Hints and challenges in heavy flavor physics

Shoji Hashimoto (KEK,SOKENDAI)

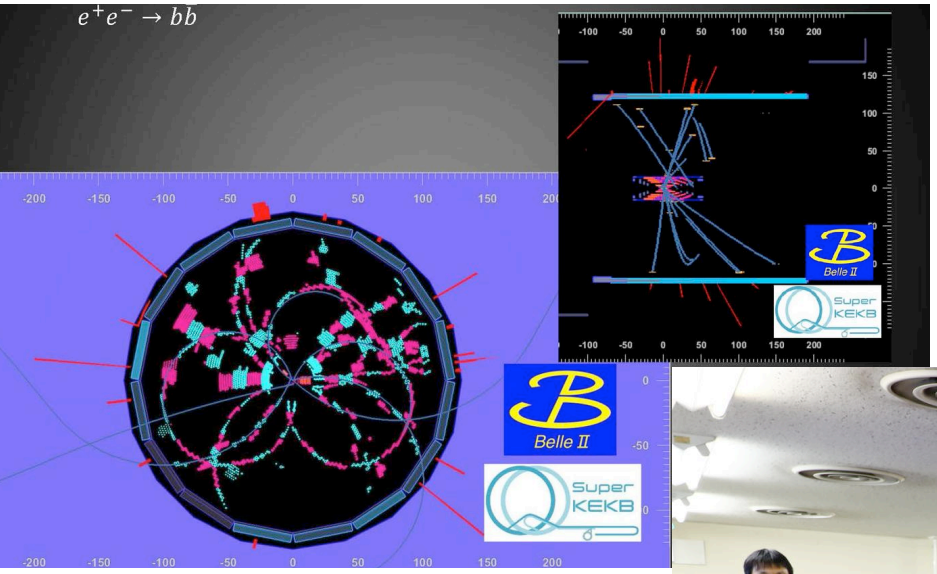
Jul 24, 2018

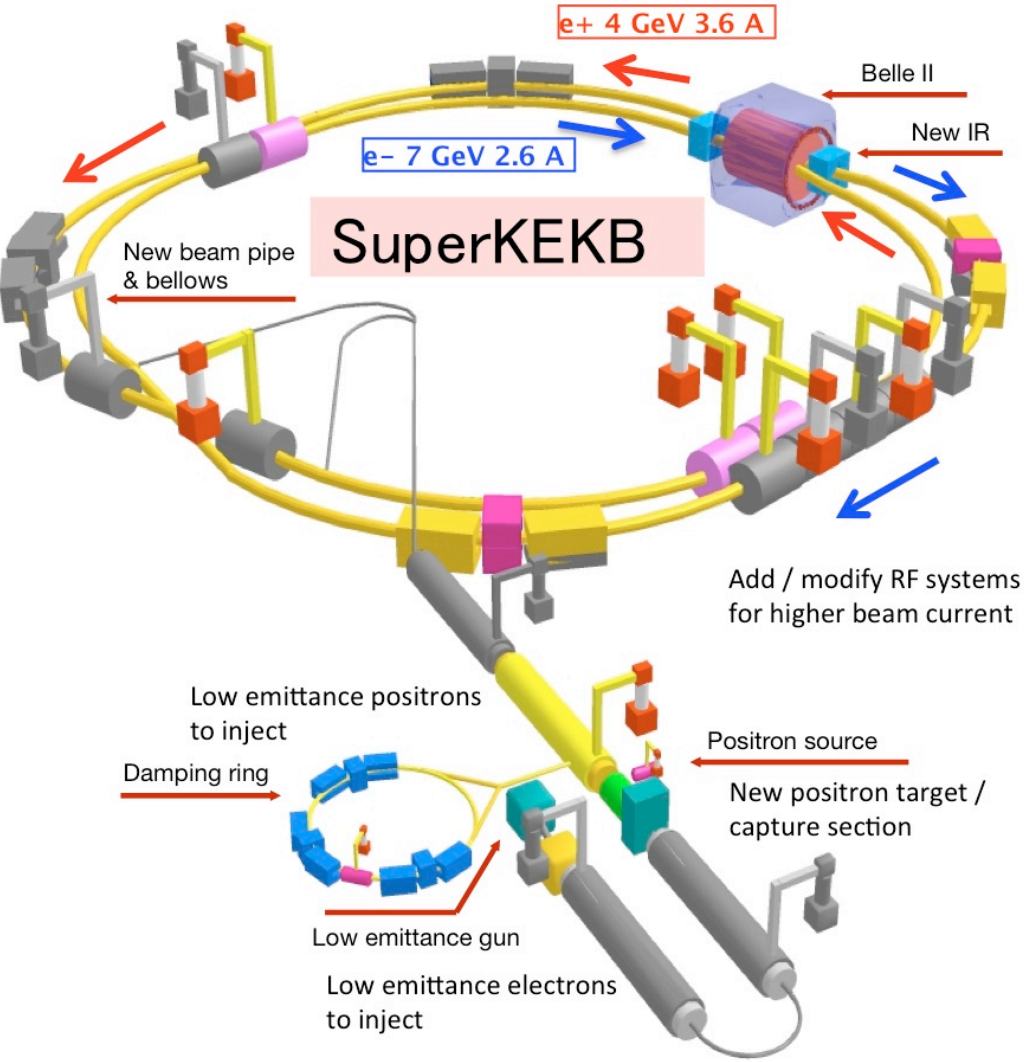
@ Lattice 2018, Michigan, USA

1. Beginning of new era

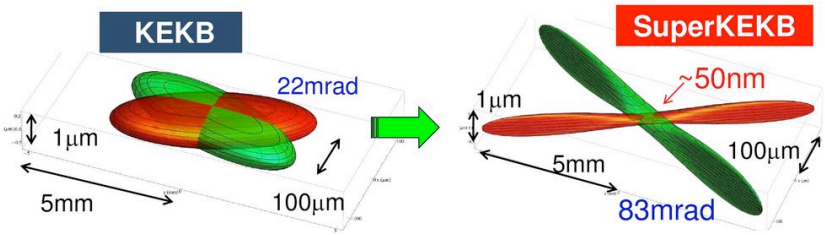
First collision at Belle II

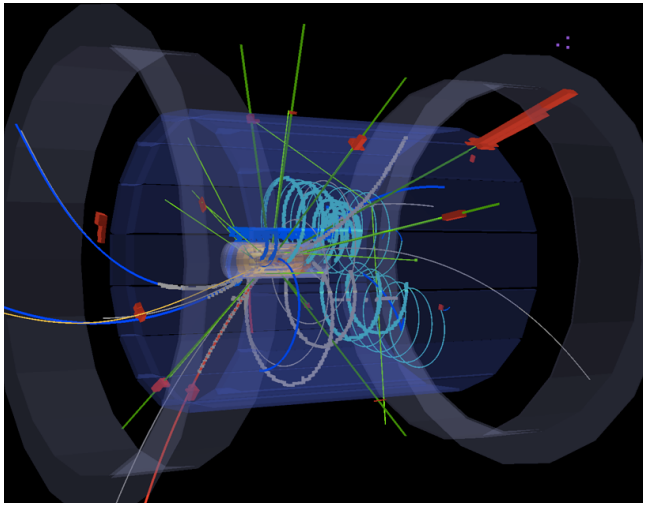
April 26, 2018

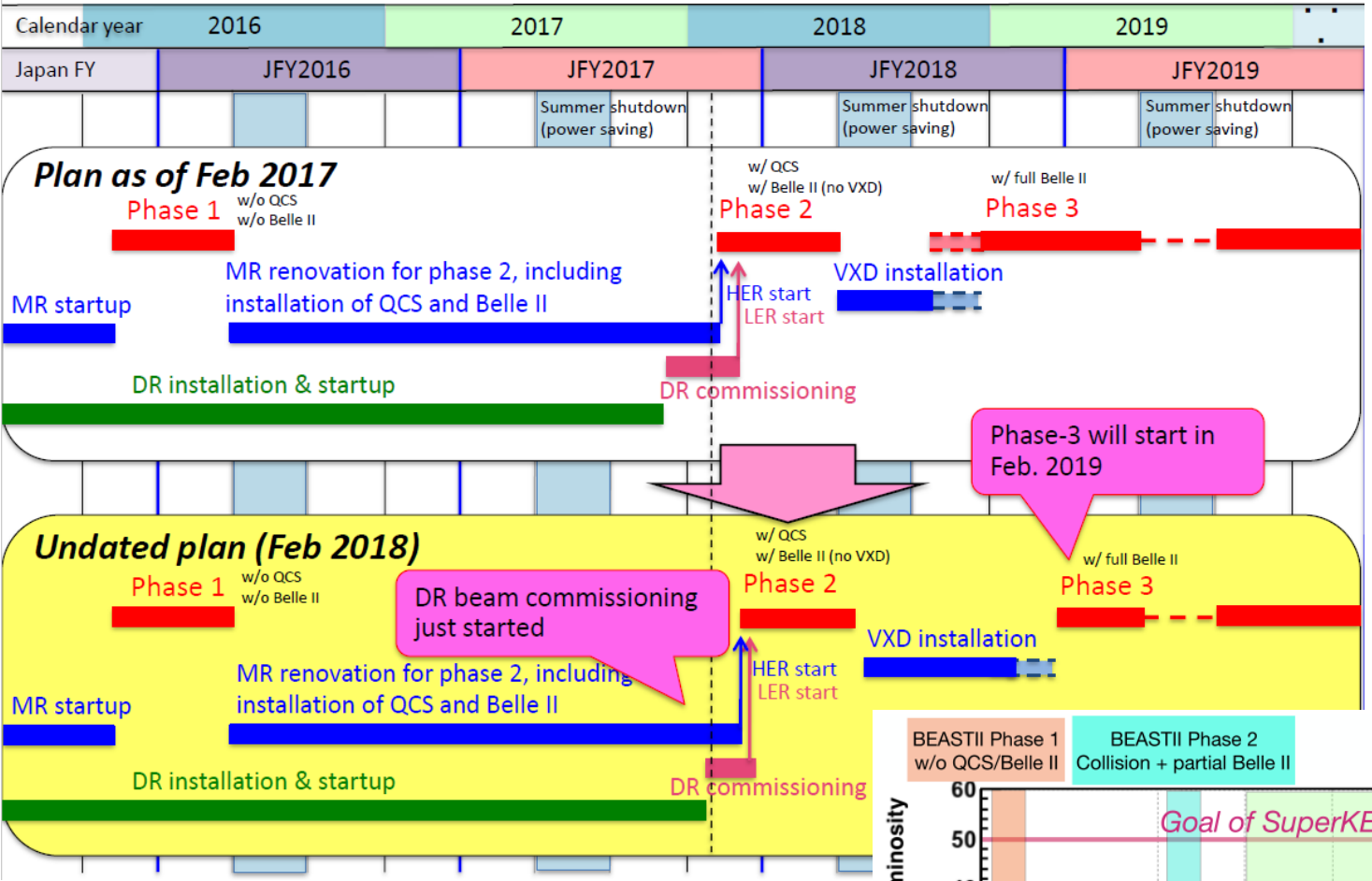


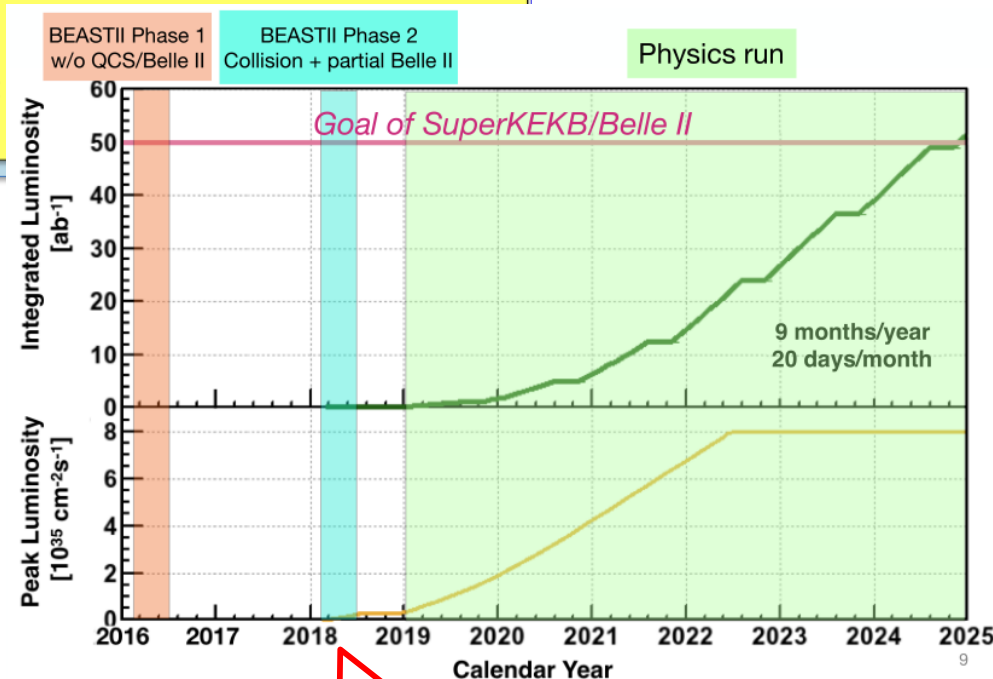


50x luminosity by "nano beam"



