Strange nucleon form factors with $N_f = 2 + 1$ O(a)-improved Wilson fermions

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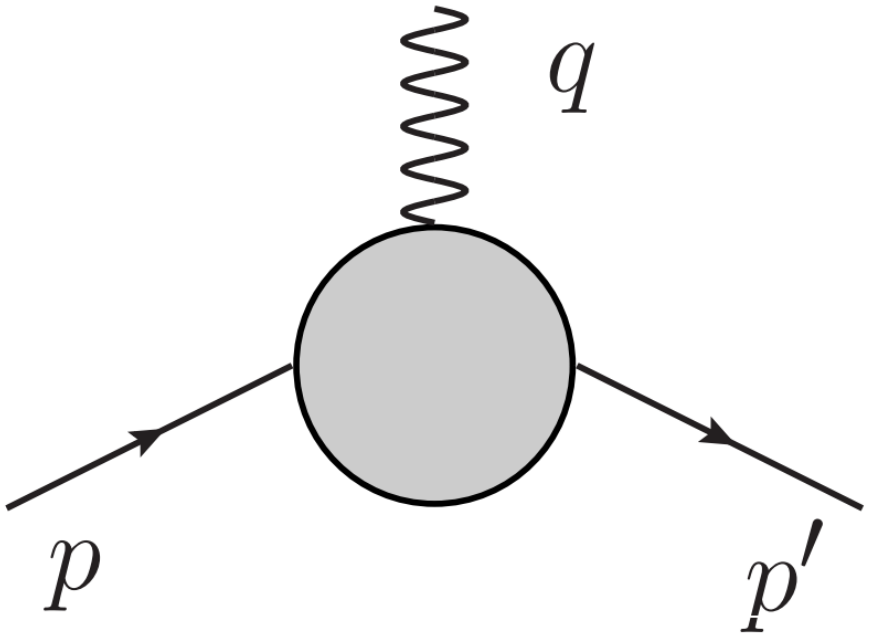
nucleon matrix elements → form factors

$$\left\langle N, \vec{p}, s \,|A_\mu(x)|\, N, \vec{p}\,', s' \right\rangle = \bar{u}^s(\vec{p})\ \widetilde{O}_{A_\mu}\ u^{s'}(\vec{p}\,')e^{\mathrm{i}q\cdot x}$$

$$\widetilde{O}_{A_\mu} = \gamma_\mu\gamma_5 G_\mathrm{A}(Q^2) + \gamma_5 \frac{q_\mu}{2m} G_\mathrm{P}(Q^2)$$



- calculate form factors on the lattice

axial vector & induced pseudoscalar form factors

- at $Q^2 = 0$:
  - contribution of intrinsic spin of quark flavor f

$$G_A^f(0) = g_A^f = \Delta f$$

$$\frac{1}{2} = \sum_f \left(\frac{1}{2} \Delta f + L_f \right) + J_g \quad [1]$$

- at $Q^2 \neq 0$:
  - isoscalar form factors and strange quark contribution

$$G_A^{u+d}(Q^2), G_A^s(Q^2) \text{ and } G_P^{u+d}(Q^2), G_P^s(Q^2)$$

$\Rightarrow$ appearance of quark-disconnected contributions

[1] X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), arXiv:hep-ph/9603249

published work

- Strange quark contributions to nucleon mass and spin from lattice QCD

  M. Engelhardt, Phys. Rev. D 86, 114510 (2012)
- Strangeness Contribution to the Proton Spin from Lattice QCD

  Bali et al., Phys. Rev. Lett. 108, 222001 (2012)
- Up, down, and strange nucleon axial form factors from lattice QCD

  Green et al., Phys. Rev. D 95, 114502 (2017)
- Nucleon axial form factors using $N_f = 2$ twisted mass fermions with a physical value of the pion mass

  Alexandrou et al., Phys. Rev. D 96, 054507 (2017)
- Quark contribution to the proton spin from 2+1+1-flavor lattice QCD

  Lin et al., arXiv:1806.10604v1 (2018)

three-point functions

$$C^N_{3,A_\mu}(\vec{q}, z_0; \vec{p}', y_0; \Gamma) = \sum_{\vec{y},\vec{z}} e^{i\vec{q}\vec{z}} e^{-i\vec{p}'\vec{y}} \; \Gamma_{\beta\alpha} \left\langle N_\alpha(\vec{y}, y_0) A_\mu(\vec{z}, z_0) \bar{N}_\beta(0) \right\rangle$$

