Constraint HMC Algorithms for gauge-Higgs models

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Motivation

Gauge-Higgs Unification

- Origin of the potential responsible for the Brout-Englert-Higgs mechanism is unknown.
- In Gauge-Higgs Unification (GHU) models the Higgs field is associated with some extra-dimensional components of a gauge field [Manton, 1979].
- In the case of one extra dimension the Higgs potential is zero at tree level and is generated only through quantum effects.
- What happens non-perturbatively? $\longrightarrow$ lattice study.
- Measure the effective potential with respect to the Higgs field using constraint HMC algorithms

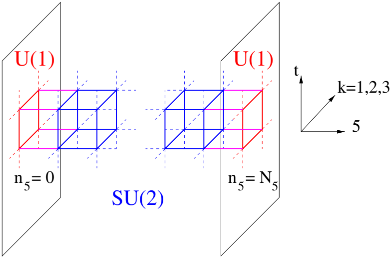
5D Orbifold Model Lattice Formulation

Wilson Gauge action for bulk gauge group $\mathrm{SU}(2)$:

$$S_W^{orb} = \frac{\beta_4}{2} \sum_{P_4} w \cdot \mathrm{tr}\{1 - P_4\} + \frac{\beta_5}{2} \sum_{P_5} \mathrm{tr}\{1 - P_5\}$$

Boundary links satisfy $gUg^{-1} = U$ with $g = -i\sigma^3$

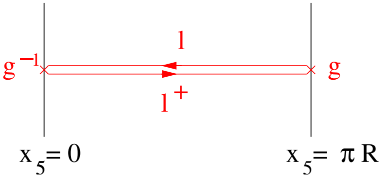
$$w = \begin{cases} \frac{1}{2} & \text{plaquette on boundary} \\ 1 & \text{otherwise} \end{cases}$$



5D Orbifold

- bare anisotropy is $\gamma = \sqrt{\beta_5/\beta_4} \simeq a_4/a_5$
- stick-symmetry $g_s = -i(\cos\theta\sigma_1 + \sin\theta\sigma_2)$
- $N_5 a_5 = \pi R$, R radius of extra dimension.

5D Orbifold Model Higgs operators



Scalar Polyakov loop (defined at $n = (n_\mu, 0)$)

$$p(n) \;=\; l(n)\, g\, l^\dagger(n)\, g^\dagger$$

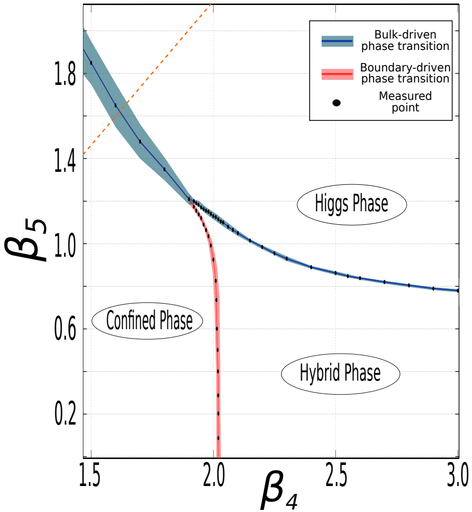
Higgs field

$$h(n) \;=\; [p(n) - p^\dagger(n), g]/(4N_5) \;\sim\; A_5^1\sigma^1 + A_5^2\sigma^2$$

Higgs operators

$$\mathcal{H}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[hh^\dagger] \quad , \quad \mathcal{P}(n_0) = \sum_{n_1,n_2,n_3} \mathrm{tr}\,[p]$$

5D Orbifold Model Phase Diagram



Notes

On the torus

- confined and Coulomb phase.

On the orbifold

- One more phase: $U(1)$ gauge links deconfine separately.
- No compactification observed at $\gamma > 1$.
- Interesting physics is found at $\gamma < 1$.

DR to 4D Abelian Higgs [Alberti, Irges, Knechtli and Moir, 1506.06035]

5D Orbifold Model Summary

Non-perturbative Gauge-Higgs Unification

- $\mathrm{SU}(2)$ pure gauge theory on a 5D orbifold has a Higgs phase with the Higgs mechanism realized as a quantum and bosonic effect.
- A Standard-Model like spectrum can be reproduced.
- Localization on the 4D boundaries is observed.
- Cut-off effects appear to be small. Excited state energies are 2–5 times larger than the ground states.
- Review on lattice works on extra dimensions

  [Knechtli and Rinaldi, arXiv:1605.04341]
- ...
- Study connection to 4D Abelian gauge-Higgs model...

