INVESTIGATING VOLUME EFFECTS FOR $N_f = 2$ AND $N_f = 2+1+1$ TWISTED CLOVER FERMIONS AT THE PHYSICAL POINT

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Abstract: We present a study of nucleon quantities, such as the axial and tensor charges, the quark momentum fraction and the first moment of the helicity distribution. We use three ensembles of twisted mass fermions with a clover term, with the quark masses fixed at their physical values. The ensembles correspond to different number of dynamical quarks (N_f=2, N_f=2+1+1) and volumes ($48^3\times96$, $64^3\times128$), which allows one to study volume and quenching effects. All ensembles have lattice spacing below 0.1fm.

Nucleon charges and moments on the Lattice

One of the most important endeavors in particle and nuclear physics is to understand the structure of the nucleon from first principles. Lattice QCD offers an ideal *ab initio* formulation that can be used to study the properties of fundamental particles numerically, at the hadronic scale, where perturbative tools fail. Nucleon structure is of particular interest, with the nucleon charges being studied for many years now.

The axial charge, g_A, has been well measured experimentally in β-decay experiments, and can be used as a benchmark quantity. The tensor charge, g_T, is needed in order to set bounds on novel CP-violating processes in theories beyond the standard model. Unlike g_A, its value is not well known experimentally with only limits on its value coming from radiative pion decay $\pi \to e\nu\gamma$ so Lattice QCD offers a method to accurately calculate it. The momentum fraction carried by the quarks, $\langle x \rangle$, does not only provide important information of the nucleon structure, but has implications also for the spin.

Parton distribution functions (PDFs) describe, to leading twist, how the probability of finding a specific parton in a hadron depends on the hadron's momentum and spin. PDFs are in the light-cone frame so it is difficult to calculate them in Euclidean space. One solution is to calculate moments of PDFs which are related to the original PDF through the operator product expansion. The two moments studied here, the quark momentum fraction and helicity, are defined as:

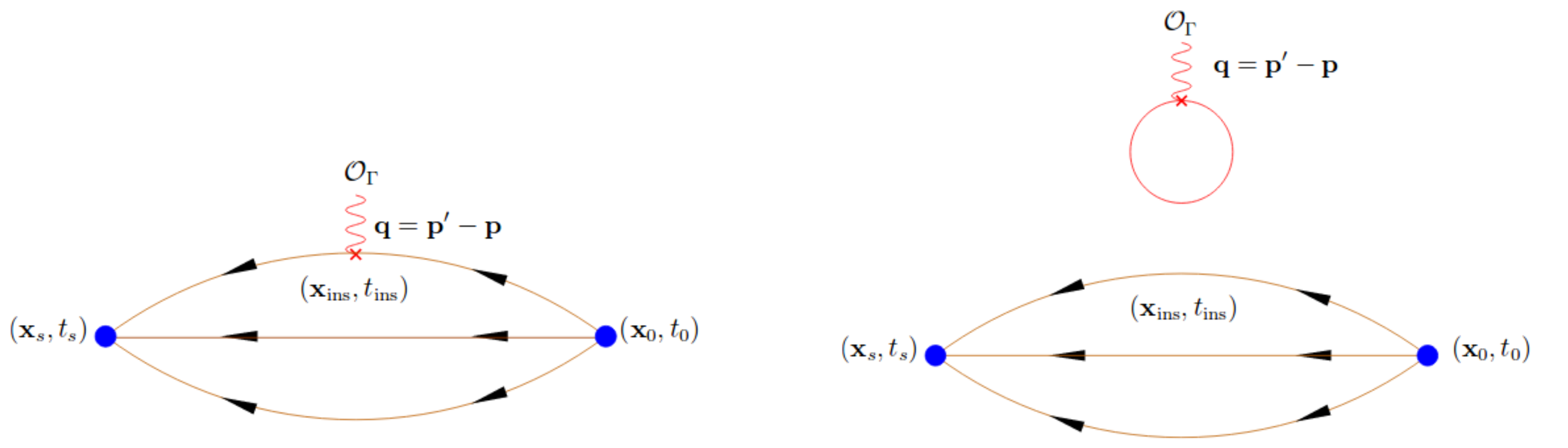
$$\langle x \rangle_q = \int_0^1 x\left[q(x) + \bar{q}(x)\right] dx, \quad \langle x \rangle_{\Delta q} = \int_0^1 x\left[\Delta q(x) - \Delta\bar{q}(x)\right] dx$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow,$$