- nucleon interpolator $N_\alpha(x)$
- flavor-diagonal axial current $A_\mu(x)$

- spectral decomposition
  - for the ground state $(z_0, (y_0 - z_0) \gg 0)$

$$C^N_{3,A_\mu}(\vec{q}, z_0; \vec{p}', y_0; \Gamma) = f(\vec{p}', \vec{q}, y_0, z_0) \; T\left(\widetilde{O}_{A_\mu}, \Gamma, \vec{q}, \vec{p}'\right)$$

$$T\left(\widetilde{O}_{A_\mu}, \Gamma, \vec{q}, \vec{p}'\right) = \mathrm{tr}\Big[\Gamma\left(E_{\vec{p}'}\gamma_0 - i\vec{p}'\vec{\gamma} + m\right) \widetilde{O}_{A_\mu}(-\vec{q}) \cdot \left(E_{\vec{p}'-\vec{q}}\gamma_0 - i(\vec{p}' - \vec{q})\vec{\gamma} + m\right)\Big]$$

ratio

- construct a ratio to cancel overlap factors and time dependence in $f(\vec{p}', \vec{q}, y_0, z_0)$

$$R_{A_\mu}(\vec{q}; \vec{p}'; \Gamma) = \frac{C^N_{3,A_\mu}(\vec{q}, z_0; \vec{p}', y_0; \Gamma)}{C^N_2(\vec{p}', y_0)} \sqrt{\frac{C^N_2(\vec{p}', y_0) C^N_2(\vec{p}', z_0) C^N_2(\vec{p}'\text{-}\vec{q}, y_0\text{-}z_0)}{C^N_2(\vec{p}'\text{-}\vec{q}, y_0) C^N_2(\vec{p}'\text{-}\vec{q}, z_0) C^N_2(\vec{p}', y_0\text{-}z_0)}}$$

$$\overset{\text{s.d.}}{=} \frac{1}{4\sqrt{(E_{\vec{p}'-\vec{q}} + m)(E_{\vec{p}'} + m) E_{\vec{p}'} E_{\vec{p}'-\vec{q}}}} T\left(\widetilde{O}_{A_\mu}, \Gamma, \vec{q}, \vec{p}'\right)$$

form factors

- used projectors: $\Gamma_0 = \frac{1}{2}(1+\gamma_0)$, $\Gamma_k = \Gamma_0 \, \mathrm{i}\gamma_5\gamma_k$
- determine T for all combinations of A_μ and Γ_ν

$$\frac{1}{4\sqrt{\cdots}} T\left(\widetilde{O}_{A_\mu}, \Gamma_\nu, \vec{q}, \vec{p}'\right) = M^A_{\nu\mu}(\vec{q},\vec{p}')G_A(Q^2) + M^P_{\nu\mu}(\vec{q},\vec{p}')G_P(Q^2)$$

- (overdetermined) system of equations at each Q^2

$$M\,\vec{G} = \vec{R}\ ;\ M = \begin{pmatrix} M^A_1 & M^P_1 \\ \vdots & \vdots \\ M^A_N & M^P_N \end{pmatrix}\ ,\ \vec{G} = \begin{pmatrix} G_A \\ G_P \end{pmatrix},\ \vec{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix}$$

- minimizing the least-squares function[2]

$$\chi^2 = (\vec{R} - M\vec{G})^T\ C^{-1}\ (\vec{R} - M\vec{G})$$

[2] Capitani et al., arXiv:1705.06186v2

averaging

- drop all non-contributing equations ($M^A = 0$ & $M^P = 0$)
- separate real and imaginary contributions
- average equivalent contributions

1. $\Gamma_1, A_2, p_a = (1\ 0\ 0)^T, q_a = (0\ 1\ 0)^T, p'_a = (1\ 1\ 0)^T$
2. $\Gamma_3, A_2, p_b = (0\ 0\ 1)^T, q_b = (0\ 1\ 0)^T, p'_b = (0\ 1\ 1)^T$

$\Rightarrow\ M^A_{12}(\vec{q}_a, \vec{p}'_a) = M^A_{32}(\vec{q}_b, \vec{p}'_b)$ & $M^P_{12}(\vec{q}_a, \vec{p}'_a) = M^P_{32}(\vec{q}_b, \vec{p}'_b)$

