Kaon Distribution Amplitude from Lattice QCD

Rui Zhang
LP^3 Collaboration

ITP,CAS

Michigan State University

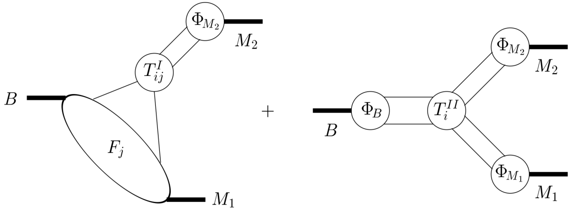
The 36th Annual International Symposium on Lattice Field Theory
East Lansing, MI, USA 07/22-28, 2018

[Chen et al., 2017]

Overview

Meson DA in exclusive processes

Meson lightcone distribution amplitudes are important inputs in exclusive processes, such as $B \to \pi K$, at large momentum transfer $Q^2 >> \Lambda_{QCD}$, where the scattering amplitude can be factorized into hard parts and soft parts:



$$< \pi K|Q_i|B >= F_0^{B\to\pi} T^I_{K,i} * f_K \Phi_K + F_0^{B\to K} T^I_{\pi,i} * f_\pi \Phi_\pi + T^{II}_i * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi$$

quasi-DA

Instead of calculating the PDA directly, we are actually calculating the quasi-DA in LaMET [Ji,2013]

$$\tilde{\phi}_M(x,\mu_R,P_z)=\frac{i}{f_M}\int\frac{dz}{2\pi}e^{-i(x-1)P_z z}\langle M(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\lambda^a\psi(z)|0\rangle$$

after a matching procedure [Ji et al., 2015]

$$\tilde{\phi}_M(x,\mu_R,P_z)=\int_0^1 dy\, Z_\phi(x,y,\mu,\mu_R,P_z)\phi_M(y,\mu)+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2},\frac{m_M^2}{P_z^2}\right),$$

where the matching kernel Z_ϕ can be expanded to one-loop level as:

$$Z_\phi(x,y)=\delta(x-y)+\frac{\alpha_s}{2\pi}(Z_\phi^{(1)}(x,y)-\delta(x-y)\int_{-\infty}^{\infty}dx'\,Z_\phi^{(1)}(x',y))+O(\alpha_s^2)$$

quasi-DA

The observable we compute on lattice is the correlator

$$\tilde{C}(z, P_z, \tau) = \left\langle \int d^3x\, e^{i\vec{P}\cdot x} \bar{\psi}(\vec{x}, \tau)\gamma^z\gamma_5\Gamma(\vec{x}, \vec{x}+z)\lambda^{a\,\dagger}\psi(\vec{x}+z, \tau)\ \bar{\psi}^S(0,0)\gamma_5\lambda^a\psi^S(0,0) \right\rangle ,$$

where we use the gauge invariant Gaussian smeared source

$$\psi^S(x) = \int d^3y\, e^{-\frac{|x-y|^2}{2\sigma^2} - i\vec{k}\cdot(\vec{x}-\vec{y})} U(x, y)\psi(y),$$

which can be related to the matrix element

$$\tilde{h}_M(z, P_z) = \langle M(P)|\bar{\psi}(0)\gamma^z\gamma_5\lambda^a\Gamma(0, z)\psi(z)|0\rangle$$

by extracting the ground-state coefficient of the correlator

$$\tilde{C}(z, P_z, \tau) = \frac{Z_{\text{src}}\tilde{h}_M(z, P_z)}{2E_0} e^{-E_0\tau} + \sum_{i>0} B_i(z, P_z)e^{-E_i\tau}$$

Parameters

The two-point correlators are obtained by running the chroma program with following parameters:

- Lattice spacing $a = 0.12fm$
- $24^3 \times 64$ lattice with $2 + 1 + 1$ flavors of HISQ
- Pion mass 310MeV
- Smeared sources and sinks with smearing mom $k = 0.73P_z$
- Meson momentum $P_z = (4\pi/6\pi/8\pi)/L = (0.77/1.15/1.53)GeV$
- 4 source locations
- 967 hypercubic smearing configurations