where $q_\uparrow$ and $q_\downarrow$ are quarks whose helicities are aligned and anti-aligned to that of a longitudanlly polarized target. To calculate each charge and moment, we calculate the matrix elements

$$\langle N(p)|\mathcal{O}_\Gamma|N(p)\rangle$$

where the form of the operator $\mathcal{O}_\Gamma$ depends on the charge or moment being calculated. These matrix elements, known as three-point functions, are represented by the diagrams below.



The left diagram is the connected contributions and the right the disconnected. In this study, we focus on isovector quantities that receive contributions only from the connected diagram (the disconnected contributions cancel out up to lattice cut-off effects). For g_A and g_T, $\mathcal{O}_\Gamma$ is the local axial-vector and tensor operators, respectively, which are:

$$\mathcal{O}^{\mu}_{A^a} = \bar{q}\gamma_5\gamma^\mu\frac{\tau^a}{2}q\,, \quad \mathcal{O}^{\mu\nu}_{T^a} = \bar{q}\sigma^{\mu\nu}\frac{\tau^a}{2}q\,.$$

and for $\langle x \rangle_q$ and $\langle x \rangle_{\Delta q}$, the $\mathcal{O}_\Gamma$ is the one-derivative vector and axial-vector operators, respectively, which are:

$$\mathcal{O}^{\mu\nu}_{V^a} = \bar{q}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\frac{\tau^a}{2}q, \quad \mathcal{O}^{\mu\nu}_{A^a} = \bar{q}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\gamma_5\frac{\tau^a}{2}q,$$

where $\bar{q} = (\bar{u}, \bar{d})$,

$$\overleftrightarrow{D}_\mu = \frac{1}{2}\left(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu\right), \quad D_\mu = \frac{1}{2}\left(\nabla_\mu + \nabla^*_\mu\right)$$

and ∇_μ (∇^*_μ) is the usual forward (backward) derivative on the lattice. The curly (square) brackets represent a symmetrization (anti-symmetrization) over indice pairs, with the symmetrization accompanied by subtraction of the trace.

Simulation Details

All three ensembles use the twisted-clover action

$$S_F\left[\chi, \overline{\chi}, U\right] = a^4\sum_x \overline{\chi}(x)\left(D_W[U] + m_{\rm cr} + i\mu_l\gamma_5\tau^3 - \frac{1}{4}c_{\rm SW}\sigma^{\mu\nu}\mathcal{F}^{\mu\nu}[U]\right)\chi(x),$$

with quarks tuned to maximal twist. This action gives an automatic $\mathcal{O}(a)$-improvement and does not require any operator modifications, simplifying the renormalization.
The table shows the different parameters and statistics for each ensemble.