- but treat each $n^2_{\vec{p}'} \in \{0, 1, 2\}$ separately

$\Rightarrow$ number of independent equations N

$$N \in [1, 2, 3, 5, 6, 8, 10, 11, 12, 13, 14, 19, 21, 25, 33]$$

setup

- in this work:
  1. average equivalent three- and two-point functions
  2. construct ratios
  3. apply asymptotic fits/summation method to isolate the ground state
  4. fit (overd.) system which contains the fit results from step 3

ensembles

- **C**oordinated **L**attice **S**imulations (CLS)[3]
  - $N_f = 2 + 1$ O(a)-improved Wilson fermions
  - tree-level improved Lüscher-Weisz gauge action
  - open boundary conditions
  - $\mathrm{tr}\ M = \text{const}$

	β	a [fm]	$N_s^3 x N_t$	m_π[MeV]	m_K[MeV]	N_{cfg}
H105	3.40	0.086	$32^3 x 96$	280	460	1020
N203	3.55	0.064	$48^3 x 128$	340	440	772
N200	3.55	0.064	$48^3 x 128$	280	460	856
D200	3.55	0.064	$64^3 x 128$	200	480	138

[3] CLS, arXiv:1411.3982v2

interpolators

- nucleon interpolator

$$N_\alpha(x) = \epsilon_{abc} \left(u^a_\beta(x) \ (C\gamma_5)_{\beta\gamma} \ d^b_\gamma(x) \right) \ u^c_\alpha(x)$$

- improved local axial current $(f = l, s)$

$$A^f_\mu(\vec{z}, z_0)^{Imp.} = A^f_\mu(\vec{z}, z_0) + ac_A \ \partial_\mu P^f(\vec{z}, z_0)$$
$$A^f_\mu(\vec{z}, z_0) = \bar{f}(\vec{z}, z_0)\gamma_5\gamma_\mu f(\vec{z}, z_0)$$

- non-perturbative determination of:[4][5]

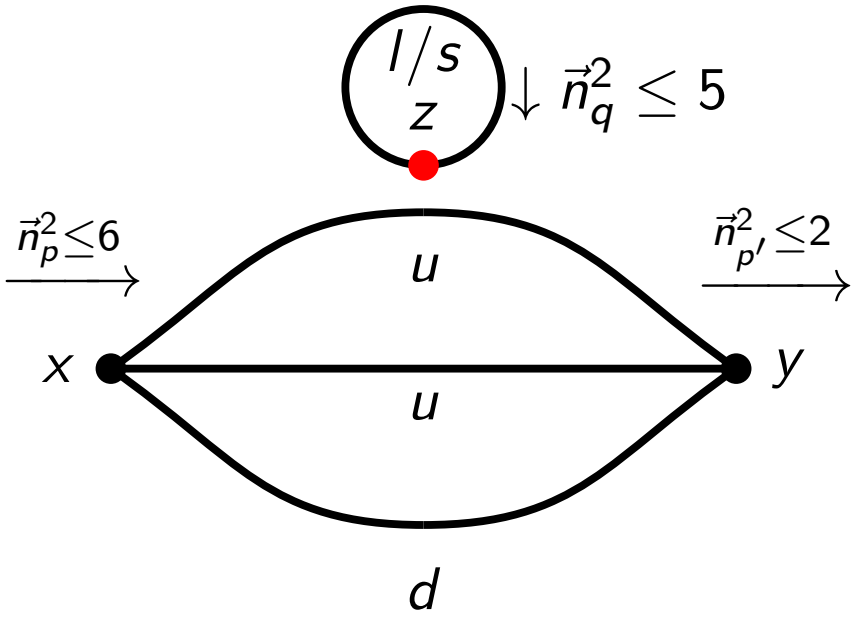
$$c_A \ \ \& \ \ \text{flavor non-singlet } Z_A$$

  - but no flavor singlet renormalization so far!

[4] Alpha Collaboration, Nucl. Phys. B 896 (2015) 555
[5] Alpha Collaboration, Phys. Rev. D 93, 114513 (2016)

isolated disconnected contributions

- light/strange quark



$$\Rightarrow G_A^{l/s}(Q^2), G_P^{l/s}(Q^2)$$

- three-point function factorizes

$$C_{3,A_\mu}^{N,l/s}(\vec{q}, z_0; \vec{p}', y_0; \Gamma_\nu) = \left\langle \mathcal{L}_{A_\mu}^{l/s}(\vec{q}, z_0) \cdot \mathcal{C}_2^N(\vec{p}', y_0; \Gamma_\nu) \right\rangle_G$$

two-point functions

- smeared-smeared quark propagators
- truncated solver method

$$C_2^N = \underbrace{\frac{1}{N_{\text{src}}^S} \sum_{n=1}^{N_{\text{src}}^S} C_2^N(x_n)^{LP}}_{\text{biased estimate}} + \underbrace{\left(\frac{1}{N_{\text{src}}^E} \sum_{n=1}^{N_{\text{src}}^E} \left(C_2^N(x_n)^{HP} - C_2^N(x_n)^{LP} \right) \right)}_{\text{bias correction}}$$