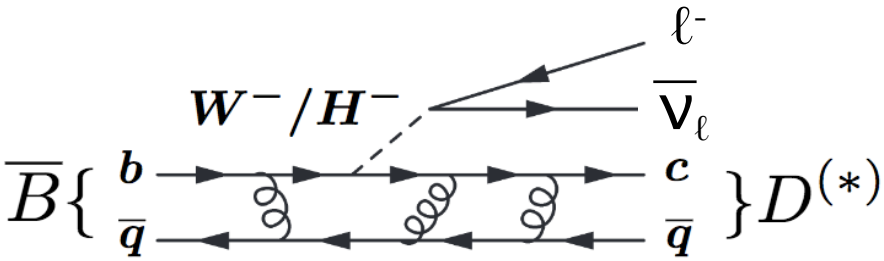


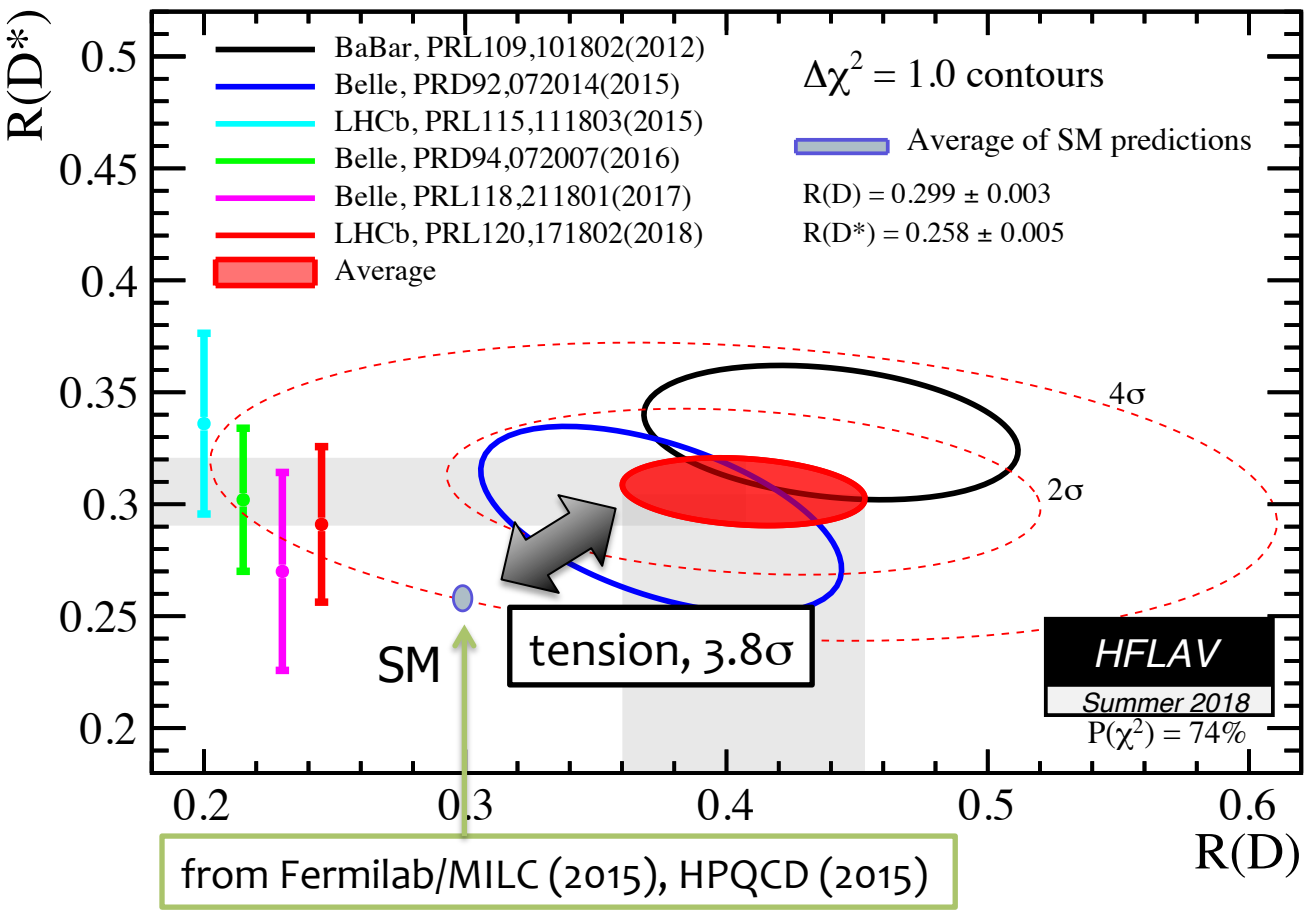

Calendar year
2016
2017
2018
2019
Japan FY
JFY2016
JFY2017
JFY2018
JFY2019
Summer shutdown (power saving)
Summer shutdown (power saving)
Summer shutdown (power saving)
Plan as of Feb 2017
Phase 1
w/o QCS
w/o Belle II
w/ QCS
w/ Belle II (no VXD)
Phase 2
w/ full Belle II
Phase 3
MR renovation for phase 2, including installation of QCS and Belle II
MR startup
HER start
LER start
VXD installation
DR installation & startup
DR commissioning
Phase-3 will start in Feb. 2019
Undated plan (Feb 2018)
Phase 1
w/o QCS
w/o Belle II
w/ QCS
w/ Belle II (no VXD)
Phase 2
w/ full Belle II
Phase 3
DR beam commissioning just started
VXD installation
MR renovation for phase 2, including installation of QCS and Belle II
HER start
LER start
MR startup
DR installation & startup
DR commissioning


BEASTII Phase 1
w/o QCS/Belle II
BEASTII Phase 2
Collision + partial Belle II
Physics run
Goal of SuperKEKB/Belle II
Integrated Luminosity [ab^{-1}]
9 months/year
20 days/month
Peak Luminosity [10^{35} cm^{-2}s^{-1}]
Calendar Year
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025

2. Hints of new physics in B decays

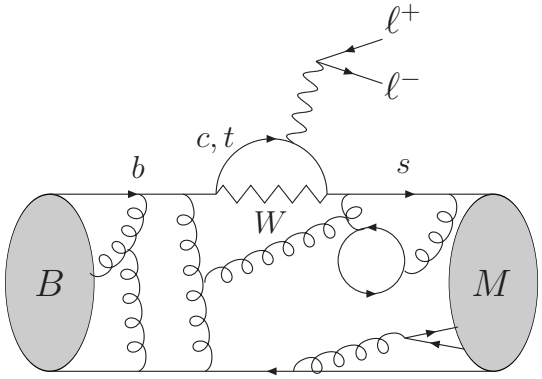
$$R(D^{(*)}) = \frac{\Gamma(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\Gamma(\bar{B} \to D^{(*)}\mu\bar{\nu})}$$

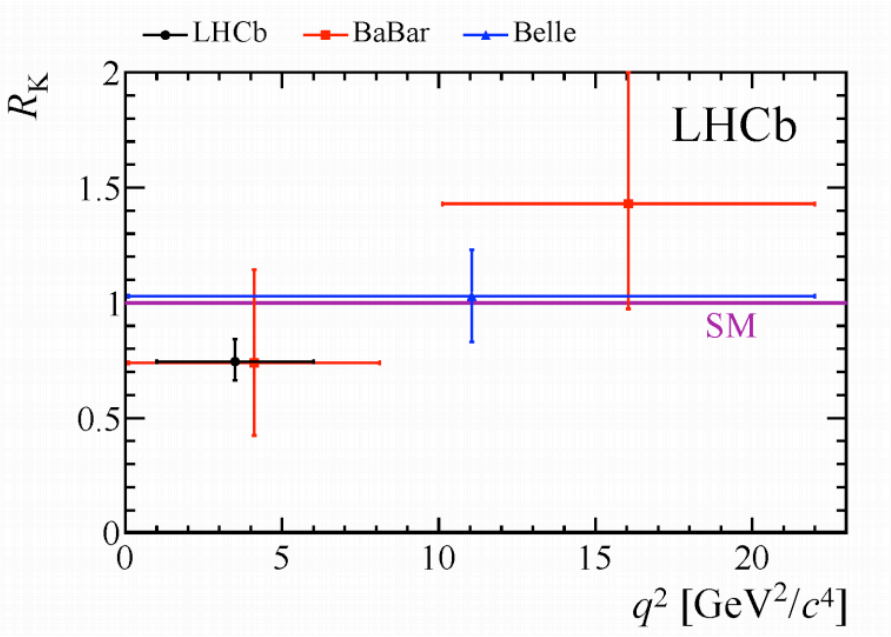


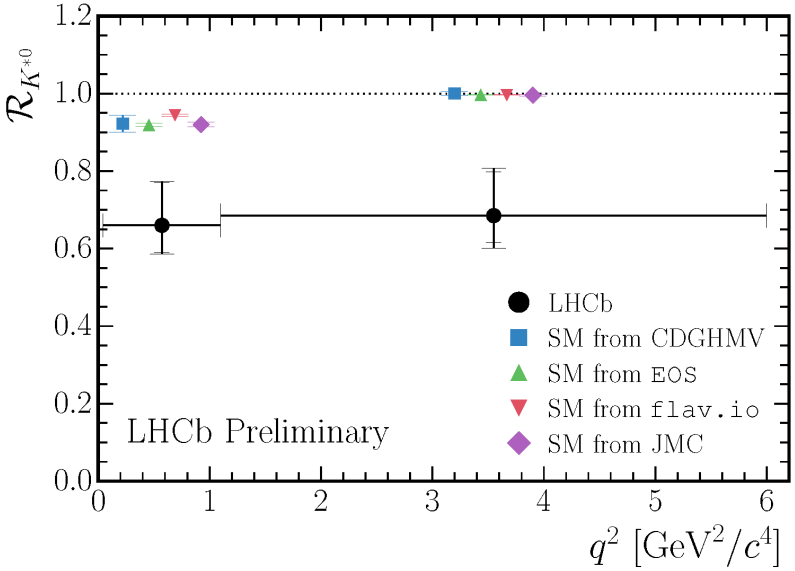


Lattice inputs used for form factors (together with exp data)

$$R(K^{(*)}) = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$



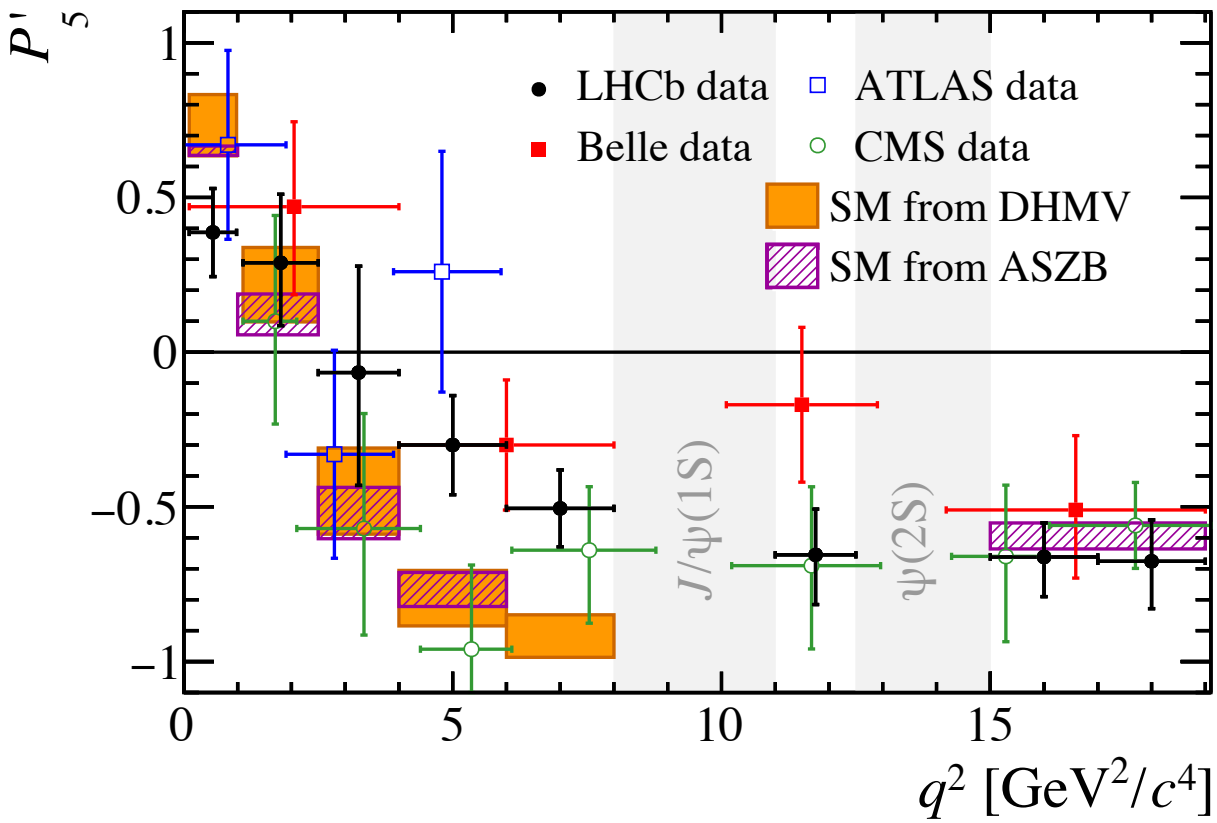




Lepton flavor universality:
not so much room to help from lattice (?)

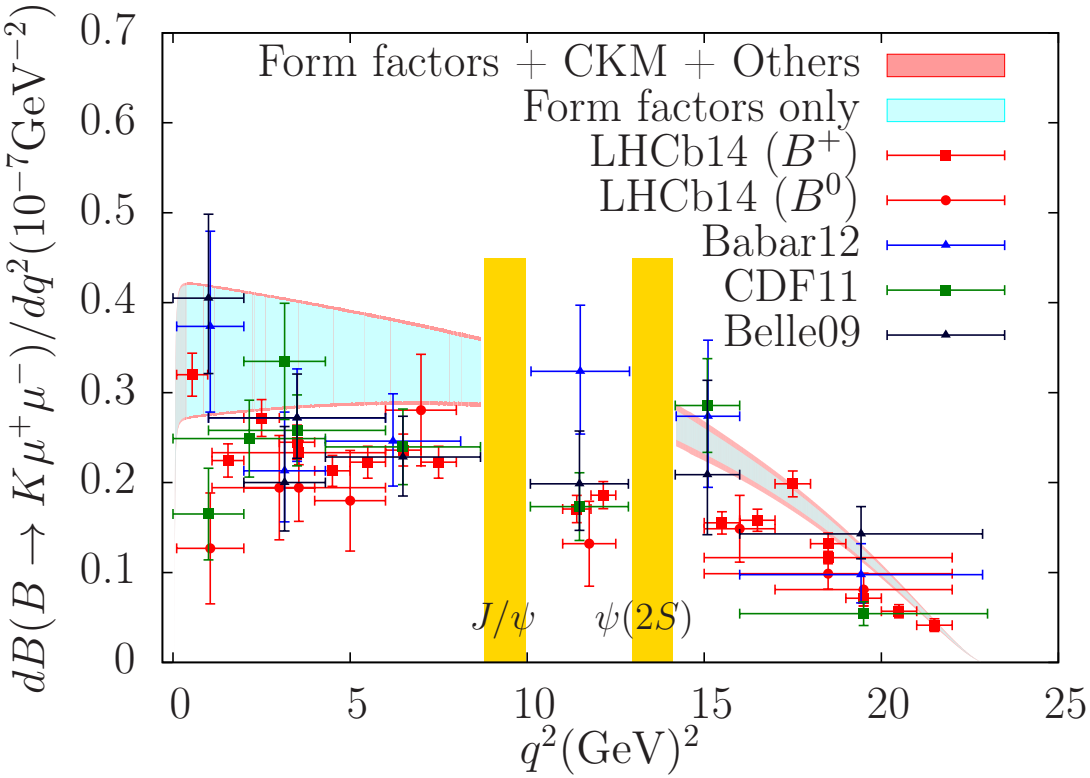
“ P_5' ”:
defined for angular distribution of $B \to K^* \mu^+ \mu^-$

a combined plot from Prisciandaro @ FPCP 2017



Uncertainty from form factors is supposed to be small…

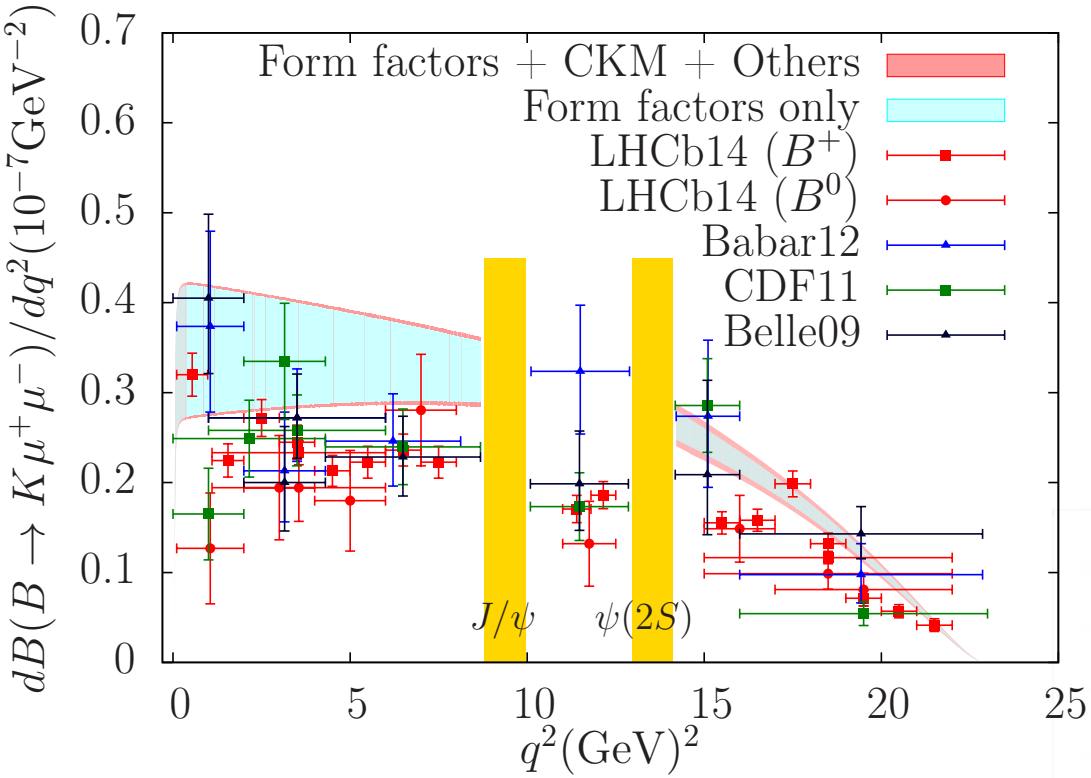
Also for branching fractions:



$B \to Kl^+l^-$

a plot from Fermilab-MILC (Du et al.), PRD93, 034005 (2016).

