Bag representation for composite degrees of freedom in theories with fermions

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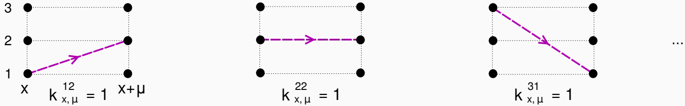
Introduction and motivation

- Rewriting theories in terms of new degrees of freedom is a powerful tool
- For abelian bosonic theories dual representations in terms of worldlines and worldsheets solve the complex action problem
- Monte Carlo simulations in terms of worldlines and worldsheets become possible
- We discuss worldline/worldsheet representation for non-abelian symmetries and theories with fermions
- We explore resummation of worldline contributions in terms of bags (space-time domains for composite d.o.f.s)

ACC and ACF dualization methods

- Key for the dualization of non-abelian systems is the decomposition of the action into its minimal units
- Example:

$$e^{\sum_{a,b}\overline{\psi}^a_x U^{ab}_{x,\mu}\psi^b_{x+\hat{\mu}}} = \prod_{a,b} e^{\overline{\psi}^a_x U^{ab}_{x,\mu}\psi^b_{x+\hat{\mu}}} = \prod_{a,b}\sum_{k^{ab}_{x,\mu}=0}^{1} (\overline{\psi}^a_x \psi^b_{x+\hat{\mu}})^{k^{ab}_{x,\mu}} (U^{ab}_{x,\mu})^{k^{ab}_{x,\mu}}$$



  - Grassmann bilinears, called **A**belian **C**olor **F**luxes, for fermions
  - complex numbers, called **A**belian **C**olor **C**ycle, for gauge theories

- This decomposition of the action allows to proceed as in the abelian case
  - $\Rightarrow$ reordering of terms
  - $\Rightarrow$ integrate out the original fields

ACC and ACF dualization methods

- Dual partition function sums over admissible configurations of the dual variables
- The configurations contributing to the long range physics are:
  - worldlines for matter degrees of freedom
  - worldsheets for gauge degrees of freedom

$$Z = \sum_{L,S} C[L,S] W[L,S] \mathrm{sign}[L,S]$$

Dualization methods for non-abelian lattice field theories

- SU(2) principal chiral model with chemical potentials
  $\Rightarrow$ ACF completely solves the sign problem

Gattringer, Göschl, Marchis, Phys. Lett. B778 (2018)

- gauge theories with fermions
  $\Rightarrow$ worldlines and worldsheet representation of QC_2D and QCD in closed form

Gattringer, Marchis, Nucl. Phys. B916 (2017), Phys. Rev. D97 (2018)

ACC and ACF dualization methods

Characteristic features of dual form of QCD:

- Exact rewriting of the partition sum in terms of wordlines and worldsheets
- The partition sum is a strong coupling expansion where all terms are known in closed form
- Sign factors for fermionic nature $\Rightarrow$ resummation strategies need to be found!

Observations:

- At strong coupling the worldline elements for joint propagation of 3 quarks behave like free fermion worldlines
- Try to reformulate in terms of effective fermion bags

  Chandrasekharan, PRD 82 (2010), PRL 108 (2012), EPJ A 49 (2013), ...

- These "**Baryon bags**" implement a resummation of contributions

Baryon bags in strong coupling QCD

Idea: direct construction of baryon bag representation without dualization

Gattringer, PRD 97 (2018)

Strong coupling QCD with one flavor of staggered fermions

$$Z = \int D[U] D[\psi, \overline{\psi}] \, e^{S_F[\psi, \overline{\psi}, U]}$$

Staggered action

$$S_F[U, \psi, \overline{\psi}] = \sum_x \left(2m \, \overline{\psi}_x \psi_x + \sum_\nu \gamma_{x,\nu} \left[\overline{\psi}_x U_{x,\nu} \psi_{x+\hat{\nu}} \; - \overline{\psi}_{x+\hat{\nu}} U^\dagger_{x,\nu} \psi_x \right] \right)$$

Factorization of the Boltzmann weight

$$e^{S_F[U, \psi, \overline{\psi}]} \; = \prod_x e^{2m \, \overline{\psi}_x \psi_x} \prod_{x,\nu} e^{\gamma_{x,\nu} \overline{\psi}_x U_{x,\nu} \psi_{x+\hat{\nu}}} \; e^{-\gamma_{x,\nu} \overline{\psi}_{x+\hat{\nu}} U^\dagger_{x,\nu} \psi_x}$$

Baryon bags in strong coupling QCD

Taylor expansion of the hopping terms

$$\begin{aligned} e^{\gamma\overline{\psi}U\psi} &= 1 + \gamma\,(\overline{\psi}U\psi) + \frac{1}{2!}(\overline{\psi}U\psi)^2 + \frac{\gamma}{3!}(\overline{\psi}U\psi)^3 \\ &= \left[1 + \frac{\gamma}{3!}(\overline{\psi}U\psi)^3\right]\left[1 + \gamma\,(\overline{\psi}U\psi) + \frac{1}{2!}(\overline{\psi}U\psi)^2\right] \\ &= e^{\frac{\gamma}{3!}(\overline{\psi}U\psi)^3} \sum_{k=0}^{2} \frac{(\gamma\overline{\psi}U\psi)^k}{k!} \end{aligned}$$