- number of sources per timeslice
  - H105: $N_{\text{src}}^E = 4,\ N_{\text{src}}^S = 32$
  - N200/N203/D200: $N_{\text{src}}^E = 1,\ N_{\text{src}}^S = 32$
- used timeslices ($T_{\text{H105}} = 96, T_{\ldots} = 128$):
  - H105: $24 \triangleright\ \triangleleft 32 \triangleright\ \triangleleft 40 \triangleright\ \triangleleft 48 \triangleright\ \triangleleft 56 \triangleright\ \triangleleft 64 \triangleright\ \triangleleft 72$
  - N200/N203/D200: $\triangleleft 40 \triangleright\ \triangleleft 48 \triangleright\ \triangleleft 56 \triangleright\ \triangleleft 64 \triangleright\ \triangleleft 72 \triangleright\ \triangleleft 80 \triangleright\ \triangleleft 88 \triangleright$
- number of measurements

H105	N203	N200	D200
391680	345856	383488	61824

quark loops

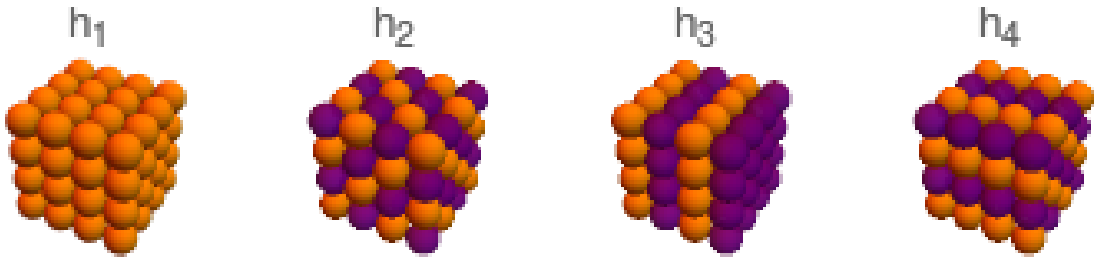
$$L^{l/s}_{A_\mu}(\vec{q}, z_0) = -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \left\langle tr\left[S^{l/s}(z;z)\ \gamma_5\gamma_\mu\right]\right\rangle_G$$

$$= -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}} \left\langle \eta^\dagger(z)\ \gamma_5\gamma_\mu\ s^{l/s}(z)\right\rangle_{G,\eta}$$

- hierarchical probing[6]