Matrix Elements

We fit the resulting correlators to the sum of first two terms

$$\tilde{C}(z, P_z, \tau) = A(z, P_z)e^{-E_0\tau} + B(z, P_z)e^{-E_1\tau}$$

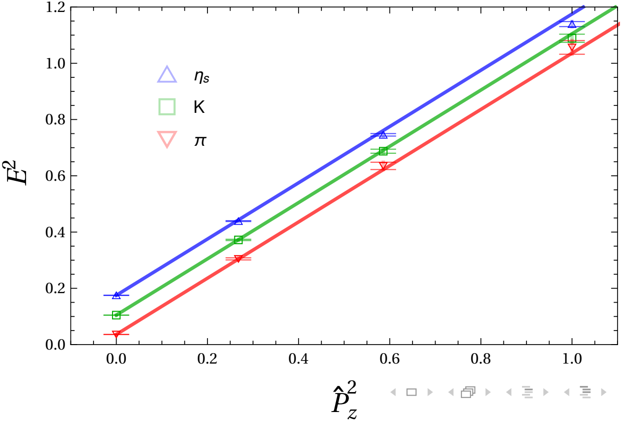
Average $\chi^2 = 1.3$.
Normalize the coefficient to obtain

$$h_M(zP_z, P_z) = \frac{\tilde{h}_M(z, P_z)}{P_z f_M} = \frac{A(z, P_z)}{A(0, P_z)}$$

so that $h(0, P_z) = 1$.
We also checked the 3-term fit results. They're consistent with our 2-term fit, thus we can safely exclude the excited-state effect here.

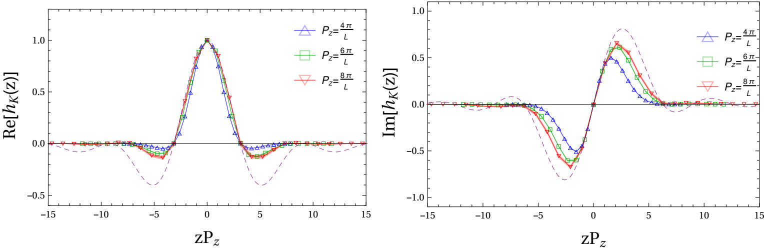
Dispersion Relation

The dispersion relations for π, K and η_s (with the connected diagram contribution only). The lines are $E^2(P_z) = m^2 + \hat{P}_z^2$, with $\hat{P}_z = 2/a \sin(P_z a/2)$, which are satisfied within two sigmas of the statistical uncertainties.



Bare quasi-DA ME

The kaon bare quasi-DA matrix elements. The dashed lines are the asymptotic forms. The bare results for pion and η_s are quite similar to the kaon's.



Renormalization

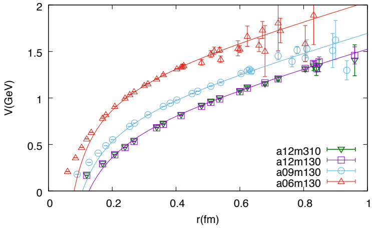
The gauge-invariant quark Wilson line operator contributes to power divergences. It can be renormalized multiplicatively in the coordinate space:

$$\tilde{O}_\Gamma(z) = \bar{\psi}(z)\Gamma W(z,0)\psi(0) = Z_{\psi,Z}e^{-\delta m|z|}(\bar{\psi}(z)\Gamma W(z,0)\psi(0))^R$$

[Ji et al., 2017; Green et al., 2017; Ishikawa et al., 2017] where δm captures the linear power divergence, and Z is a logarithmic renormalization constant. The power divergence has to be nonperturbatively renormalized.

δm counterterm

The δm can be determined by computing the $q - \bar{q}$ static potential $V(r) = \frac{c_{-1}}{r} + c_0 + c_1 r$



where $c_0 = \frac{c_{0,1}}{a} + c_{0,2}$, $\delta m = -\frac{c_{0,1}}{2a} = 0.154(2)/a = 225(3)MeV$. The improved quasi-DA is [Zhang et al., 2017]

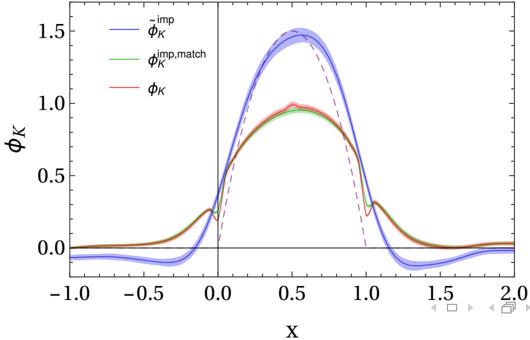
$$\tilde{\phi}_M^{\text{imp}}(x, P_z) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-i(x-1)zP_z + \delta m|z|} P_z h_M(z, P_z).$$

Matching and mass corrections

Final DAs are obtained by applying the one-loop matching kernel

$$\phi_M^{\text{imp,match}}(x,P_z) \simeq \tilde{\phi}_M^{\text{imp}}(x,P_z) - \frac{\alpha_s}{2\pi}\int_{-\infty}^{\infty} dy\, \left[Z_\phi^{(1)}(x,y)\,\tilde{\phi}_M^{\text{imp}}(y,P_z) - Z_\phi^{(1)}(y,x)\,\tilde{\phi}_M^{\text{imp}}(x,P_z)\right]$$

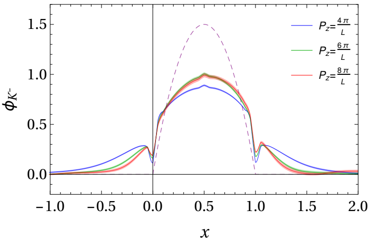
and then the mass corrections to the improved DAs. The dashed line is the asymptotic form, the green band is DA without mass correction.