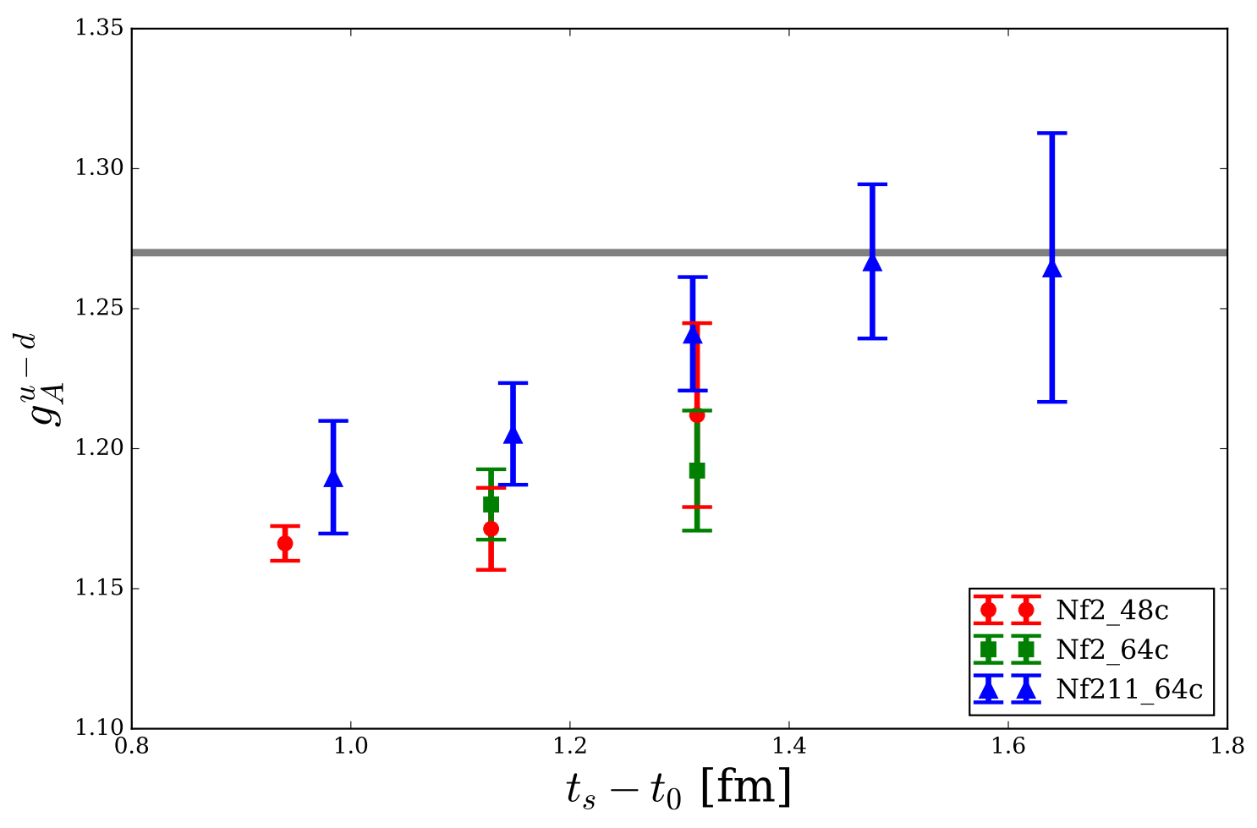
Parameters

	N_f	Volume	a (fm)	m_π (MeV)
Nf2_48c	2	$48^3\times96$	0.094	130
Nf2_64c	2	$64^3\times128$	0.094	130
Nf211_64c	2+1+1	$64^3\times128$	0.082	137