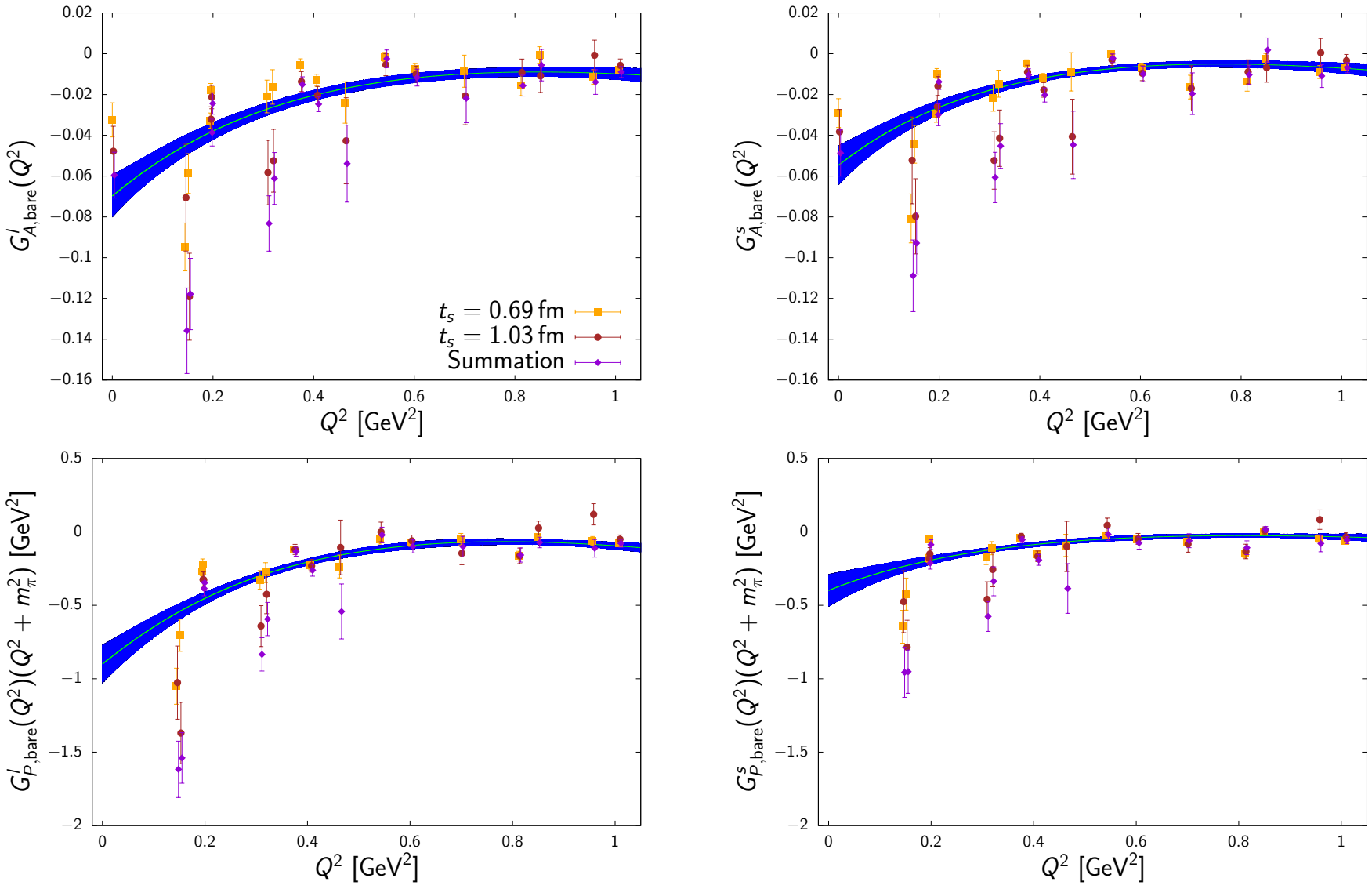
$$\eta_n \to h_n \odot \eta$$

- 4D noise vectors, 2×512 hadamard vectors/configuration

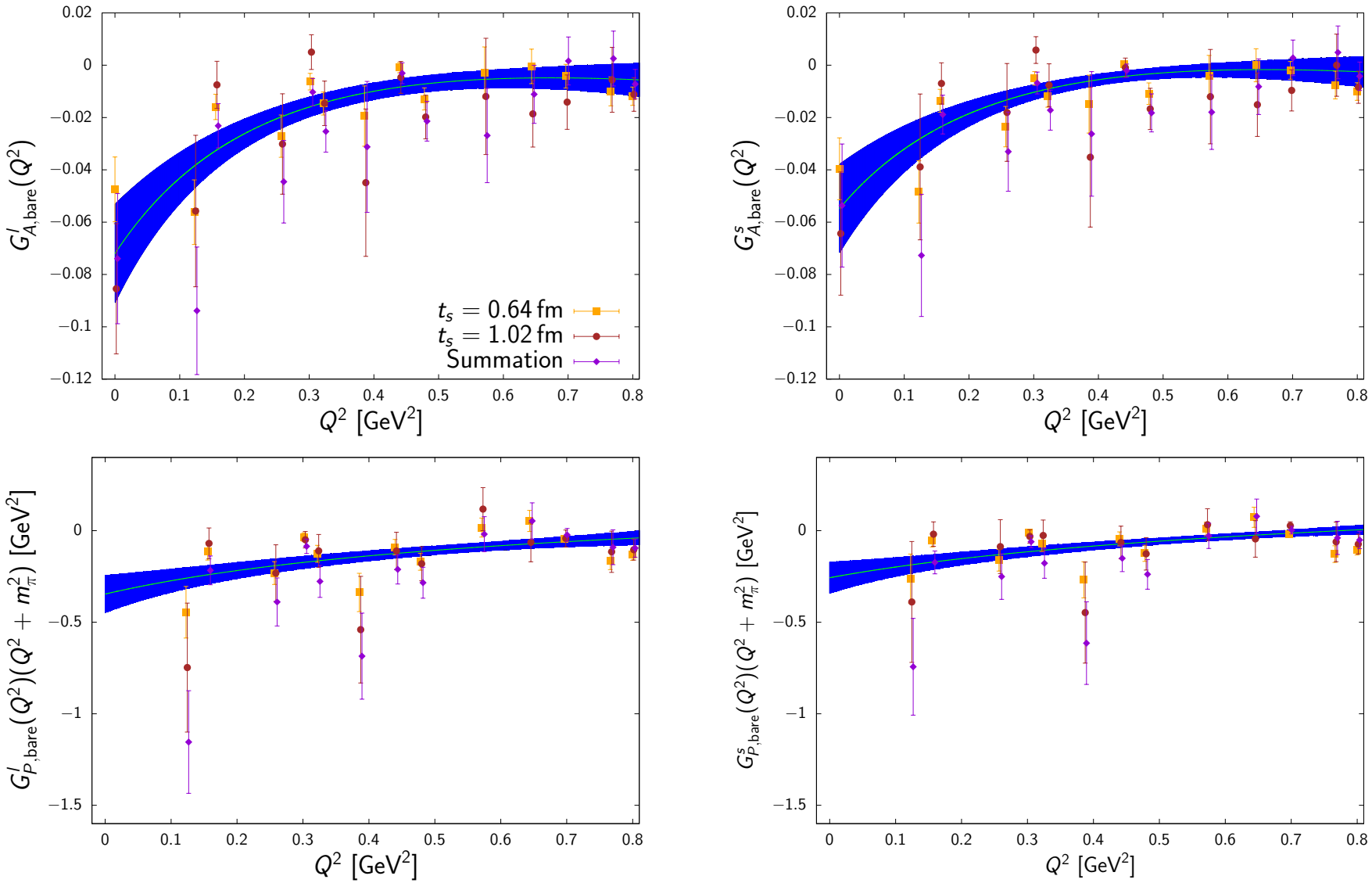


