ANOMALOUS MAGNETIC MOMENT OF THE MUON WITH DYNAMICAL QCD+QED

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QCDSF Collaboration

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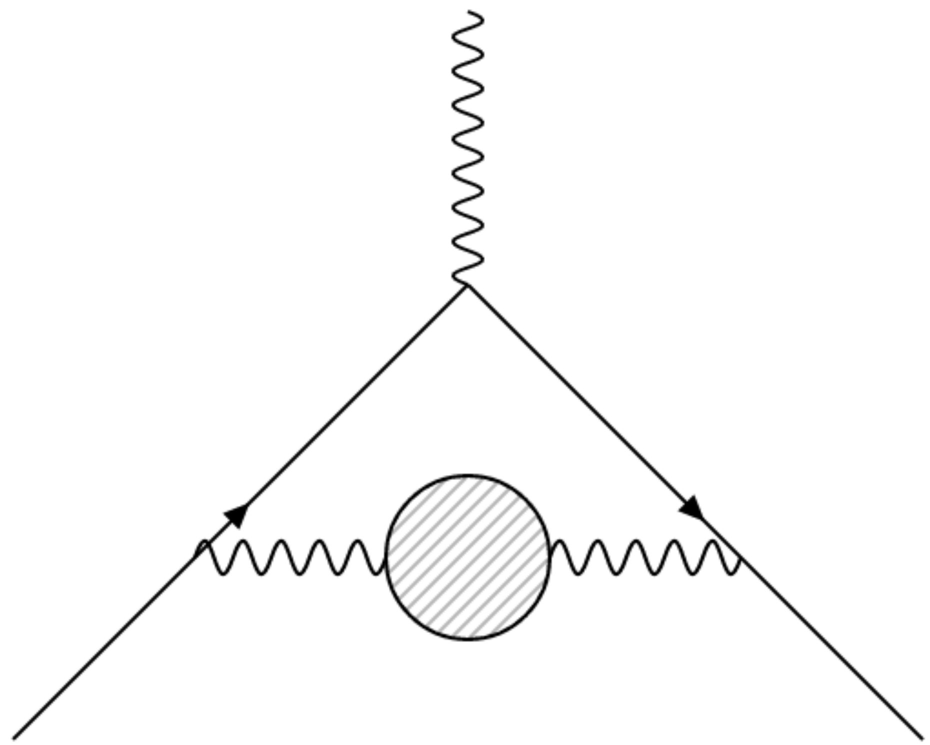
INTRODUCTION TO a_μ^{HVP}



See previous talks

This talk:

- Apply QCDSF's flavour-breaking procedure to a_μ^{HVP}
- Employing recent dynamical QCD+QED configurations

ACCESSING a_μ^{HVP}

- **Traditional**:

$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dQ^2 K(Q^2; m_\mu^2)\hat{\Pi}(Q^2),$$

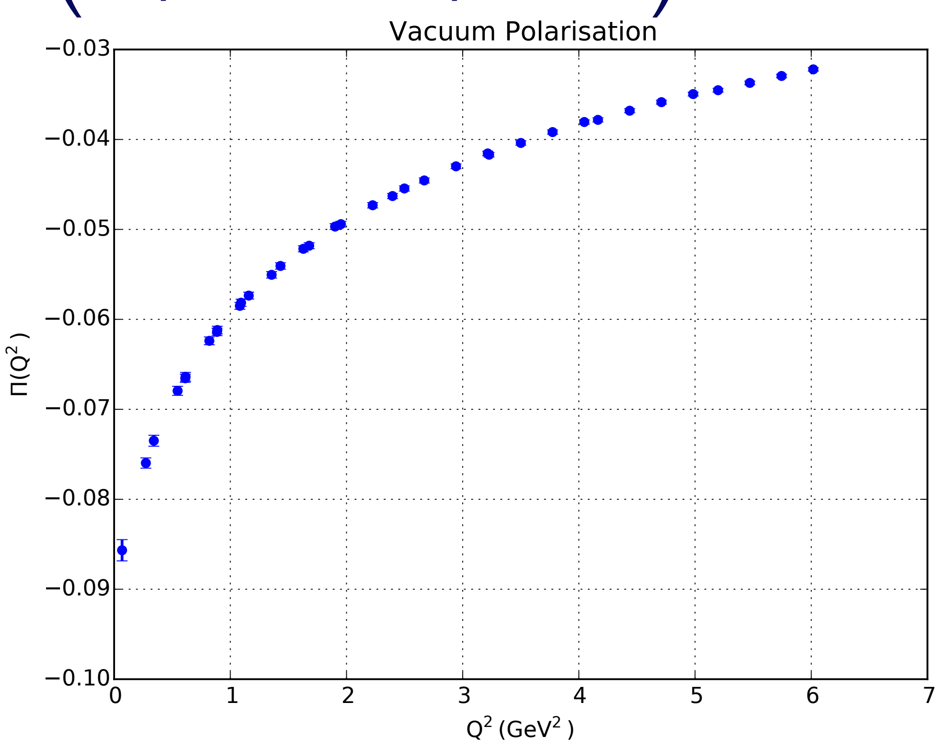
Known kernel

$$\Pi(Q^2) - \Pi(0)$$

- polarisation tensor:

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x}\langle J_\mu(x) J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2\right)\Pi(Q^2)$$

- fit with Padé, VMD, …
- put back into integral



ACCESSING a_μ^{HVP}

- **Time-moment(um) representation**:

$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dt\, G(t) \tilde{K}(t; m_\mu)$$

vector-vector 2-pt function

Known kernel

[Bernecker-Meyer (2011)]

$$G(t) = \frac{1}{3} \sum_{i=1,2,3} \int d^3x \, \langle J_i(x) J_i(0) \rangle$$

- Note:
  - requires long-time integral ($\to \infty$)
  - lattice data have finite t and suffer from large noise at large t
  - lots of progress - see other talks

[QCDSF (2011)]

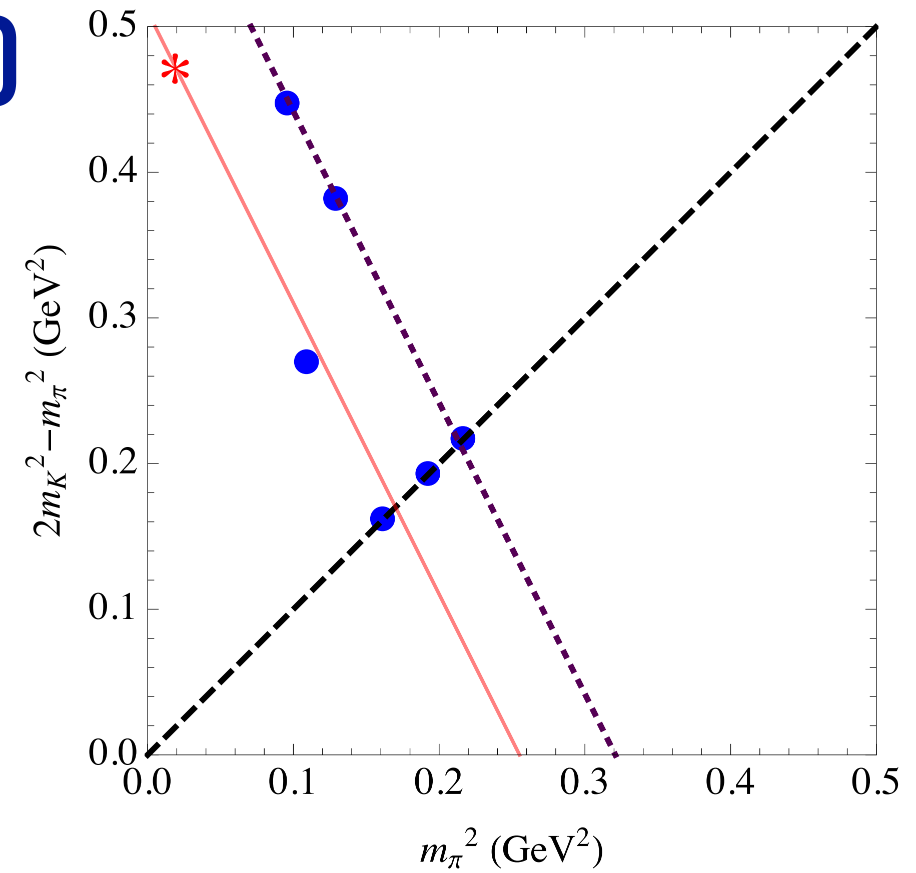
RECALL: QCDSF QUARK MASS TUNING (QCD

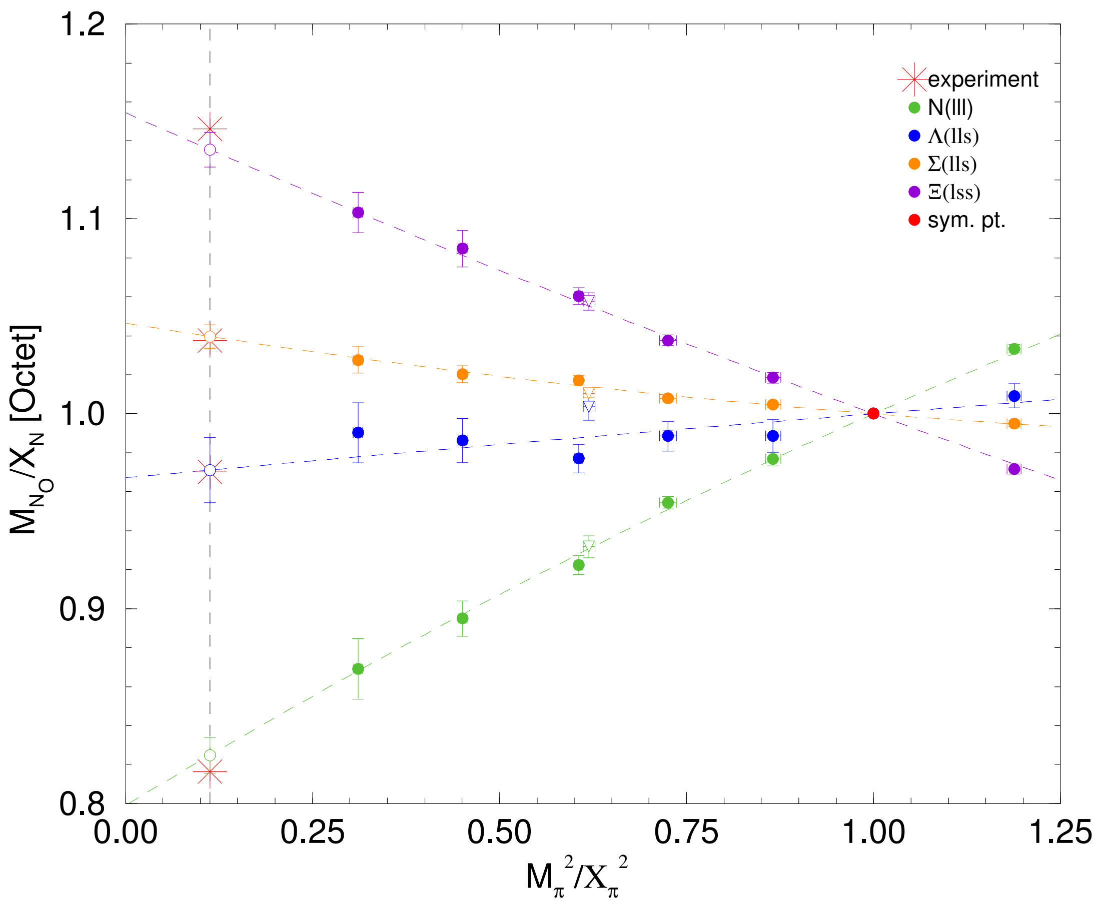
- $N_f = 2+1$ $O(a)$-improved Clover ("SLiNC")
- Tree-level Symanzik gluon action
- Novel method for tuning the quark masses
  - keep the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}(2m_l^R + m_s^R)$$

  - at its physical value $\overline{m}^{R*}$

- Multiple *V, a, m_q*





FLAVOUR-BREAKING EXPANSIONS (QCD)

[QCDSF (2011)]

- Using properties of SU(3)
- e.g. light octet vector mesons with flavour $(a\bar{b})$ *[partially quenched]*

$$M(a\bar{b}) = M_0 + \alpha(\delta\mu_a + \delta\mu_b) + \frac{1}{2}c(\delta m_u + \delta m_d + \delta m_s) + \frac{1}{6}\beta_0(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$$
$$+\beta_1(\delta\mu_a^2 + \delta\mu_b^2) + \beta_2(\delta\mu_a^2 - \delta\mu_b^2)$$

SU(3)-symmetric point

$$(\delta\mu_q, \delta m_q) = (\mu_q, m_q) - m_0 = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)$$

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$\delta m_u + \delta m_d + \delta m_s = 0$ *on our trajectory*

$$(\delta\mu_q, \delta m_q) = (\mu_q, m_q) - m_0 = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)$$

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- Flavour-diagonal (*with* $\bar{m}$ = constant) :

$$M(a\bar{a}) = M_0 + 2\alpha\delta\mu_a + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_a^2$$

LATTICE QCD+QED SET-UP

QCDSF, JHEP 1604, 093 (2016)

- Non-compact QED

gauge-fixing of Uno & Hayakawa (2008) — on valence quarks

- Gauge coupling corresponding to $\alpha_{QED} = 0.1$
- $SU(3)_f$ symmetric point?
  - QCD: trivial — input $am_u = am_d = am_s \Rightarrow m_u^R = m_d^R = m_s^R$
  - +QED: with $Q_u = +\frac{2}{3},\ Q_d = Q_s = -\frac{1}{3}$

$$am_u = am_d = am_s \Rightarrow m_u^R \neq m_d^R = m_s^R$$

- Define the *"Dashen Scheme"*
  - Tune quark masses to $SU(3)_{sym}$ point via $m_\pi^{u\bar{u}} = m_\pi^{d\bar{d}} = m_\pi^{s\bar{s}}$
    - *n : 0* $\quad m_\pi^{n\bar{n}} = 408(3)\ \mathrm{MeV}$
    - *d : -1/3* $\quad m_\pi^{d\bar{d}} = 409(1)\ \mathrm{MeV}$
    - *u : +2/3* $\quad m_\pi^{u\bar{u}} = 407(3)\ \mathrm{MeV}$

$V=32^3x64$, $a=0.068fm$

FLAVOUR-BREAKING EXPANSIONS (QCD+QED)

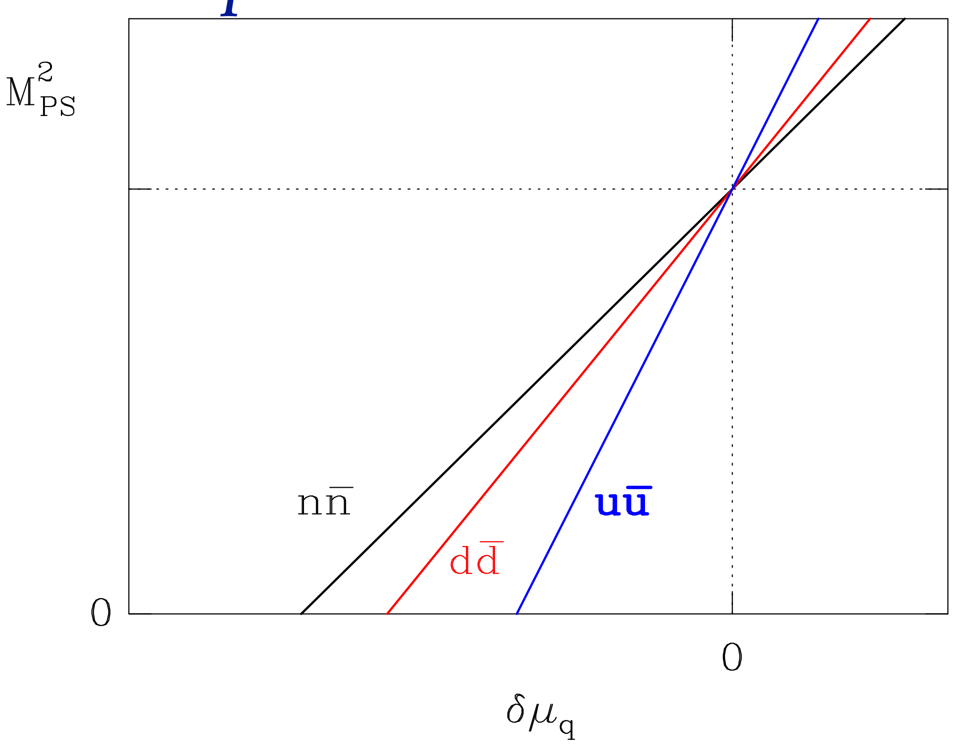
- Extend to include quark charges, e.g. flavour-diagonal (*with* $\bar{m}$ = constant)

$$M(a\bar{a}) = M_0 + 2\alpha\delta\mu_a + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_a^2 + \beta_0^{EM}(e_u^2 + e_d^2 + e_s^2) + 2\beta_1^{EM}\, e_a^2$$
$$+\gamma_0^{EM}(e_u^2\delta m_u + e_d^s\delta m_d + e_s^2\delta m_s) + 2\gamma_1^{EM}\, e_a^2\delta\mu_a$$
$$+2\gamma_4^{EM}(e_u^2 + e_d^2 + e_s^2)\delta\mu_a + 2\gamma_5^{EM} e_a(e_u\delta m_u + e_d\delta m_d + e_s\delta m_s)$$

