WWU MÜNSTER

Z_S / Z_P from three-flavour lattice QCD

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LATTICE 18
East Lansing, MI, USA

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Outline

1. Motivation: Why is Z_S/Z_P important in the context of quark mass calculations
2. A new method to determine Z_S/Z_P based on Ward identities in the Schrödinger functional framework
3. Preliminary results and crosschecks

Calculation of quark masses

Why are we interested in heavy quark masses?

- Fundamental parameters of the Standard Model
  - Main source of uncertainty in Higgs partial widths comes from m_c, m_b and α_s (e.g. arXiv:1404.0319)
- Matching parameters for Heavy Quark Effective Theory

Challenges

- Large discretization effects due to the high mass

$\Longrightarrow$ Systematic uncertainties have to be treated carefully

How can we calculate quark masses from LQCD

We work on $N_f = 2 + 1$ CLS based ensembles with Wilson-clover fermions

1. **Tune the hopping parameter κ such that a particle containing the desired quark has it's physical mass.**
2. **Use an appropriate renormalization pattern to relate this to the renormalized quark mass.**

How is the quark mass renormalized in our setup

The standard ALPHA Collaboration method uses the PCAC mass

$$m_{12} = \frac{\left\langle \left[\tilde{\partial}_0 A_0^{12}(x_0) + a c_A \partial_0^* \partial_0 P^{12}(x_0)\right] P^{21}(0)\right\rangle}{2\left\langle P^{12}(x_0)\, P^{21}(0)\right\rangle}$$

and it's renormalization and improvement pattern to calculate the renormalized quark mass

$$m_{12R} = \frac{M}{\bar{m}(L)} \frac{Z_A}{Z_P(L)} m_{12}\left[1 + a(b_A - b_P) m_{q12} + a(\bar{b}_A - \bar{b}_P)\mathrm{tr}(M)\right] + \mathcal{O}(a^2)$$

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$$m_{12R} = \frac{M}{\bar{m}(L)} \frac{Z_A}{Z_P(L)} m_{12} \left[1 + a(b_A - b_P) m_{q12} + a(\bar{b}_A - \bar{b}_P)\mathrm{tr}(M)\right] + \mathcal{O}(a^2)$$

$$\frac{m_{3R}}{m_{1R}} = 2\frac{m_{13}}{m_{12}} \left[1 + (b_A - b_P)\frac{(am_{q3} - am_{q2})}{2}\right] - 1 + \mathcal{O}(a^2)$$

How is the quark mass renormalized in our setup

- Method used in $N_f = 2$, arXiv:1205.5380
- **Light and strange quark masses for $\mathrm{N_f} = 2 + 1$ simulations with Wilson fermions**
  Jonna Koponen, tomorrow 2:40 pm
- We want to use a complementary method as a cross check for the future computation of the charm quark's mass (CLS based, partly joint with the Regensburg group)

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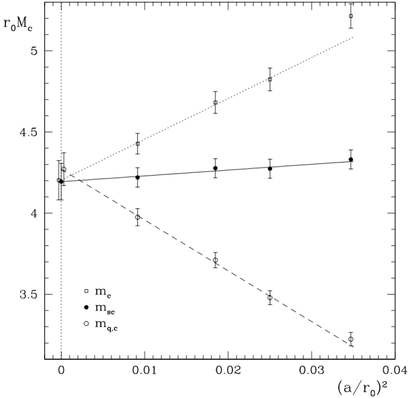

Rolf, Sint, $N_f = 0$

How is the quark mass renormalized in our setup

1. **Subtractive renormalization**

$$am_{\mathrm{q,f}} = \frac{1}{2\kappa_{\mathrm{f}}} - \frac{1}{2\kappa_{\mathrm{crit}}}$$

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2. **Multiplicative renormalization** (plus $\mathcal{O}(a)$ improvement)

$$m_{\mathrm{fR}} = \frac{1}{Z_{\mathrm{S}}(L)}\left\{\left[m_{\mathrm{q,f}} + (r_m - 1)\frac{\mathrm{tr}(M)}{N_{\mathrm{f}}}\right]\right.$$
$$\left. + a\left[b_m m_{\mathrm{q,f}}^2 + \bar{b}_m m_{\mathrm{q,f}}\mathrm{tr}(M) + (r_m d_m - b_m)\frac{\mathrm{tr}(M^2)}{N_{\mathrm{f}}} + (r_m \bar{d}_m - \bar{b}_m)\frac{\mathrm{tr}(M)^2}{N_{\mathrm{f}}}\right]\right\} + \mathcal{O}(a^2)$$