[6] Stathopoulos et al., arXiv:1302.4018v1

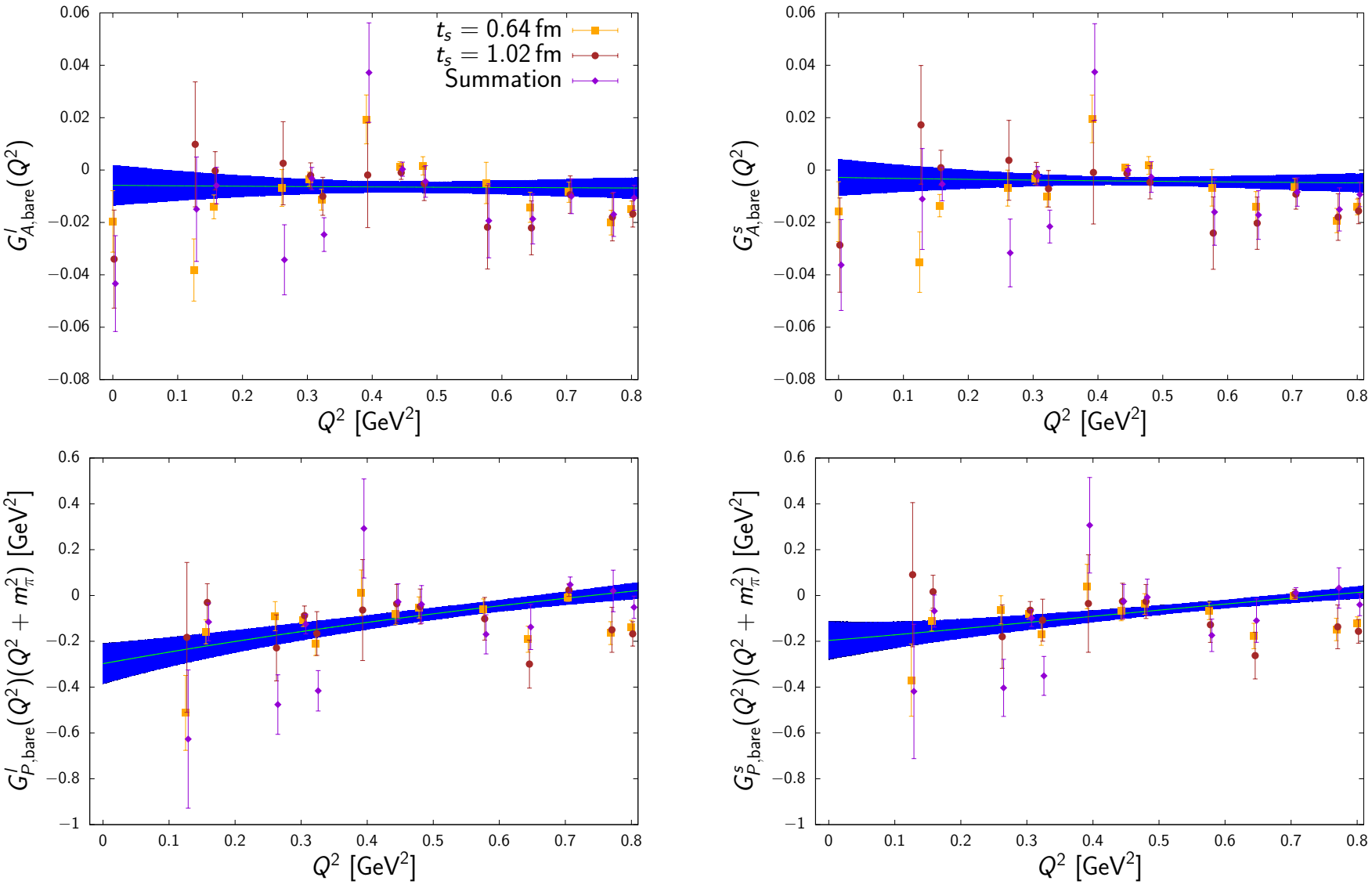
$m_\pi = 280\,\text{MeV},\ \ a = 0.086\,\text{fm}$



