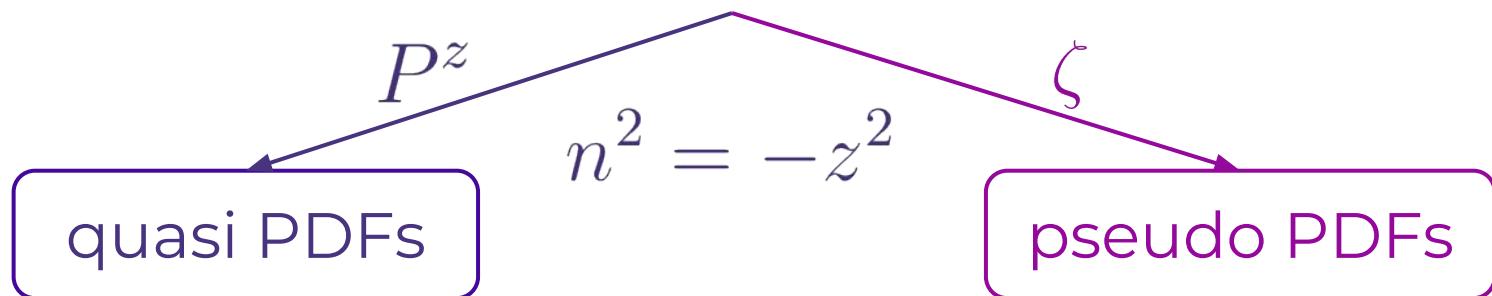
A panoply of distributions: PDFs

$$f_{j/H}^{(0)}(x) = \int_{-\infty}^{\infty} \frac{d\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \langle P | \bar{\psi}(0, \omega^-, \mathbf{0}_T) W(\omega^-, 0) \Gamma \psi(0) | P \rangle$$

PDFs

$$\xi^- \quad n^2 = 0$$

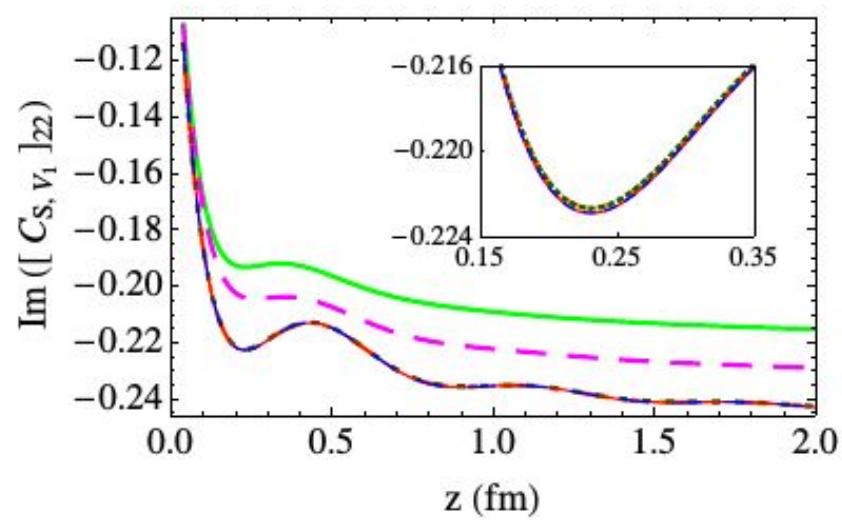
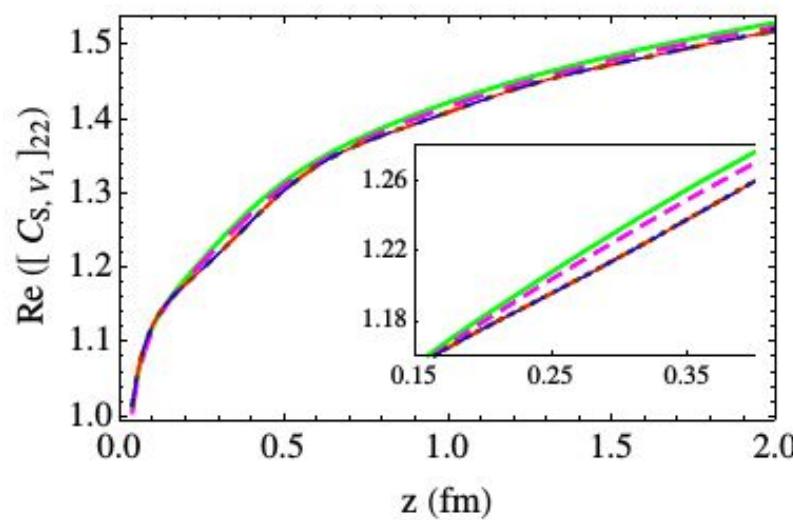
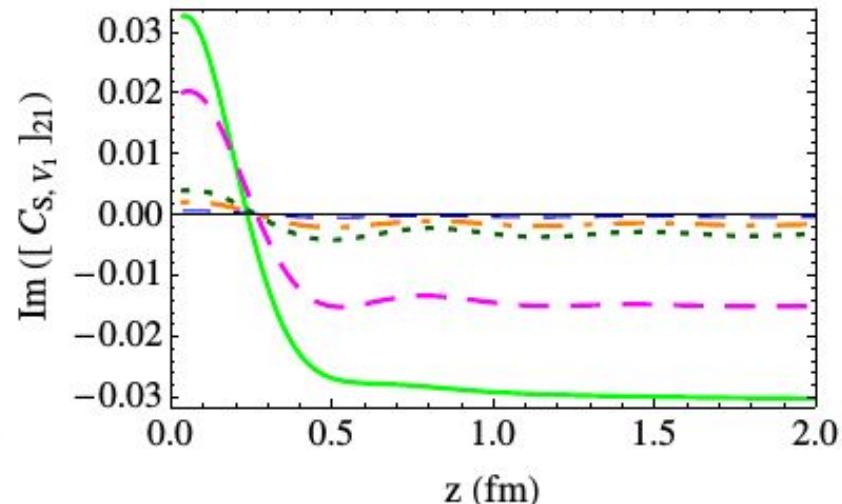
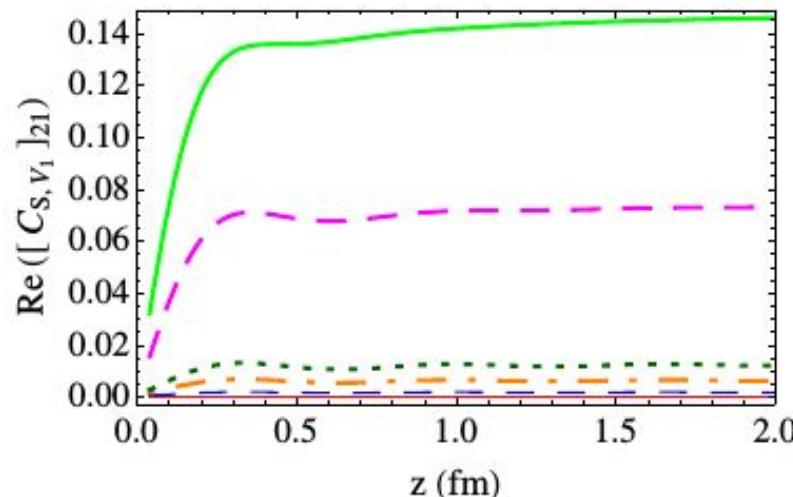
$$I^{(0)}(\zeta = P \cdot n, n^2) = \frac{1}{2P^\mu} \langle P | \bar{\psi}(n) W(n, 0) \Gamma_\mu \psi(0) | P \rangle$$



$$\tilde{f}_{j/H}^{(0)}(\xi, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{i\xi P_z z} \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle$$