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How is the quark mass renormalized in our setup

We have two different methods to arrive at the **renormalized quark mass**:

$$\frac{m_{3R}}{m_{1R}} = 2\frac{m_{13}}{m_{12}}\left[1 + (b_A - b_P)\frac{(am_{q3} - am_{q2})}{2}\right] - 1 + \mathcal{O}(a^2)$$

$$m_{1R} - m_{2R} = \frac{1}{Z_S(L)}\left[m_{q1} - m_{q2}\right]\left[1 + ab_m(m_{q1} + m_{q2}) + a\bar{b}_m \mathrm{tr}(M)\right] + \mathcal{O}(a^2)$$

Progress on non-perturbative determination of renormalization constants and parameters

- **Renormalization and improvement factors for the axial current were previously determined in our group**
  [ALPHA Collaboration, Bulava, Della Morte, Heitger, Wittemeier; arXiv:1502.04999, arXiv:1604.05827]
  (There is also an approach to Z_A in the chirally rotated Schrödinger functional)
- **Results for Z_P and the RGI factor $\frac{M}{\bar{m}(L)}$ were published recently by our collaboration**
  [ALPHA Collaboration, Campos, Fritzsch, Pena, Preti, Ramos, Vladikas; arXiv:1802.05243]
- **Determination of b_m and $b_A - b_P$ is almost finished**
  [arXiv:1710.07020 & in progress: De Divitiis, Fritzsch, Heitger, Köster, Kuberski, Vladikas]
- **Z_S is undetermined so far**

Our method for the determination of Z_S/Z_P

We start with the general **axial Ward identity**:

$$\int_{\partial R} d\sigma_\mu(x) \left\langle A_\mu^a(x)\mathcal{O}_{int}^b(y)\mathcal{O}_{ext}^c(z)\right\rangle - 2m\int_R d^4x \left\langle P^a(x)\mathcal{O}_{int}^b(y)\mathcal{O}_{ext}^c(z)\right\rangle$$
$$= -\left\langle [\delta_A^a\mathcal{O}_{int}^b(y)]\mathcal{O}_{ext}^c(z)\right\rangle$$

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And use the transformation property of the **pseudoscalar density** under small chiral rotations:

$$\delta_\mathrm{A}^a \mathrm{P}^b(x) = d^{abc}\mathrm{S}^c(x) + \frac{\delta^{ab}}{N_\mathrm{f}}\bar{\psi}(x)\psi(x)$$

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- $d^{abc} \neq 0$, for $SU(N_\mathrm{f})$ with $N_\mathrm{f} \geq 3$

Our method for the determination of Z_S/Z_P

Inserting a pseudoscalar density into the chiral Ward identity leads to:

$$\int \mathrm{d}^3\mathbf{y} \int \mathrm{d}^3\mathbf{x} \left\langle \left[A_0^a(y_0+t,\mathbf{x}) - A_0^a(y_0-t,\mathbf{x})\right] P^b(y_0,\mathbf{y})\mathcal{O}_{\mathrm{ext}} \right\rangle$$

$$-2m \int \mathrm{d}^3\mathbf{y} \int \mathrm{d}^3\mathbf{x} \int_{y_0-t}^{y_0+t} \mathrm{d}x_0 \left\langle P^a(x_0,\mathbf{x})P^b(y_0,\mathbf{y})\mathcal{O}_{\mathrm{ext}} \right\rangle$$

$$= -d^{abc} \int \mathrm{d}^3\mathbf{y} \left\langle S^c(y)\mathcal{O}_{\mathrm{ext}} \right\rangle$$

Our method for the determination of Z_S/Z_P

When the Ward identity is evaluated on a **lattice with Schrödinger functional boundary conditions** we end up with:

$$
\begin{aligned}
& Z_A Z_P \left[1 + ab_A m_q + a\bar{b}_A \mathrm{tr}(M)\right]\left[1 + ab_P m_q + a\bar{b}_P \mathrm{tr}(M)\right] \times \\
& \left[f_{AP}^{I,abcd}(y_0 + t, y_0) - f_{AP}^{I,abcd}(y_0 - t, y_0) - 2m\,\tilde{f}_{PP}^{abcd}(y_0 + t, y_0 - t)\right] \\
= & - Z_S \left[1 + ab_S m_q + a\bar{b}_S \mathrm{tr}(M)\right] f_S^{abcd}(y_0) + \mathcal{O}(a^2) + \mathcal{O}(am)
\end{aligned}
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= & -Z_S\, f_S^{abcd}(y_0) + \mathcal{O}(a^2)
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= & -Z_S \qquad f_S^{abcd}(y_0) + \mathcal{O}(a^2)
\end{aligned}
$$