Exp data are consistently lower than the SM prediction + lattice (and other) form factors.



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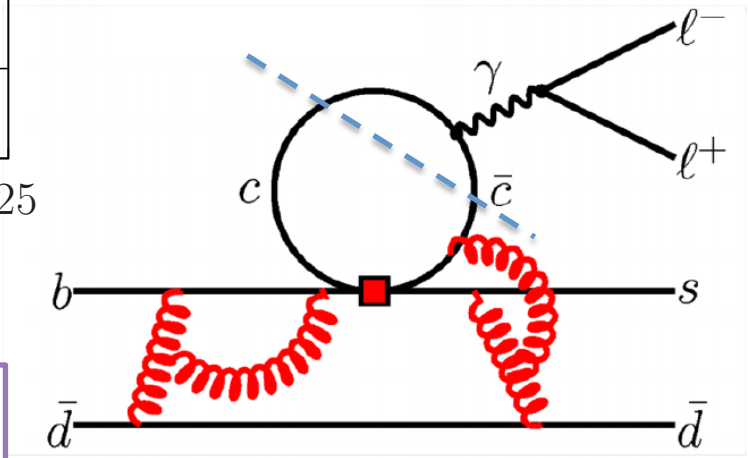
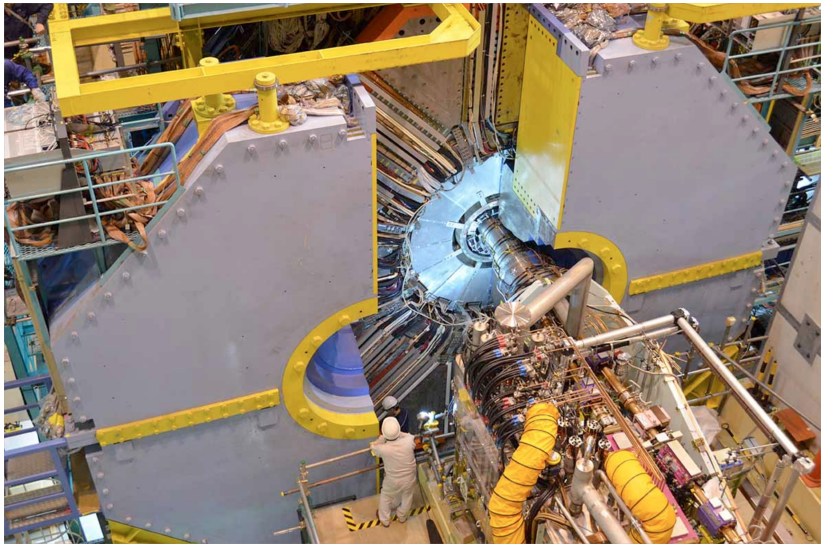


Long-distance effects, such as this, well understood?

Challenges for LQCD

Precision frontier



Going to and beyond 1%!

Understanding unknowns

HINTS… REALLY?

PUZZLES…
INCLUSIVE VS EXCLUSIVE

LONG DISTANCE?

Even 20% helps…

Plans

1. Beginning of new era
2. Hints of new physics in B decays
3. Precision frontier
   - Decay constant reached the 1% milestone. How?
4. A $|V_{cb}|$ story
   - exclusive vs inclusive puzzle? Or, problems in form factors, fits, expansions…?
5. More challenges
   - inclusive from lattice?
   - long-distance in $B \to K\, l^+ l^-$

Disclaimer

- Not comprehensive:
  - Sorry if your important contribution is missed.
- No averages:
  - See FLAG3, arXiv:1607.00299 (or EPJC), and its on-line update
    - B and D decay-const, mixing: M. Della Morte, Y. Aoki, D. Lin
    - B and D form factors: E. Lunghi, D. Becirevic, S. Gottlieb, C. Pena
  - And, FLAG4, appearing early 2019.

- Rather, in this talk, I want to share my (sketchy) understanding with you.

maybe a bit provocative…

Many thanks to…

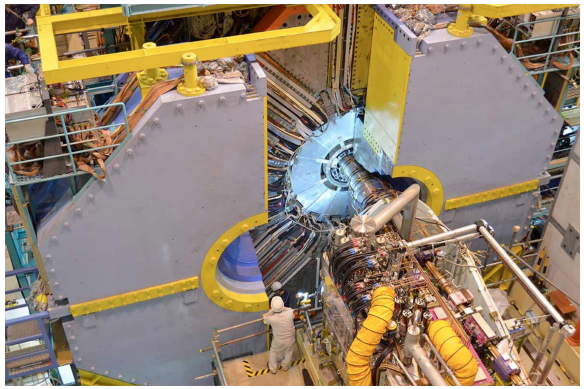
- Huey-Wen Lin, and local organizers!
- Chris Bouchard, Brian Colquhoun, Christine Davies, Aida El-Khadra, Paolo Gambino, Andreas Juttner, Takashi Kaneko, Javed Komijani, Zoltan Ligeti, Katsumasa Nakayama, Chris Sachrajda, Amarjit Soni, Alejandro Vaquero

3. Precision frontier

3.1 B meson decay constant

Precision deserves the cost

experimental result = CKM (or NP) × form factor (= lattice input)



Lattice calculations are equally valuable as experimental data!

A big news in the precision front…

B meson decay constant

- FLAG average (2016): f_B = 186 ± 4 MeV (2+1+1 flavors)

- Fermilab-MILC (Bazavov et al.), arXiv:1712.09262

  f_B = 189.4 ± 1.4 MeV (2+1+1 flavors)

✔ Precision has finally reached sub-percent (0.7%). What does make it possible?

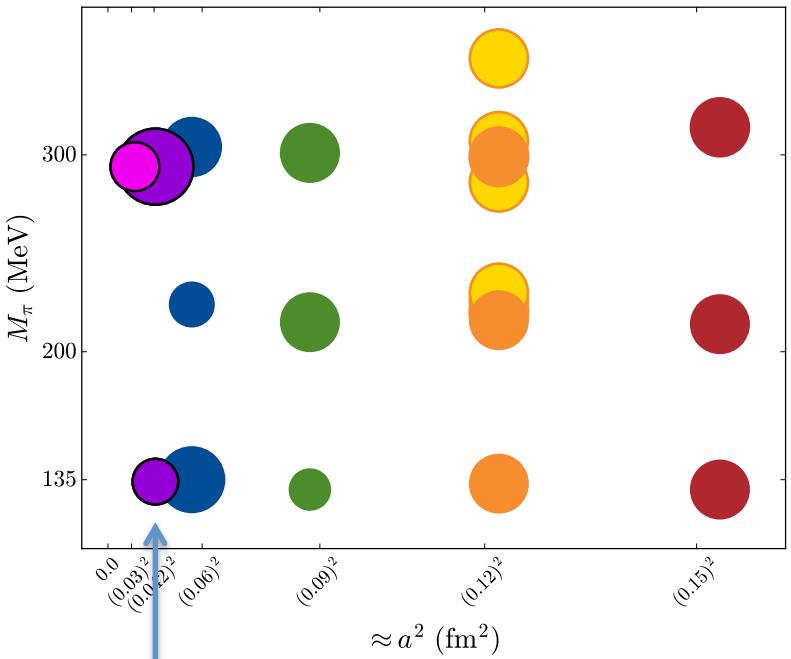
Common wisdom:

"Heavy quark is much harder because its Compton wavelength is shorter than the lattice grid, so …"

A big news in the precision front…

- Fermilab-MILC (Bazavov et al.), arXiv:1712.09262

f_B = 189.4 ± 1.4 MeV (2+1+1 flavors)



$1/a$ = 4.7 GeV

- **HISQ** for both heavy and light
- Down to a = 0.042 fm at the physical pion mass.
- The computational effort:
  - 70 TFlops*yr (generation)
  - 70 TFlops*yr (measurement),

  is a big factor. But, it cannot explain everything.

Discretization effects

El-Khadra, Kronfeld, Mackenzie, PRD55 (1997) 3933.
Follana et al. PRD75 (2007) 054502.

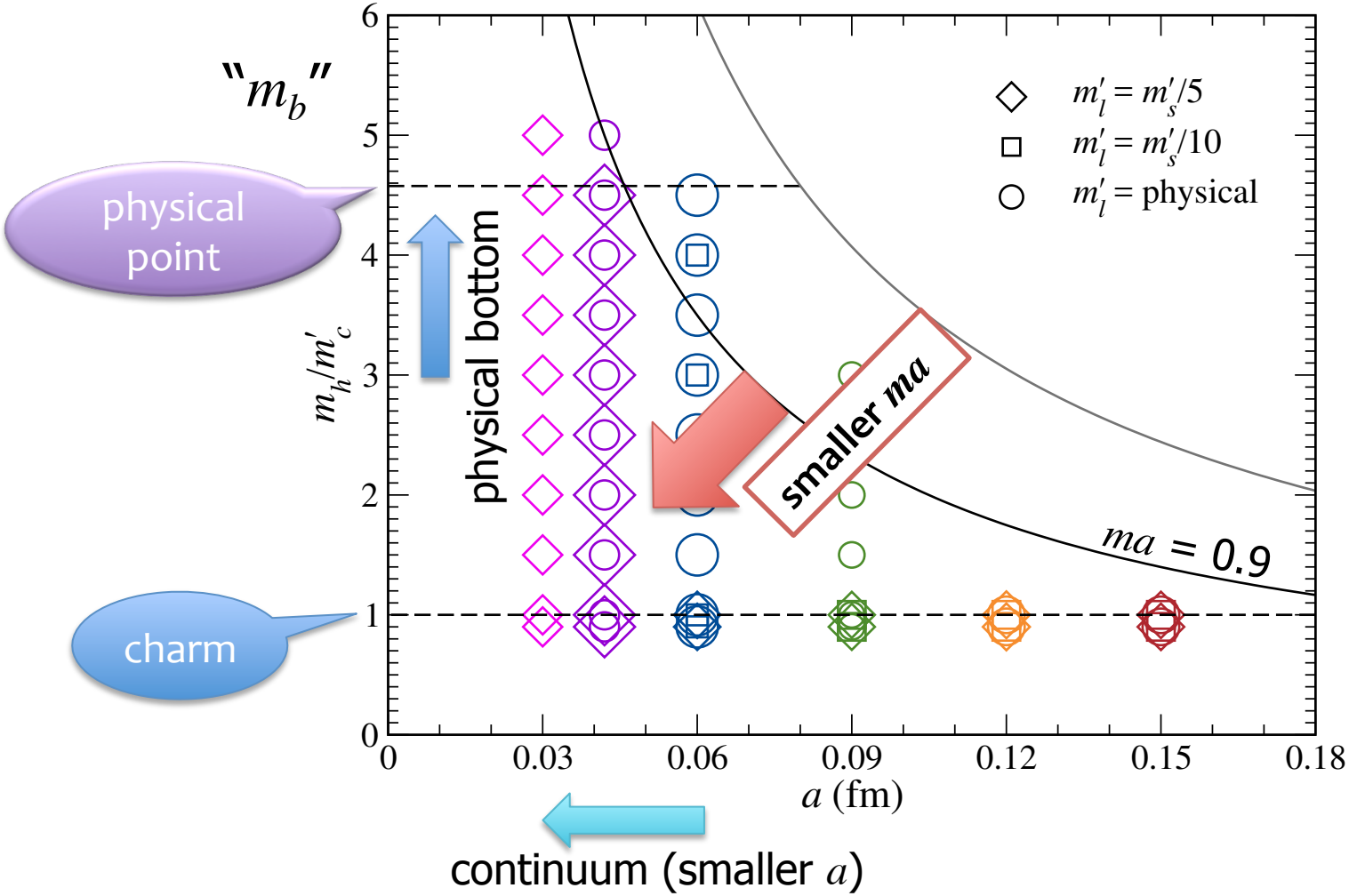
Relative errors (for $am < 1$)

	naïve guess: O(a)-improved	Fermilab interpret. (based on clover)	HISQ (for heavy)	rough estimate for "m_b"
light	$O(a^2\Lambda^2)$ $O(a^2m^2)$	$O(a^2\Lambda^2)$	$O(\alpha_s a^2\Lambda^2)$	~ 0.2%
$O(1/m^0)$		$O(a^2m^2)$	$O(\alpha_s a^2m^2)$	~ 7%
$O(1/m)$		$O(\alpha_s a\Lambda)$	$O(\alpha_s a^2m\Lambda)$	~ 1%
$O(1/m^2)$		$O(\alpha_s a\Lambda^2/m)$	$O(\alpha_s a^2\Lambda^2)$	~ 0.2%

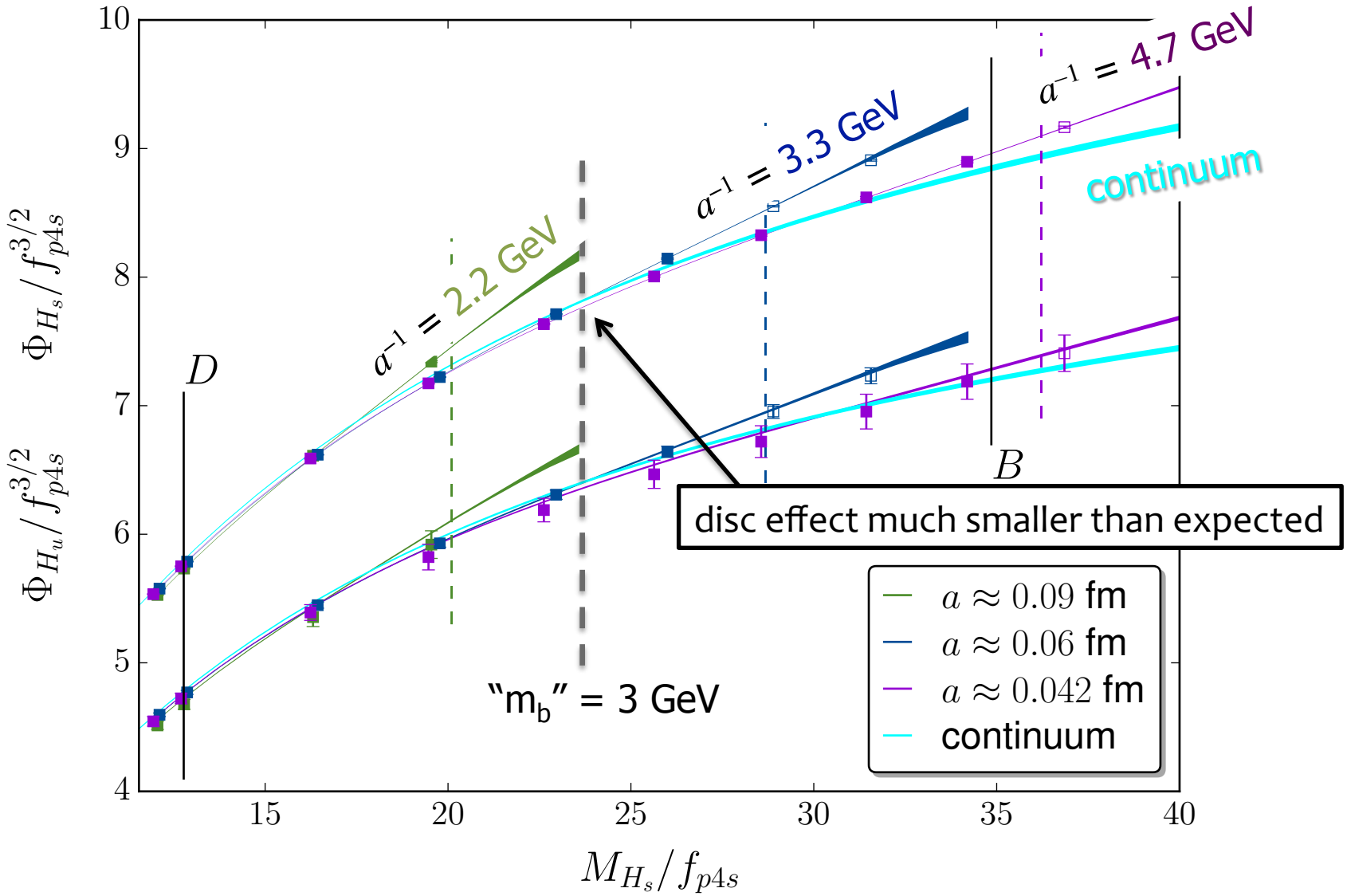
HQET estimate (a la Fermilab)

assuming lattice ensembles of a ~ 0.04 fm (a^{-1} ~ 5 GeV) and taking "m_b" = 3 GeV, Λ_{QCD} ~ 0.5 GeV, α_s ~ 0.2.