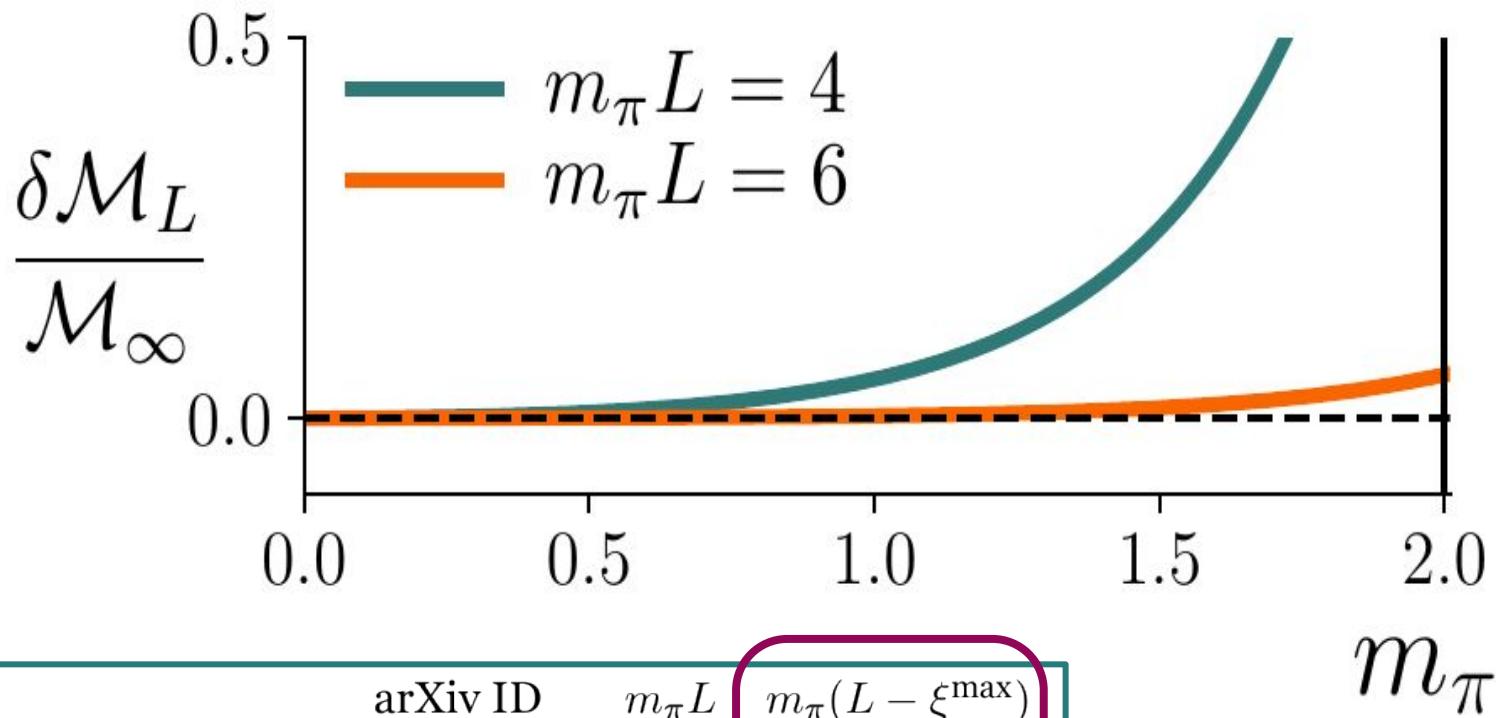
$$\tilde{p}_{j/H}^{(0)}(\xi, z^2) = \int_{-\infty}^{\infty} \frac{d\xi}{4\pi} e^{i\xi \zeta} \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle$$

Renormalisation



| | | |
|----------------------|-----------------------------------|---------------------------------|
| $m_1 = m_2 = 0$ MeV | $m_1 = m_2 = 13.2134$ MeV | $m_1 = 2.3$ MeV, $m_2 = 95$ MeV |
| $\cdots \cdots$ | $- - -$ | $- \cdot -$ |
| $m_1 = m_2 = 95$ MeV | $m_1 = 2.3$ MeV, $m_2 = 1275$ MeV | $\textcolor{red}{—}$ |

Finite volume effects



| | arXiv ID | $m_\pi L$ | $m_\pi(L - \xi^{\max})$ |
|--------------------------|------------|-----------|-------------------------|
| LP ³ nucleons | 1807.07431 | 4.0 | 2.7 |
| LP ³ mesons | 1804.01483 | 4.6 | 1.7 |
| ETMC | 1803.02685 | 3.0 | 1.4 |
| Bali <i>et al.</i> | 1807.06671 | 3.4 | 2.8 |
| JLab | 1706.05373 | 9.1 | 5.2 |