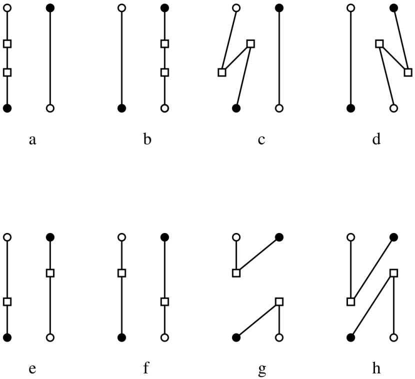

Figure: Graphical representation of the Wick contractions contributing to $f_{\Gamma\tilde{\Gamma}}$ (arXiv:hep-lat/9611015).

$$
\begin{aligned}
f^{abcd}_{\Gamma\tilde\Gamma}(x,y) \propto & -\langle \mathcal{O}'^{a}\, \bar\psi(x)\Gamma T^b \psi(x)\, \bar\psi(y)\tilde\Gamma T^c \psi(y)\, \mathcal{O}^d \rangle \\
= & - a^{12}\,\mathrm{Tr}\left(T^a T^b T^c T^d\right) \sum_{\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \left\langle \mathrm{tr}\left\{ [\zeta'(\mathbf{v}')\bar\psi(x)]\Gamma[\psi(x)\bar\psi(y)]\tilde\Gamma[\psi(y)\bar\zeta(\mathbf{u})]\gamma_5[\zeta(\mathbf{v})\bar\zeta'(\mathbf{u}')]\gamma_5 \right\}\right\rangle \\
& - a^{12}\,\mathrm{Tr}\left(T^a T^b T^d T^c\right) \sum_{\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \left\langle \mathrm{tr}\left\{ [\zeta'(\mathbf{v}')\bar\psi(x)]\Gamma[\psi(x)\bar\zeta(\mathbf{u})]\gamma_5[\zeta(\mathbf{v})\bar\psi(y)]\tilde\Gamma[\psi(y)\bar\zeta'(\mathbf{u}')]\gamma_5 \right\}\right\rangle \\
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& + a^{12}\mathrm{Tr}(T^a T^b)\mathrm{Tr}(T^d T^c) \sum_{\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \left\langle \mathrm{tr}\left\{ [\zeta'(\mathbf{v}')\bar\psi(x)]\Gamma[\psi(x)\bar\zeta'(\mathbf{u}')]\gamma_5 \right\} \mathrm{tr}\left\{ [\psi(y)\zeta(\bar{\mathbf{u}})]\gamma_5[\zeta(\mathbf{v})\bar\psi(y)]\tilde\Gamma \right\}\right\rangle \\
& + a^{12}\mathrm{Tr}(T^a T^c)\mathrm{Tr}(T^d T^b) \sum_{\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \left\langle \mathrm{tr}\left\{ [\zeta'(\mathbf{v}')\bar\psi(y)]\tilde\Gamma[\psi(y)\bar\zeta'(\mathbf{u}')]\gamma_5 \right\} \mathrm{tr}\left\{ [\psi(x)\zeta(\bar{\mathbf{u}})]\gamma_5[\zeta(\mathbf{v})\bar\psi(x)]\Gamma \right\}\right\rangle
\end{aligned}
$$

Flavour choices

- We are **free to choose** the flavour indices a, b, c, d to arrive at **different Ward identities** differing only by ambiguities proportional to the lattice spacing.
- Two specific choices seem to be beneficial from a numerical point of view:
  - **WI(83)-WI(41):** $[\mathbf{a} = \mathbf{c} = \mathbf{8}, \mathbf{b} = \mathbf{d} = \mathbf{3}] - [\mathbf{a} = \mathbf{c} = \mathbf{4}, \mathbf{b} = \mathbf{d} = \mathbf{1}]$
    leads to a Ward identity where **disconnected** and **one kind of connected** diagrams contribute
  - **WI(2568):** $\mathbf{a} = \mathbf{2}, \mathbf{b} = \mathbf{5}, \mathbf{c} = \mathbf{6}, \mathbf{d} = \mathbf{8}$
    leads to a Ward identity where **all connected** but **no disconnected** diagrams contribute