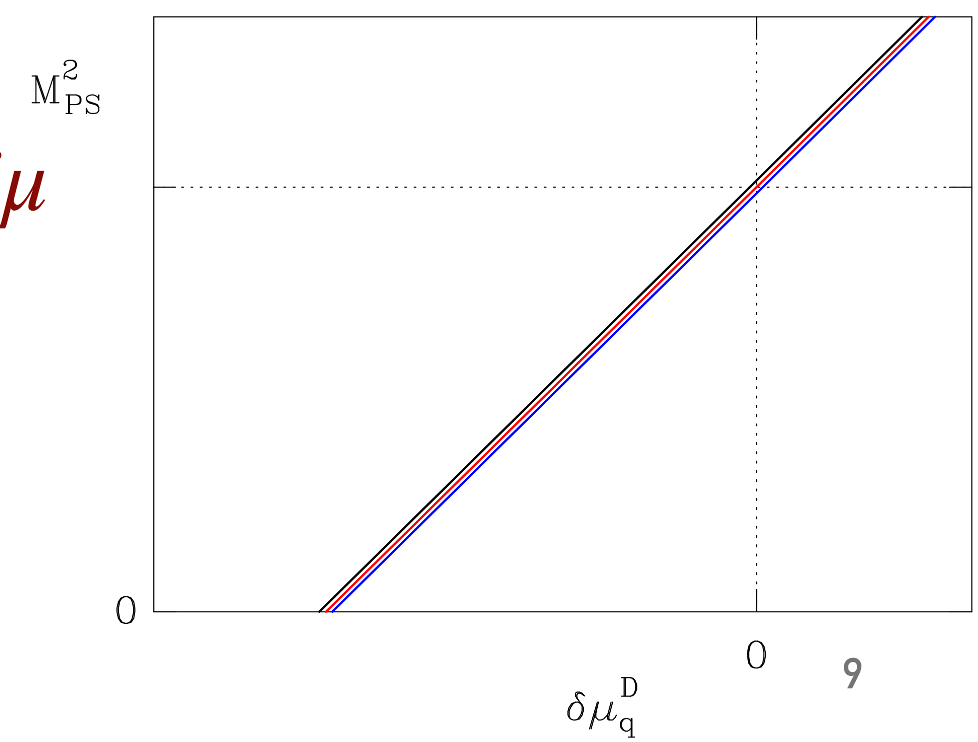
Dashen scheme:

QCDSF, JHEP 1604, 093 (2016)

- *absorb all EM effects of neutral PS mesons into q masses*
- *rescale the horizontal axis so that all meson masses depend on the "Dashen mass" in the same way*



$$\delta\mu_q^D = (1 + KQ_q^2e^2)\delta\mu$$

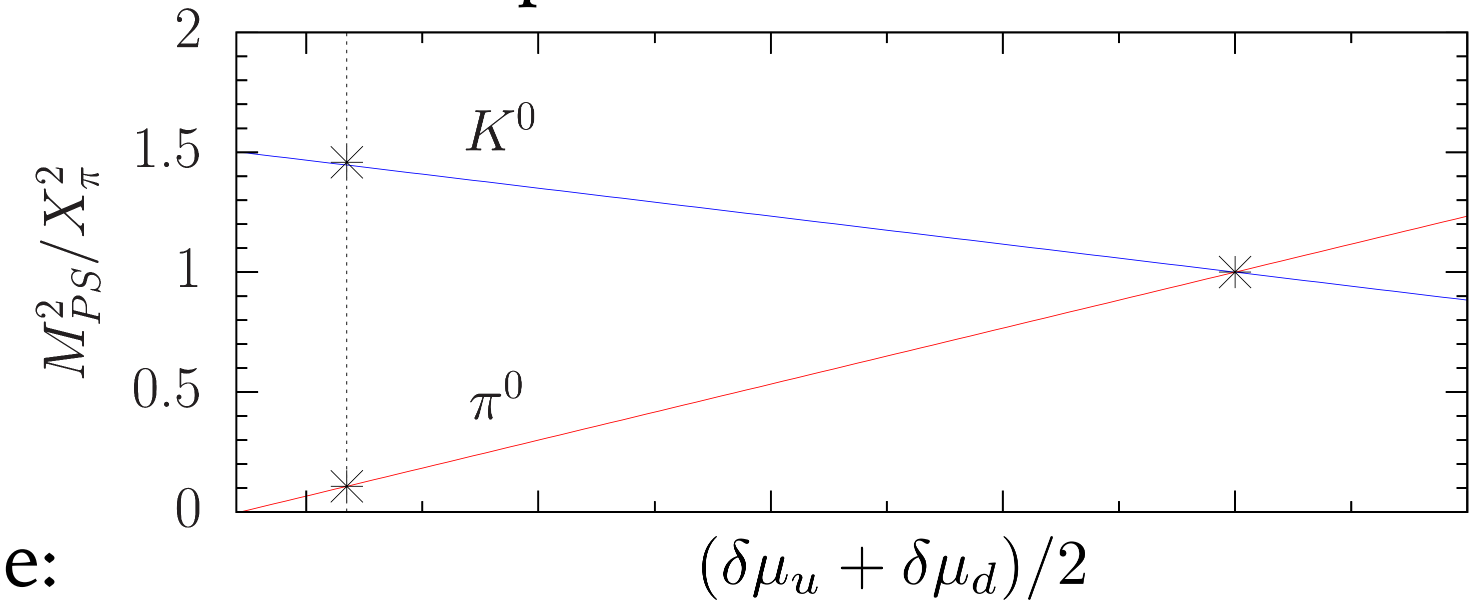


QCD+QED SPECTRUM

QCDSF, JHEP 1604, 093 (2016)

- Physical point determination. Constrain to experimental masses:

$$M_{\pi^0} = 134.977\,\text{MeV},$$
$$M_{K^0} = 497.614\,\text{MeV},$$
$$M_{K^+} = 493.677\,\text{MeV}$$



Physical point, and lattice scale:

	$32^3 \times 64$	$48^3 \times 96$
$a\delta m_u^\star$	$-0.00834(8)$	$-0.00791(4)$
$a\delta m_d^\star$	$-0.00776(7)$	$-0.00740(4)$
$a\delta m_s^\star$	$0.01610(15)$	$0.01531(8)$
a^{-1}/GeV	$2.89(5)$	$2.91(3)$

- Prescription for switching Dashen $\rightarrow \overline{\text{MS}}$

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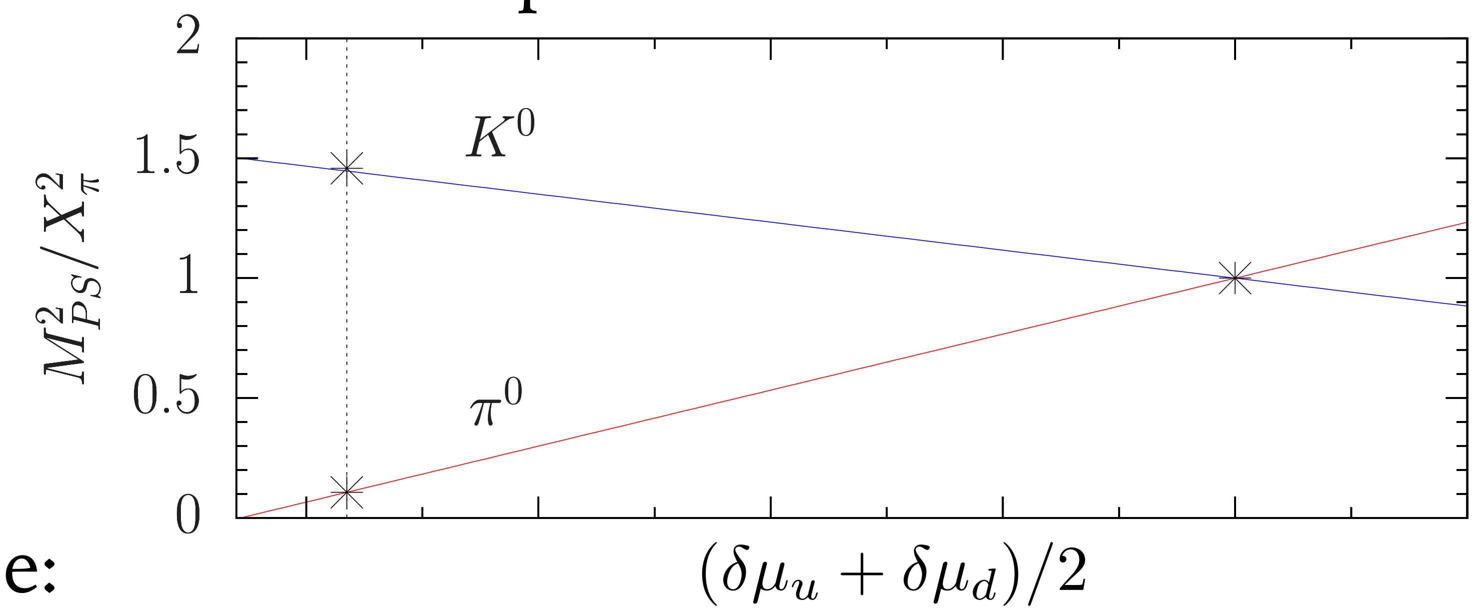

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a^{-1}/GeV	$2.89(5)$	$2.91(3)$

Sum = 0

- Prescription for switching Dashen $\to \overline{\mathrm{MS}}$

G-2: LATTICE QCD+QED SET-UP

$a=0.068fm$ $\alpha_{\mathrm{QED}} = \frac{e^2}{4\pi} \simeq 0.1$

- Simulate with 5 ensembles

#	(L,T)	N_f	$m_{u\bar{u}}$	$m_{d\bar{d}}$	$m_{s\bar{s}}$	$m_{q\bar{q}}^{min}L$	m_{π^+}	m_{K^+}
1	(32,64)	$2+1$	430	405	405	4.4	435	435
2	(32,64)	$2+1$	360	435	435	4.0	415	415
3	(32,64)	$1+1+1$	290	300	570	3.2	320	470
4	(48,96)	$2+1$	430	405	405	6.7	435	435
5	(48,96)	$2+1$	360	435	435	5.9	420	420

- partially-quenched with masses

$$260 \leq m_{q\bar{q}} \leq 770\, MeV$$

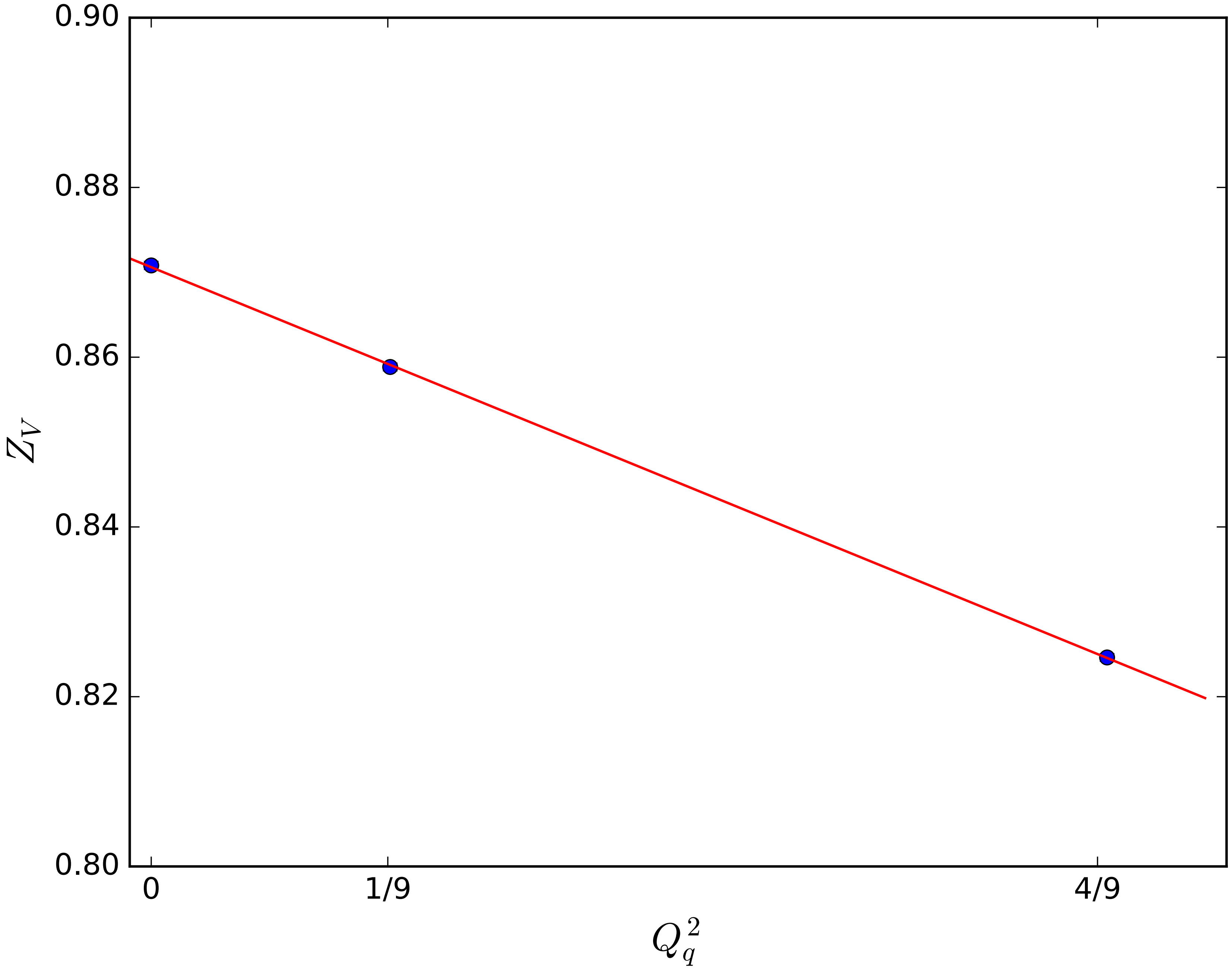
- and charges

$$Q_q \in \left(0, -\frac{1}{3\sqrt{13}}, +\frac{2}{3\sqrt{13}}, \pm\frac{1}{3}, \pm\frac{\sqrt{2}}{3}, \pm\frac{2}{3}\right) e$$

ZV

- Z_V determined from nucleon 3pt functions at tuned symmetric point *(uud, uun, nnd, …)*
- Z_V depends on the charge of the active quark

[Also observed in Boyle et al., 1706.05293]



$$m_\pi^{n\bar{n}} = 408(3)\ \mathrm{MeV}$$

$$m_\pi^{d\bar{d}} = 409(1)\ \mathrm{MeV}$$

$$m_\pi^{u\bar{u}} = 407(3)\ \mathrm{MeV}$$

TIME-MOMENT CALCULATION

- Recall we need 2-point function at large times (noisy)

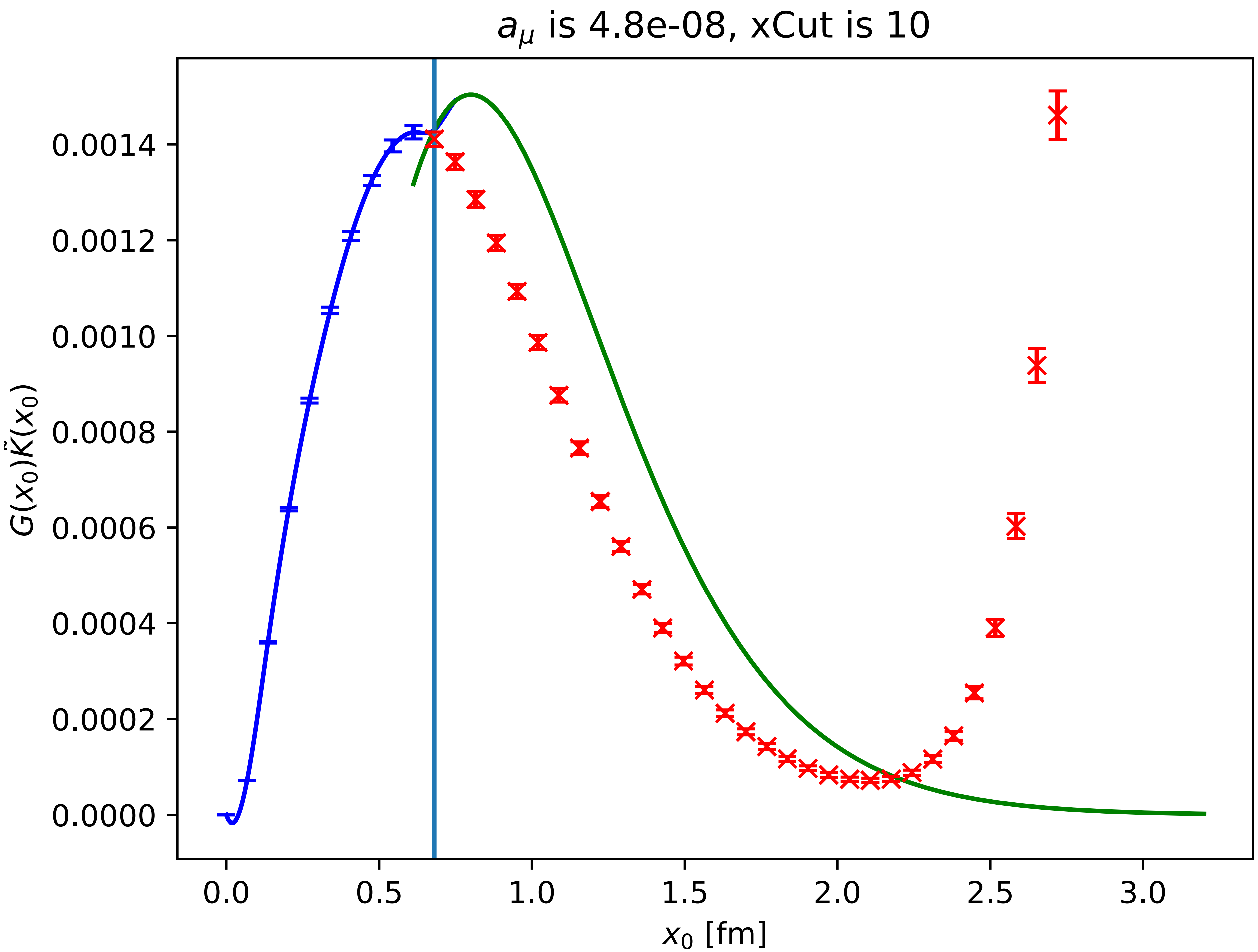
$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dt\, G(t)\tilde{K}(t; m_\mu) \qquad G(t) = \frac{1}{3}\sum_{i=1,2,3}\int d^3x\, \langle J_i(x)J_i(0)\rangle$$