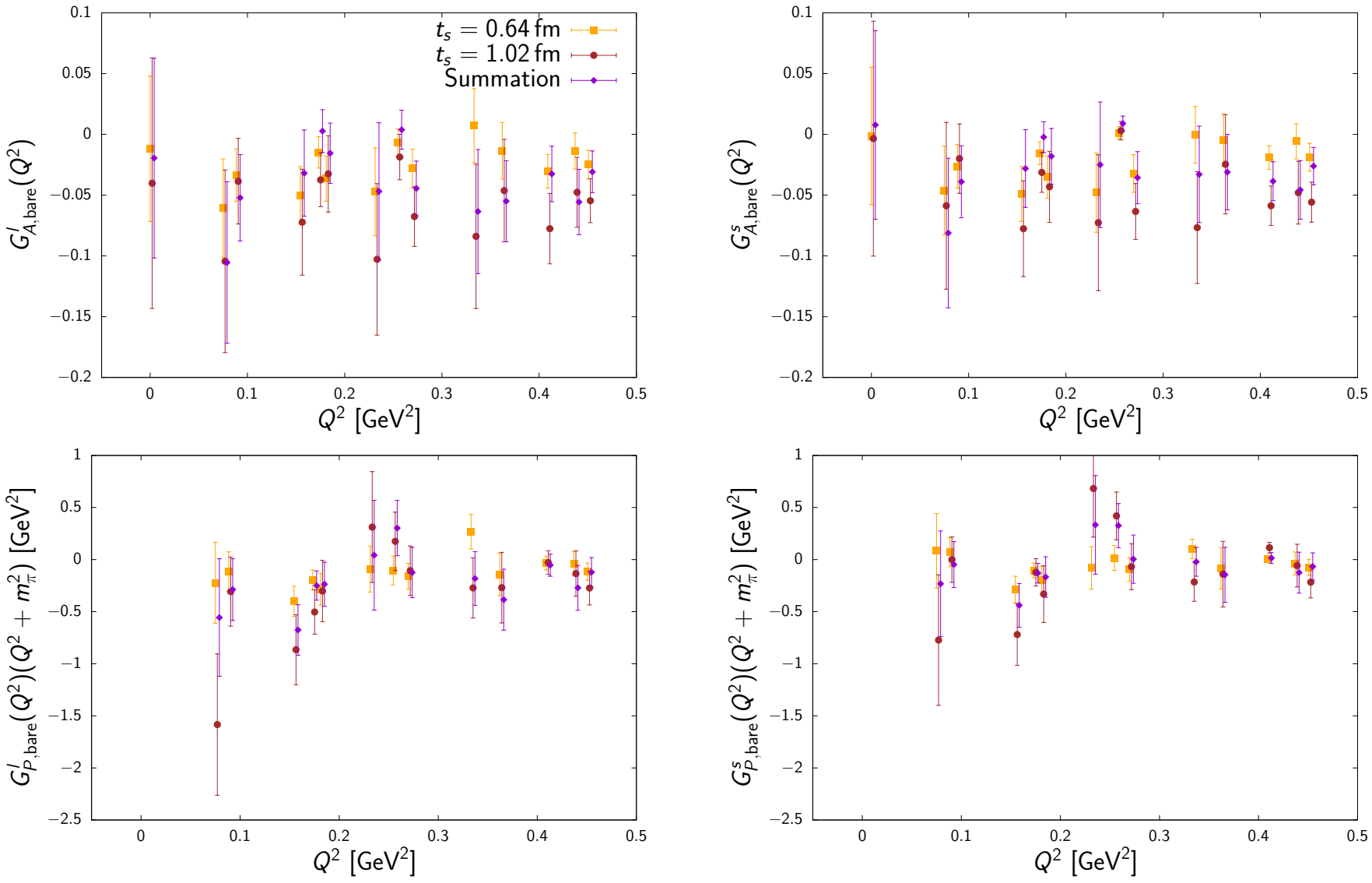
$m_\pi = 280\,\text{MeV},\ \ a = 0.064\,\text{fm}$