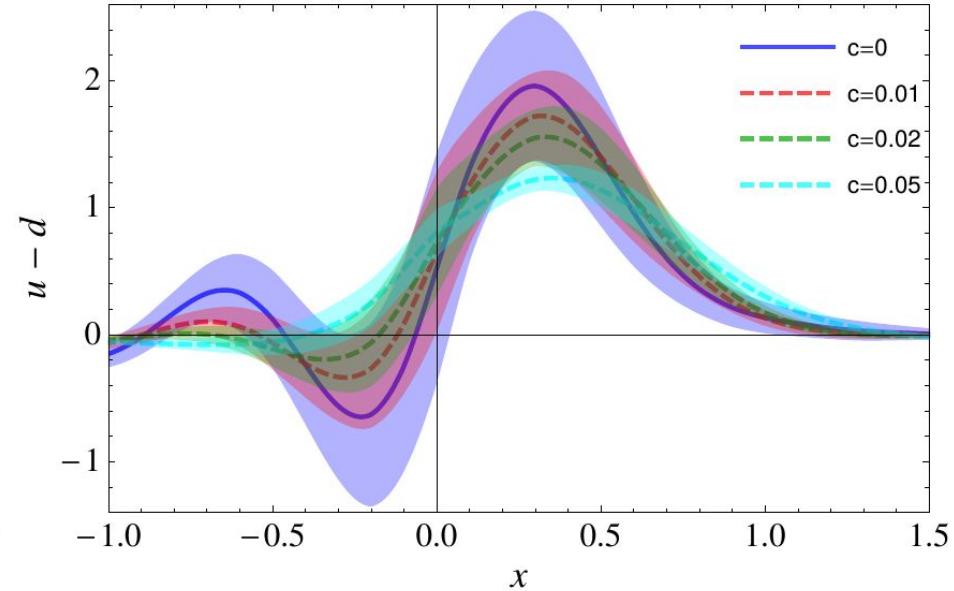
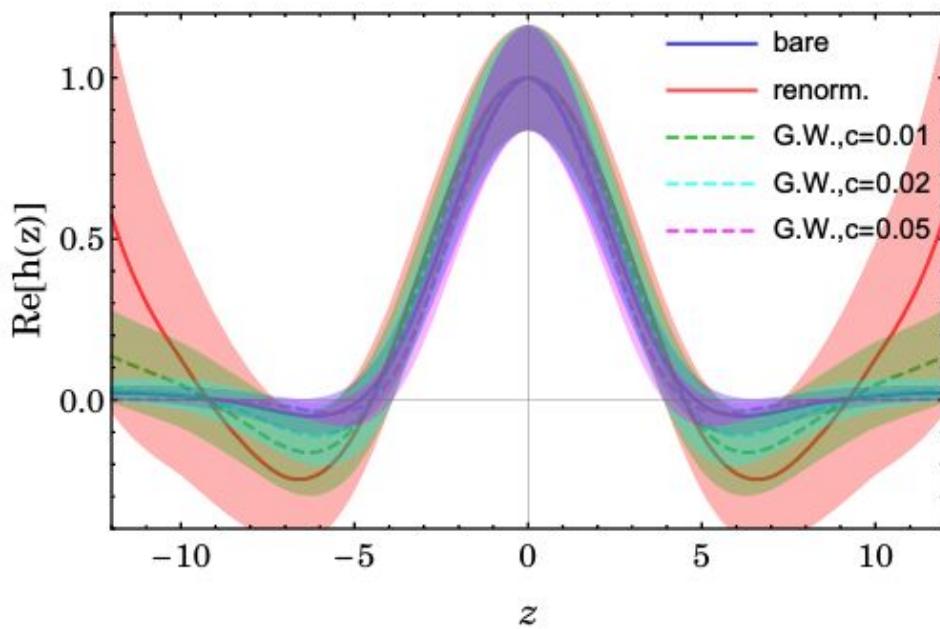
$m_\pi \xi$

C. Lauer poster

Spurious oscillations

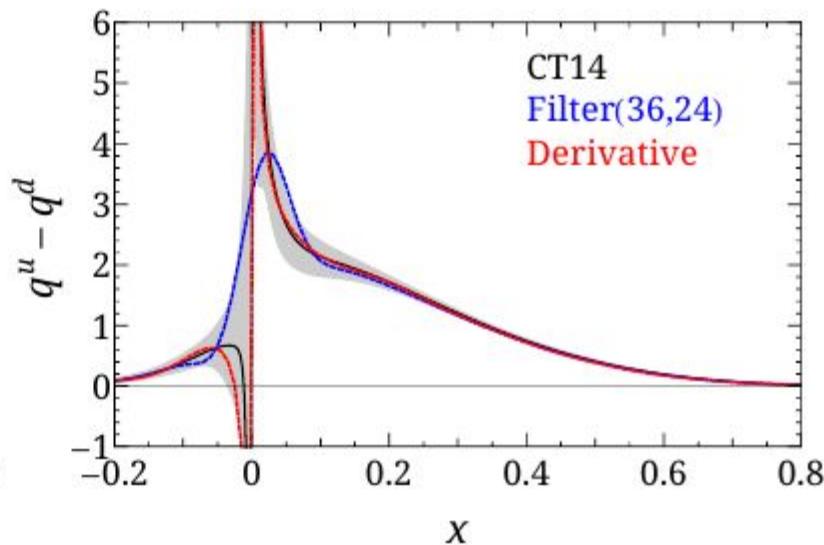
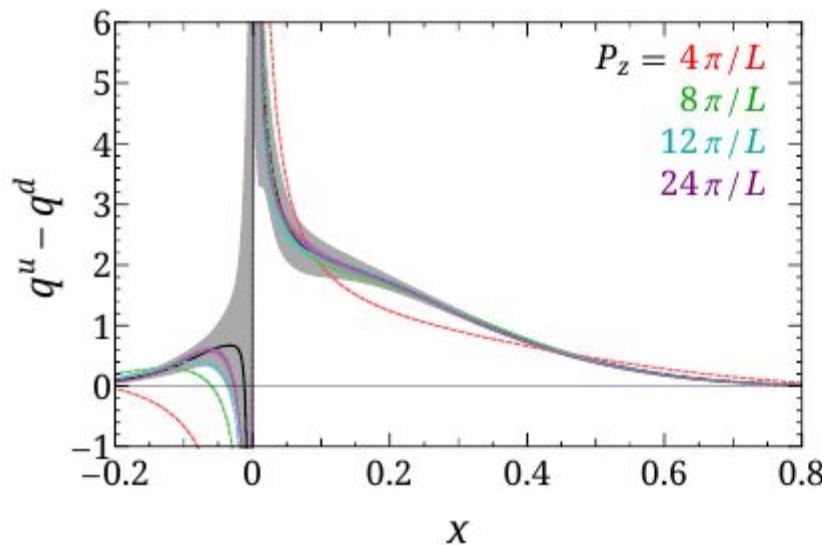
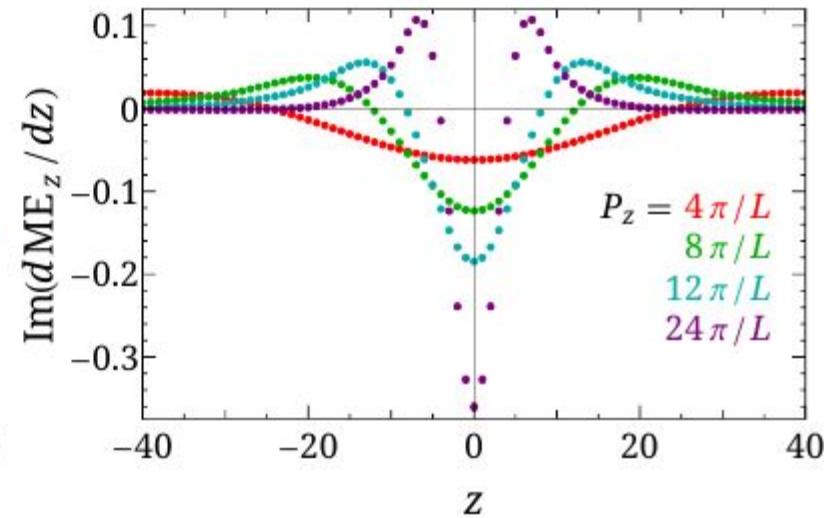
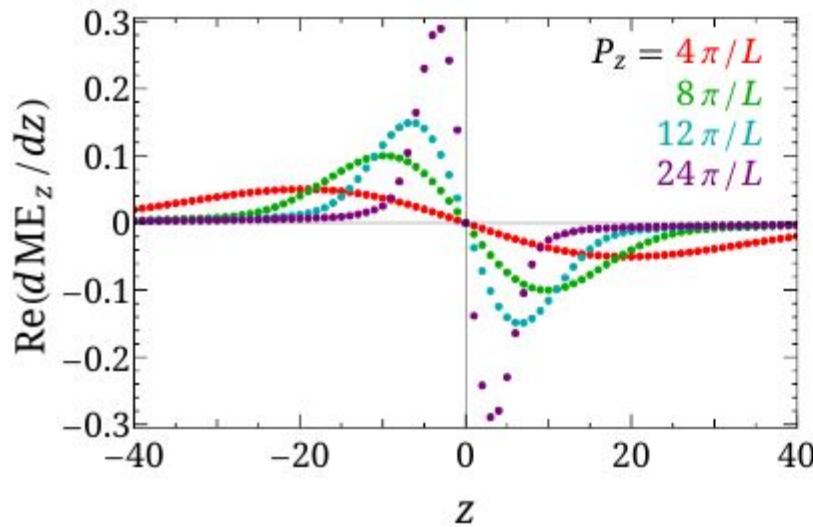
Multiple methods studied by LP³, e.g. Gaussian weighting