$L^3 \times T/a^4$	β	κ	#REP	#MDU	ID
$12^3 \times 17$	3.3	0.13652	20	10240	A1k1
		0.13660	10	13672	A1k2
		0.13648	5	6876	A1k3
$14^3 \times 21$	3.414	0.13690	32	25600	E1k1
		0.13695	48	38400	E1k2
$16^3 \times 23$	3.512	0.13700	2	20480	B1k1
		0.13703	1	8192	B1k2
		0.13710	3	22528	B1k3
$16^3 \times 23$	3.47	0.13700	3	29560	B2k1
$20^3 \times 29$	3.676	0.13700	4	15232	C1k2
		0.13719	4	15472	C1k3
$24^3 \times 35$	3.810	0.13712	6	10272	D1k1
		0.13701	3	5672	D1k2
		0.137033	7	6488	D1k4

- Schrödinger functional boundary conditions
- Line of constant physics (system size $L \approx 1.2$ fm)
- Use of wavefunctions to maximize the overlap with the ground state

Table: Summary of simulation parameters of the gauge configuration ensembles used in this study, as well as the number of (statistically idependent) replica per ensemble 'ID' and their total number of molecular dynamics units.

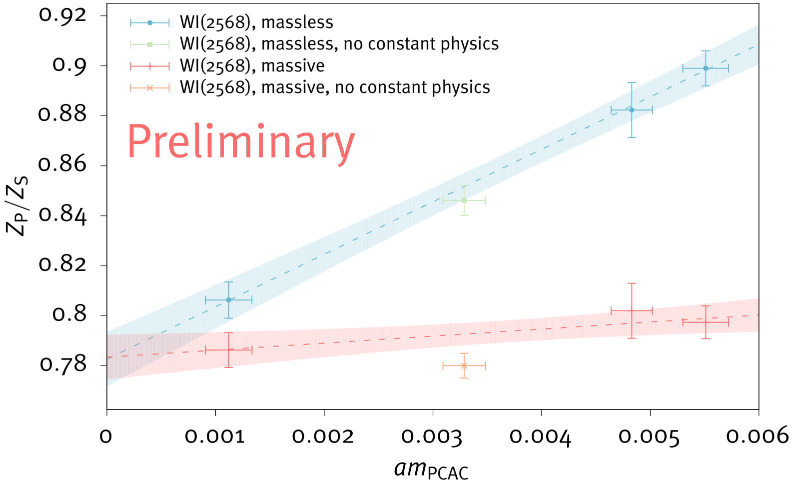

Figure: Preliminary chiral extrapolation of Z_P/Z_S derived from WI(2568) without and with mass term for $g_0^2 = 1.7084$. Data points from Ensemble B2k1, which violates the constant physics condition, for comparison.

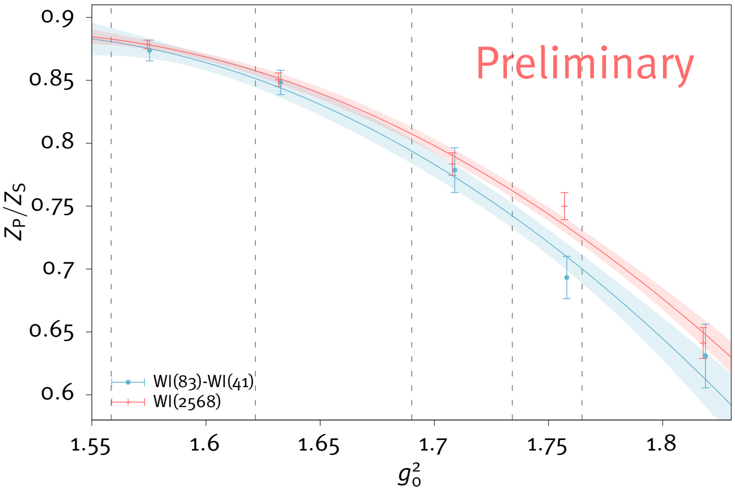

Figure: Preliminary results for Z_P/Z_S from WI(83)-WI(41) and WI(2568) with interpolating Padé fits. Dashed lines indicate the bare couplings used in CLS simulations.

Thank you for your attention!