- Instead only use 2-point function up to some t_{cut}
- Then use ground state vector meson mass in single exponential

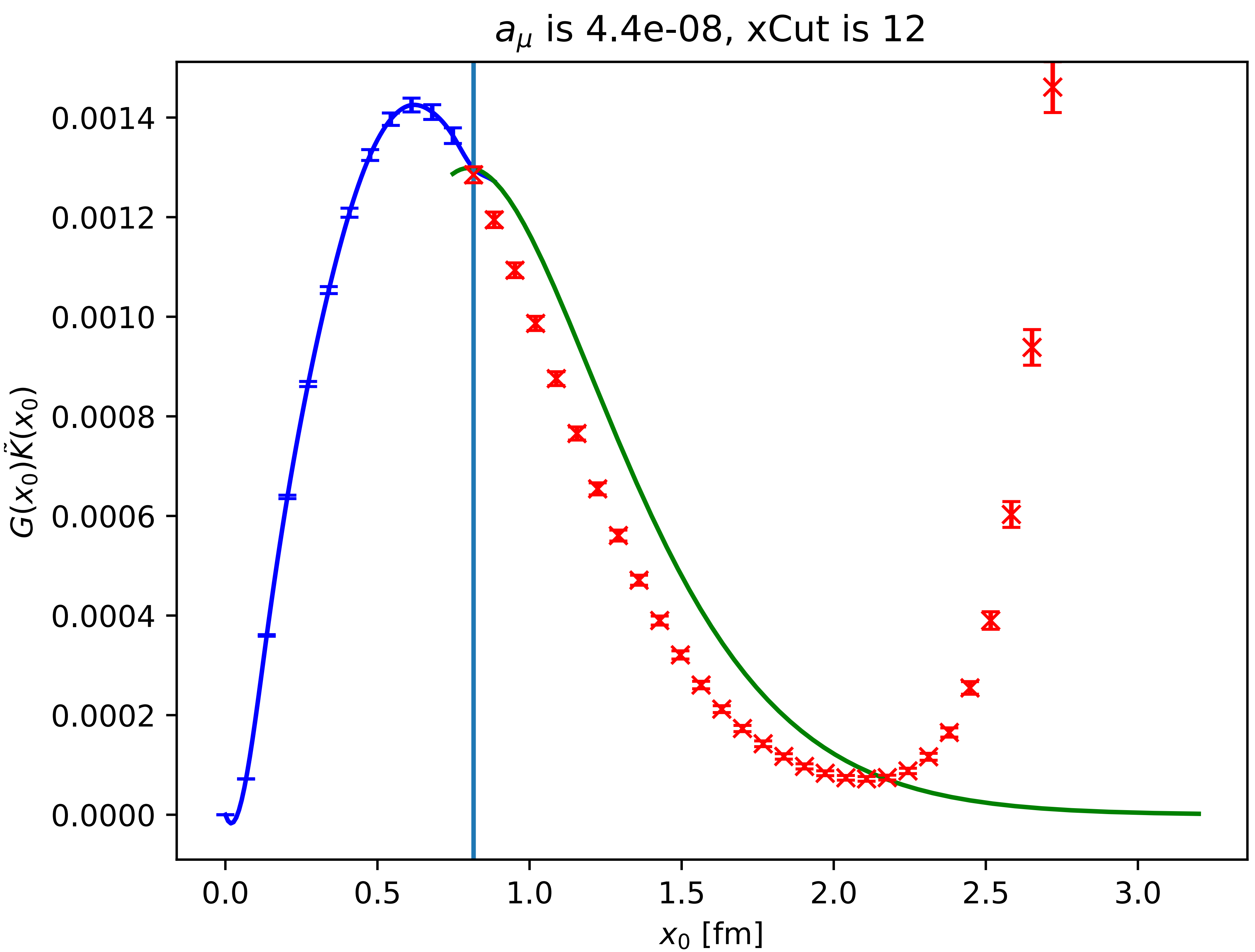
$$G(t) = \begin{cases} G(t) & t \le t_{cut} \\ Ae^{-M_v t} & t > t_{cut} \end{cases}$$

➡ remaining systematic error in description of correlator at large t using single state

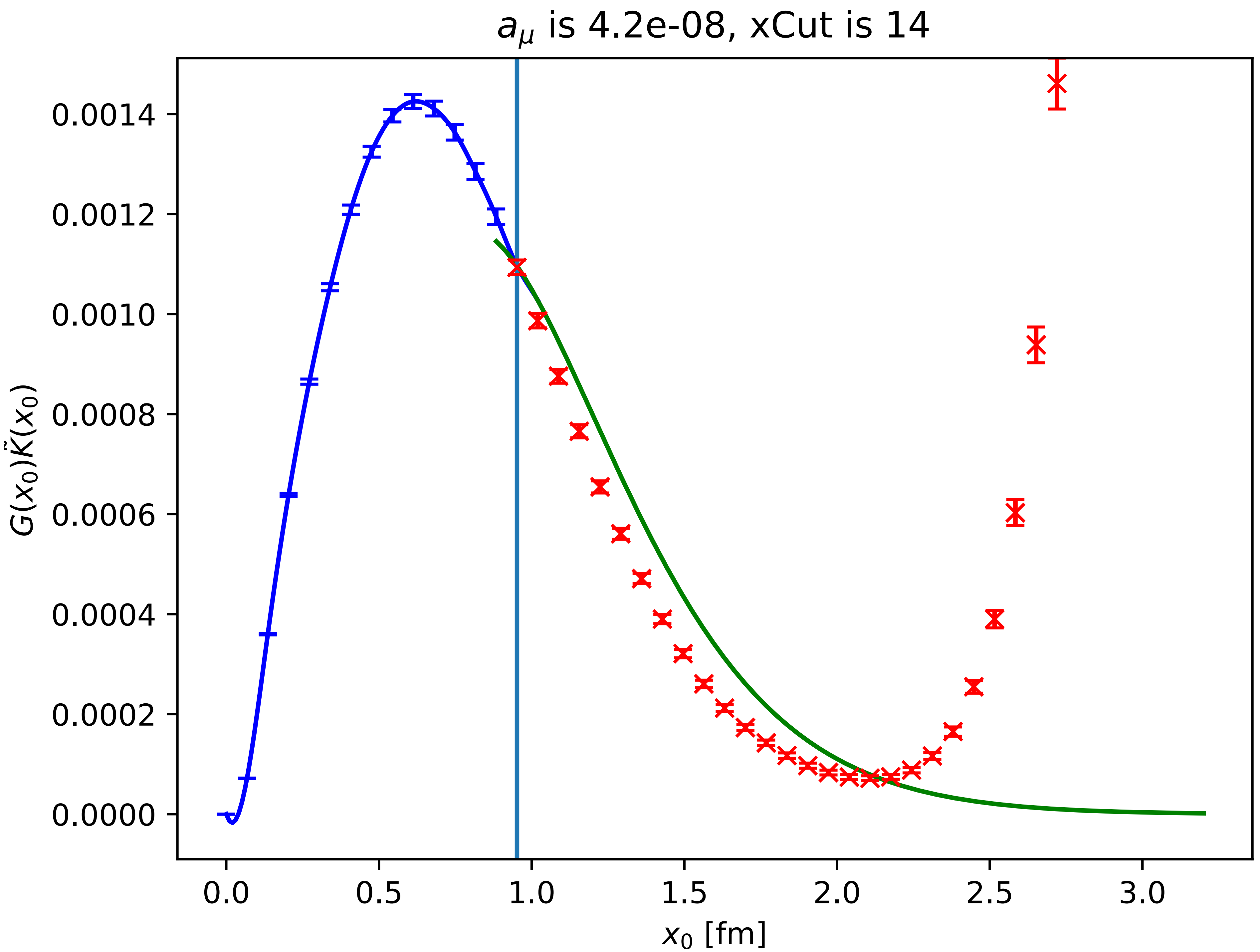
TIME-MOMENT CALCULATION $(t_{cut}$ *dependence)*



TIME-MOMENT CALCULATION *(t_{cut} dependence)*



TIME-MOMENT CALCULATION (t_{cut} *dependence*)



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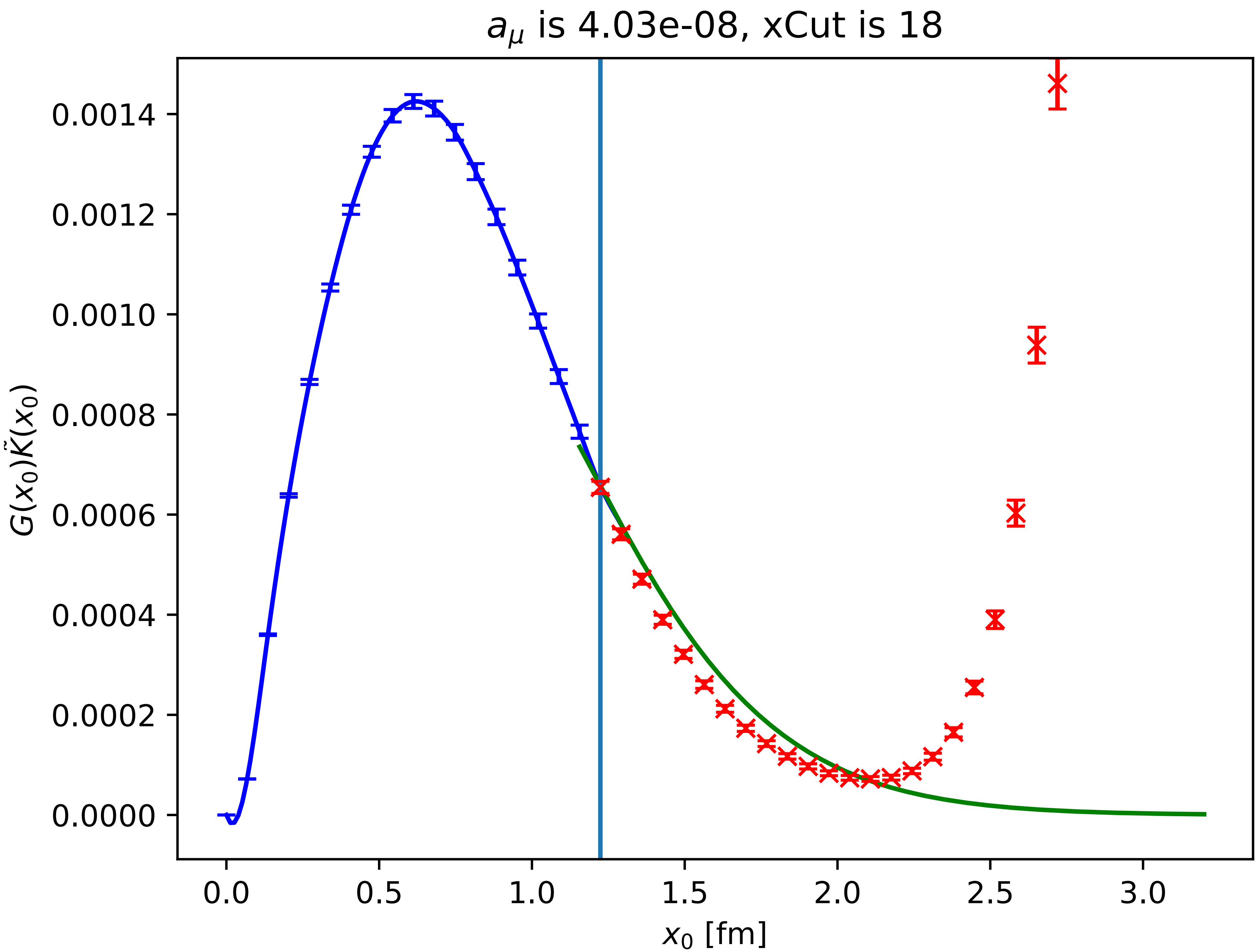

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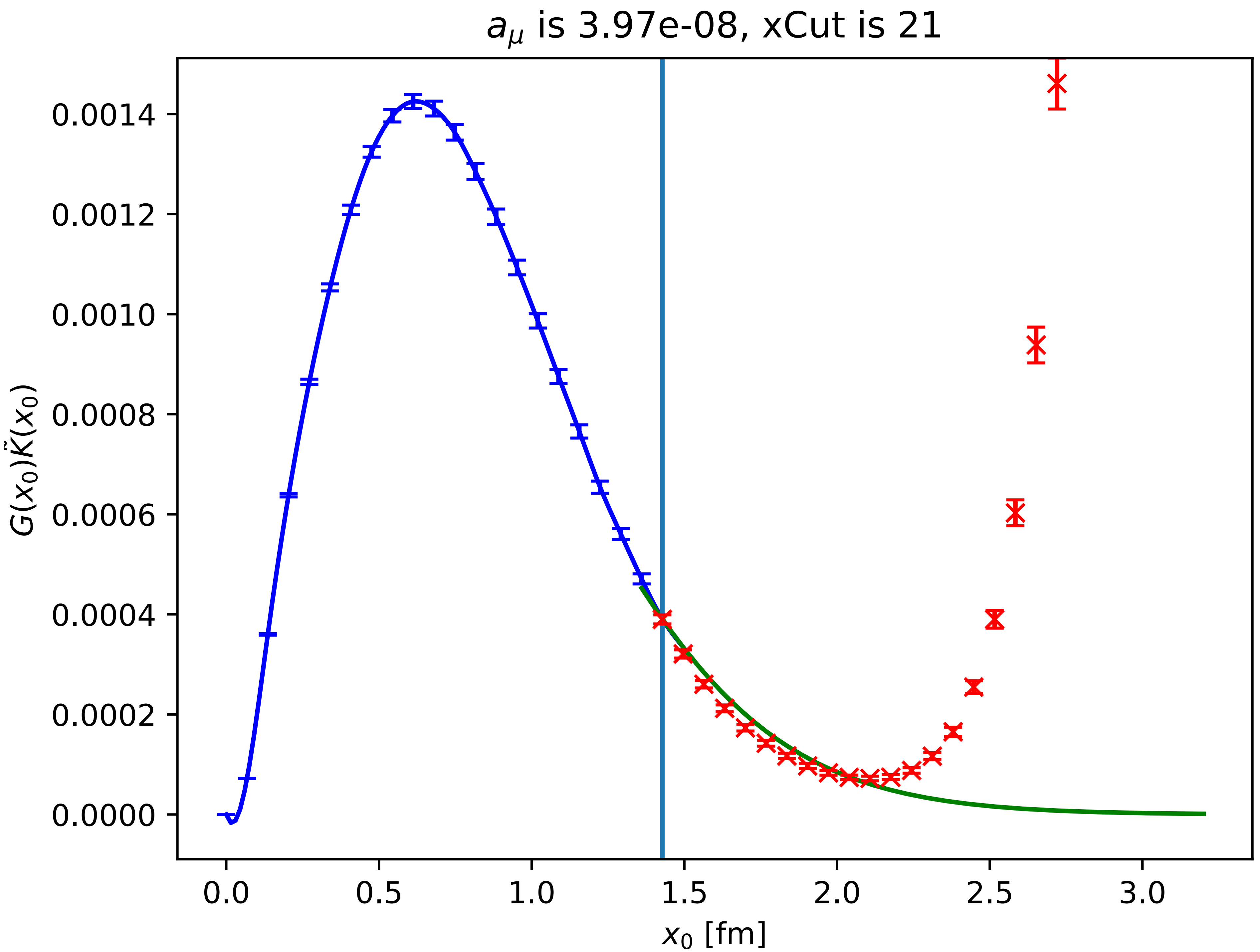