Cubic term is independent of gauge fields: $(\overline{\psi}U\psi)^3 = 3!\overline{\psi}_3\overline{\psi}_2\overline{\psi}_1\psi_1\psi_2\psi_3 = 3!\overline{B}B$

Baryon fields $B = \psi_1\psi_2\psi_3$, $\overline{B} = \overline{\psi}_3\overline{\psi}_2\overline{\psi}_1$ nilpotent, anti-commuting

Factorization of the baryon contributions

$$e^{S_F[U,\psi,\overline{\psi}]} \;=\; e^{S_B[\overline{B},B]}\, W_{QD}[\overline{\psi},\psi,U]$$

Baryon bags in strong coupling QCD

Baryons propagate as free fermions

$$S_B[B,\overline{B}] = \sum_x \left(2M\,\overline{B}_x B_x + \sum_\nu \gamma_{x,\nu}\left[\overline{B}_x B_{x+\hat{\nu}} - \overline{B}_{x+\hat{\nu}} B_x\right]\right)$$

Lattice factorizes into disjoint dynamically determined space-time regions

- **baryon bags** $\mathcal{B}_i$, $\mathcal{B} = \bigcup \mathcal{B}_i$

$$Z_{\mathcal{B}_i} = \int D_{\mathcal{B}_i}[\overline{\psi},\psi]\; e^{\sum_{x,y} \overline{B}_x D^{(i)}_{x,y} B_y} = \det D^{(i)}$$

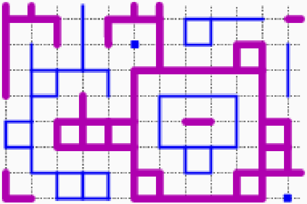
- **complementary domain** $\overline{\mathcal{B}} = \mathcal{B}/\Lambda$

$$Z_{\overline{\mathcal{B}}} = \int D[U]\, D_{\overline{\mathcal{B}}}[\overline{\psi},\psi]\;\, W_{QD}[\overline{\psi},\psi,U]$$

Baryon bags in strong coupling QCD

Partition sum

$$Z = \sum_{\{\mathcal{B}\}} \prod_i \det D^{(i)} \times Z_{\overline{\mathcal{B}}}$$



- sum over configurations of baryon bags
- baryon bag contributions $\mathcal{B}_i \Rightarrow$ dynamics described by free fermions for baryons
- complementary domain $\overline{\mathcal{B}} \Rightarrow$ monomers and dimers for quarks and diquarks

Can we find a description in terms of effective bags for composite degrees of freedom also for other theories?

- Strong coupling QC_2D
  - only bosonic bound states (mesons and baryons made of 2 quarks)
  - effective boson bags?

- Strong coupling QCD with flavors
  - different types of composite strong coupling baryons
  - work in preparation

Composite boson bags in QC_2D

Taylor expansion of the hopping terms

$$\begin{aligned} e^{\gamma\overline{\psi}U\psi} &= 1 + \gamma\,(\overline{\psi}U\psi) + \frac{1}{2!}(\overline{\psi}U\psi)^2 \\ &= e^{\frac{1}{2!}(\overline{\psi}U\psi)^2}\sum_{k=0}^{1}\frac{(\gamma\overline{\psi}U\psi)^k}{k!} \end{aligned}$$

Quadratic term is independent of gauge fields: $(\overline{\psi}U\psi)^2 = 2!\overline{\psi}_2\overline{\psi}_1\psi_1\psi_2 = 2!\overline{B}B$

- Inside the bags $\mathcal{B}_i$ the dynamical degrees of freedom are composite bosons:

$$B_x = \psi_x^1\psi_x^2\,, \quad \overline{B}_x = \overline{\psi}_x^2\overline{\psi}_x^1 \qquad \text{nilpotent but commuting}$$

- Composite boson bag contributions as permanents

$$Z_{\mathcal{B}_i} = \int D_{\mathcal{B}_i}[\overline{\psi},\psi]\ e^{\sum_{x,y}\overline{B}_x D^{(i)}_{x,y} B_y} = \mathrm{perm}\, D^{(i)}$$

- Partition sum

$$Z = \sum_{\{\mathcal{B}\}}\prod_i \mathrm{perm}\, D^{(i)} \times Z_{\overline{\mathcal{B}}}$$

Bags for theories with 2 flavors

2 flavors strong coupling $\mathbb{Z}_3$-QCD

- simpler gauge integrals
- triality constraints same as QCD

Work in progress...

$\Rightarrow$ we expect richer dynamics in the complementary domain due to the existance of propagating mesons (not only dimers)

Summary

Bag reformulation of theories with fermions

- Goal: resum terms in worldline representation
- At strong coupling the partition sum factorizes into disjoint dynamically determined regions: bags $\mathcal{B}_i$ and complementary domain $\overline{\mathcal{B}}$
- Inside the bags the dynamics is described by free propagating composite degrees of freedom
- Inside the complementary domain monomer and dimer system

Outlook

- Extend the baryon bag approach to strong coupling QCD with 2 flavors
- Find a systematic approach to include gauge corrections

Thank you for your attention!