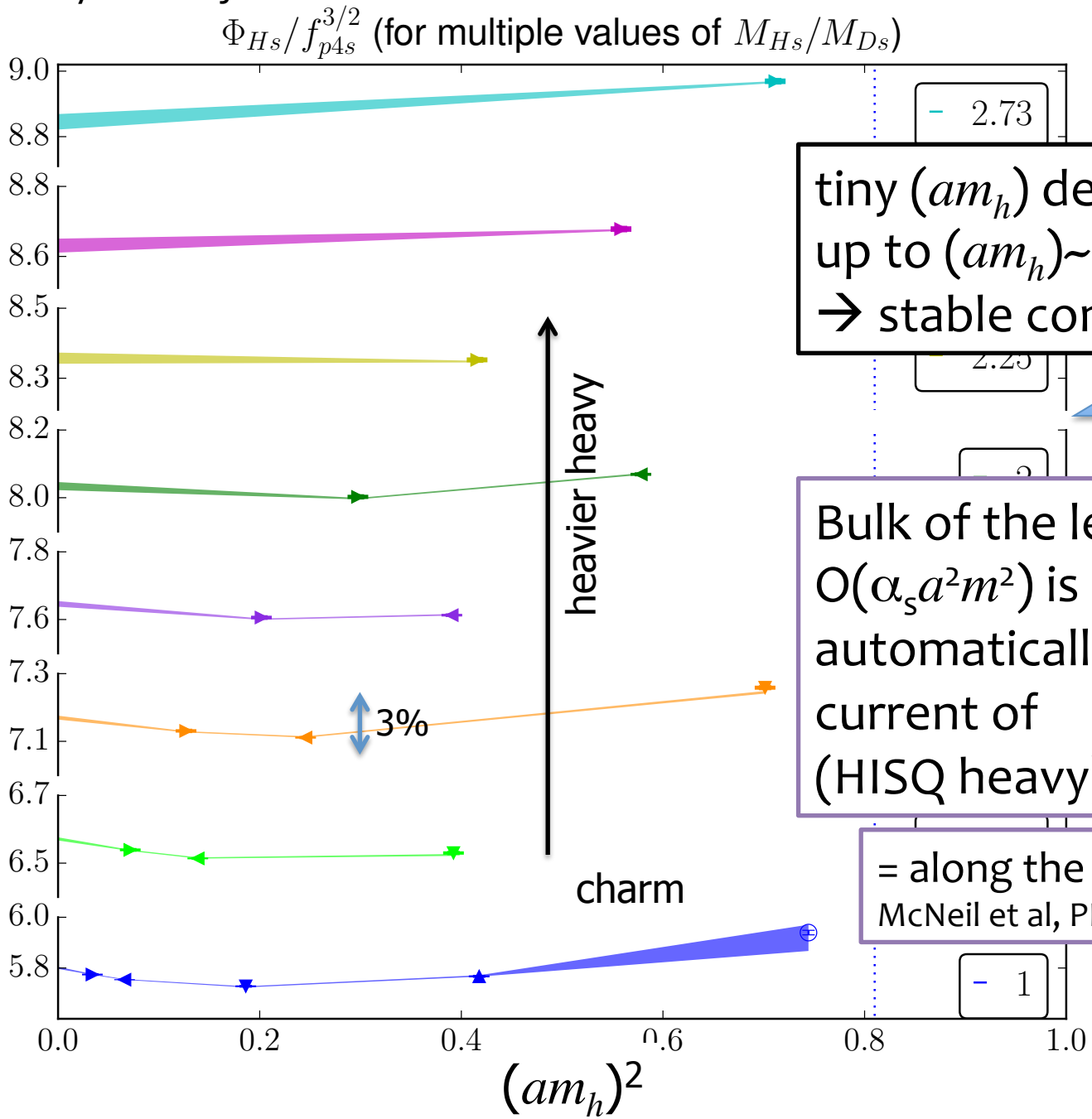
Need to be extrapolated away


"m_b"
physical point
charm
physical bottom
smaller ma
ma = 0.9
$m'_l = m'_s/5$
$m'_l = m'_s/10$
m'_l = physical
m_h/m'_c
a (fm)
0
0.03
0.06
0.09
0.12
0.15
0.18
1
2
3
4
5
6
continuum (smaller a)

"Global fit" (allowing terms of $1/m^k \times \alpha_s(ma)^n$)



courtesy of Komijani



tiny (am_h) dependence, ~ 3%, up to (am_h)~0.8
→ stable continuum limit

Bulk of the leading error of $O(\alpha_s a^2 m^2)$ is eliminated by automatically renormalized current of
(HISQ heavy) x (HISQ light).

= along the strategy of HPQCD
McNeil et al, PRD85, 031503 (2012)

Strategies for heavy quarks

Effective theories [HPQCD, Fermilab/MILC, RBC/UKQCD, Seoul, …]
- NRQCD, Fermilab (and related)
- Need operator matching → $O(\alpha_s^2)$

Effective theory [ALPHA]
- HQET + 1/m corrections
- Non-perturbative matching through step scaling

Relativistic [HPQCD, RBC/UKQCD, JLQCD, ETM, …]
- HISQ, DW, … any
- Need small lattice spacing, with improved action, renormalization done

+ Fermilab/MILC

Brute force
= simple solution…

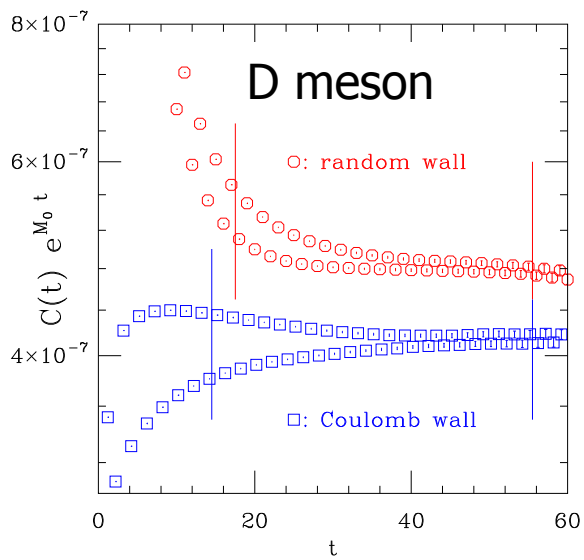
Statistical error

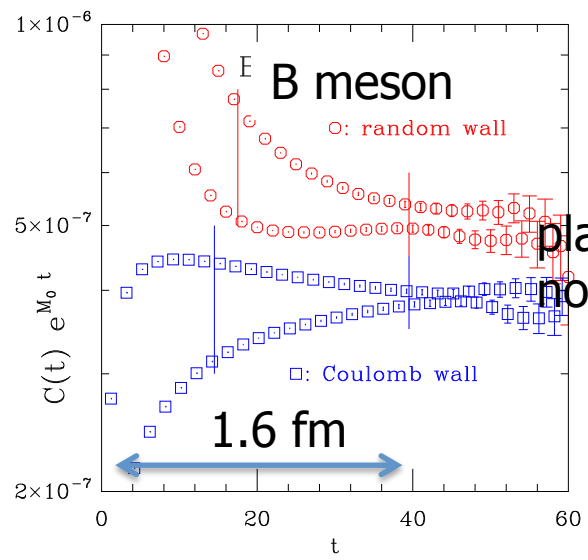
Growing noise of correlators (Lepage (1989))

$$\frac{\delta C_{HL}(t)}{C_{HL}(t)} \propto \exp\left[\left(m_B - \frac{m_{\eta_b} + m_\pi}{2}\right) t\right]$$

~ 0.5 GeV = (0.4 fm)$^{-1}$

- "Doubling (x2.7) time" much worse than for B_s (0.6 fm) or D (0.65 fm).

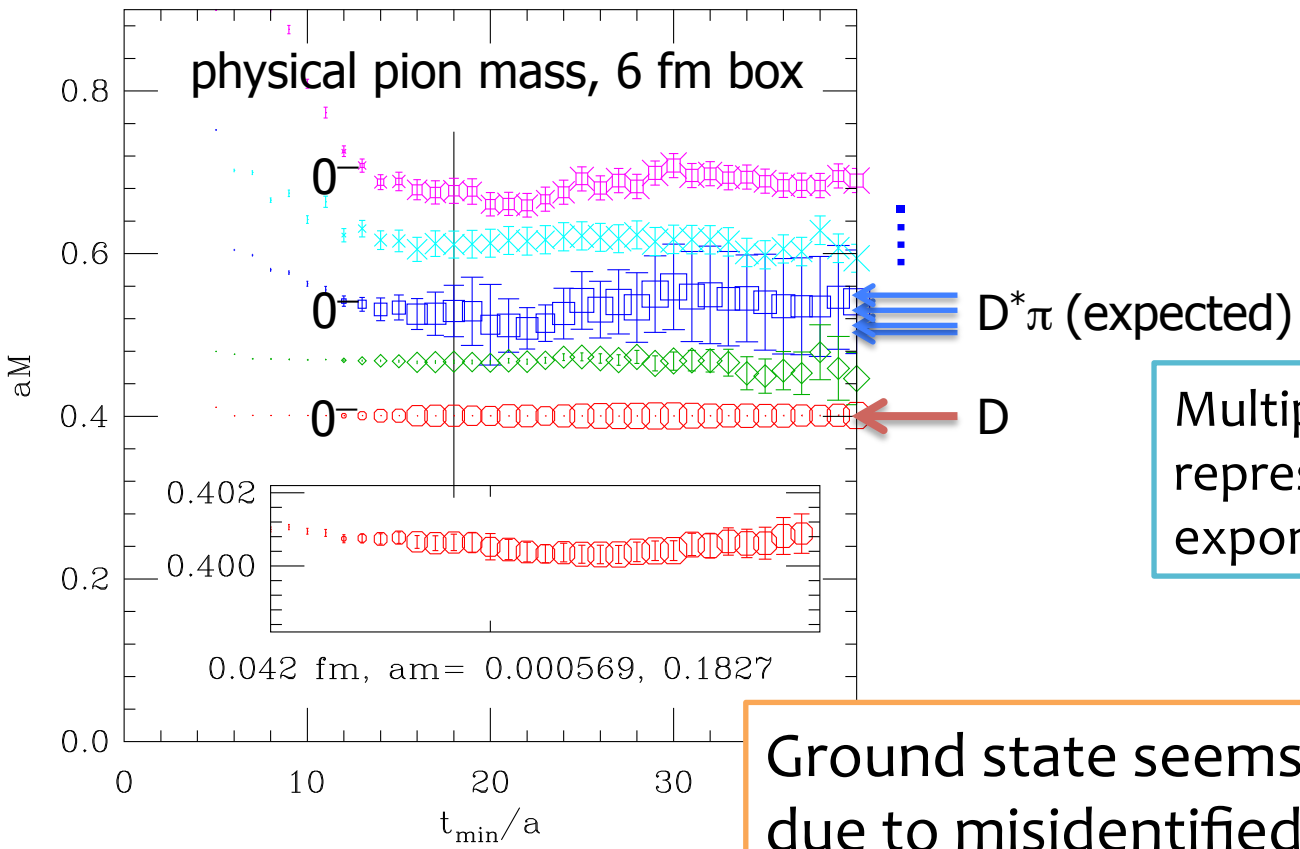




plateau not reached

MUST fit including excited states

ex) D meson + $D^*\pi$ continuum (discrete on the lattice)



Multiple states are represented by a single exponential, or two exp's.

Ground state seems stable. Any bias due to misidentified excited states…?

Physical point lattice is a ~~nightmare~~ big challenge

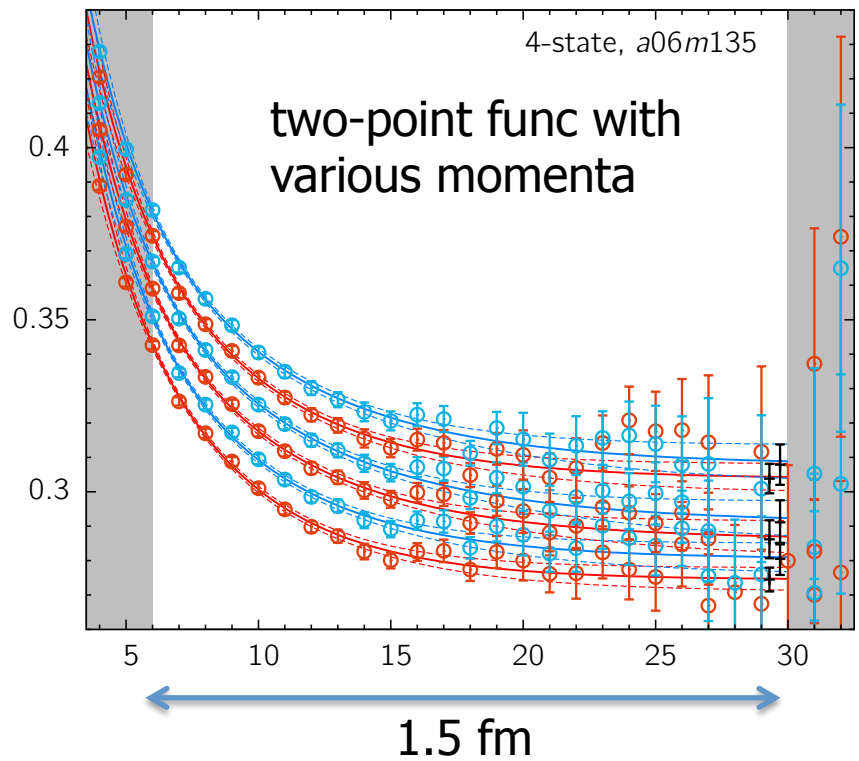
- Bad signal-to-noise; fit before plateau.
- Excited state energy is lower ($\sim m_\pi$).
- Excited states are denser ($\sim 1/L$).

Two- (or three-) exponential model is not really correct.
The ground state might be affected. Small, maybe?

noise reduction techniques: all-mode averaging (RBC), …
+ Multi-level (Giusti @ Lattice 2017; Nada, Mon, 15:00)

Lessons from nucleon?