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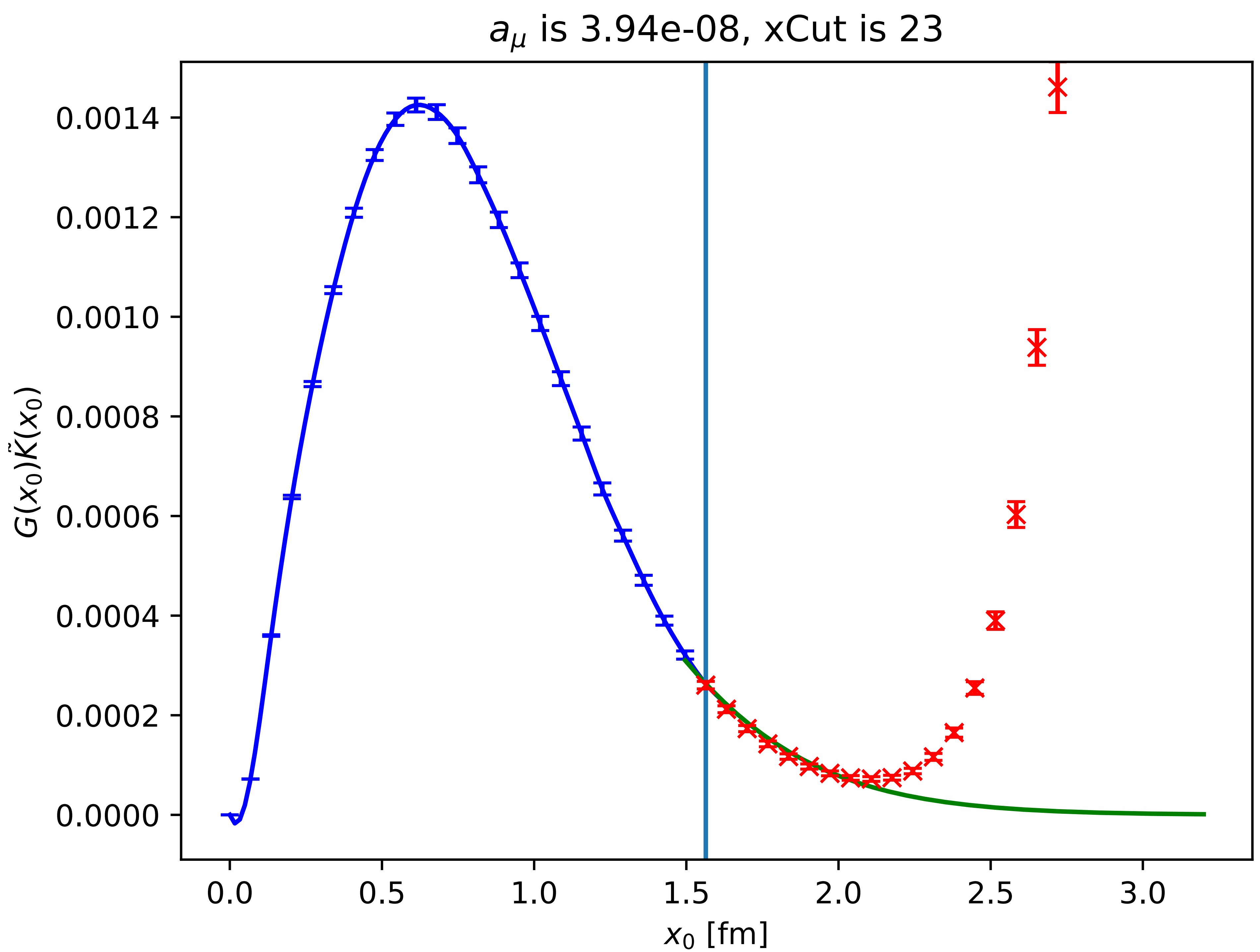

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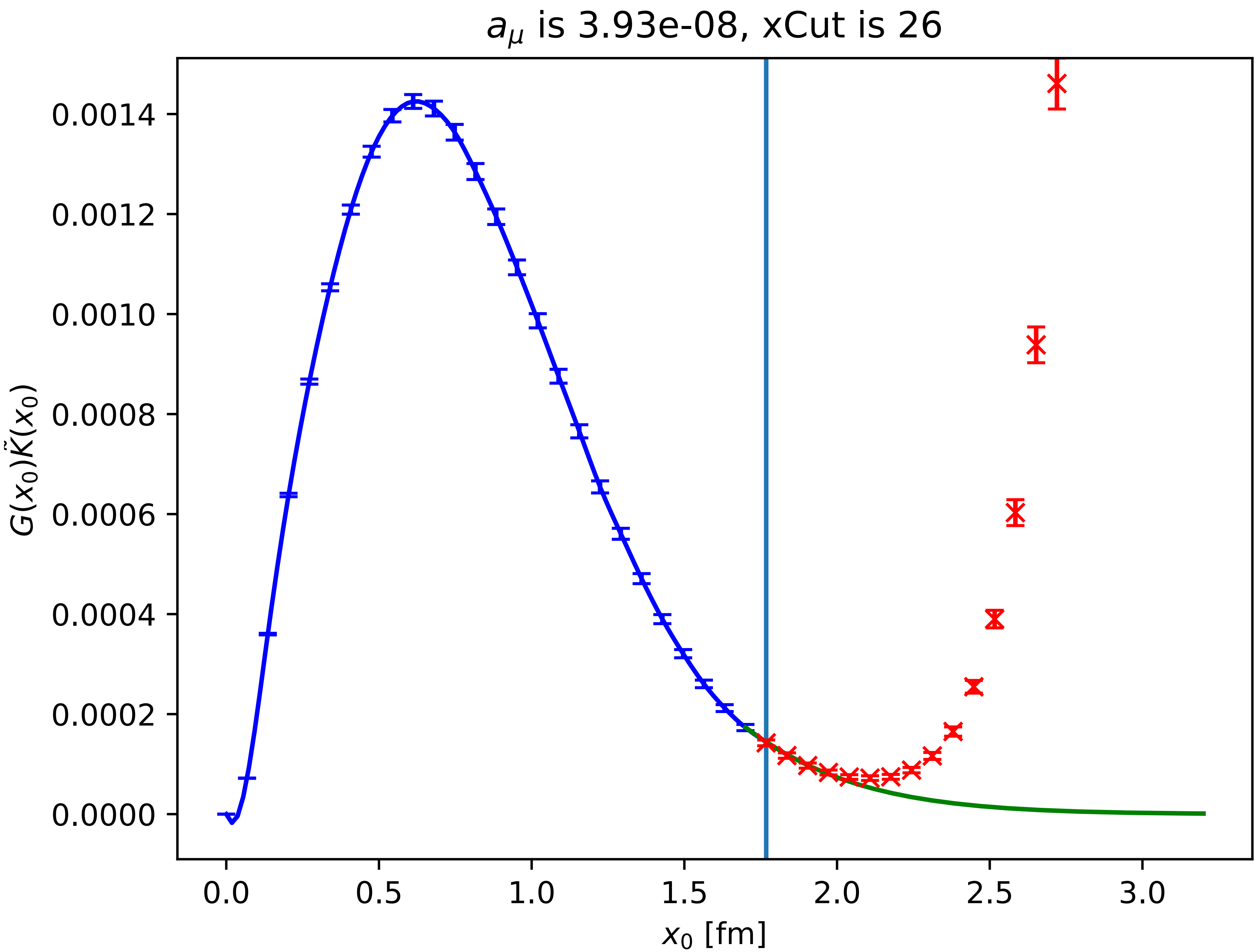


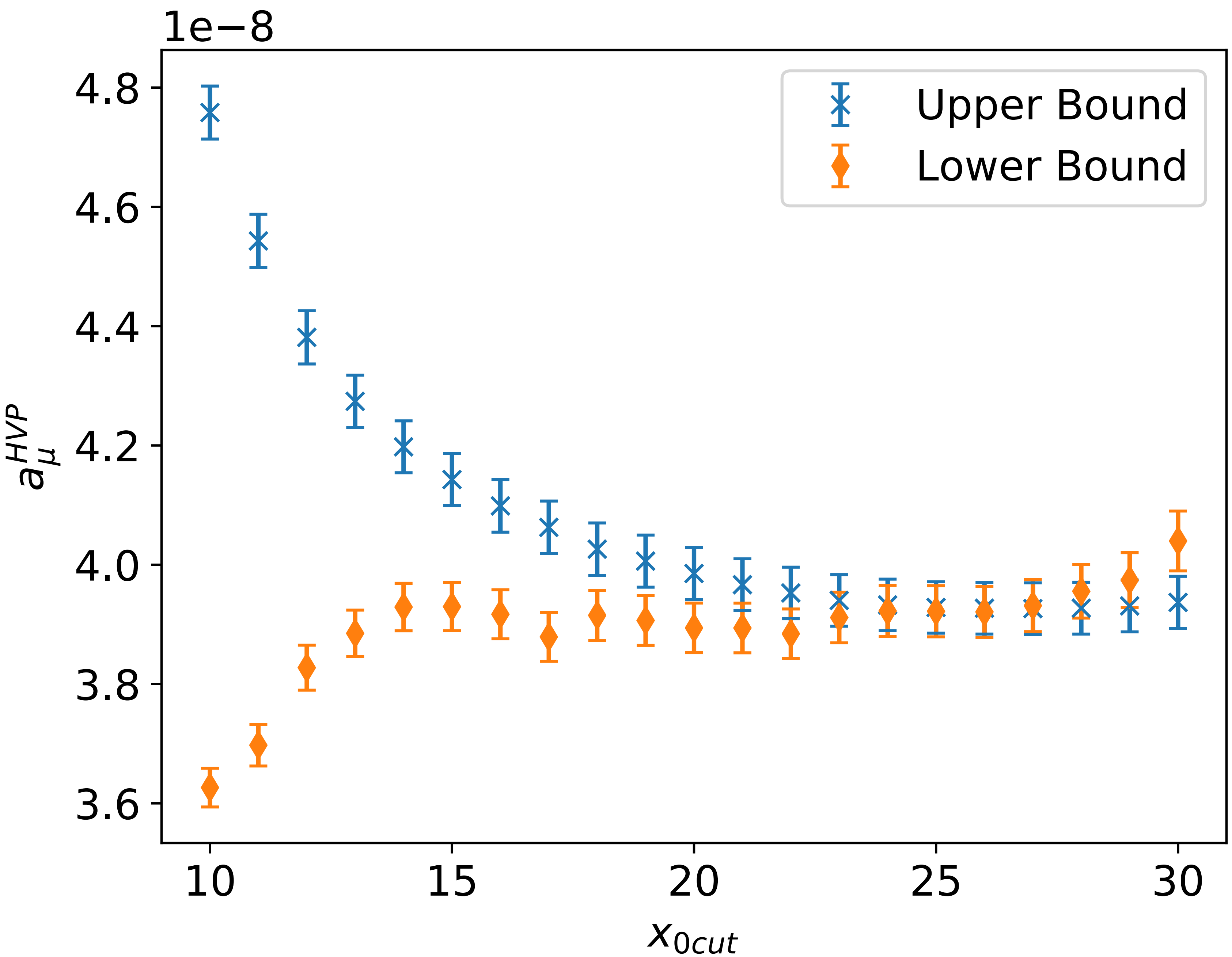
BOUNDING METHOD

[see e.g. A.Meyer(RBC/UKQCD) @ g-2, Mainz]

$$\tilde{C}(t; t_{cut}, E) = \begin{cases} C(t) & t < t_{cut} \\ C(t_{max})\, e^{-E(t-t_{cut})} & t \geq t_{cut} \end{cases}$$

upper bound : $E = E_0$

lower bound : $E = \log\left[\frac{C(t_{max})}{C(t_{max}+1)}\right]$



FLAVOUR-BREAKING EXPANSIONS (QCD+QED)

- Recall flavour-diagonal vector meson $(with\ \bar{m} = \text{constant})$

$$M(a\bar{a}) = M_0 + 2\alpha\delta\mu_a + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_a^2 + \beta_0^{EM}(e_u^2 + e_d^2 + e_s^2) + 2\beta_1^{EM}\, e_a^2$$
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$$+2\gamma_4^{EM}(e_u^2 + e_d^2 + e_s^2)\delta\mu_a + 2\gamma_5^{EM}e_a(e_u\delta m_u + e_d\delta m_d + e_s\delta m_s)$$

- same expansion for a_μ

$$a_{\mu,a} = a_{\mu,0} + 2\alpha\delta\mu_a + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_a^2 + \beta_0^{EM}(e_u^2 + e_d^2 + e_s^2) + 2\beta_1^{EM}\, e_a^2$$
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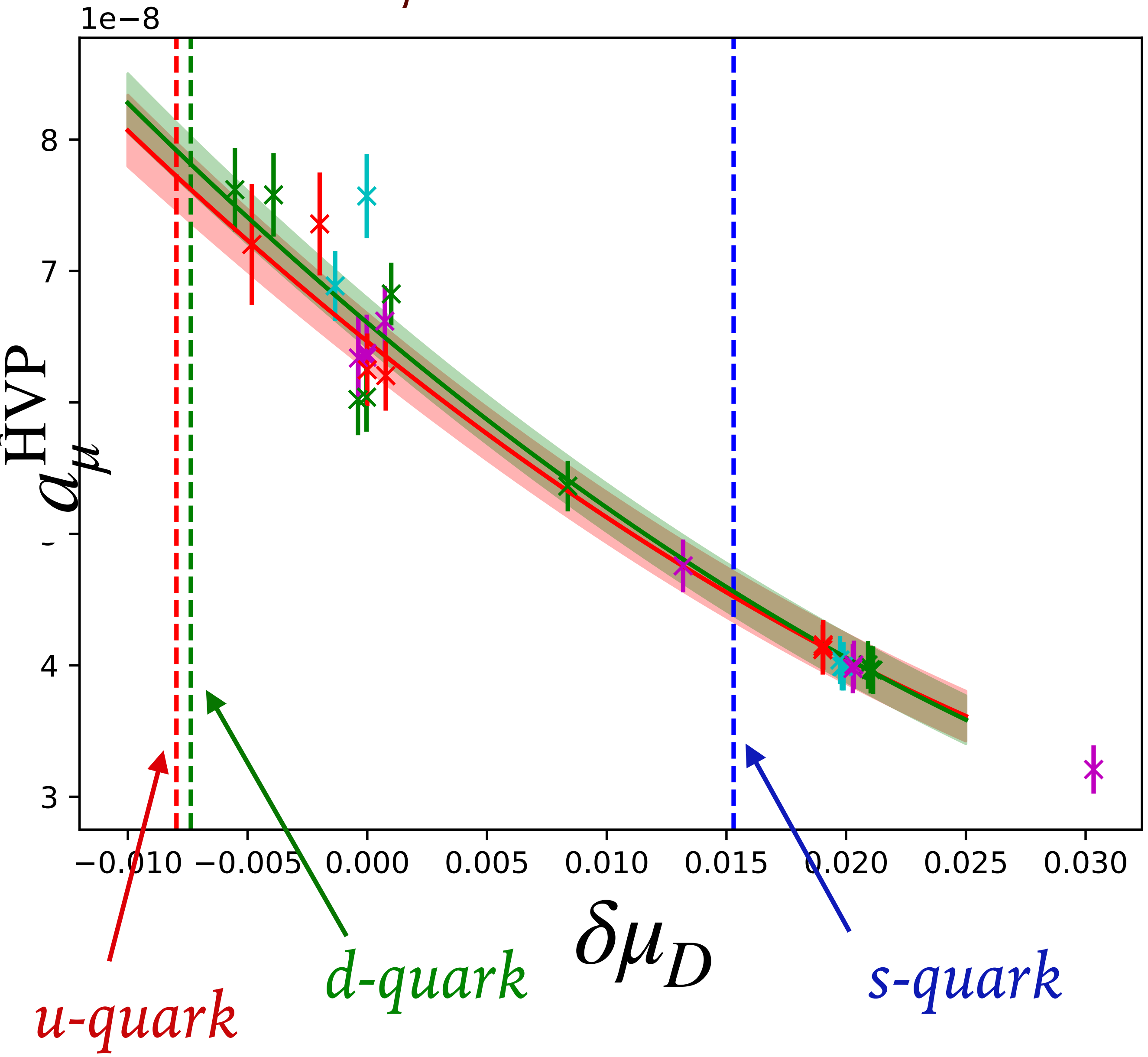
FLAVOUR EXPANSION