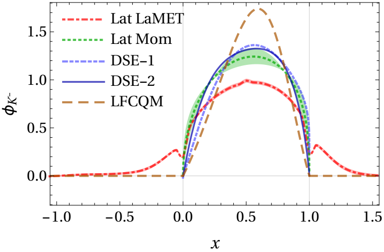


$P_z = 1.53 GeV$

Improved kaon DA

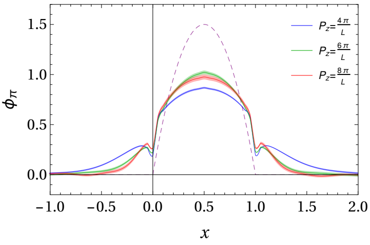
We then obtain the kaon distribution amplitudes for $P_z = (0.77/1.15/1.53) GeV$, with statistical errors only. The purple dashed line is the asymptotic form.

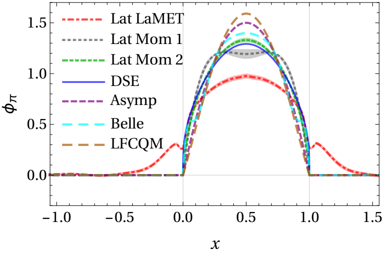




Improved pion DA

the pion distribution amplitudes for $P_z = (0.77/1.15/1.53) GeV$, with statistical errors only. The purple dashed line is the asymptotic form. The η_s result is similar, with smaller errors.





SU(3) relations

It was shown in ChPT that the DAs satisfied the SU(3) relation

$$\phi_{K^+}(x,\mu) - \phi_{K^-}(x,\mu) = \phi_{K^0}(x,\mu) - \phi_{\bar{K}^0}(x,\mu) \propto m_s - m_{u/d},$$
$$\phi_{\pi}(x,\mu) + 3\phi_{\eta}(x,\mu) - 2\phi_{K^+}(x,\mu) - 2\phi_{K^-}(x,\mu) = \mathcal{O}(m_q^2),$$

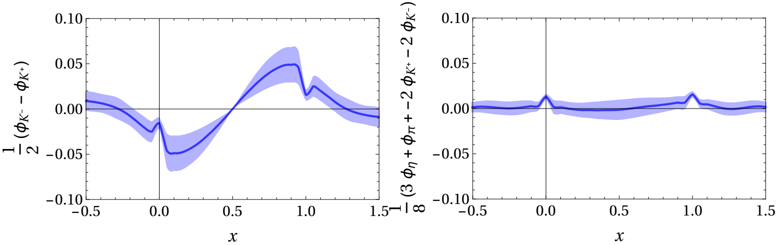
[Chen and Stewart, 2004] where the ϕ_η can be obtained by

$$\phi_\eta = (2\phi_{\eta_s} + \phi_\pi)/3.$$

Thus we can compare the two magnitudes

$$\delta_{\mathrm{SU(3)},1} = (\phi_{K^-} - \phi_{K^+})/2 = \mathcal{O}(m_q),$$
$$\delta_{\mathrm{SU(3)},2} = (\phi_{\pi} + \phi_{\eta_s} - \phi_{K^+} - \phi_{K^-})/4 = \mathcal{O}(m_q^2).$$

SU(3) relations



Summary

- We compute the quasi-DA of pion, kaon and η_s on lattice;
- Applied the δm counterterm renormalization, one loop matching kernel and mass corrections;
- Supported the $SU(3)$ relation predicted by ChPT.
- Future study: smaller lattice spacing, larger volume, physical pion mass.

The End

renormalon ambiguity

- In an OPE the leading order Wilson coefficient has an ambiguity from perturbation which requires higher order power corrections to cancel it.
- Our renormalization is done non-perturbatively, so there is no renormalon ambiguity.
- The perturbative matching could have renormalon ambiguity, but its size is the same order as the (twist-4) power correction.

disconnected diagrams for η_s

- The disconnected diagram is $O((m_s - \bar{m})^2)$ suppressed because there are two fermion loops.
- The error caused by the different values of ground-state energy E_0 is reduced when P_z increases, and is negligible at our momentum.
- The η_0 contribution is suppressed by a mixing factor $sin\theta \sim 0.08$ times a factor of $(m_s - \bar{m})$