PNDME, PRD96, 114503 (2017)
Nucleon at the physical point



Baryons are harder:

- Noise "doubling time" ~ 0.27 fm
- 4-state fit yields the first excited state energy

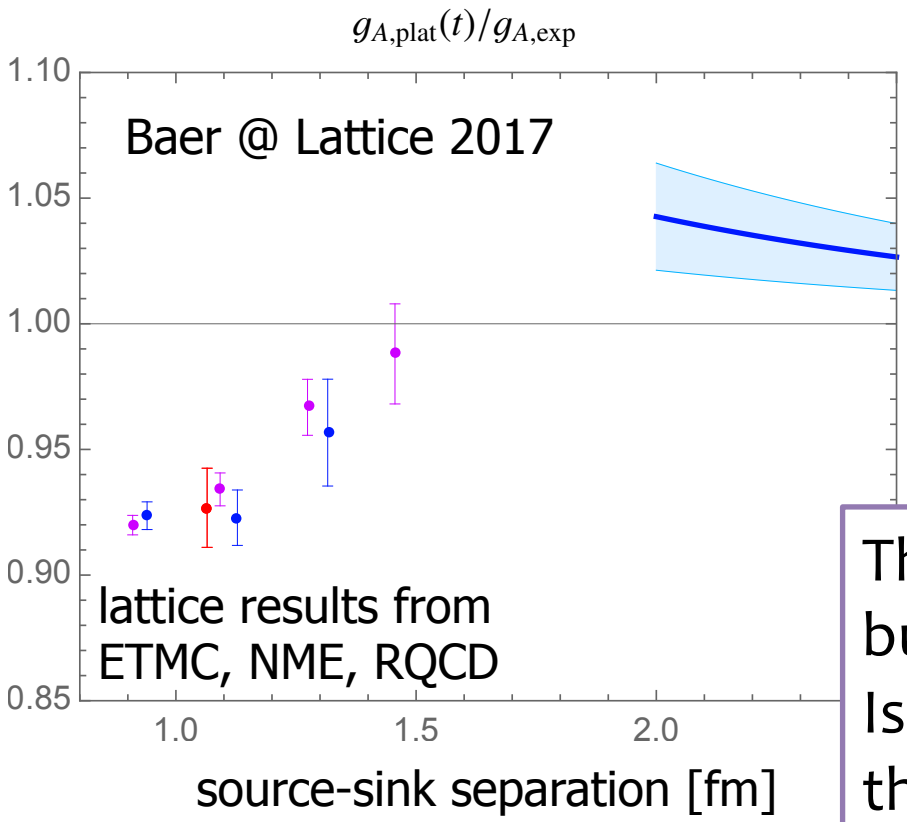
  ΔE = 0.82(11) GeV,
- while the expected $N\pi$ state should give

  ΔE = 0.29 GeV.

Source of the excited-state contamination is not well understood.

Lessons from nucleon?

Expected bias to g_A from the excited states:



ChPT estimate of the Nπ contamination, dying very slowly.

The $N\pi$ state should exist, but we are not yet there. Is the ground state robust, then?

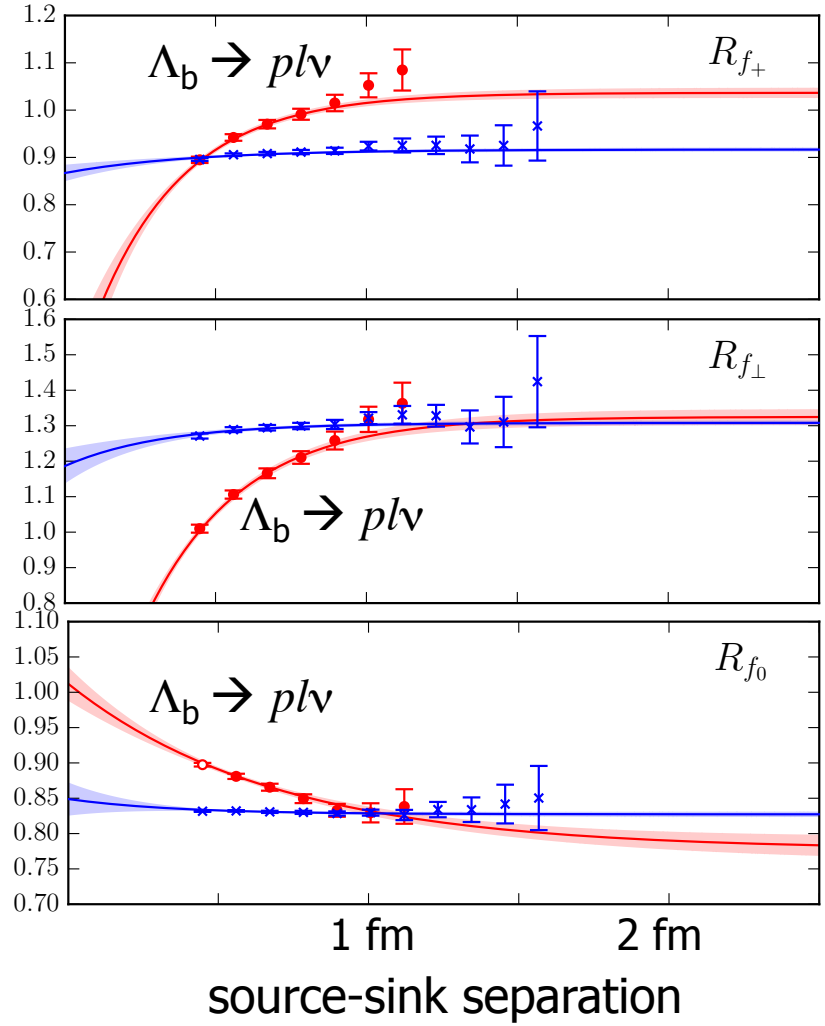
Even tougher...

$\Lambda_b \to p l \nu$ form factor:

Detmold, Lehner, Meinel, PRD92 (2015) 034503.

- Noise "doubling time" ~ 0.25 fm
- Signal dies much earlier than a plateau.
- fit-before-plateau with multi-exp

Looking at χ^2 and fit stability within a narrow fit range enough? Is the ground state extraction robust??



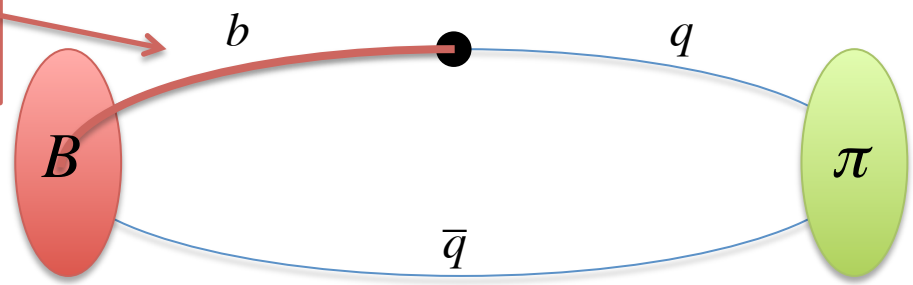
3. Precision frontier

3.2 Heavy to light decays

$B \to \pi$, $B_s \to K$, …

Doubly tougher:

Disc effect for heavy



associated Disc effect

finite mom: ~ 2.6 GeV/c

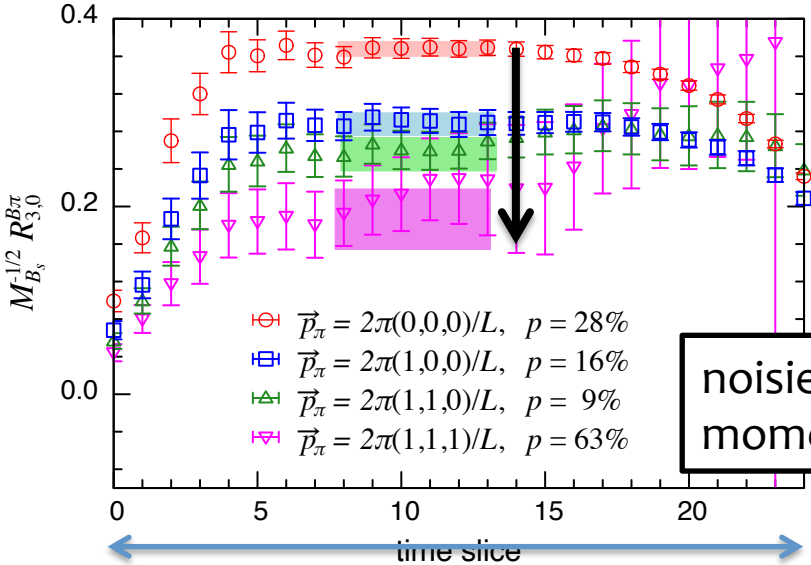
Bad S/N for heavy-light

Bad S/N due to mom

$$\frac{\delta C_\pi(t,\mathbf{p})}{C_\pi(t,\mathbf{p})} \propto \exp\left[(E_\pi(\mathbf{p}) - m_\pi)t\right]$$

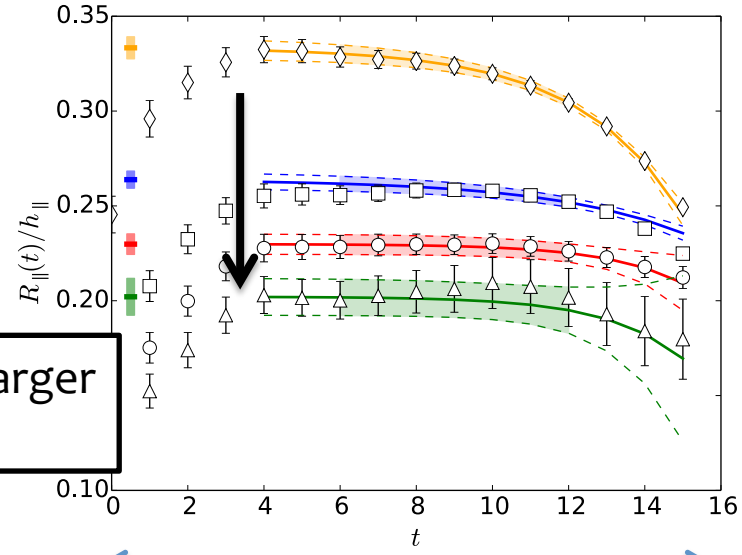
"Noise doubling time" ~ $1/\boldsymbol{p}$

RBC/UKQCD, PRD91, 074510 (2015)



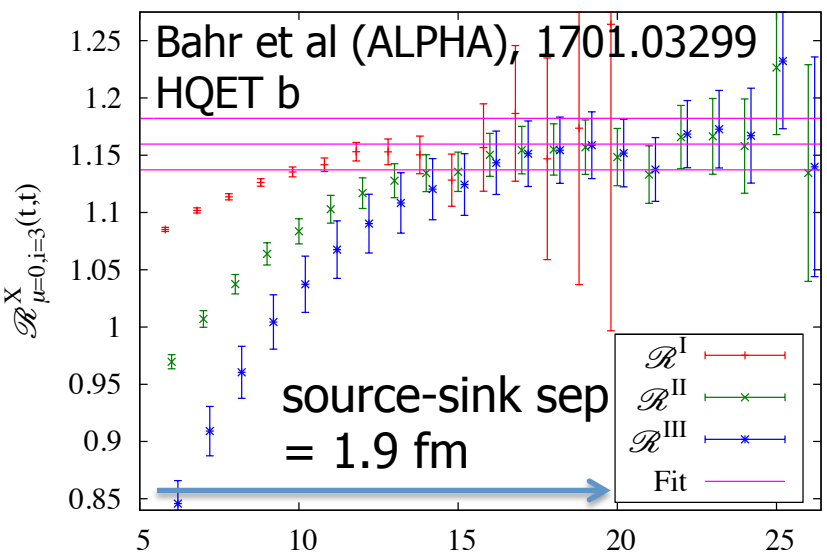
2.1 fm

Fermilab/MILC, PRD92, 014024 (2015)



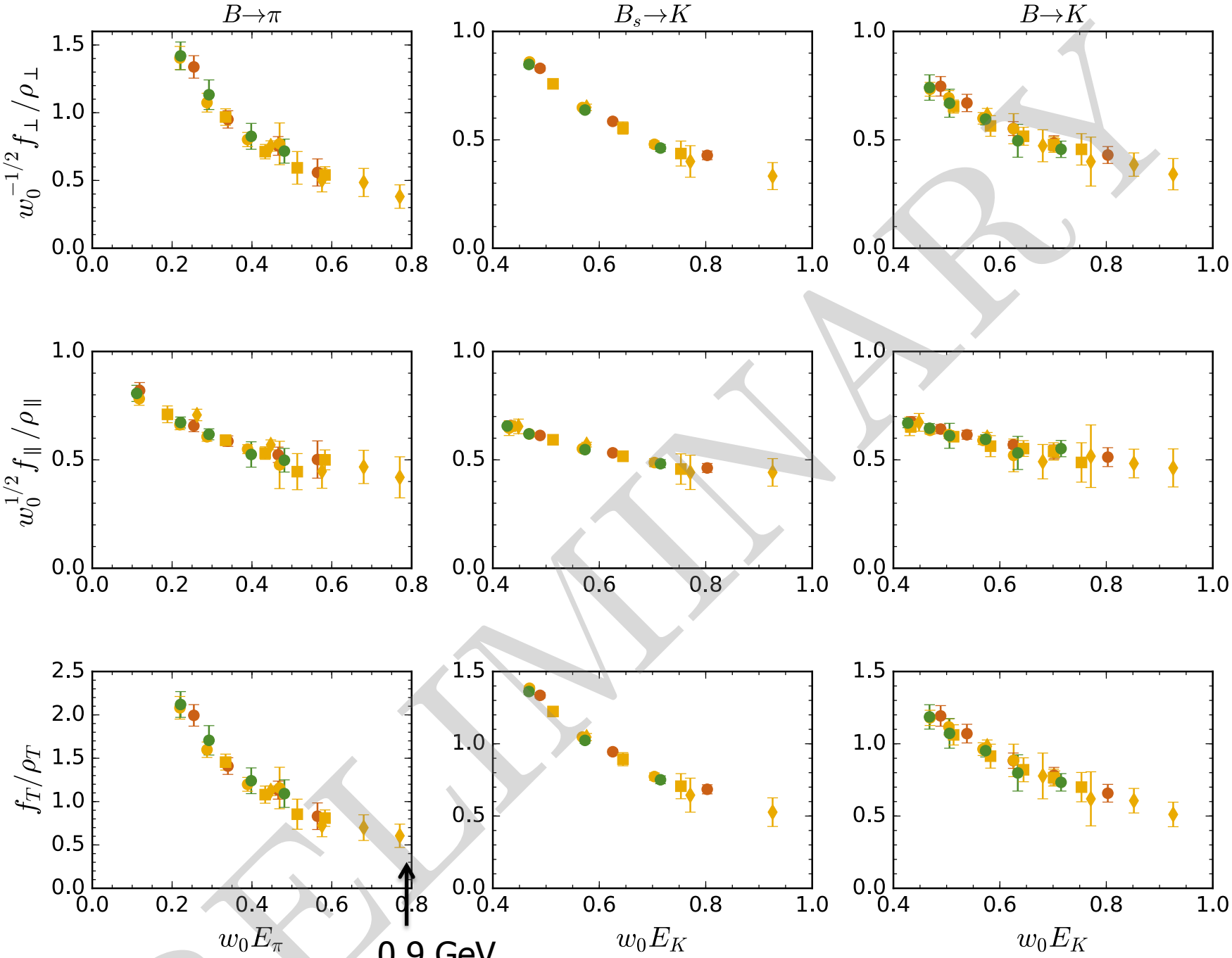
1.9 fm

noisier for larger momenta



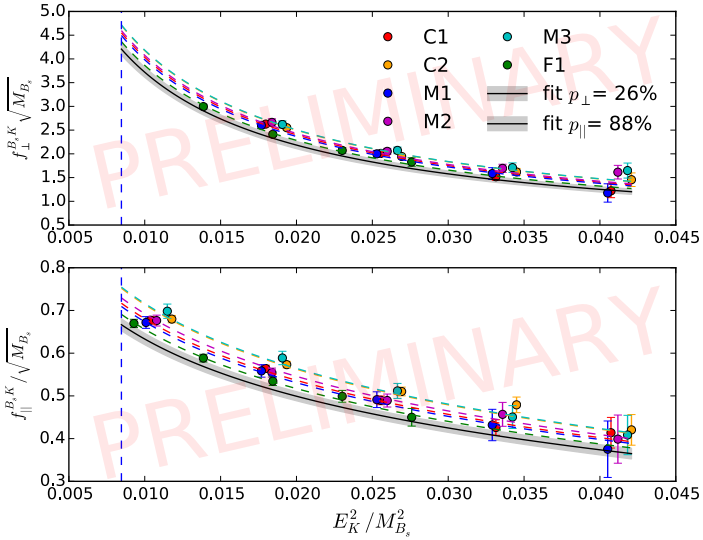
S/N sets the limit on the value of q^2 one can reach. Ground-state extraction is delicate in any case.