➤ Apply simultaneous to all quark masses/charges on each volume

$$32^3 \times 64$$

$$a_\mu \approx 480 \times 10^{-10}$$



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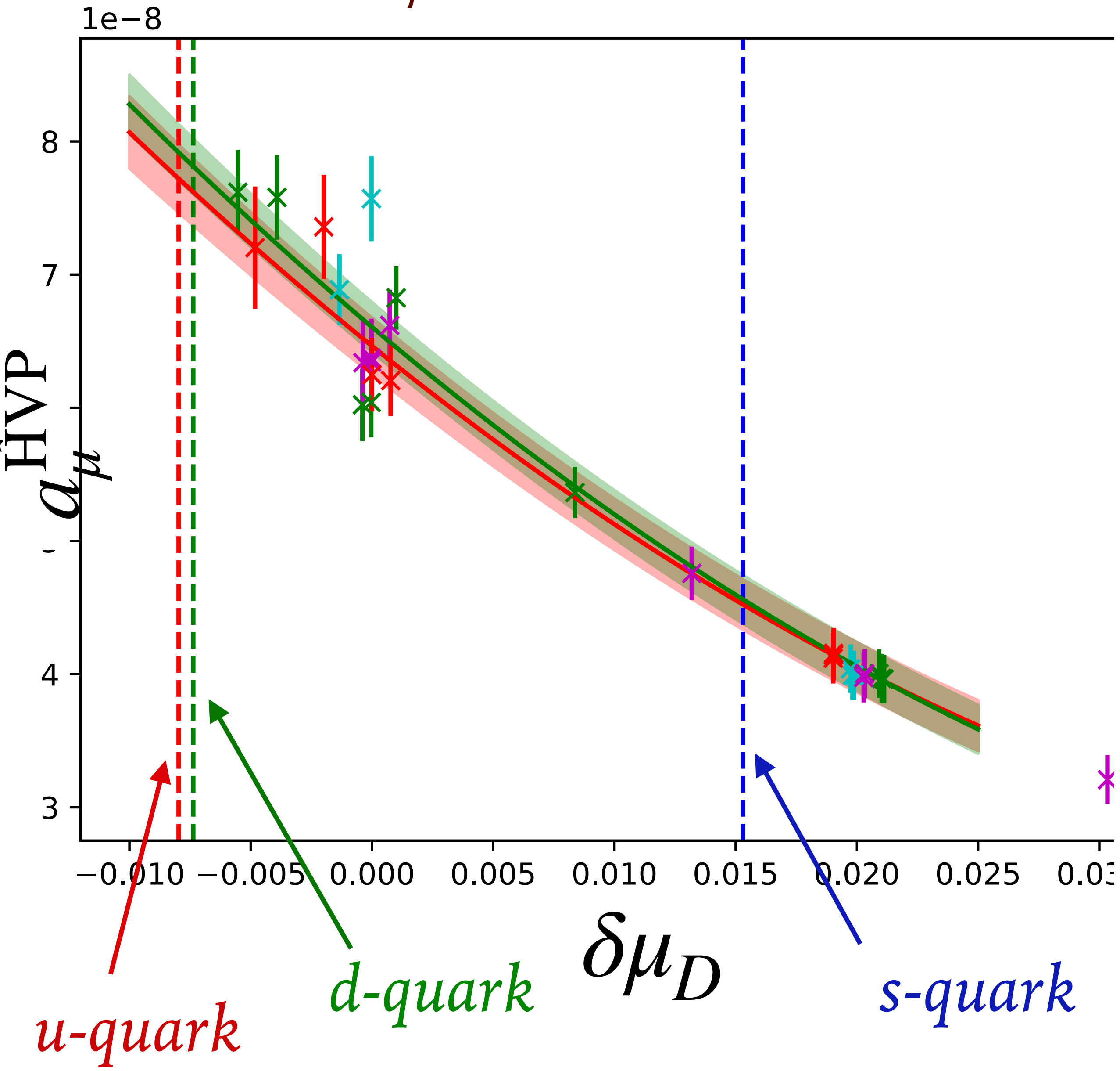
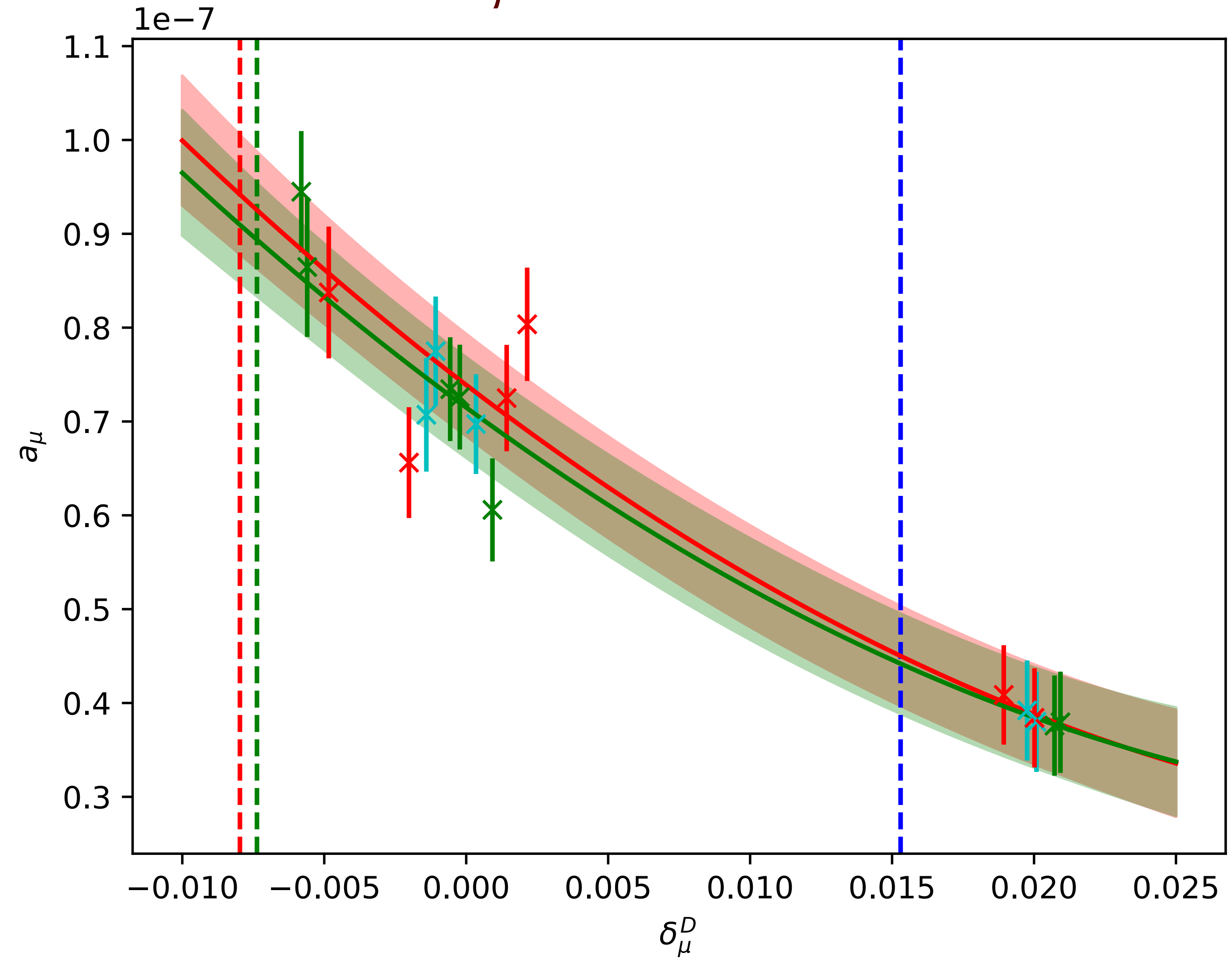




FINITE VOLUME EFFECTS

- **Aubin et al. (2016)**: $m_\pi = 220\,\mathrm{MeV},\ \mathrm{L} = 3.8\,\mathrm{fm},\ \mathrm{m}_\pi \mathrm{L} = 4.2$
  - Compare different irreducible representations

[Aubin et al. Phys.Rev. D93 (2016)]

$$a_{\mu,A_1}^{\mathrm{HVP}}[0.1\,\mathrm{GeV}^2] = 6.8(4) \times 10^{-8}$$

$$a_{\mu,A_1^{44}}^{\mathrm{HVP}}[0.1\,\mathrm{GeV}^2] = 7.5(3) \times 10^{-8}$$

$A_1\!: \sum \Pi_{ii}$

$A_1^{44}\!: \Pi_{44}$

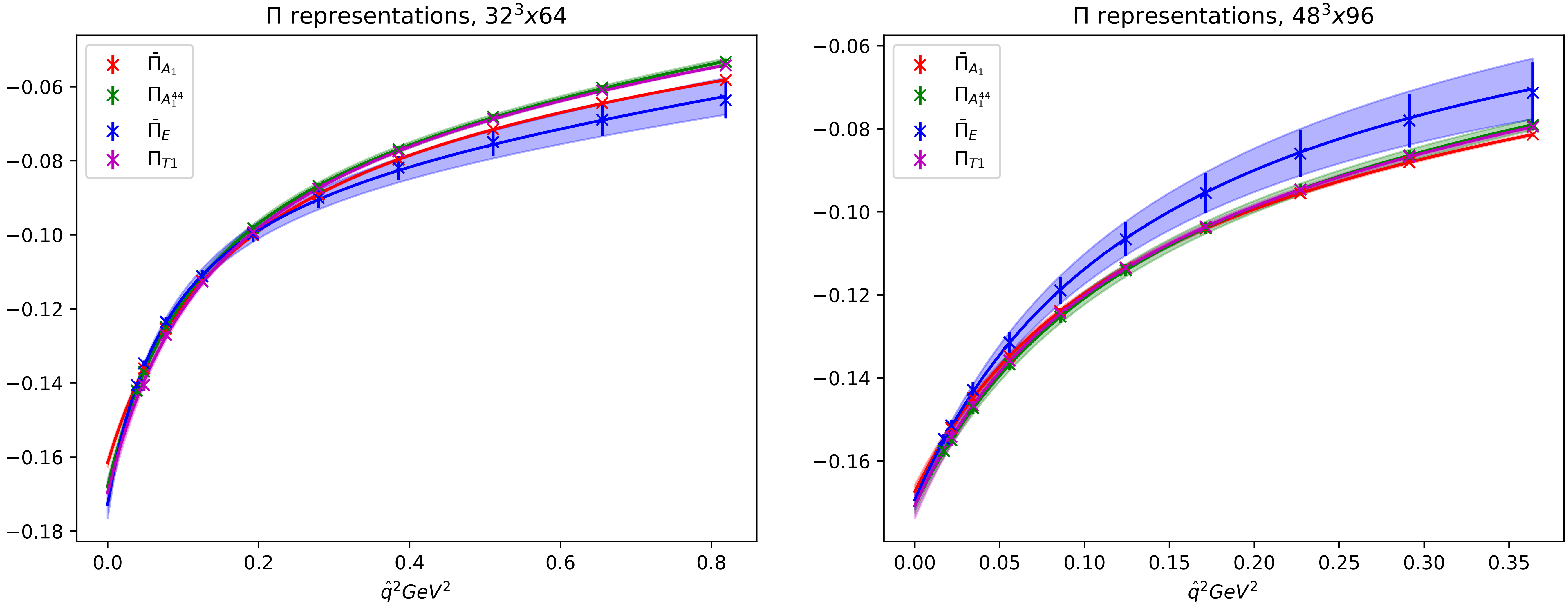
→ 10 - 15% finite volume effects

A_1: $\sum \Pi_{ii}$

A_1^{44}: Π_{44}

FINITE VOLUME EFFECTS

- Compare 4 different representations on our 2 volumes



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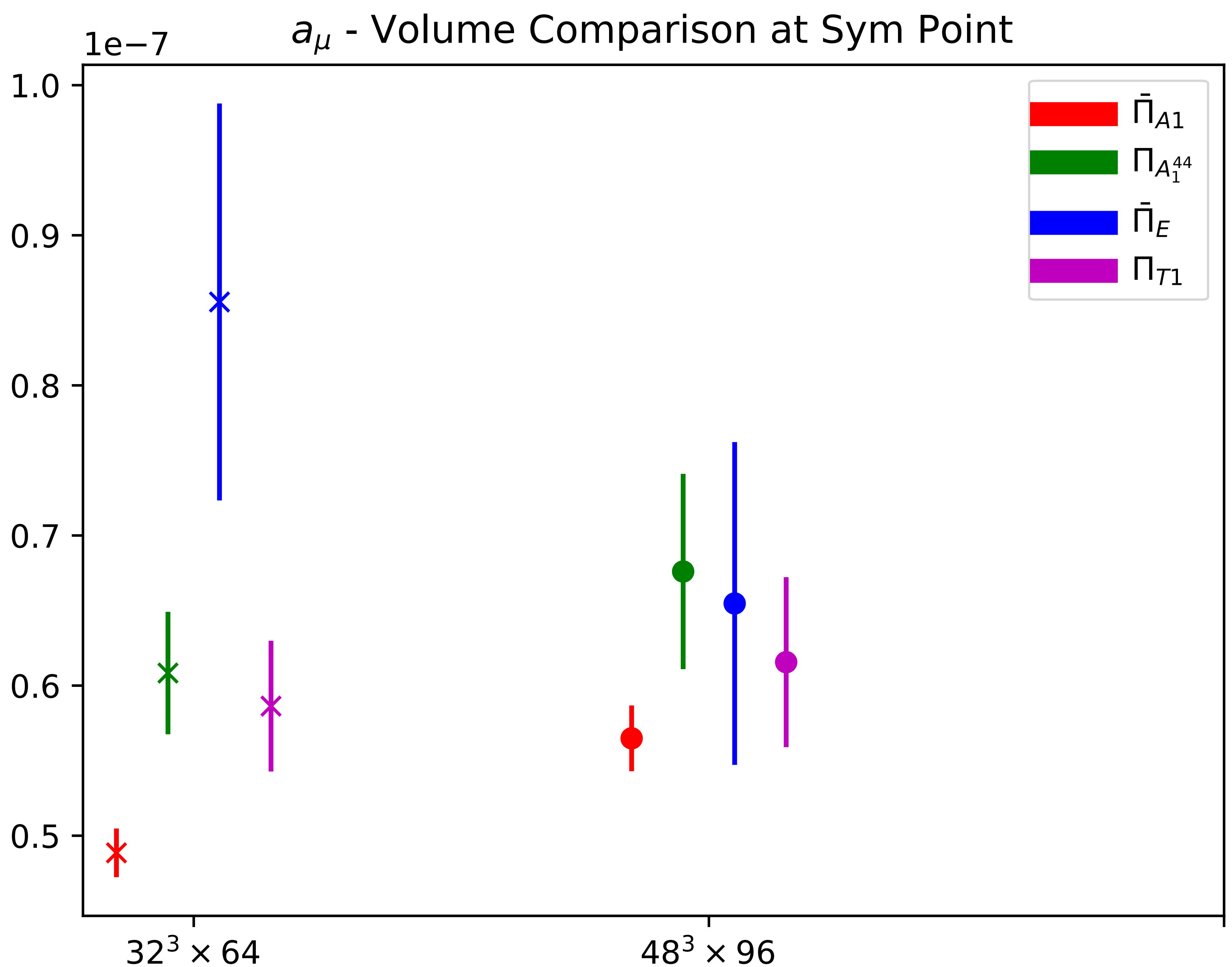
a_μ^{HVP}