$$\begin{aligned}\tilde{f}_{j/H}^{(\text{GW})}(\xi, P^z) &= \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{i\xi P_z z - z^2/\Lambda_z^2} \langle P | \bar{\psi}(0, z, \mathbf{0}_T) W(z, 0) \Gamma \psi(0) | P \rangle \\ &= \frac{\Lambda_z P^z}{2\sqrt{\pi}} \int_{-\infty}^{\infty} dy e^{-\frac{(\xi-y)^2 (\Lambda_z P^z)^2}{4}} \tilde{f}_{j/H}^{(0)}(y, P^z)\end{aligned}$$

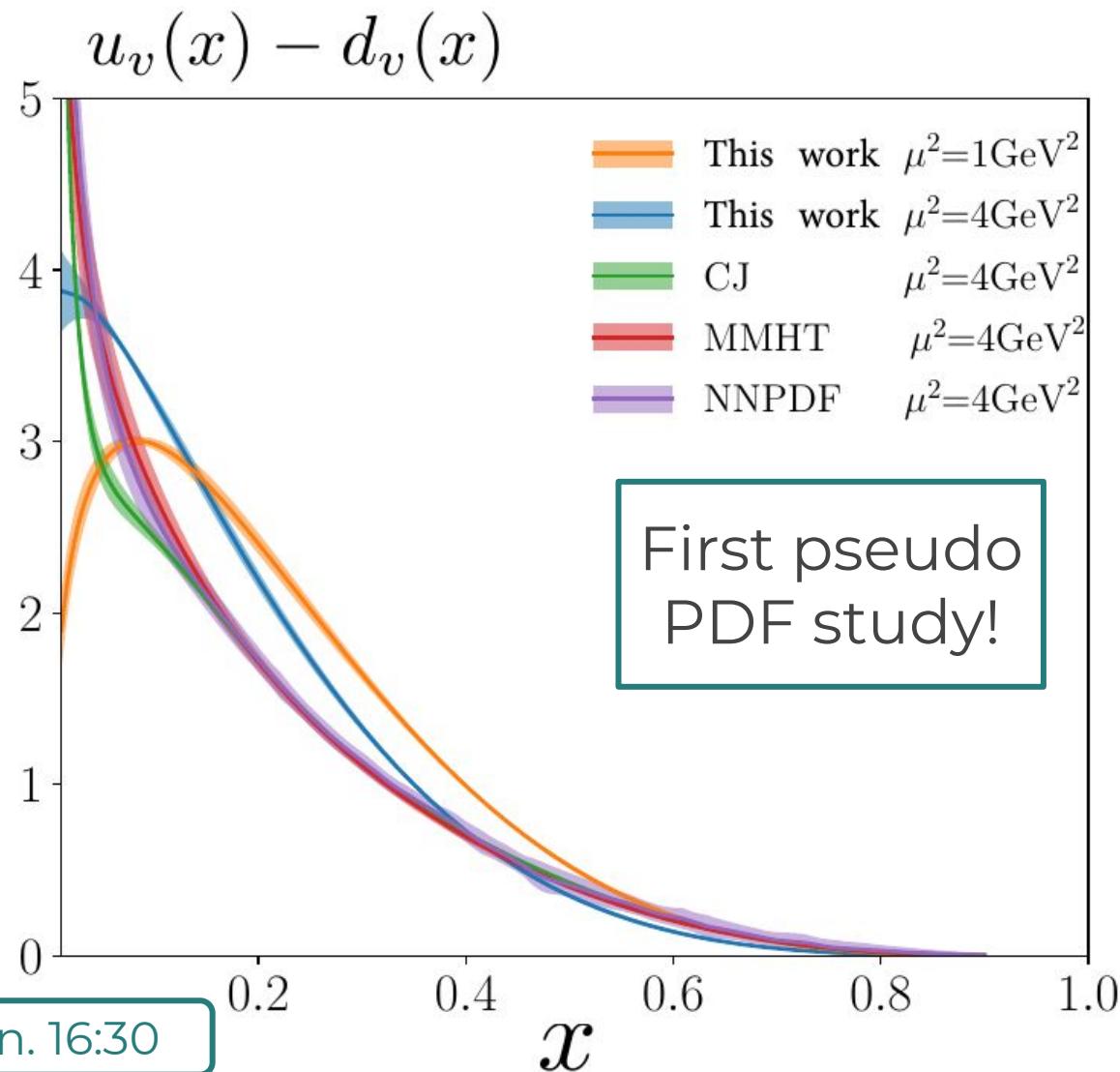


Spurious oscillations

Multiple methods studied by LP³, e.g. derivative method



Unpolarised nucleon PDF

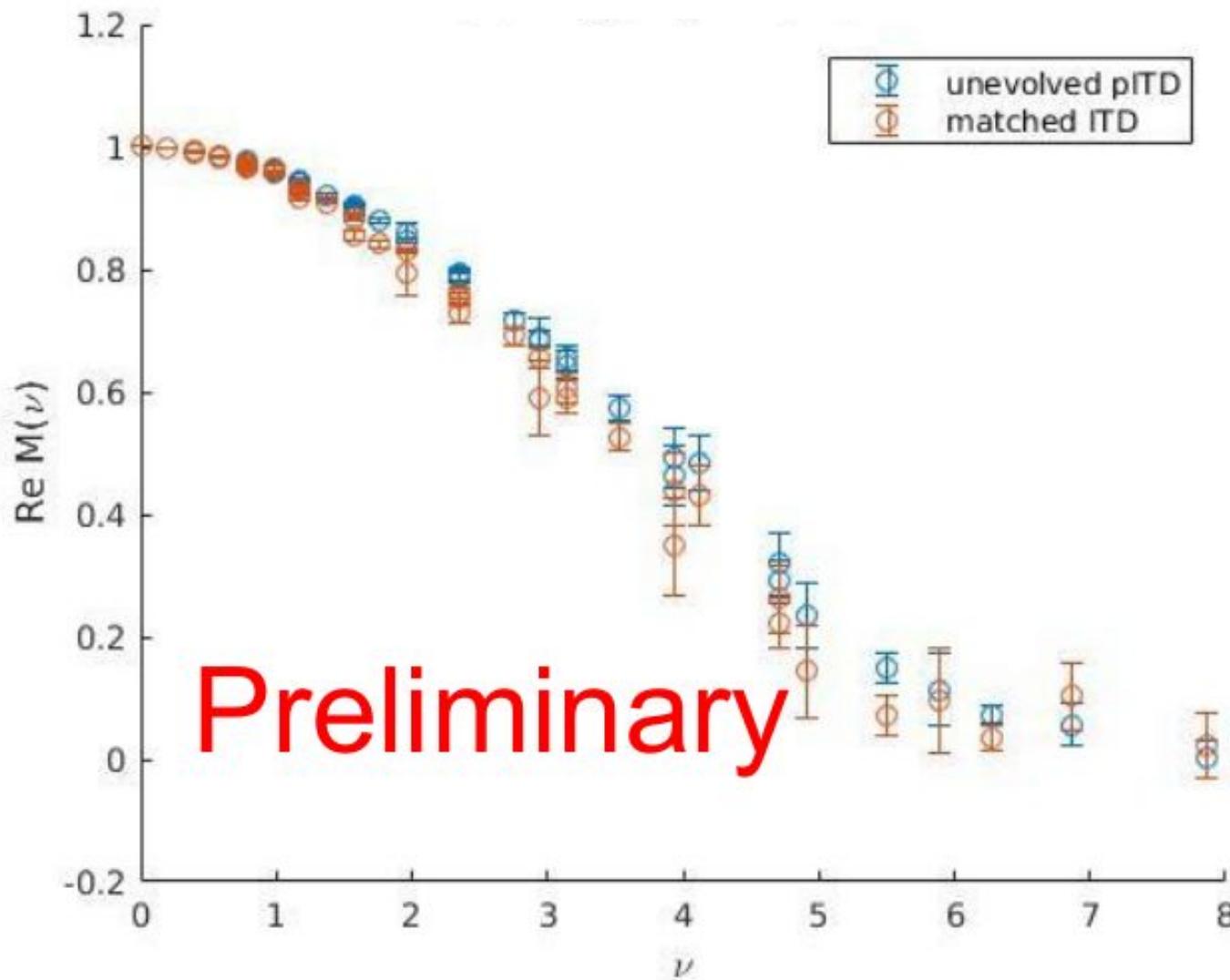


J. Karpie Mon. 16:30

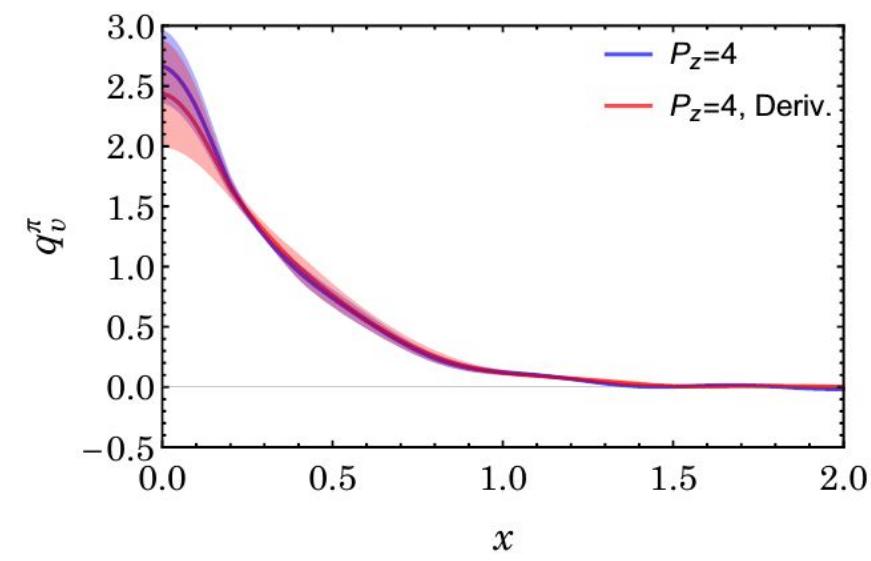
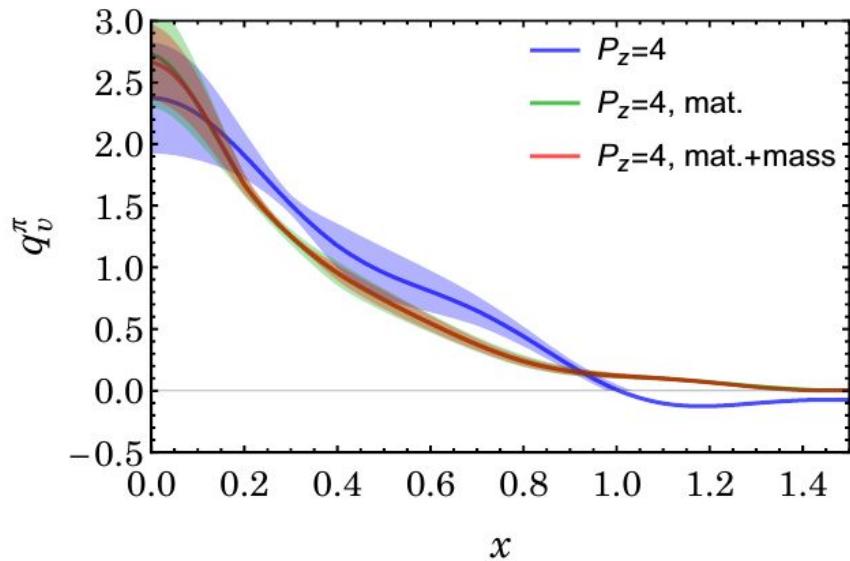
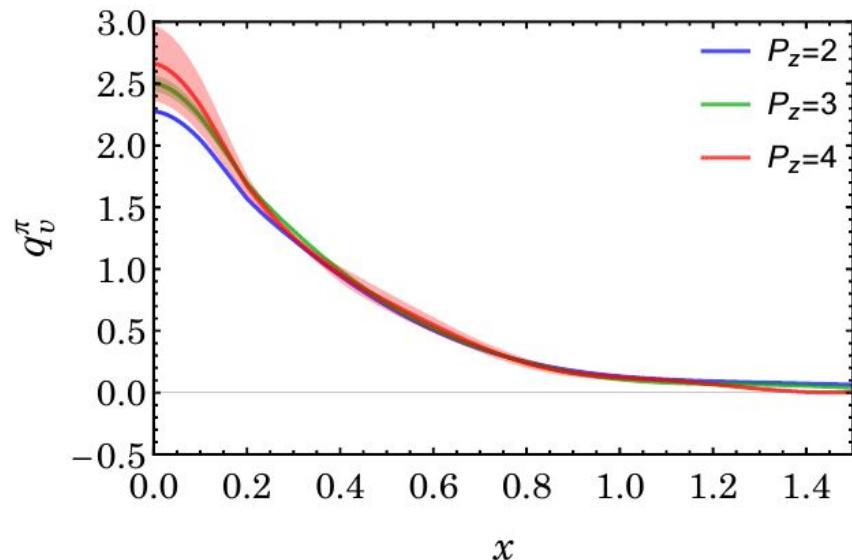
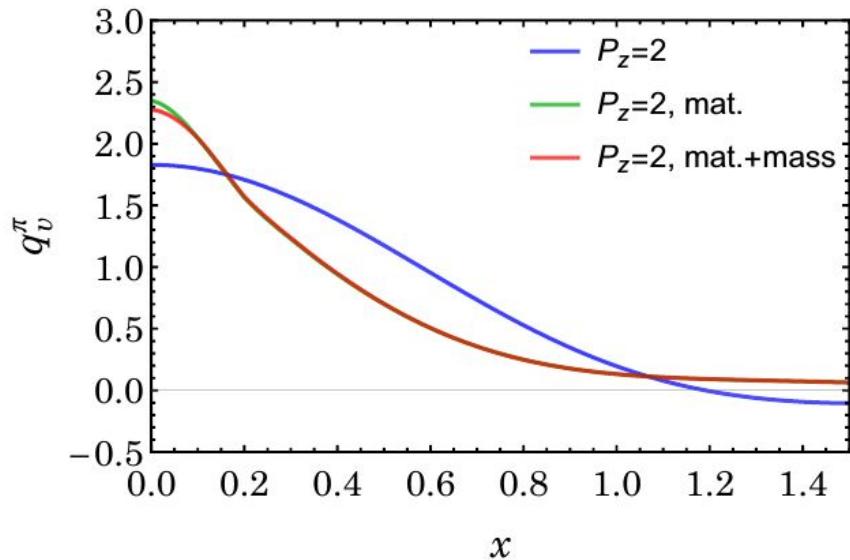
S. Zafeiropoulos Mon. 16:10