Gelzer [Fermilab/MILC] @ Lattice 2017 ➔ Fermilab/MILC (Gelzer, Fri, 17:10)

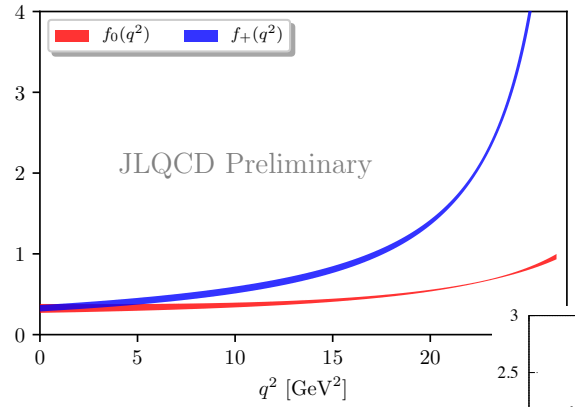


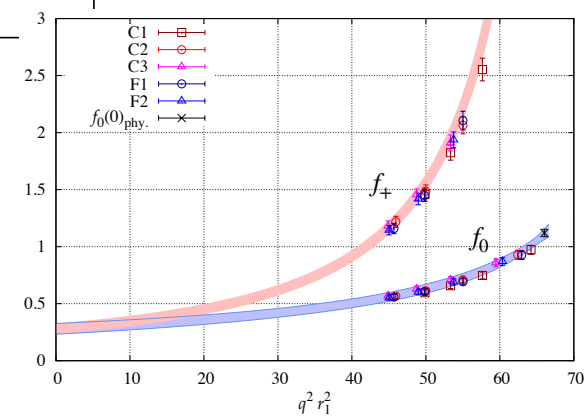
More to appear at this conference

RBC/UKQCD (Witzel, Fri, 17:30)
$B_s \to K$



JLQCD (Colquhoun, Thu, 10:10)
$B \to \pi$





ETM (Riggio, Fri, 16:30)

HPQCD (Bouchard, Fri, 16:50)

Summary of this section

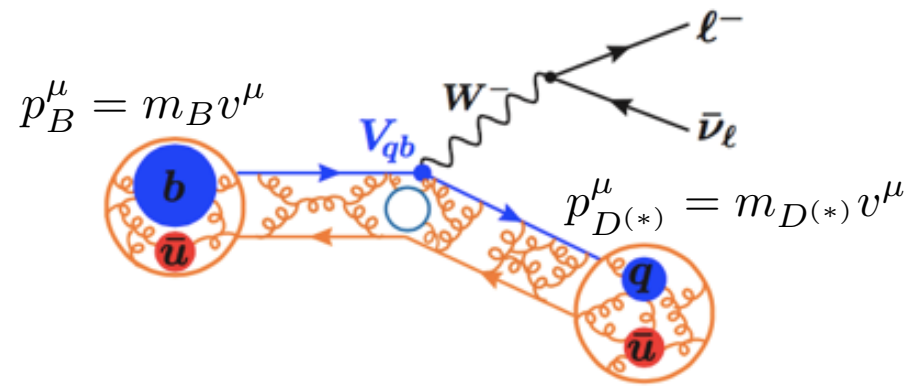
- Precise calculation is getting more realistic, even for b quark.
  - Improved action, especially HISQ. Much better than (my) naïve estimate.
  - Cross-check with other formulations highly desired.

- A serious, and general, problem
  - The enemy is the **noise**.
  - More difficult for 3-point functions.
  - "Fit before plateau" necessary. Then, we need to understand the excited states better.

4. A $|V_{cb}|$ story

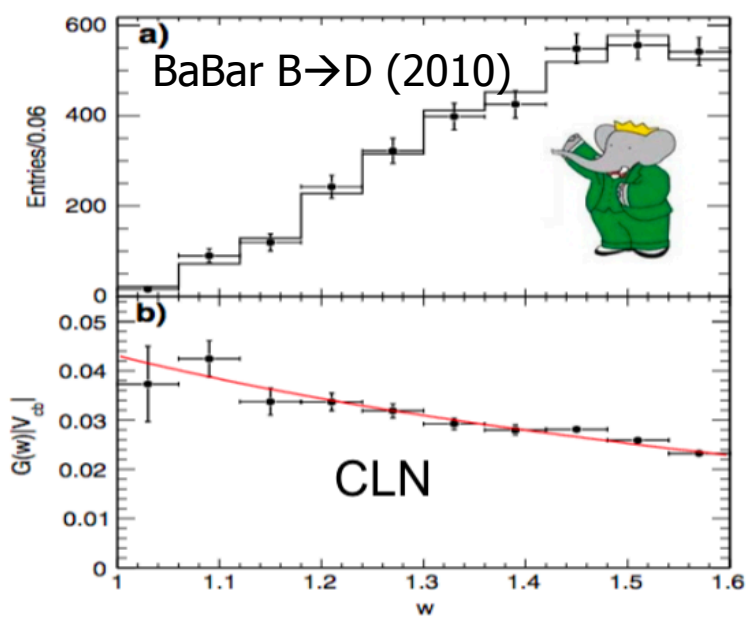
$|V_{cb}|$ from $B \to D^{(*)} l\nu$

Well-known story… :



Due to heavy quark symmetry:

- Various form factors reduce to the universal Isgur-Wise function $\xi(w=v.v')$, or zero.
- Known in the zero-recoil limit: $\xi(1)=1$
- Correction to the $m \to \infty$ limit starts from $1/m^2$ in the zero-recoil limit.



First extrapolate to the zero-recoil limit, then use the lattice input.

Zero-recoil form factor

Lattice calculation is precise, because

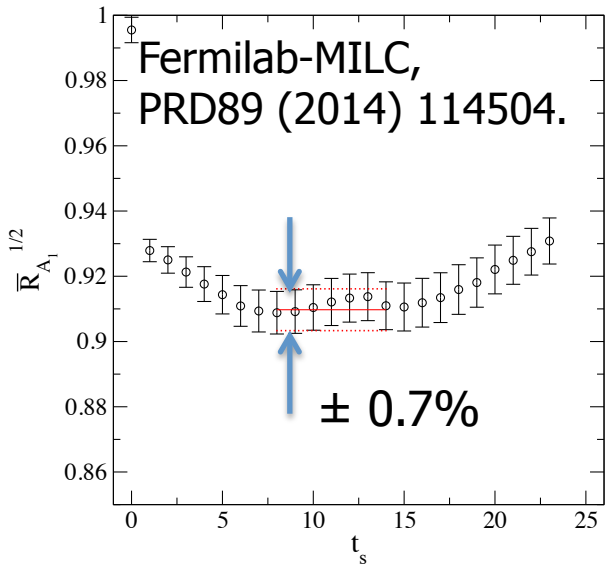
- Stat and syst errors cancel in the double ratio

$$|h_{A_1}(1)|^2 = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma_4 b|\bar{B}\rangle}$$

- Effectively, one measures the deviation from the heavy quark limit

$$|h(1)| = 1 + O(1/m_{c,b}^2)$$

= "10% deviation from 1 is calculated at a 10% accuracy."



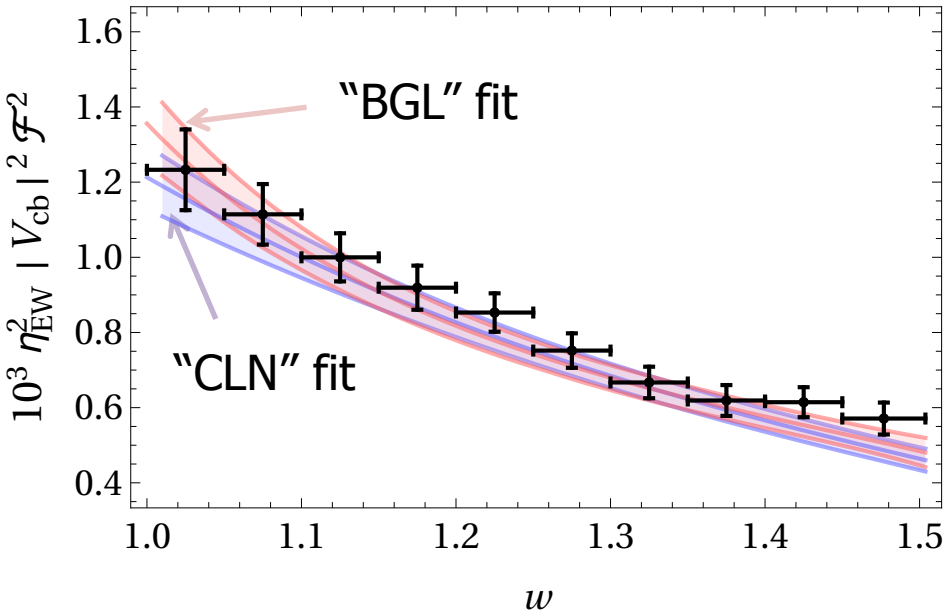
The absence of $O(1/m)$, Luke's theorem, is the key for the precision.

	HPQCD	Fermilab/MILC	$\lvert V_{cb} \rvert$
B → D	Na et al, PRD92, 054510 (2015) • NRQCD b + HISQ c G(1) = 1.035(40)	Bailey et al, PRD92, 034506 (2015) • improved Wilson for heavy (c and b) G(1) = 1.054(4)(8)	HFLAV average (2016) + lattice ave 0.0398(10)(14)
B → D*	Harrison et al, PRD97, 054502 (2018) • NRQCD b + HISQ c F(1) = 0.895(10)(24)	Bailey et al, PRD89, 114504 (2014) • improved Wilson for heavy (c and b) F(1) = 0.906(4)(12)	HFLAV average (2016) + lattice ave 0.0391(5)(5)
inclusive	N/A		HFLAV (2016) 2.9 σ 0.0422(8)

New (confusing) story

New Belle data (arXiv:1702.01521) for $B \to D^*$ triggered independent analyses by theorists.

Bigi, Gambino, Schacht, PLB769 (2017) 441.



See also,
Grinstein, Kobach, PLB771 (2017) 359;
Bernlochner, Ligeti, Papucci, Robinson, PRD96, 091503(R) (2017).

"BGL" fit yields higher $|V_{cb}|$ consistent with "inclusive". Most previous results relied on "CLN".

(for the form factor shape)

“CLN” and “BGL” ??

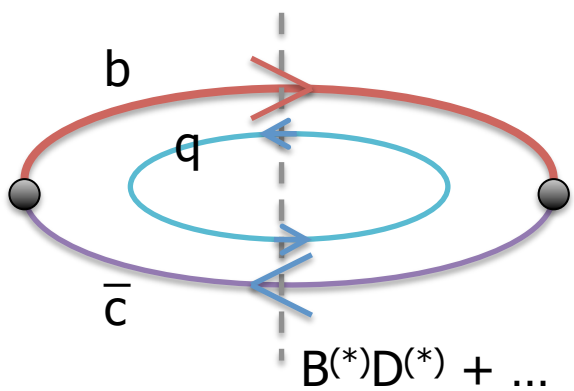
Dispersive bound:

Okubo (1971), Bourrely, Machet, de Rafael (1981);
“BGL”: Boyd, Grinstein, Lebed, PLB353 (1995) 306.

$$\langle J^{\bar{c}b} J^{\bar{b}c} \rangle = \sum_{\text{states}} \int |\langle 0|J|B^{(*)} D^{(*)}\rangle|^2 + \cdots$$

two-point func.

form factors



Relatively simple in the heavy quark limit (form factors are reduced to $\xi(w)$ or 0). At O(1/m), some estimates are involved.

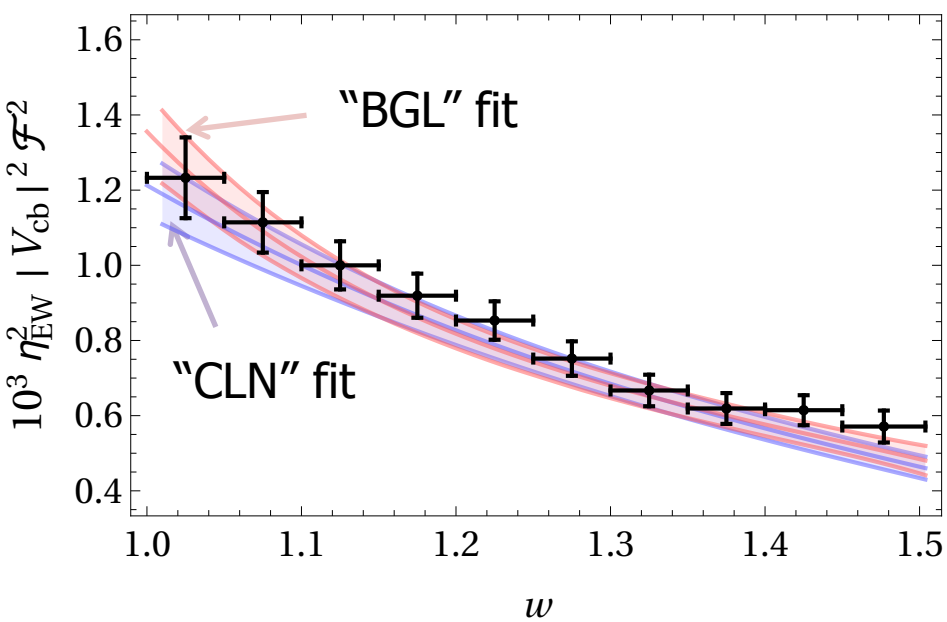
“CLN”: Caprini, Lellouch, Neubert, NPB530 (1998) 153.

"CLN" parametrization includes some estimates of O(1/m):

$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right),$$
$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$
$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}$$

$$\begin{cases} R_1(w) = \dfrac{h_V(w)}{h_{A_1}(w)} \\ R_2(w) = \dfrac{h_{A_3}(w) + r_{D^*} h_{A_2}(w)}{h_{A_1}(w)} \end{cases}$$



- Strong constraint on the curvature.
- Has been the standard in the exp analyses.

Is the 1/m estimate okay?