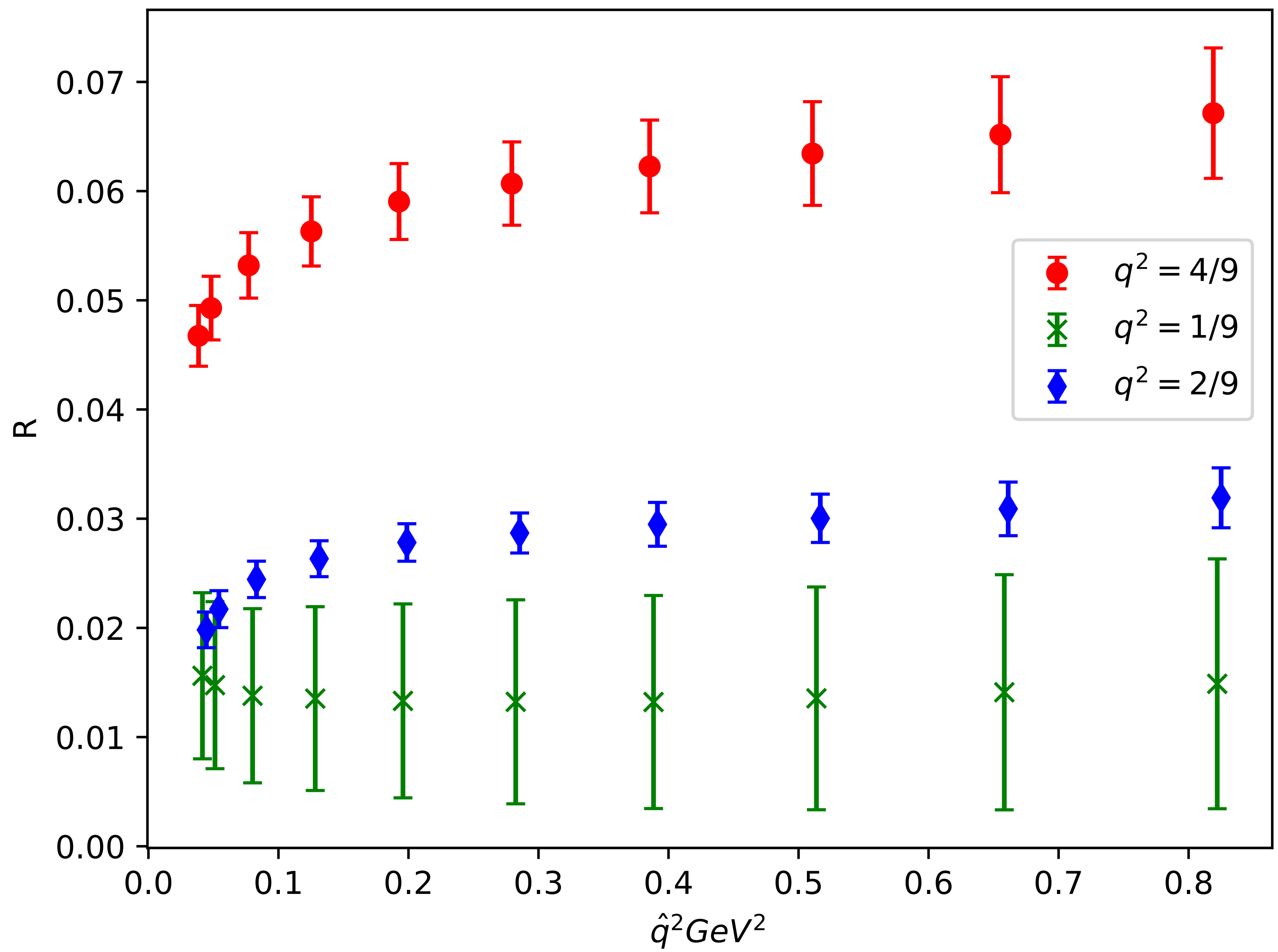
ISOLATING CHARGE EFFECTS

$$\alpha_{\text{QED}} = \frac{e^2}{4\pi} \simeq 0.1$$

- Difficult to resolve electric charge effects from fit
- Try correlated ratios

$$R = \frac{\Pi^q(Q^2) - \Pi^n(Q^2)}{\Pi^n(Q^2)}$$

Without Z_V



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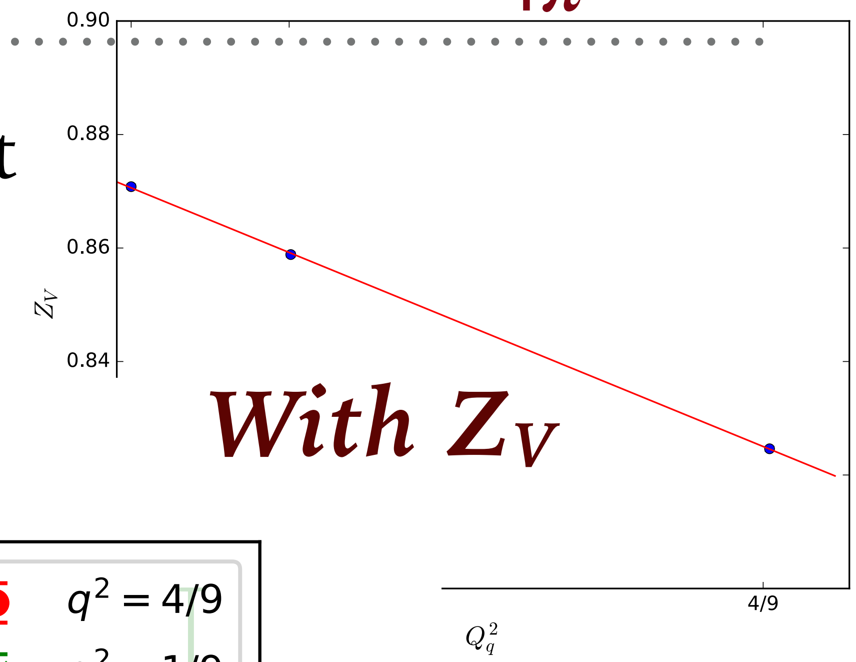

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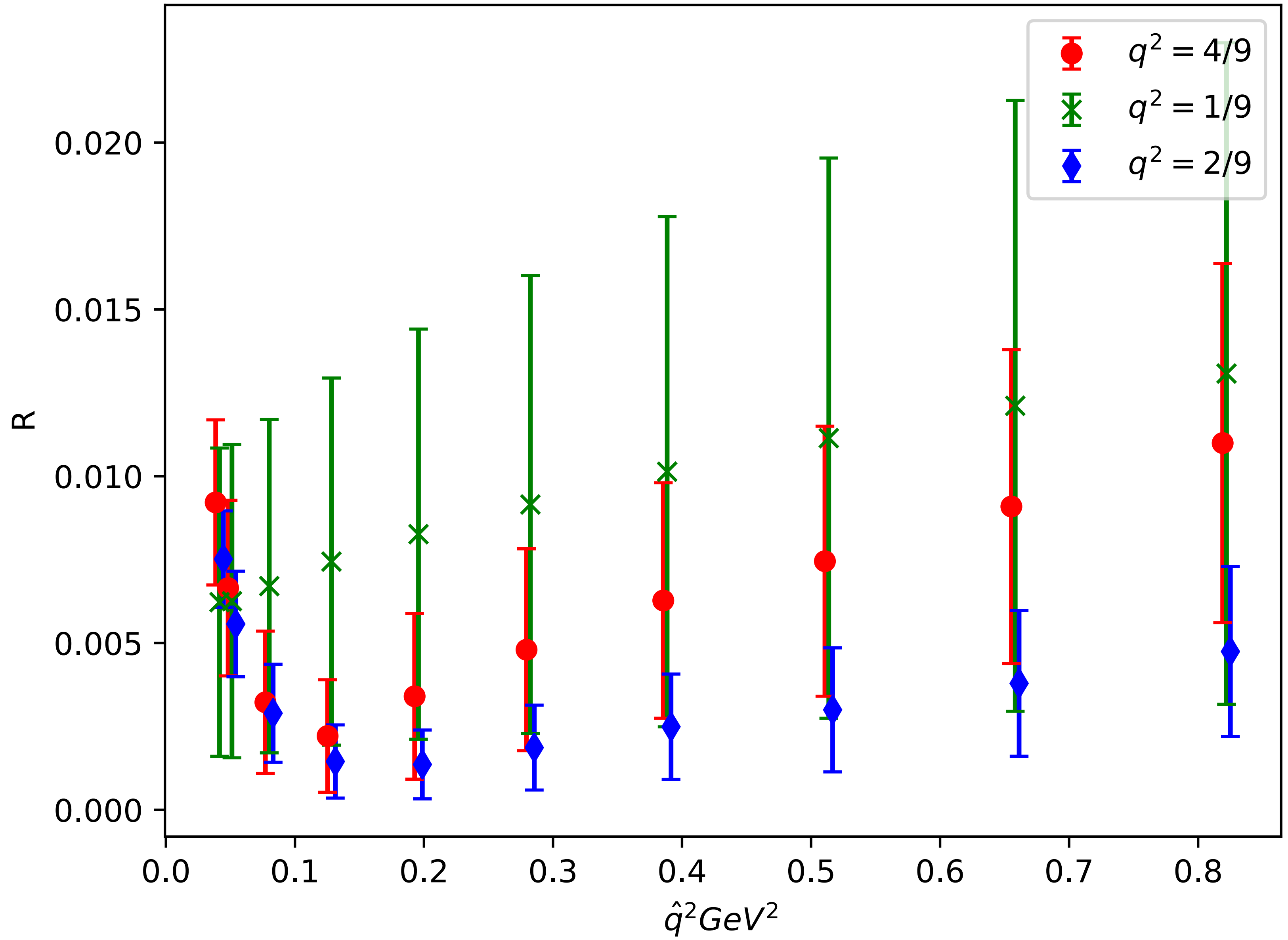

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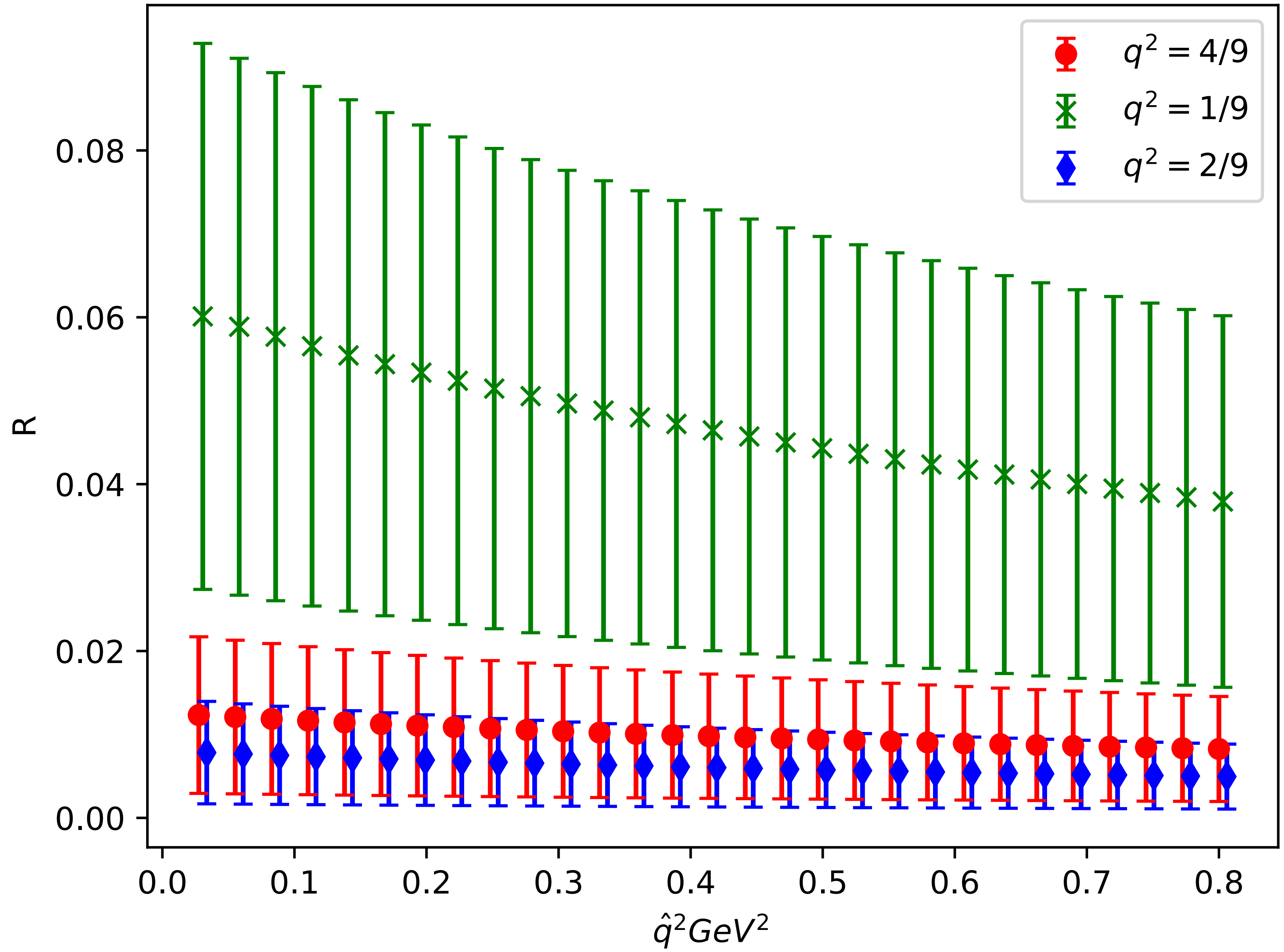

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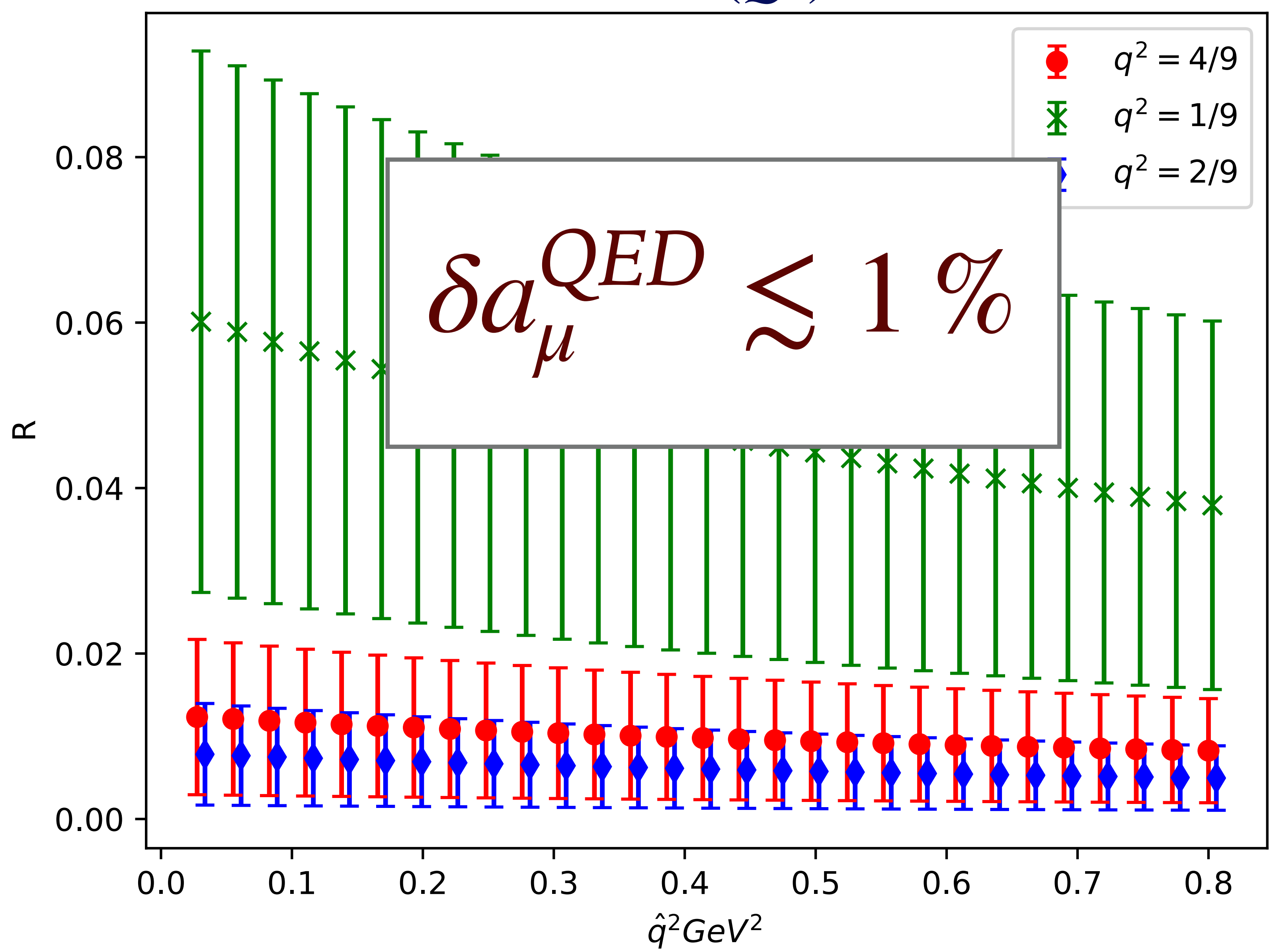


$$\delta a_\mu^{QED} \lesssim 1\,\%$$

SUMMARY

- Constraints on QED effect on a_μ becoming possible *[Miura, Wed. 9:00]*
- This work:

$$\delta a_\mu^{QED} \lesssim 1\,\%$$

- Flavour-breaking expansion can be applied to a_μ

  ➡ *Separation of strong and EM IB effects in Dashen*

- Much still to be done!