Orginos et al., PRD 96 (2017) 094503

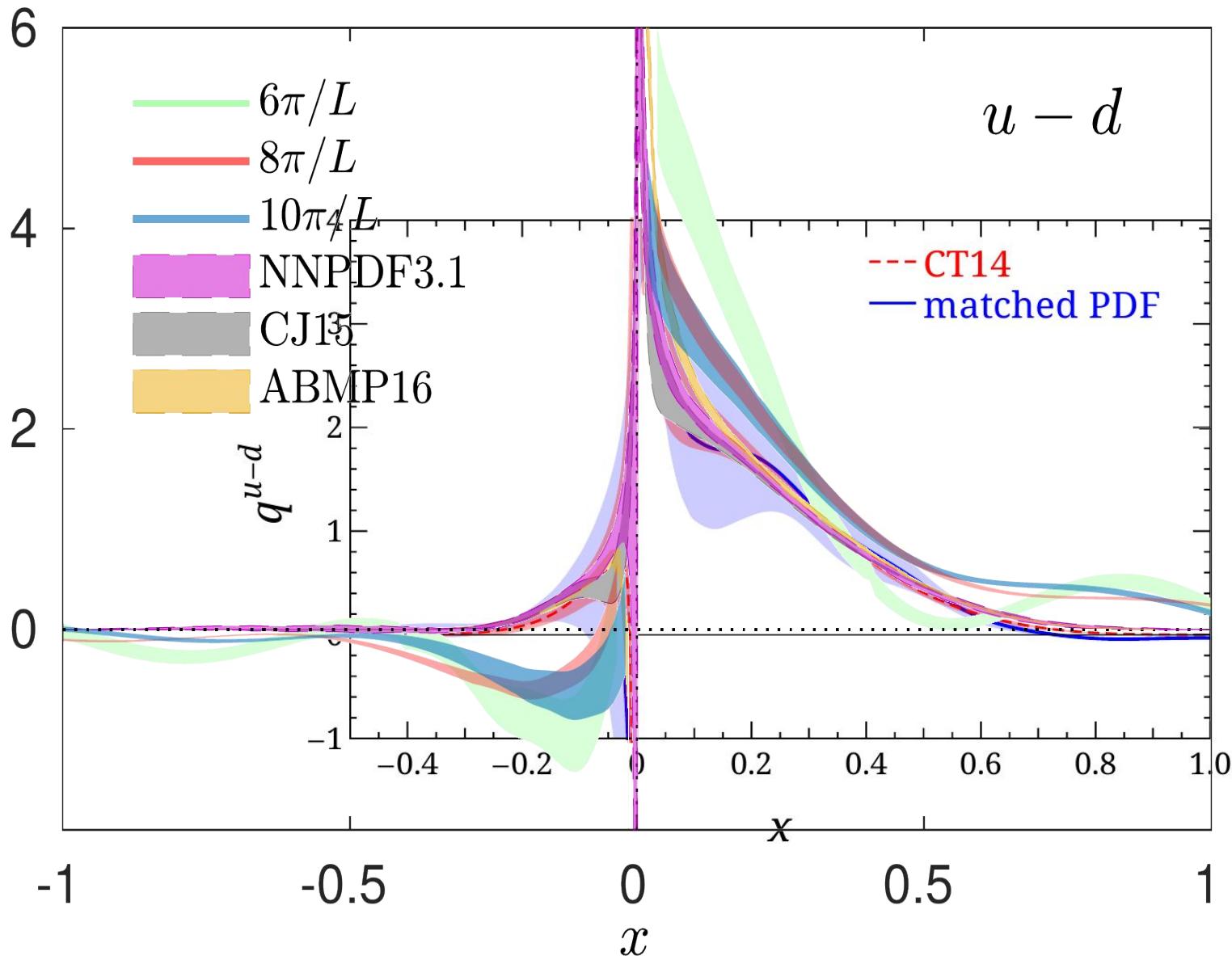
JLab results: unpolarised nucleon PDF



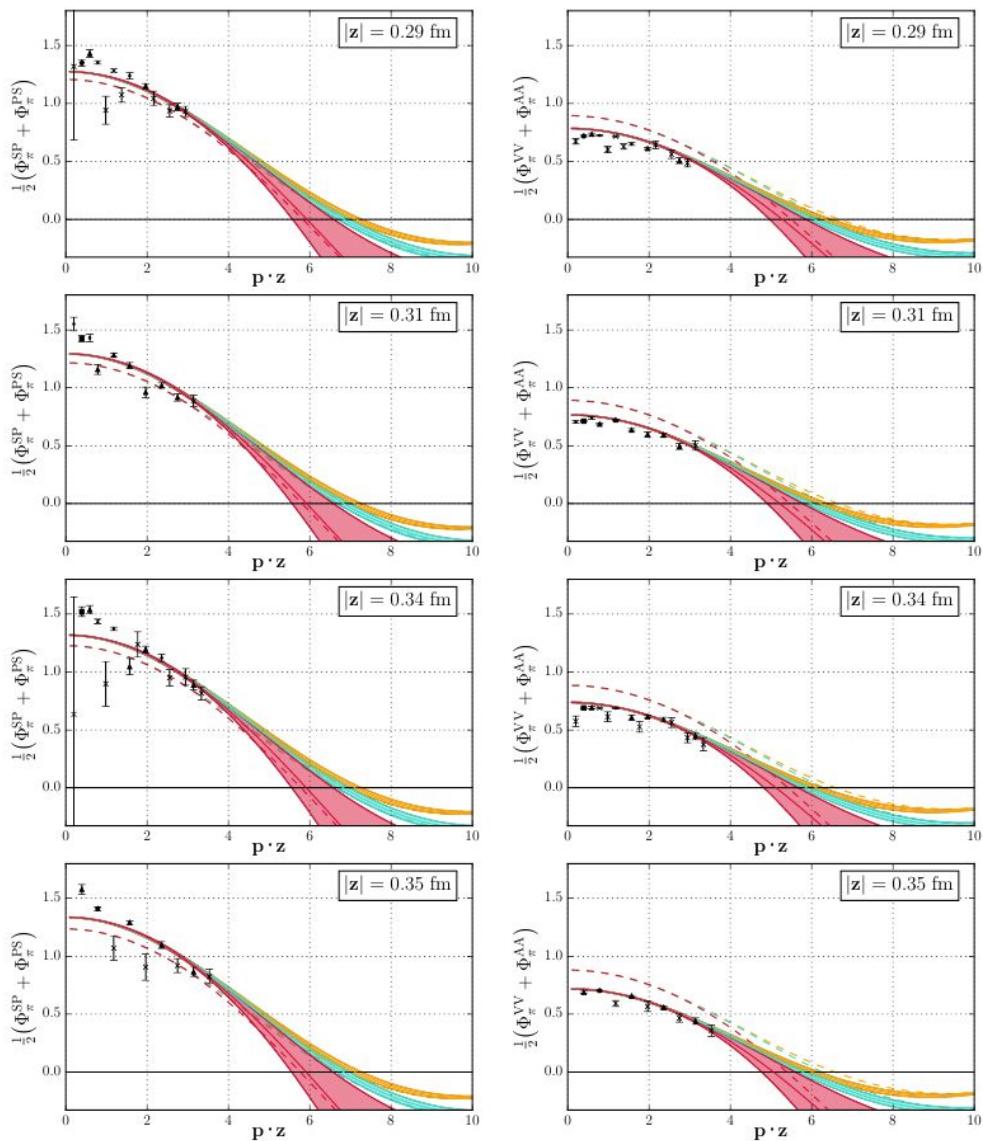
LP³ results: unpolarised pion PDF



Recent ETMC and LP³ results: comparison



Position-space correlators: results



See P. Wein's talk
Wednesday 14:40

Position-space correlators: results

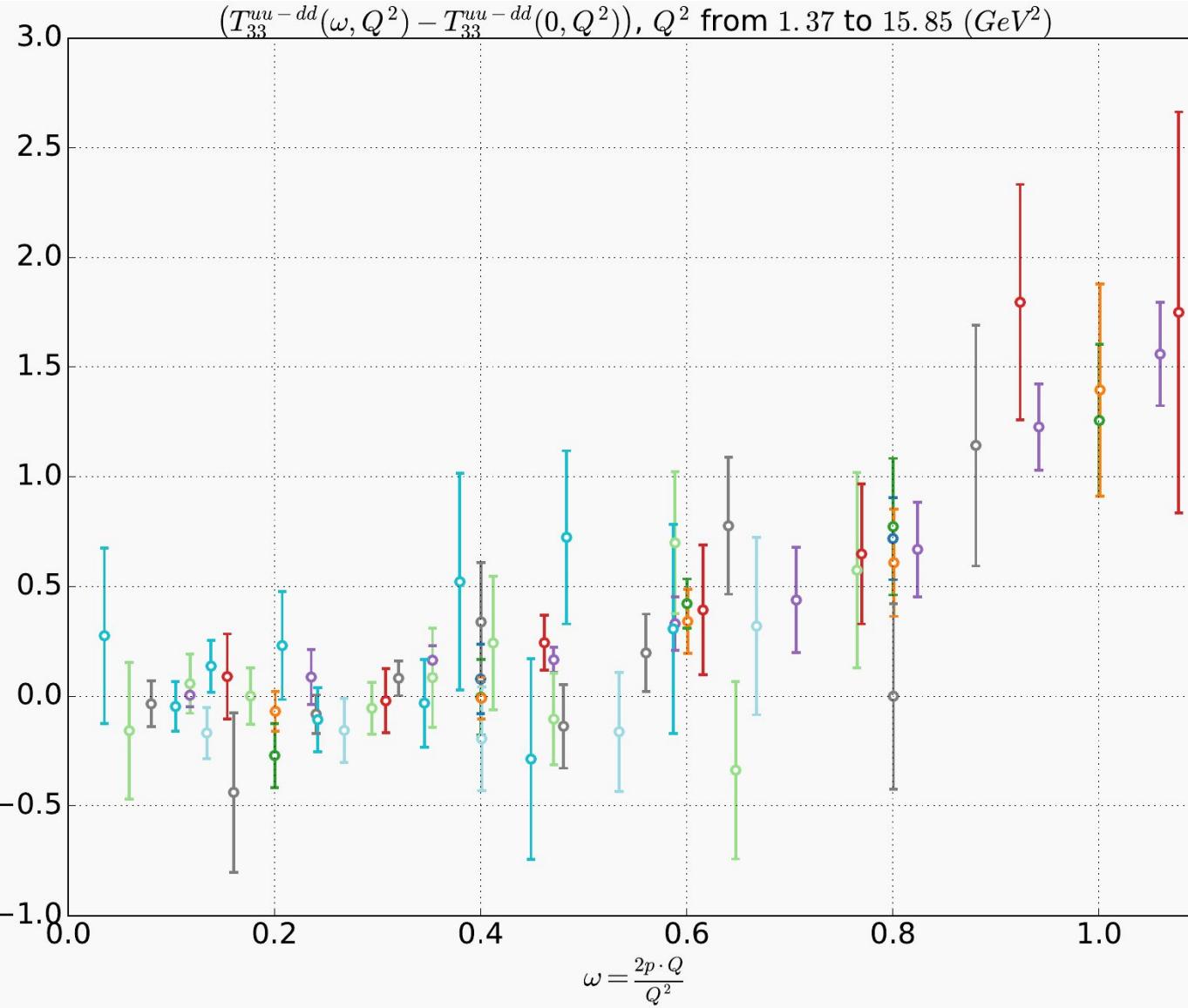
TABLE II. Fit results for three different DA parametrizations (which are all defined at the reference scale 2 GeV) and various fit ranges in $\mu = 2/|\mathbf{z}|$. Ansatz A corresponds to assuming the shape [10], while B and C use the expansion of the DAs in terms of Gegenbauer polynomials, Eq. [6], truncated at $n = 2$ and $n = 4$. The numbers in parentheses give the statistical error. As discussed in the main text, a rather generous systematic uncertainty of 30%–50% should be assigned to these results and the values for a_4^π from ansatz A and B are meaningless. The fit range corresponding to the curves plotted in Figs. [8][10] is highlighted.