$m_\pi = 340\,\text{MeV},\ \ a = 0.064\,\text{fm}$

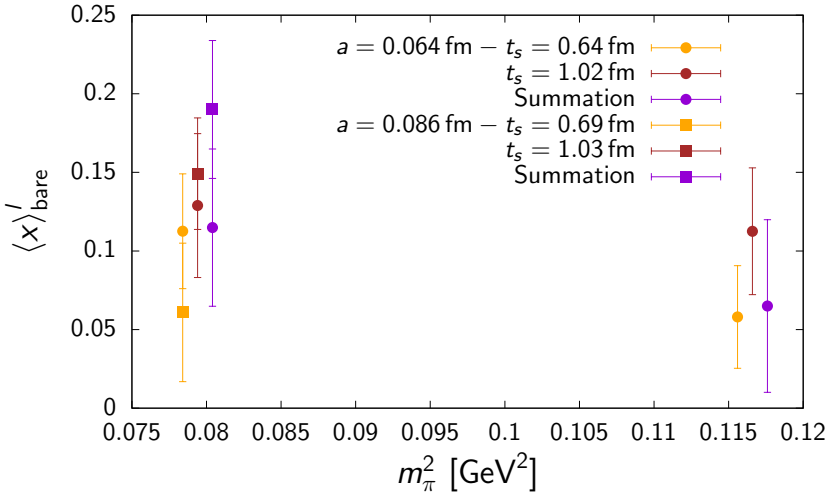


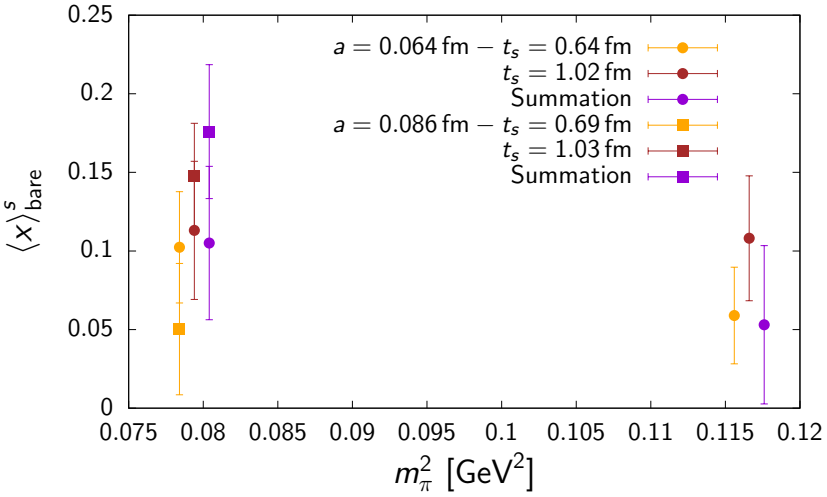
$m_\pi = 200\,\text{MeV},\ \ a = 0.064\,\text{fm}$



outlook

- more data
  - double statistics on N200 & D200
  - ensemble N302
    ($\beta = 3.7, a = 0.049\,\text{fm}, m_\pi = 360\,\text{MeV}, 48^3 \times 128$)
- non-perturbative flavor singlet renormalization
- disconnected electromagnetic form factors
- quark momentum fractions





Thank you!