Backup

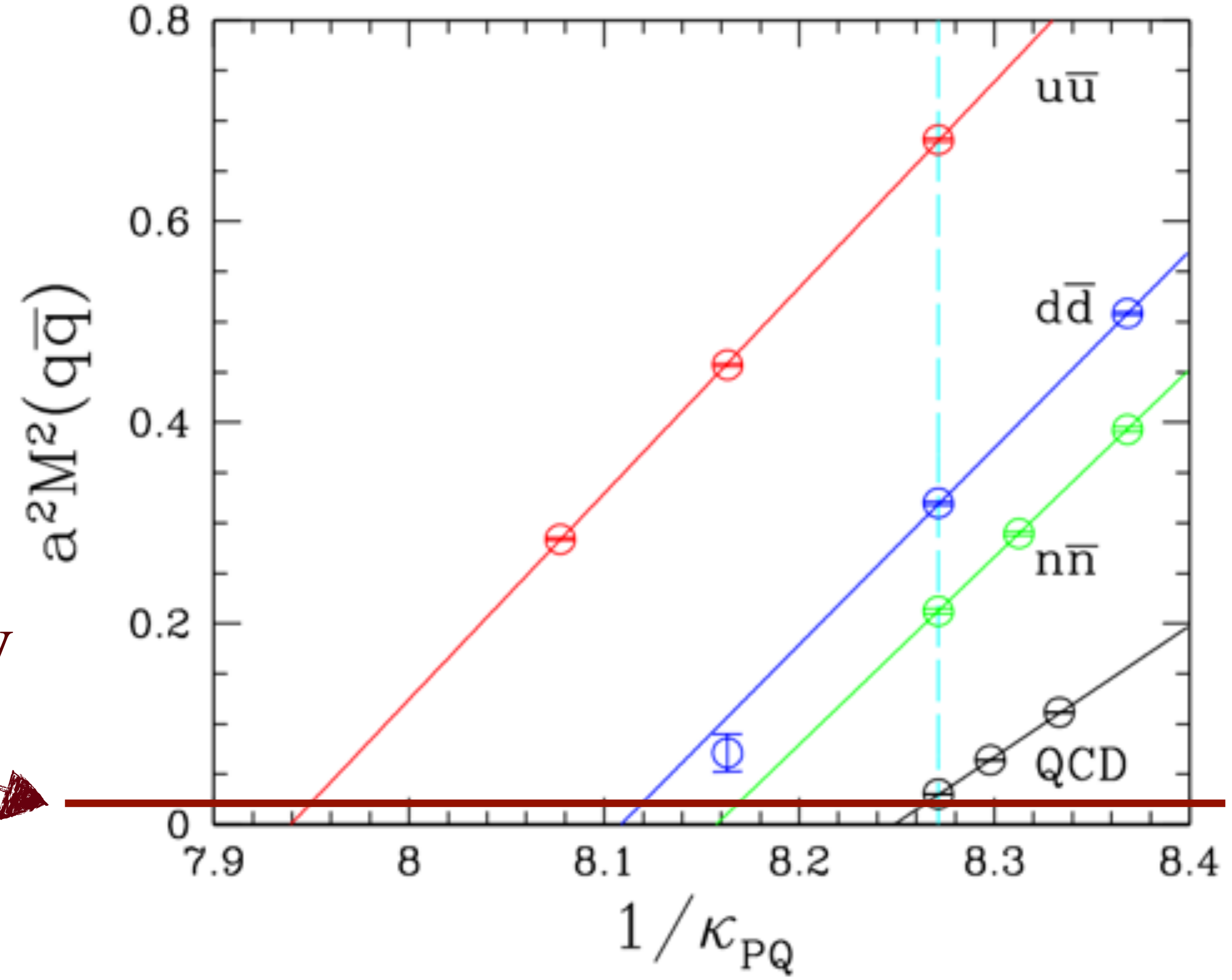
“DASHEN SCHEME”

$$am_q = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

$$\beta_{\mathrm{QCD}} = 5.50$$

$$\beta_{\mathrm{QED}} = 0.8$$

$$X_\pi = \sqrt{\frac{1}{3}(m_{K^+}^2 + m_{K^0}^2 + m_{\pi^+}^2)} = X_\pi^{phys} = 411\,\mathrm{MeV}$$



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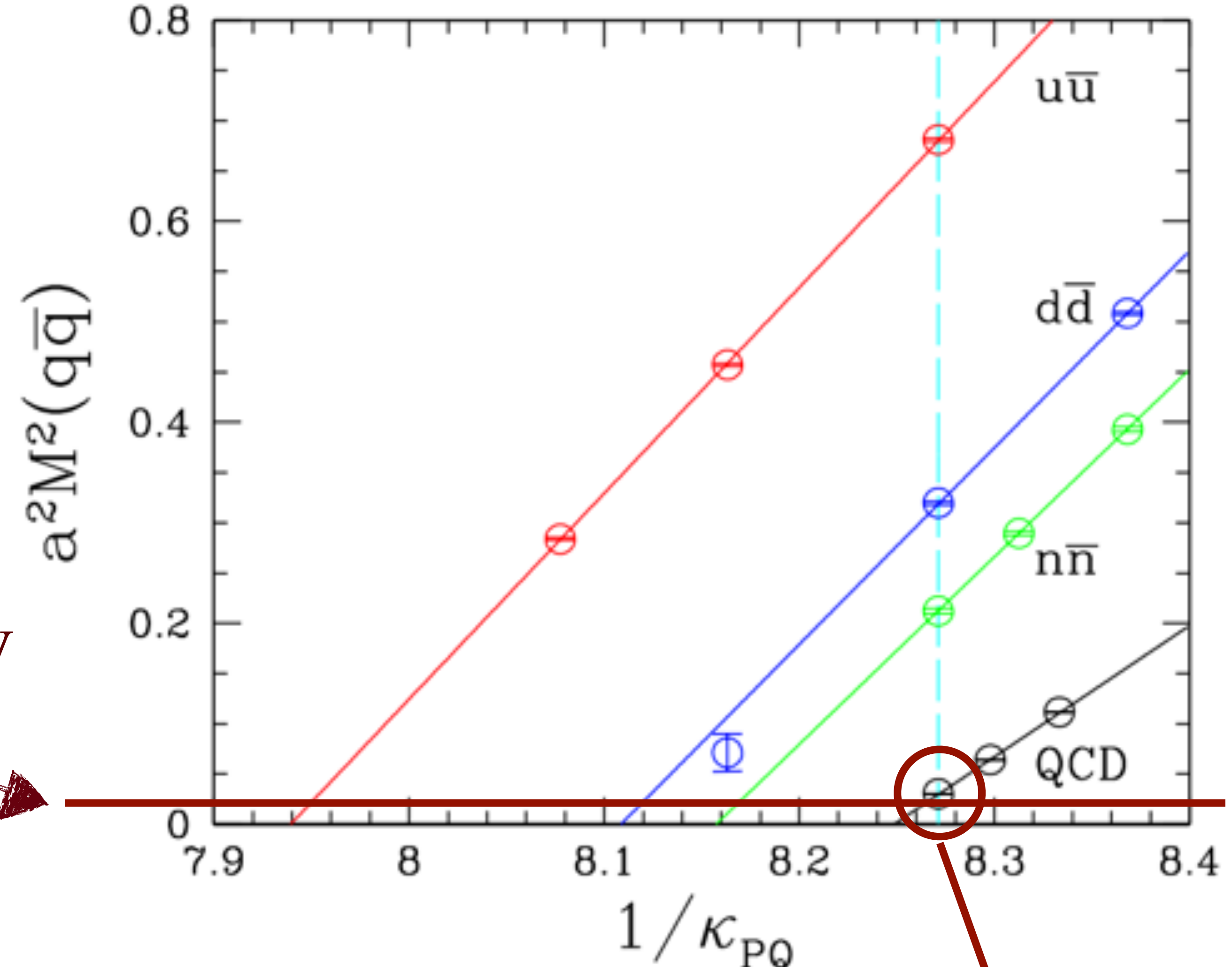

$\kappa_{\mathrm{sym}}^{\mathrm{QCD}}$

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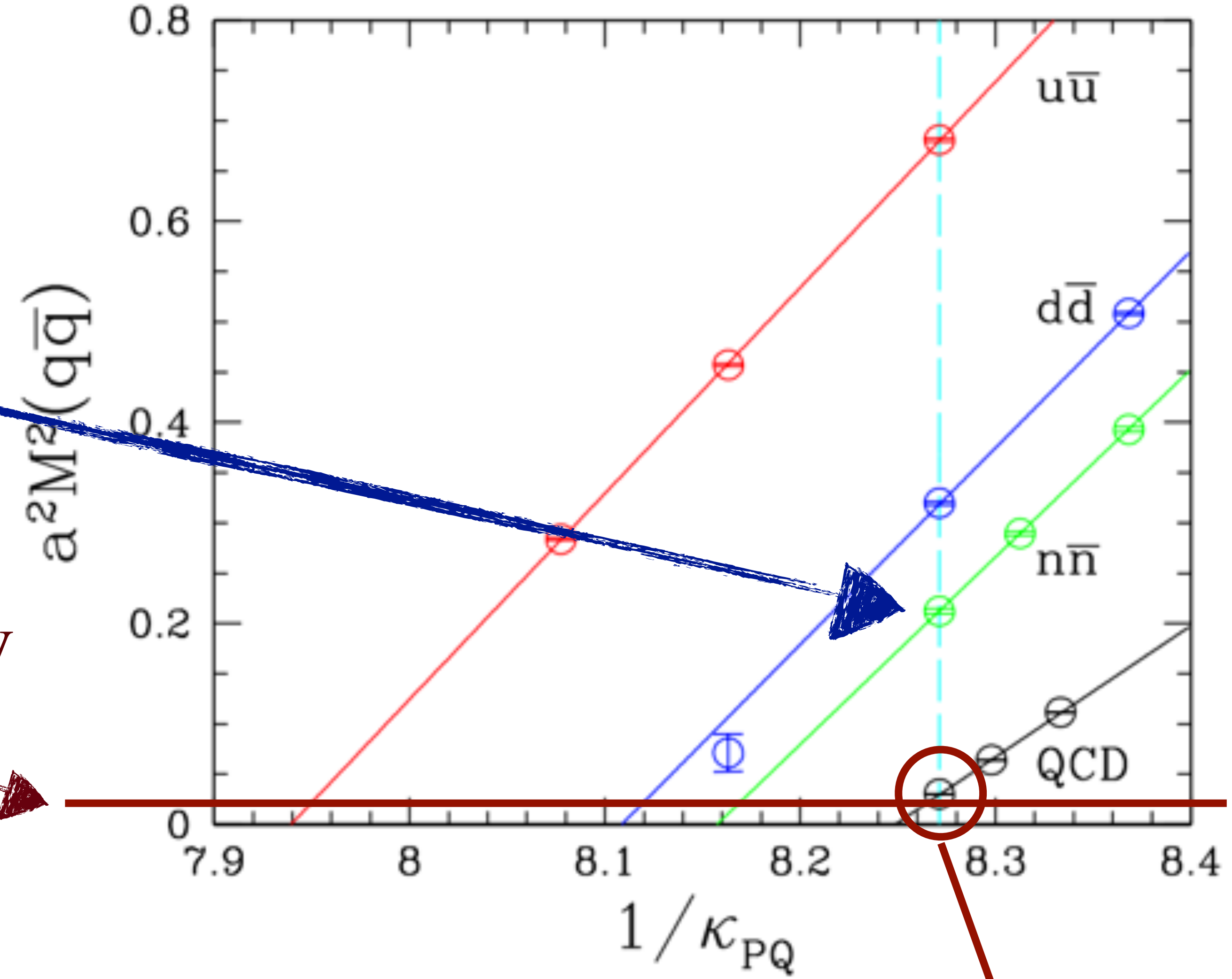

$\kappa_{\rm sym}^{\rm QCD}$

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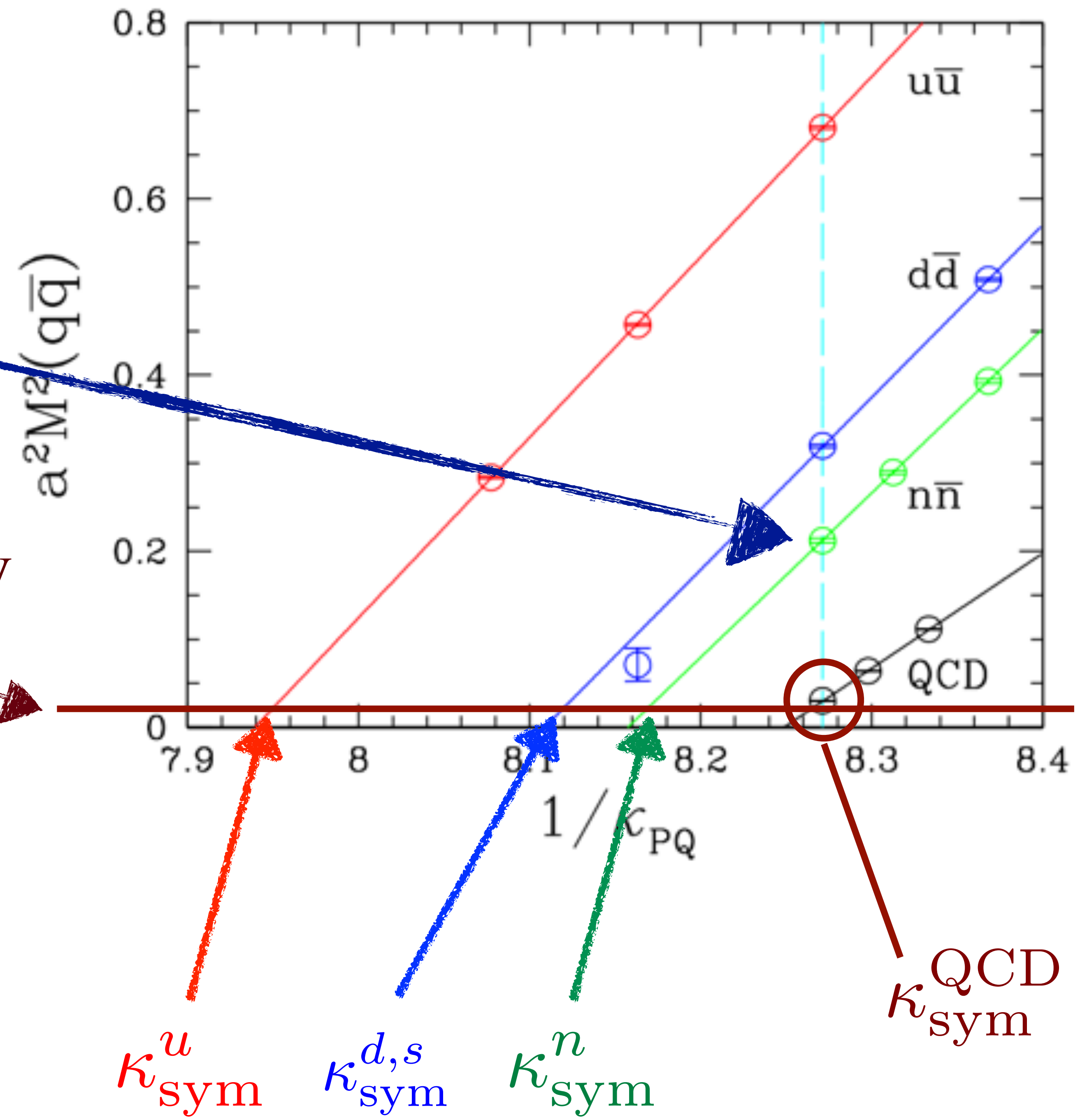

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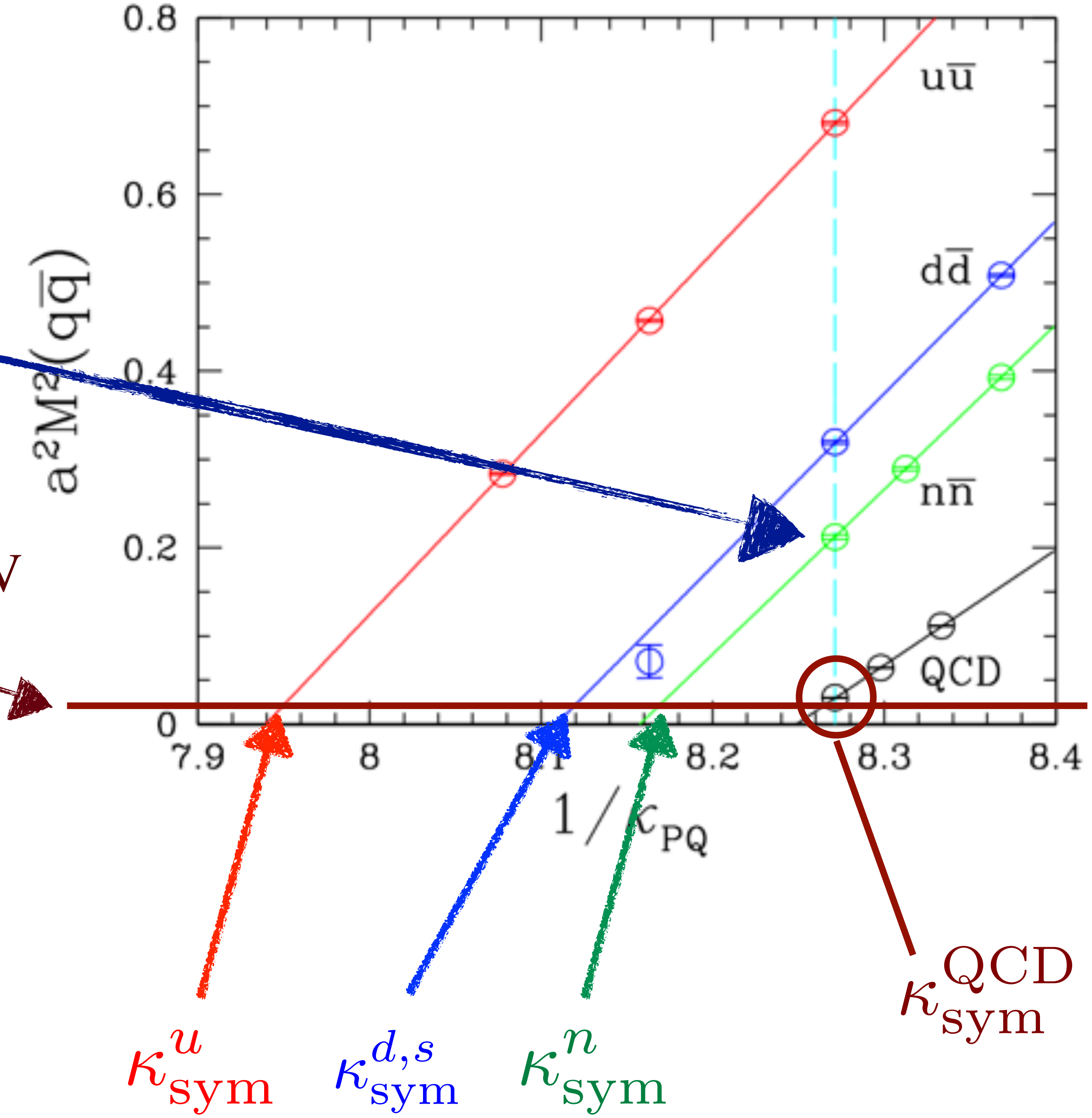