4D Abelian Higgs Unitary Gauge

$$\begin{aligned}
\phi(x) &= \rho(x)\exp i\varphi(x) \;\rightarrow\; \phi_1 = \rho\cos\varphi,\; \phi_2 = \rho\sin\varphi \\
S_\rho[V,\rho] &= \sum_x \rho_x^2 + \lambda(\rho_x^2 - 1)^2 \\
&\quad -2\kappa\rho_x \sum_\mu \rho_{x+\hat{\mu}} \mathrm{tr}\,(\underbrace{e^{-i\varphi_x} U_{x,\mu} e^{i\varphi_{x+\hat{\mu}}}}_{=V_{x,\mu}}) \\
H[V,\rho] &= S_\rho[V,\rho] + \frac{1}{2}\sum_x \pi(x)^2 + \mu\left(\frac{1}{\Omega}\sum_x \rho(x) - \Phi\right)
\end{aligned}$$

constraint EOMs: [Fodor, Holland, Kuti, Nogradi, Schroeder, 0710.3151]

$$\begin{aligned}
\dot{\rho}(x,t) &= \frac{\partial H}{\partial \pi(x,t)} = \pi(x,t) \\
\dot{\pi}(x,t) &= -\frac{\partial H}{\partial \rho(x,t)} = -\frac{\partial S_\rho}{\partial \rho(x,t)} - \frac{\mu}{\Omega}
\end{aligned}$$

4D Abelian Higgs Constraint HMC

time derivatives of the constraint to ensure it's unchanged

$$\frac{\partial}{\partial t}\left(\frac{1}{\Omega}\sum_x \rho(x) - \Phi\right) = \sum_x \dot{\rho}(x) = \sum_x \pi(x) = 0$$

$$\frac{\partial}{\partial t}\sum_x \pi(x) = \sum_x \dot{\pi}(x) = 0 \Rightarrow \mu = -\sum_x \frac{\partial S_\rho}{\partial \rho(x)}$$

to measure the effective potential we use the derivative

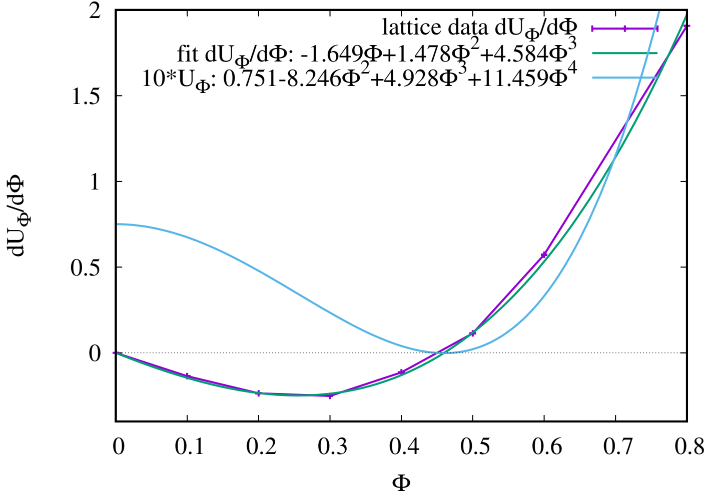
with respect to $\frac{d}{d\Phi} = \frac{1}{\Omega}\sum_x \frac{d\rho_x}{d\Phi}\frac{d}{d\rho_x}$ of $S_\rho[V,\rho]$

$$\begin{aligned}\frac{dU_\Omega}{d\Phi} = 2\Phi \quad &+ \quad 4\lambda\left\langle \frac{1}{\Omega}\sum_x (\rho(x)^2 - 1)\rho(x)\right\rangle_\Phi \\ &+ \quad 2\kappa\left\langle \frac{1}{\Omega}\sum_{x,\mu}(\rho(x) + \rho(x+\hat{\mu}))V_\mu(x)\right\rangle_\Phi\end{aligned}$$

4D Abelian Higgs Effective Potential

$$\frac{dU_\Omega}{d\Phi} \approx 2c_1\Phi + 3c_2\Phi^2 + 4c_3\Phi^3$$

[Irges, Koutroulis, 1703.10369]



5D Torus Model Axial Gauge

first step towards 5D orbifold model we work on the torus

$$S_W^{tor} = \frac{\beta_4}{2} \sum_{P_4} \mathrm{tr}\{1 - P_4\} + \frac{\beta_5}{2} \sum_{P_5} \mathrm{tr}\{1 - P_5\}$$

to simplify the Higgs operator (Polyakov loop) we use the axial gauge along the fifth dimension $V_5 = \prod_{n=0}^{N_5} [U_5(x,n)]$

$$\begin{aligned} H[U] &= S_W^{tor}[U] + \frac{1}{2} \sum_{\mathbf{x},\mu} \mathrm{tr}\,[\pi_\mu^2(\mathbf{x})] + \lambda(\frac{1}{\Omega} \sum_x \mathrm{tr}\, V_5(x) - \Phi) \\ \dot{V}_5(x) &= \pi_5(x) V_5(x) \\ \dot{\pi}_5(x) &= -\frac{\partial S[V_5]}{\partial V_5(x)} - \frac{\lambda}{4\Omega} \mathrm{tr}\,[\sigma_i V_5(x)]\sigma^i \end{aligned}$$