Nucleon Charges: g_A^{u-d}, g_T^{u-d}

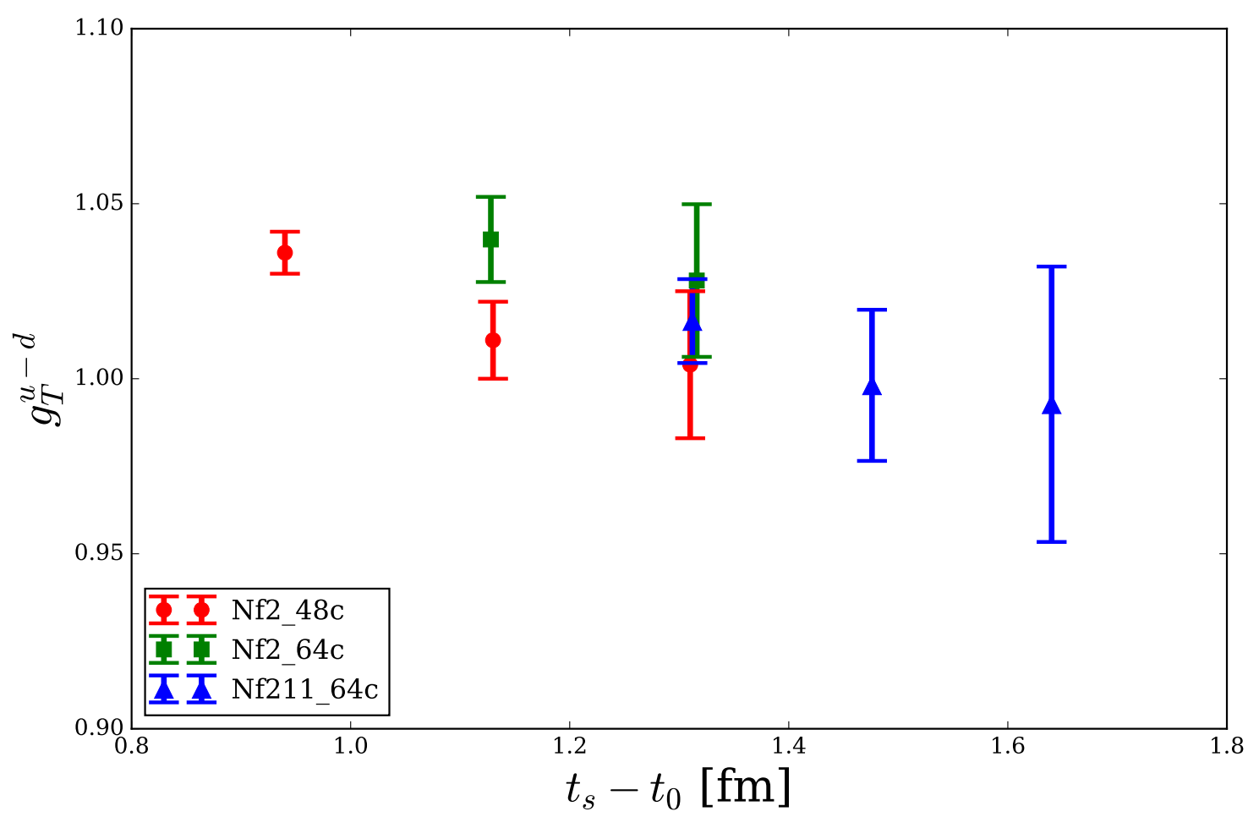
We first present g_A and g_T in the plots shown below. Data are plotted at different source-sink separations for the three different ensembles. In the plot of g_A, the experimentally measured value, shown by the gray line, is also included.

g_A^{u-d}



Statistics: Nf2_48c: 9264 Nf2_64c: 5328, 6368
Nf211_64c: 1000, 3000, 8000, 16000, 16000