- Iterate and converge to

$$\kappa^u_{\text{sym}} = 0.1243838$$

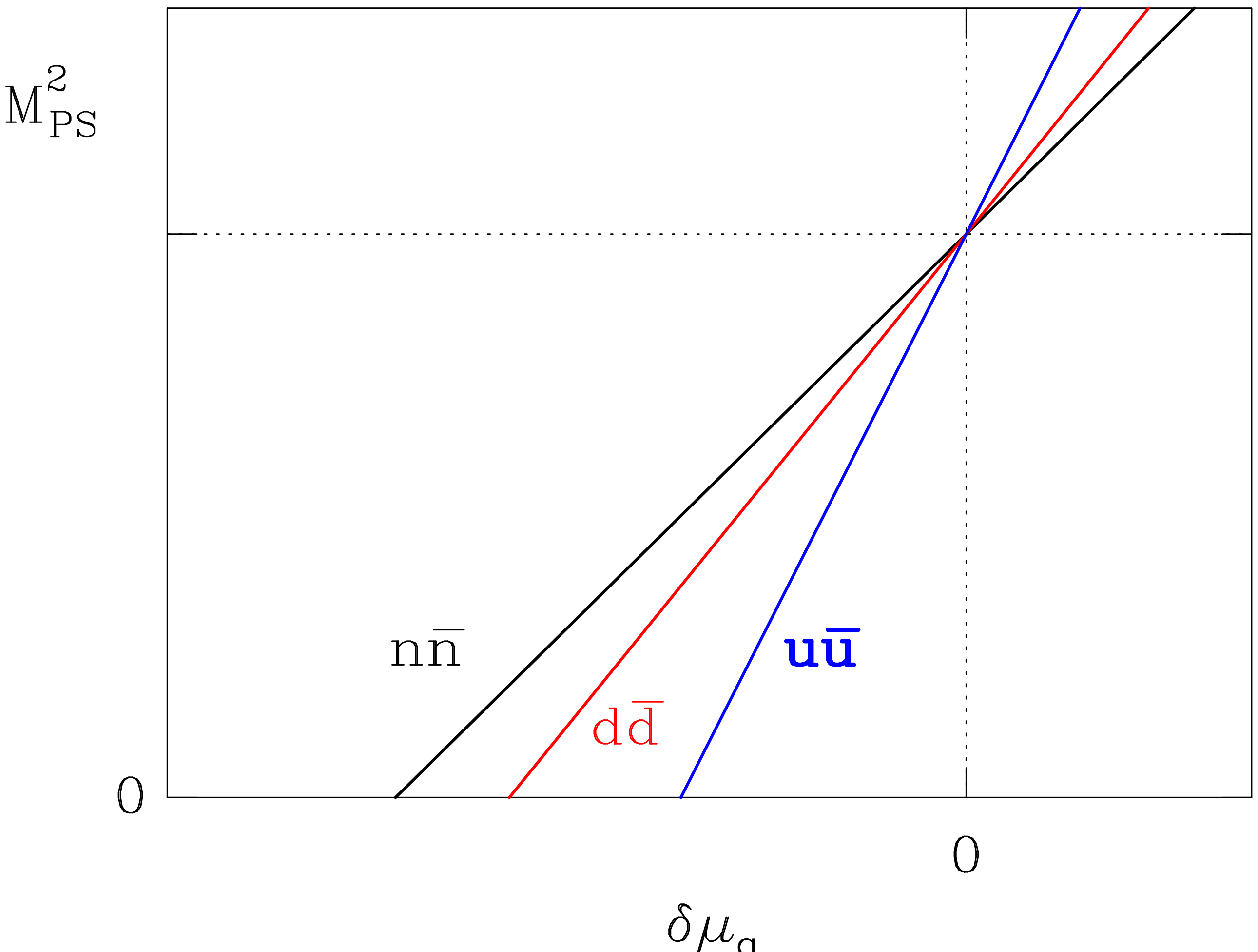
$$\kappa^{d,s}_{\text{sym}} = 0.1217026$$

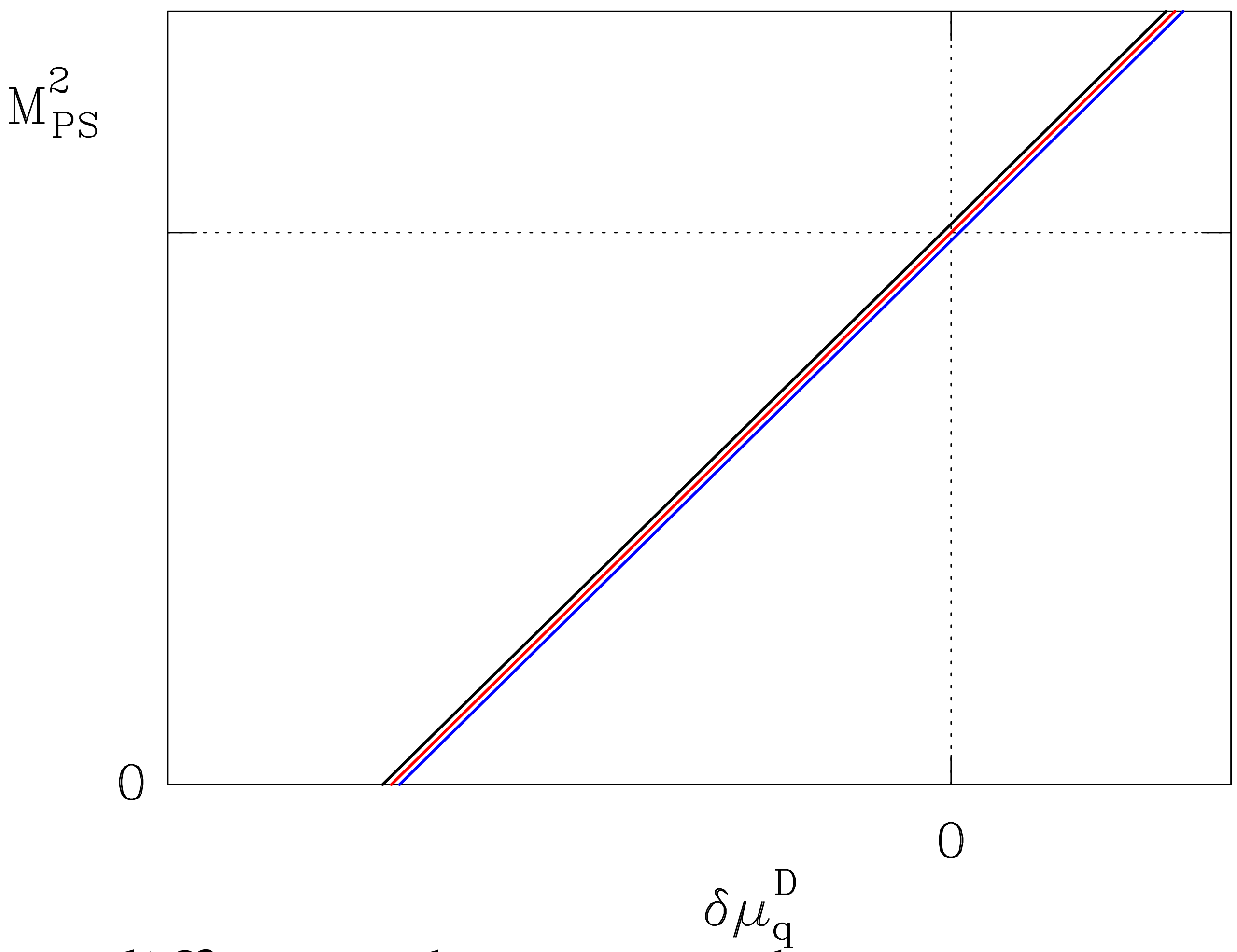
$$\kappa^n_{\text{sym}} = 0.1208142$$

"DASHEN SCHEME"

QCDSF, JHEP 1604, 093 (2016)

- Cartoon illustrating the different running of the bare quark masses





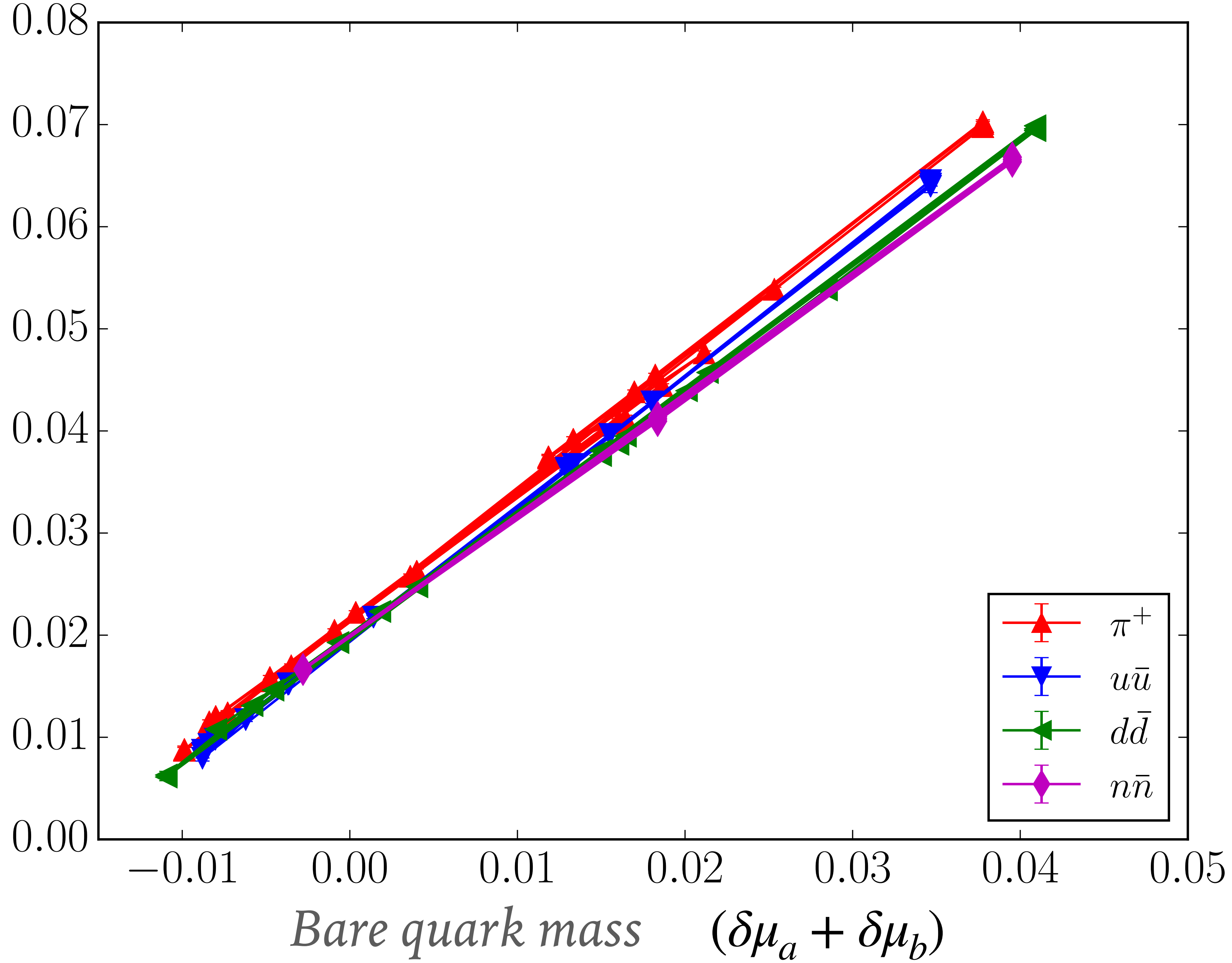
- Once tuned to the symmetric point, different charge quarks run differently to the chiral limit

Dashen scheme: rescale the horizontal axis so that all meson masses depend on the "Dashen mass" in the same way

$$\delta\mu_q^D = (1 + KQ_q^2e^2)\delta\mu_q$$

PSEUDOSCALAR MASSES

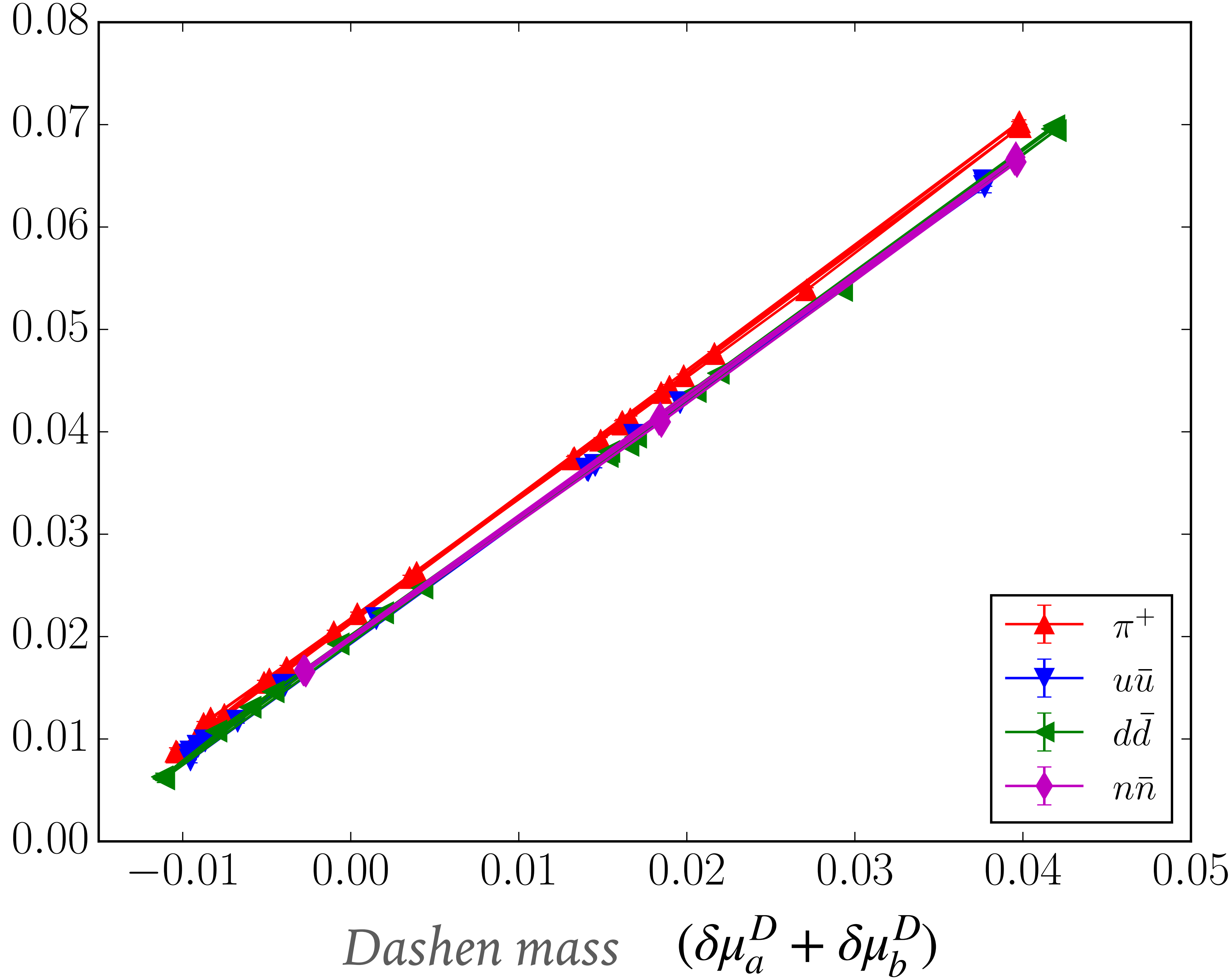
QCDSF, JHEP 1604, 093 (2016)



- Neutral mesons on different lines
- Scatter of charged pion: dependence on $\delta m_d - \delta m_u$

"DASHEN SCHEME" – PSEUDOSCALAR MASSES

QCDSF, JHEP 1604, 093 (2016)



- Neutral mesons on uniform curve
- "Scatter" removed from charged mesons

LATTICE QCD+QED – ZERO MODES

- Ground state energy of a single particle shifted from rest mass

$$E = \sqrt{m^2 + Q^2 (e\vec{B})^2}$$

- Subtract $\boldsymbol{B^2}$ contribution to find m

- Alternative: eliminate by additional gauge-fixing of Uno & Hayakawa (2008) — on valence quarks

$$\sum_{\vec{x}} A_\mu(t, \vec{x}) = 0, \quad \forall \mu \text{ and } t$$