5D Torus Model Constraint HMC

time derivatives of the constraint to ensure it's unchanged

$$\frac{\partial}{\partial t}\left(\frac{1}{\Omega}\sum V_5 - \Phi\right) = \sum \dot{V}_5 = \sum \mathrm{tr}\,\pi_5 V_5$$

$$\frac{\partial}{\partial t}\sum \mathrm{tr}\,\pi_5 V_5 = \sum \mathrm{tr}\,\dot{\pi}_5 V_5 + \sum \mathrm{tr}\,\pi_5^2 V_5$$

$$\frac{\lambda}{4\Omega} = \left(\sum \mathrm{tr}\,\pi_5^2 V_5 - \sum \mathrm{tr}\,\frac{\partial S[V_5]}{\partial V_5} V_5\right) / \sum \mathrm{tr}\,\{\mathrm{tr}\,[\sigma_i V_5]\sigma^i V_5\}$$

$\Rightarrow \sum \mathrm{tr}\,\pi_5^2 V_5$ makes the Hamiltonian non-separable

$$H(U,\pi) \neq H_1(\pi) + H_2(U)$$

as a consequence we cannot use standard leap-frog, and need to work with a new symplectic algorithm

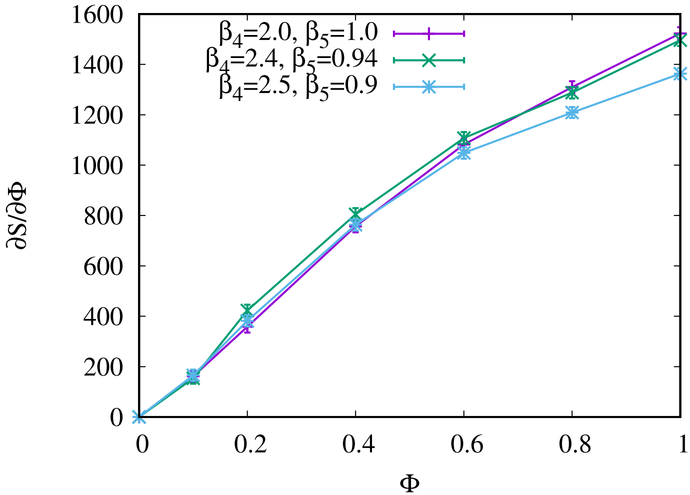
Rattle Integration Scheme

$$\begin{aligned}
\pi_{n+1/2} &= \pi_n - \frac{h}{2}\left(\frac{\partial S}{\partial V_n} + \frac{\lambda}{4\Omega}\mathrm{tr}\,[\sigma_i V_n]\sigma^i\right) \\
V_{n+1} &= e^{h\pi_{n+1/2}} V_n\ , \qquad 0 = \frac{1}{\Omega}\sum \mathrm{tr}\, V_{n+1} - v \\
\pi_{n+1} &= \pi_{n+1/2} - \frac{h}{2}\left(\frac{\partial S}{\partial V_{n+1}} + \frac{\mu}{4\Omega}\mathrm{tr}\,[\sigma_i V_{n+1}]\sigma^i\right) \\
0 &= \frac{1}{\Omega}\sum \mathrm{tr}\left\{\mathrm{tr}\,[\sigma_i V_{n+1}]\sigma^i \pi_{n+1}\right\} = -\frac{2}{\Omega}\sum \mathrm{tr}\, V_{n+1}\pi_{n+1}
\end{aligned}$$

The first three equations determine $(\pi_{n+1/2}, V_{n+1}, \lambda)$, whereas the remaining two give (π_{n+1}, μ)...

$$\frac{\mu}{4\Omega} = \left(2\sum \mathrm{tr}\,\pi_{n+1/2}V_{n+1}/h - \sum \mathrm{tr}\,\frac{\partial S}{\partial V_{n+1}}V_{n+1}\right) \Big/ \sum \mathrm{tr}\left\{\mathrm{tr}\,[\sigma_i V_{n+1}]\sigma^i V_{n+1}\right\}$$

Effective Higgs Potential of 5D Torus Model - no SSB!



evaluated Jacobian $J = \frac{\partial(V_{n+1}, \pi_{n+1})}{\partial(V_n, \pi_n)} \Rightarrow$ det$J = 1$!

Conclusions

Constraint HMC Algorithms for gauge-Higgs models

- implemented the constraint HMC for 4D Abelian gauge-Higgs model and computed the effective Higgs potential in the spontaneously broken phase
- implemented a symplectic constraint HMC algorithm for 5D torus and found no SSB in accordance with the absence of stick symmetry

Outlook

- test constraint HMC algorithm for 5D orbifold
- study connection to 4D Abelian Higgs model via
- effective potential derived from constraint HMCs
- other applications: finite temperature QCD