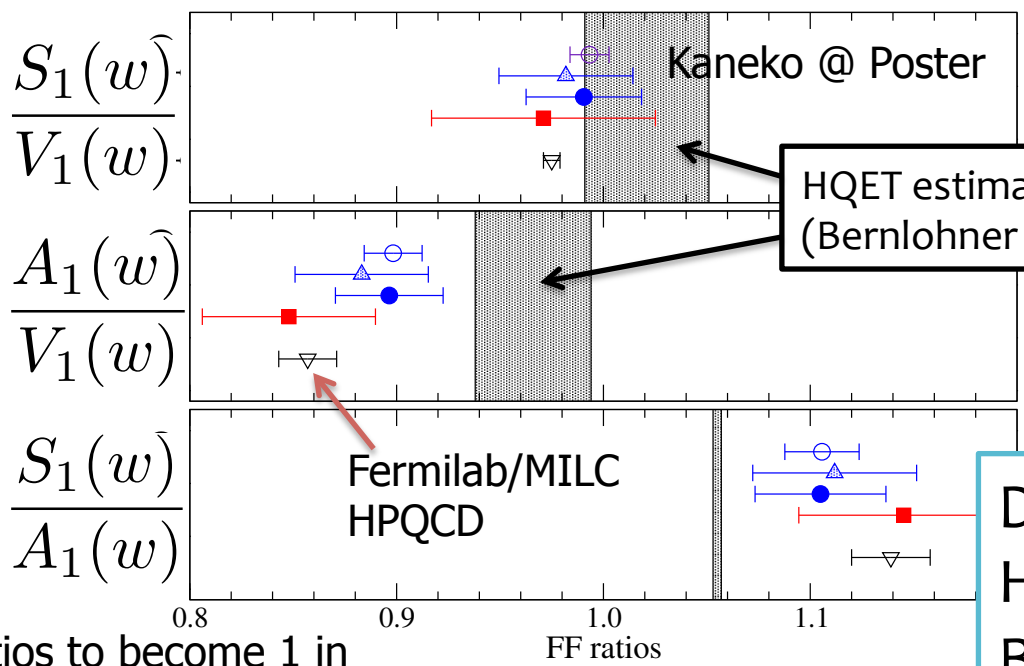
Comparison between lattice and HQET estimate at O(1/m)

Bigi, Gambino, Schacht, JHEP11 (2017) 061.

$$S_1(w) = h_+(w) - \frac{1+r}{1-r}\frac{w-1}{w+1}h_-(w)$$

$$V_1(w) = h_+(w) - \frac{1-r}{1+r}h_-(w)$$

$$A_1(w) = h_{A_1}(w)$$



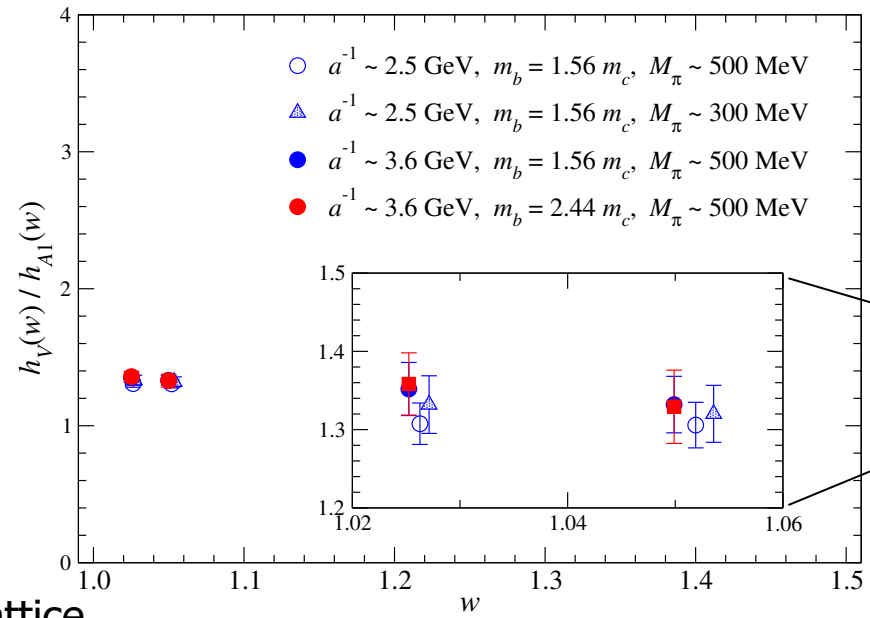
HQET estimate (at 1/m)
(Bernlohner et al, PRD95, 115008 (2017))

Ratios to become 1 in the heavy quark limit

Deviation of O(10%) from HQET estimate is found. But, not 100%.

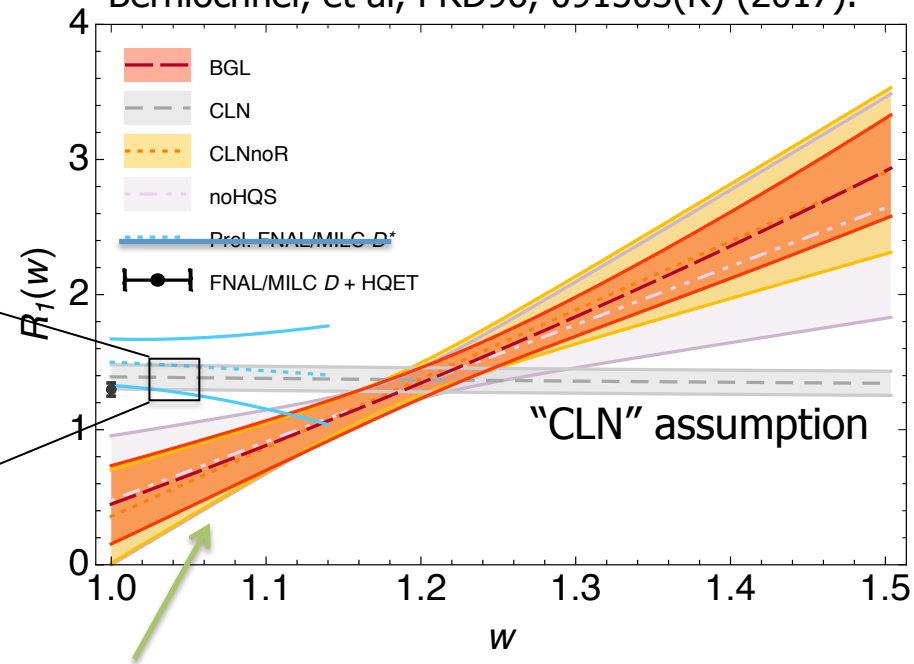
Consistency at O(1/m) ?

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}$$



Lattice
Kaneko @ Poster

Bernlochner, et al, PRD96, 091503(R) (2017).



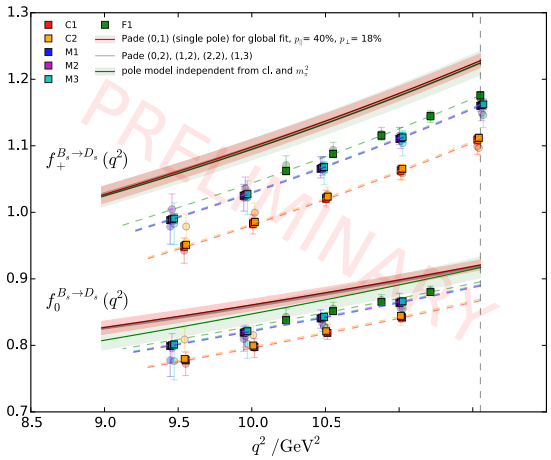
$R_1(w)$ from the "BGL" fit of exp data

See also,
Vaquero, Fri, 14:20

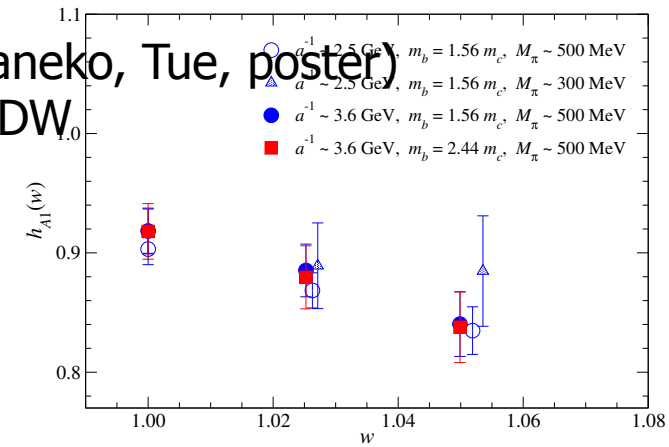
Lattice data favor the "CLN" assumption.
So, "BGL" is not the end of the story.

More to appear at this conference

RBC/UKQCD (Witzel, Fri, 17:30)
$B_s \to D_s$, DW

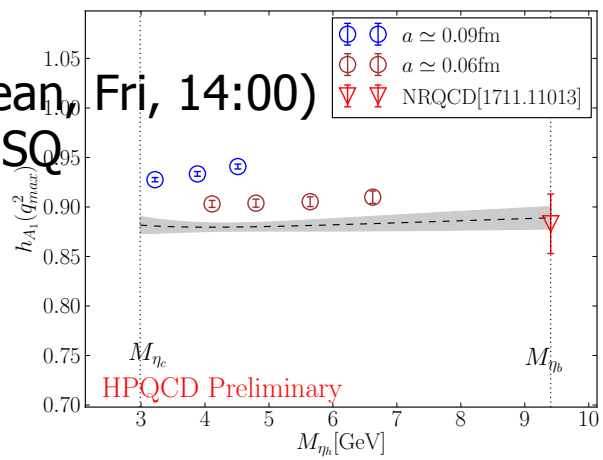


JLQCD (Kaneko, Tue, poster)
$B \to D^{(*)}$, DW

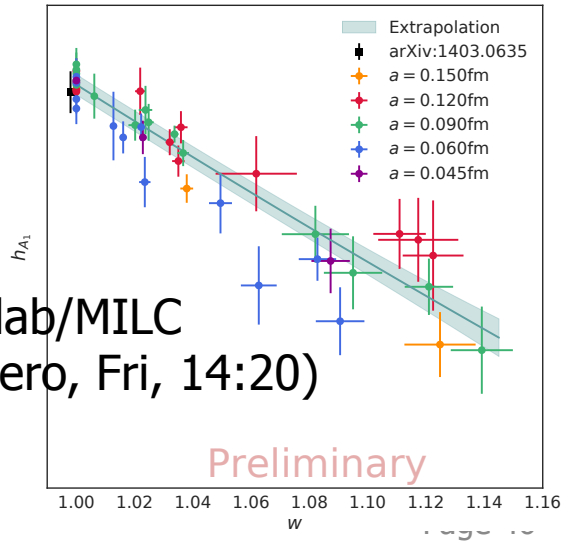


Seoul
(Park, Fri, 14:40)

HPQCD (Maclean, Fri, 14:00)
$B_s \to D_s^{(*)}$, HISQ



Fermilab/MILC
(Vaquero, Fri, 14:20)



Summary of this section

- The situation of exclusive $|V_{cb}|$ is unclear (to me).
  - Need to reanalyze the exp data (esp, BaBar's)
  - Lattice calculations of F(1) and G(1) are precise and consistent among different groups.
  - Slopes needed for direct comparison with exp.
  - Systematic comparison with HQET (and sum rules) is very interesting.

The "inclusive vs exclusive" puzzle is still there.

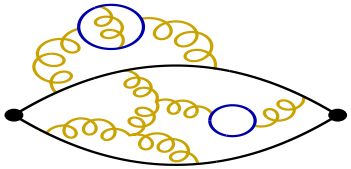
5. More challenges

5.1 Short-distance physics

Exp data at short-distances?

Ex) charmonium moments

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V^{(1)}(q^2) = i\int d^4x\, e^{ipx}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle$$



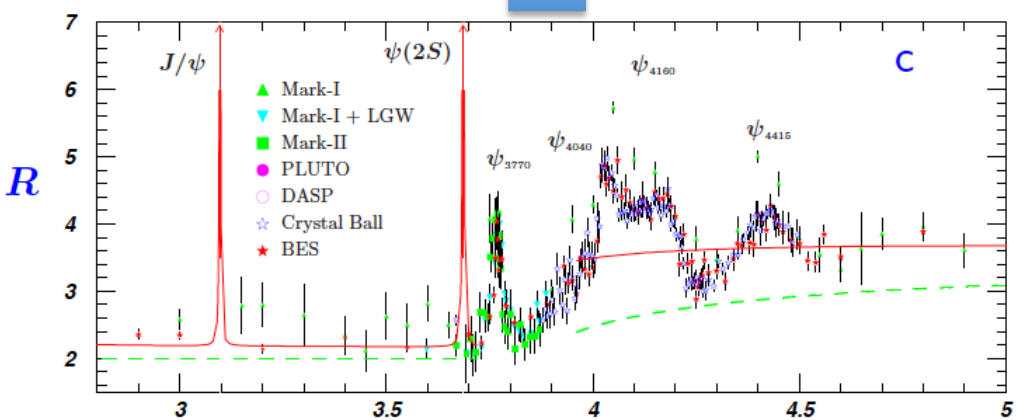
$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \left(\Pi(q^2)\right)_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s \frac{1}{s^{n+1}} R(s)_{e^+e^-\to\ \mathrm{hadron}}$$

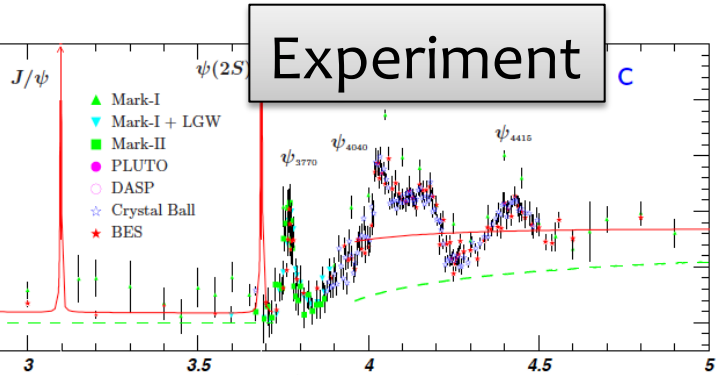
(weighted) integral over final states

Short distance: ~ $1/m_c$
- Perturbative
- Lattice

$$R_{2n} \sim \sum_t (t/a)^{2n} G_V(t)$$

(temporal moment)

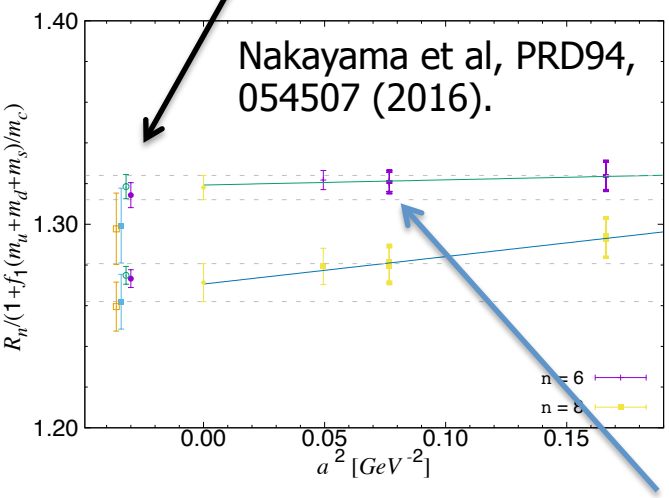




Quarkonium sum rules (1970s ~)

Perturbation theory

- Applicable at short distances
- Often combined with OPE (or Heavy Quark Expansion)



Lattice

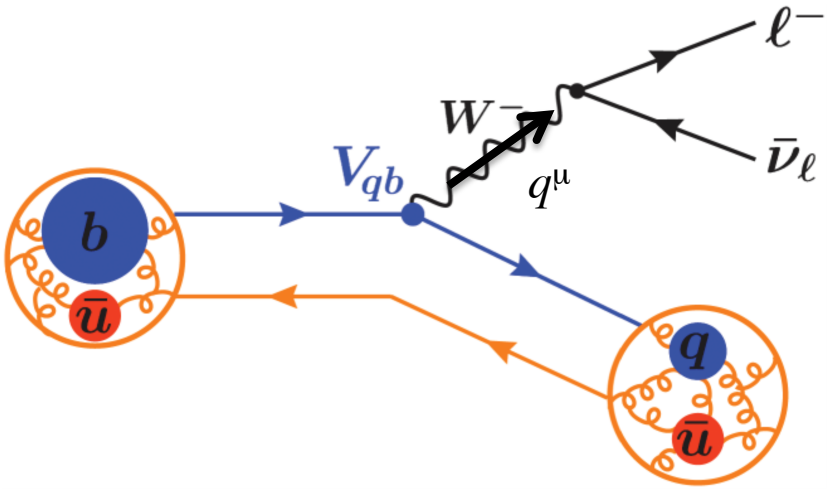
- Applicable at any distance scales (in principle)
- Space-like region

$$R_{2n} \sim \sum_t (t/a)^{2n} G_V(t)$$

Determination of charm quark mass

HPQCD (2008,2010,2015)
Nakayama et al (2016)
Maezawa, Petreczky (2016)

Inclusive B decay?



Summing all possible final states

- D, D*, Dπ, Dππ, ...

Partial decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

sum over all final states

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

Optical theorem: $-\frac{1}{\pi}\mathrm{Im}T_i = W_i$

Forward scattering matrix element:

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}|B\rangle$$

Can we calculate this on the lattice?
How?