g_T^{u-d}



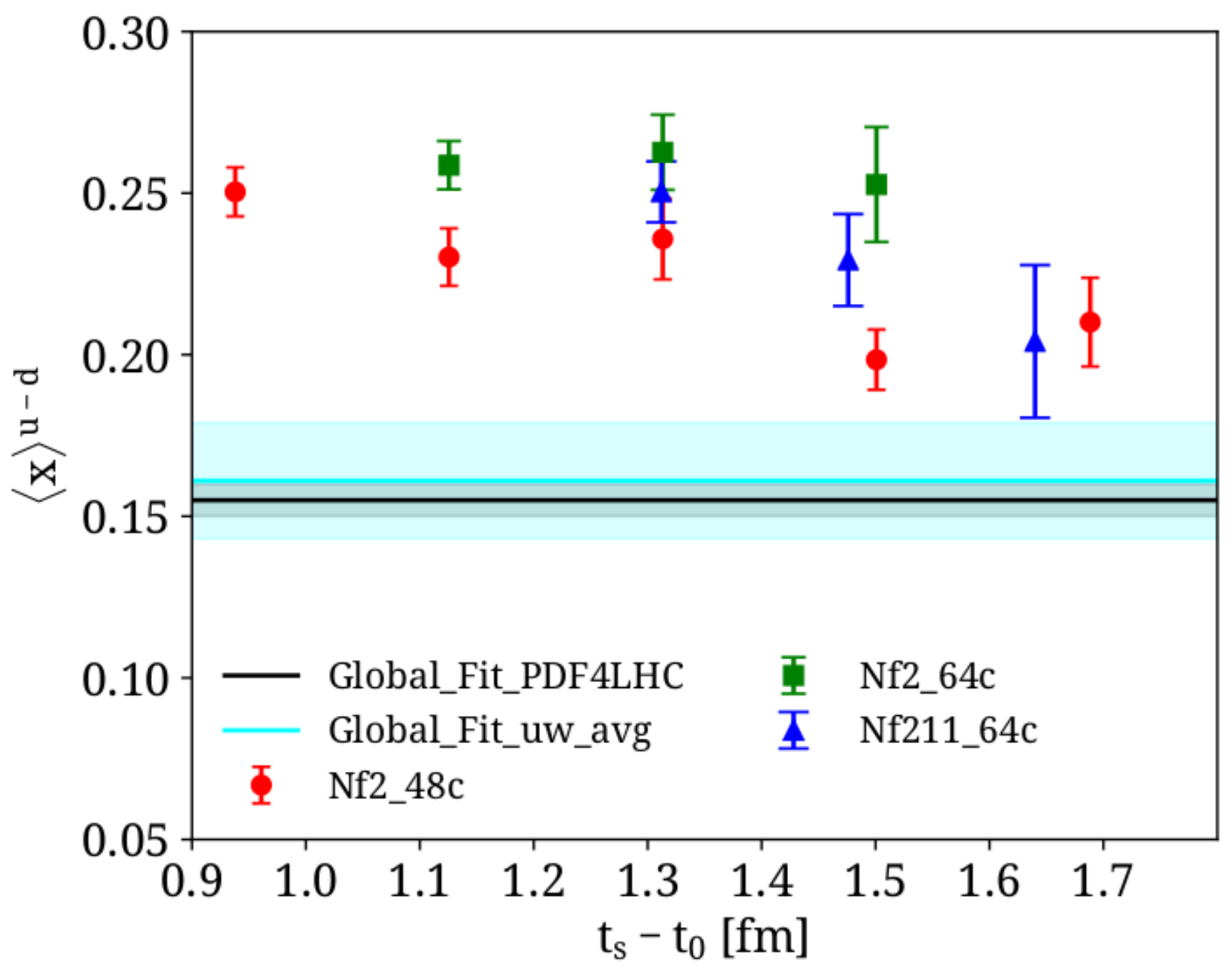
Statistics: Nf2_48c: 9264 Nf2_64c: 5056 Nf211_64c: 12000

In the left plot, it can be seen that g_A rises as the $T_{\rm sink}$ increases for Nf2_48c and Nf211_64c which shows that excited state contamination is effecting the values at lower $T_{\rm sink}$. At $T_{\rm sink}$ ~1.4 fm and ~1.6 fm, the values of g_A agree with the experimental value.
Similarly to g_A, g_T decreases in the right plot with increased $T_{\rm sink}$, again showing effects due to excited state contamination. There do not seem to be strong volume effects on g_T at these lattice volumes since all three ensembles agree within statistical uncertainty.

Moments of PDFs : $\langle x \rangle_{u-d}, \langle x \rangle_{\Delta u - \Delta d}$