| Ansatz | a_2^π | a_4^π | $\delta_2^\pi [\text{GeV}^2]$ | |
|---------------------------|-----------|-----------|-------------------------------|-----------------------------|
| 0.9 GeV < μ < 1.8 GeV | | | | |
| I | A | 0.29(2) | 0.16(2) | 0.202(3) $\alpha = 0.17(5)$ |
| | B | 0.28(2) | 0.0 | 0.202(3) |
| | C | 0.28(4) | 0.0(0.6) | 0.202(4) |
| 1.0 GeV < μ < 1.8 GeV | | | | |
| II | A | 0.31(3) | 0.17(2) | 0.223(4) $\alpha = 0.13(5)$ |
| | B | 0.30(3) | 0.0 | 0.223(4) |
| | C | 0.26(5) | -1.1(0.9) | 0.225(4) |
| 1.1 GeV < μ < 1.8 GeV | | | | |
| III | A | 0.36(3) | 0.22(3) | 0.242(4) $\alpha = 0.05(5)$ |
| | B | 0.35(3) | 0.0 | 0.242(4) |
| | C | 0.29(6) | -1.6(1.2) | 0.244(4) |
| 1.0 GeV < μ < 1.5 GeV | | | | |
| IV | A | 0.30(3) | 0.17(2) | 0.218(4) $\alpha = 0.15(5)$ |
| | B | 0.30(3) | 0.0 | 0.219(4) |
| | C | 0.22(5) | -1.7(0.9) | 0.222(4) |
| 1.0 GeV < μ < 1.3 GeV | | | | |
| V | A | 0.26(3) | 0.14(2) | 0.202(4) $\alpha = 0.22(6)$ |
| | B | 0.26(3) | 0.0 | 0.202(4) |
| | C | 0.09(5) | -3.6(0.9) | 0.209(4) |

See P. Wein's talk
Wednesday 2:40 PM

$$\begin{aligned}
 n_f &= 2 \\
 a &\simeq 0.071 \text{ fm} \\
 L &\simeq 2.3 \text{ fm} \\
 m_\pi &\simeq 295 \text{ MeV} \\
 m_\pi L &\simeq 3.4 \\
 P_z^{\max} &\simeq 1.9 \text{ GeV}
 \end{aligned}$$

Compton amplitude: results



$$n_f = 2$$

$$a \simeq 0.07 \text{ fm}$$

$$L \simeq 2.4 \text{ fm}$$

$$m_\pi \simeq 420 \text{ MeV}$$

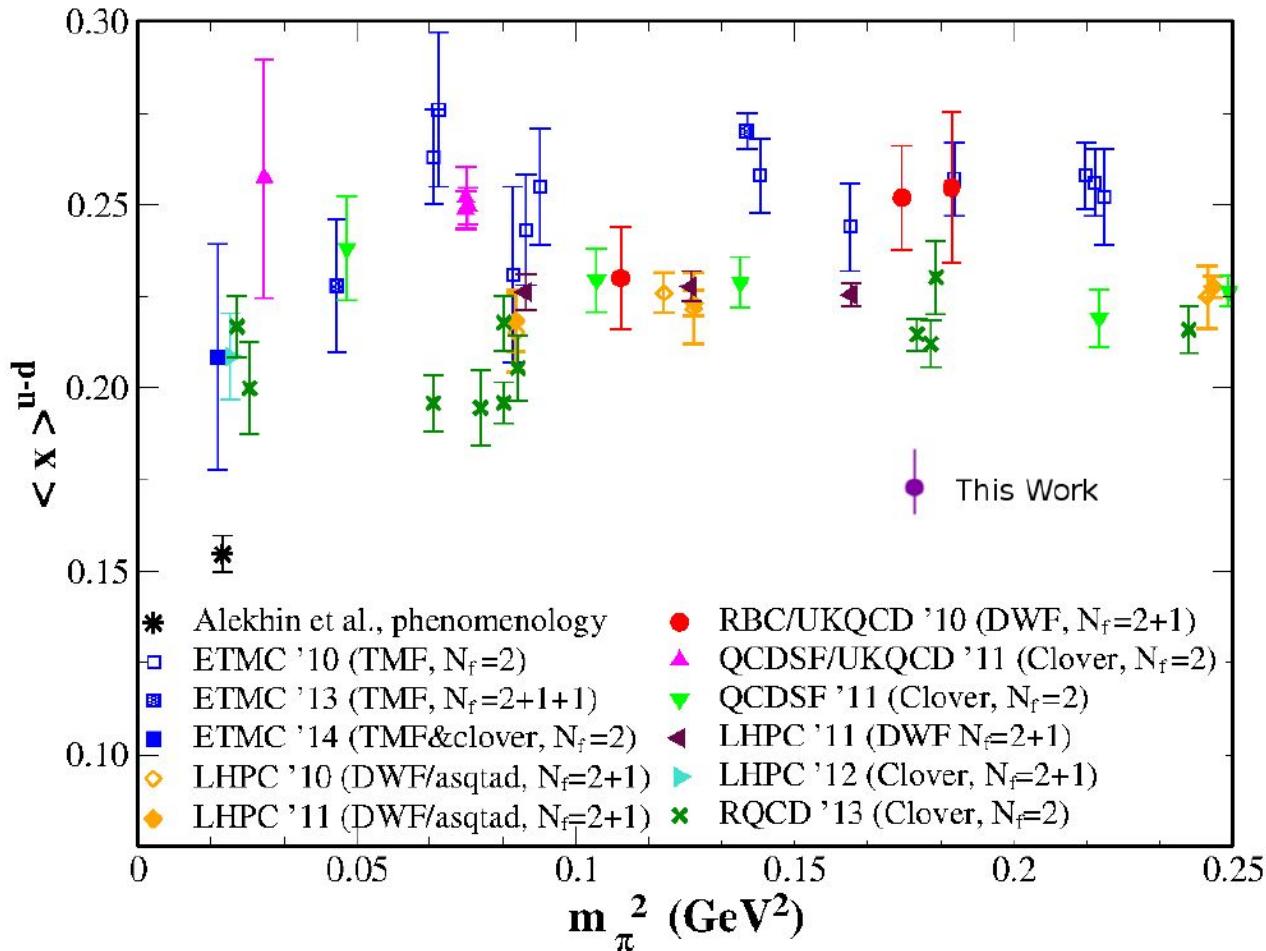
$$m_\pi L \simeq 5.0$$

$$P_z^{\max} \simeq 2.6 \text{ GeV}$$

$$\omega = \frac{2p \cdot Q}{Q^2}$$

Compton amplitude: results

Original plot: Constantinou, 1511. 00214



$$n_f = 2$$

$$a \simeq 0.07 \text{ fm}$$

$$L \simeq 2.4 \text{ fm}$$

$$m_\pi \simeq 420 \text{ MeV}$$

$$m_\pi L \simeq 5.0$$

$$P_z^{\max} \simeq 2.6 \text{ GeV}$$