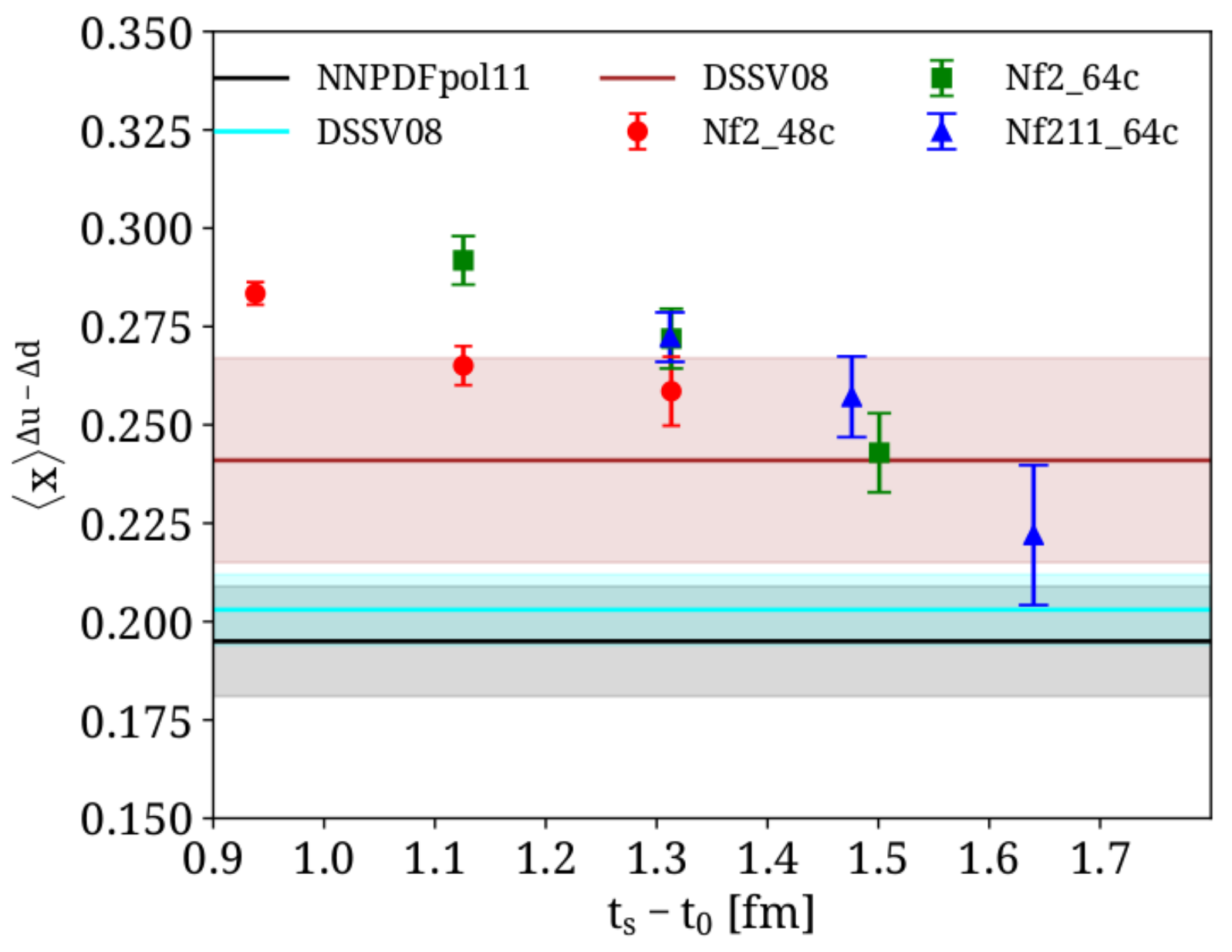
Below are two plots similar to the two above but for the nucleon moments. Phenomenological fits of experimental data are shown with bands.

$\langle x \rangle_{u-d}$



Statistics: Nf2_48c: 9264, 9264, 9264 47696, 69784
Nf2_64c: 5056 Nf211_64c: 9008

$\langle x \rangle_{\Delta u - \Delta d}$



Statistics: Nf2_48c: 9264 Nf2_64c: 5056 Nf211_64c: 9008

Both plots show a general decrease of the moments as $T_{\rm sink}$ increases except the $\langle x \rangle_{u-d}$ data for Nf2_64c which appears to stay constant over $T_{\rm sink}$. This behaviour will be better understood with higher statistics at $T_{\rm sink}$ ~1.5 fm and new data at $T_{\rm sink}$ ~1.7 fm.

Conclusion and Outlook

In this work we study the connected contributions of nucleon charges and moments of PDFs using three ensembles at the physical point. This allows us to investigate both volume and quenching effects. By increasing the source-sink separation, we suppress excited state contamination until we find convergence. We find non-negligible volume effects in some of the quantities and we are currently increasing statistics and producing higher values of $T_{\rm sink}$ to better understand the trends of these effects.

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