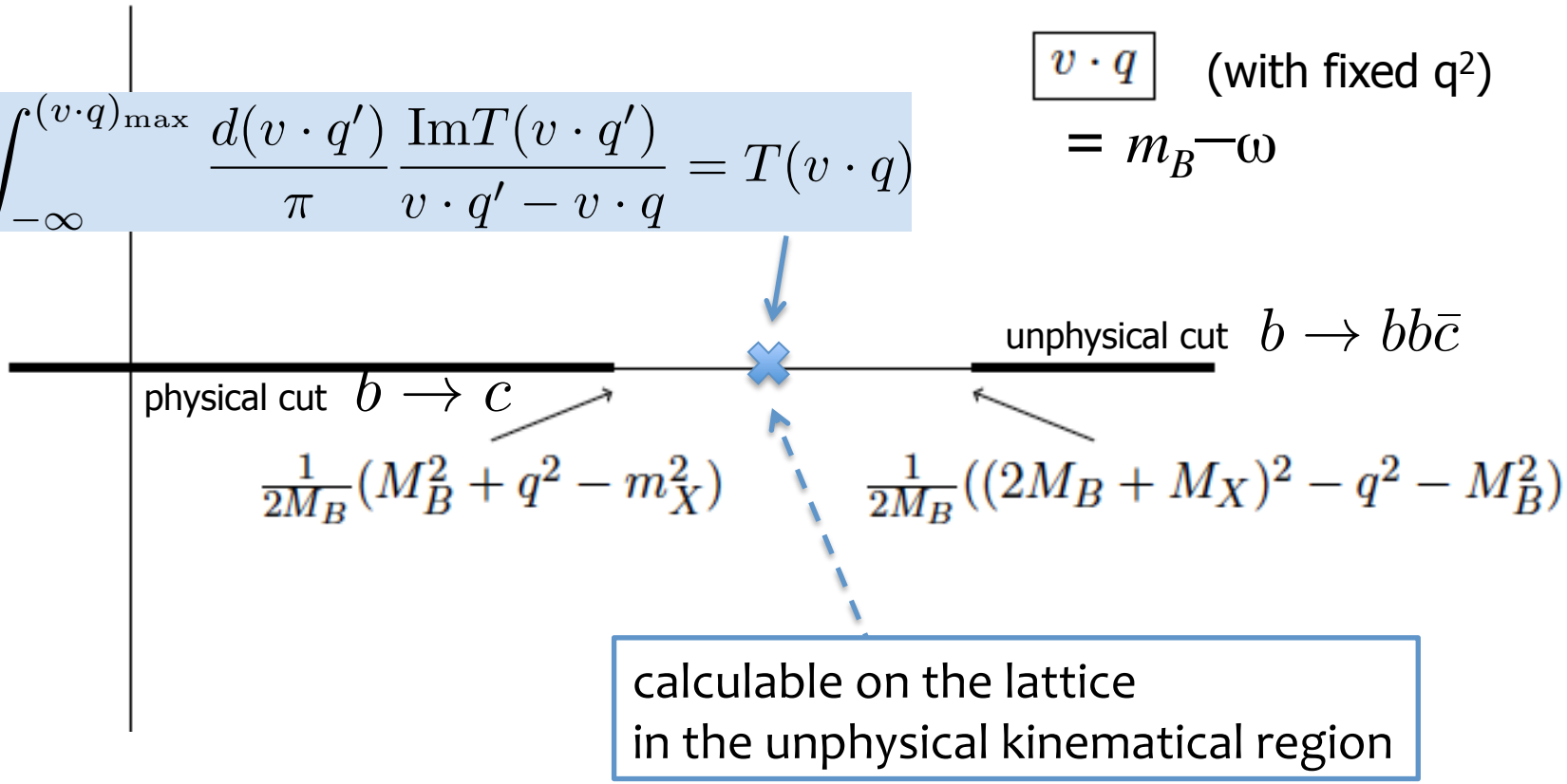
$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}|B\rangle$$

Analytic structure:

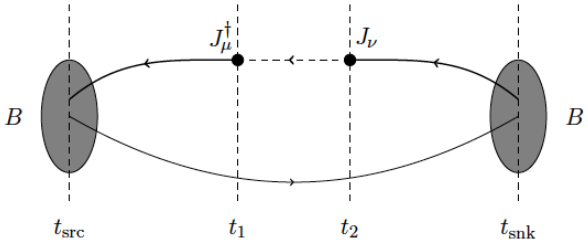
$$\int_{-\infty}^{(v\cdot q)_{\max}} \frac{d(v\cdot q')}{\pi} \frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q} = T(v\cdot q)$$

$\boxed{v\cdot q}$ (with fixed q²)

$= m_B - \omega$



Lattice calculation:



$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle$$

"Fourier" (or Laplace) transform: $T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$

Cauchy's integral:

Compare at unphysical point

SH, PTEP 2017, 053B03

$$T(\omega) = \int d\omega' \frac{\frac{1}{\pi}\mathrm{Im}T(\omega')}{\omega'-\omega}$$

input from exp

OR

lattice calc

$$C(t) = \int d\omega' e^{-\omega' t}\left(-\frac{1}{\pi}\mathrm{Im}T(\omega')\right)$$

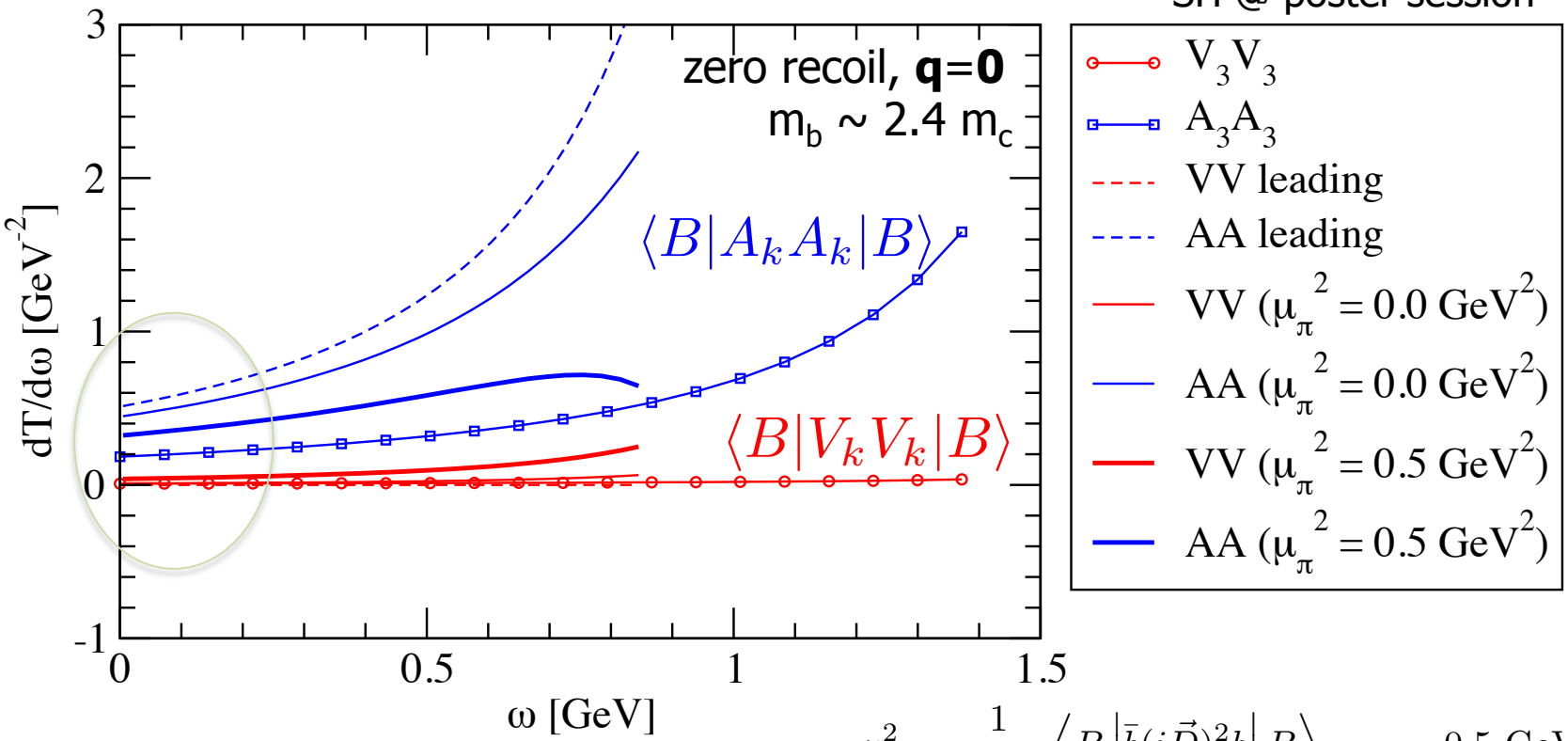
Compare with exp

Inverse problem: Backus-Gilbert method, etc
Hansen, Meyer, Robaina, PRD96, 094513 (2017)

Comparison with Heavy Quark Expansion:

- Exp data not available in the form I can use in this analysis.
- HQE to order $1/m^2$

SH @ poster session



$$\mu_\pi^2 = \frac{1}{2M_B}\left\langle B\left|\bar{b}(i\vec{D})^2 b\right|B\right\rangle \qquad \sim 0.5\ \mathrm{GeV}^2,$$

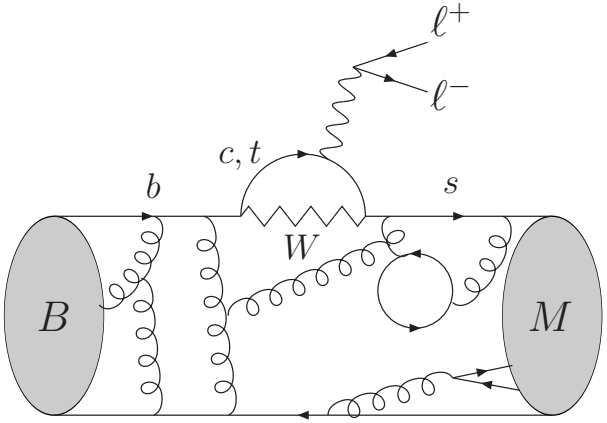
$$\mu_G^2 = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle = 0.37\ \mathrm{GeV}^2$$

Marginal agreement.
Need to include pert corrections.

5. More challenges

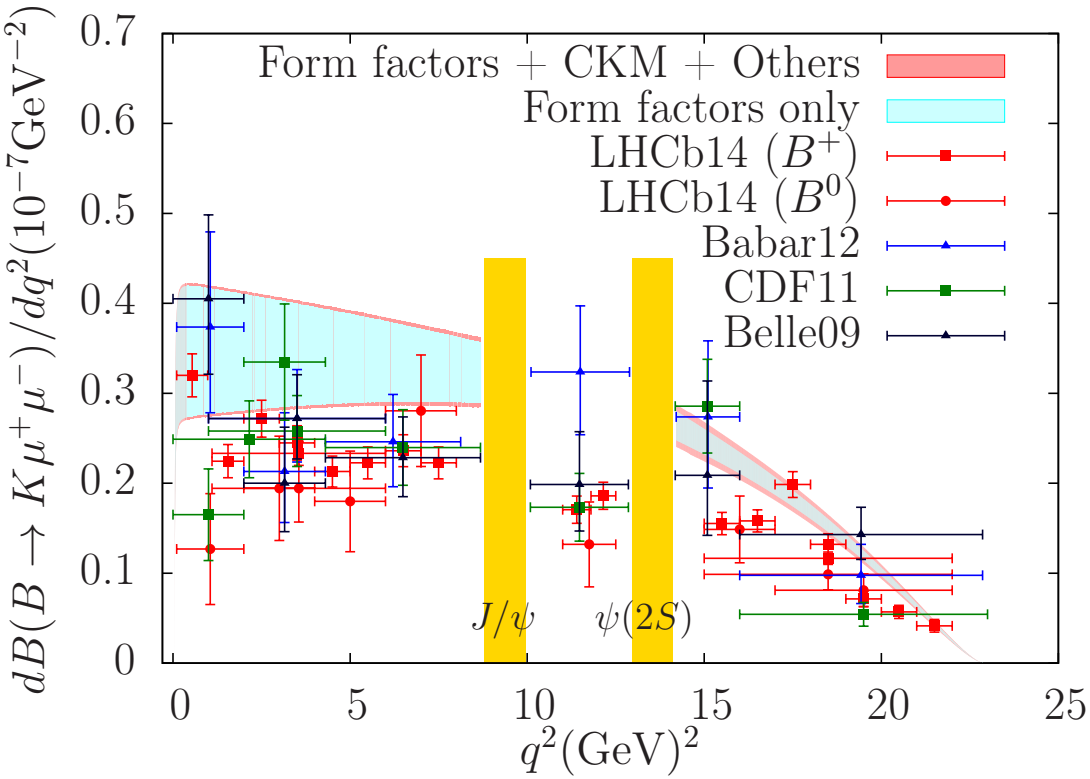
5.2 Long-distance effects

$B \rightarrow K\, l^+ l^-$



Rare process, penguin induced:

Fermilab-MILC (Du et al.), PRD93, 034005 (2016).



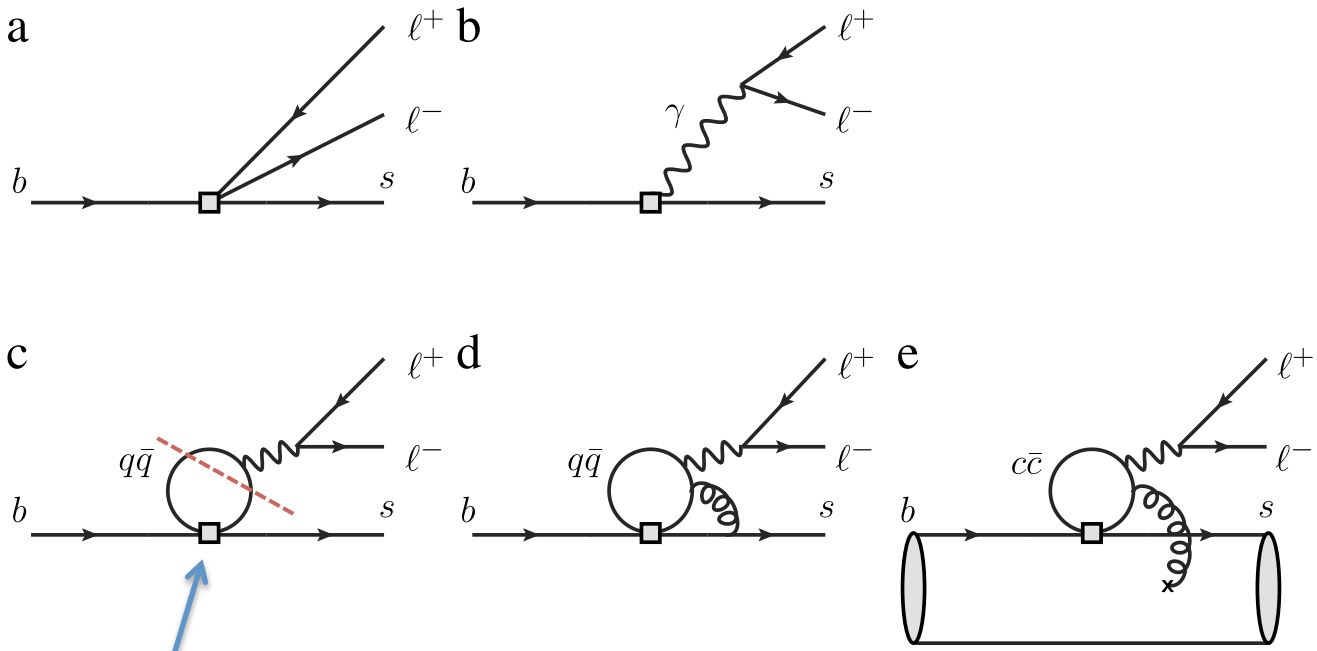
$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R,$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

The effect of charmonium resonances?

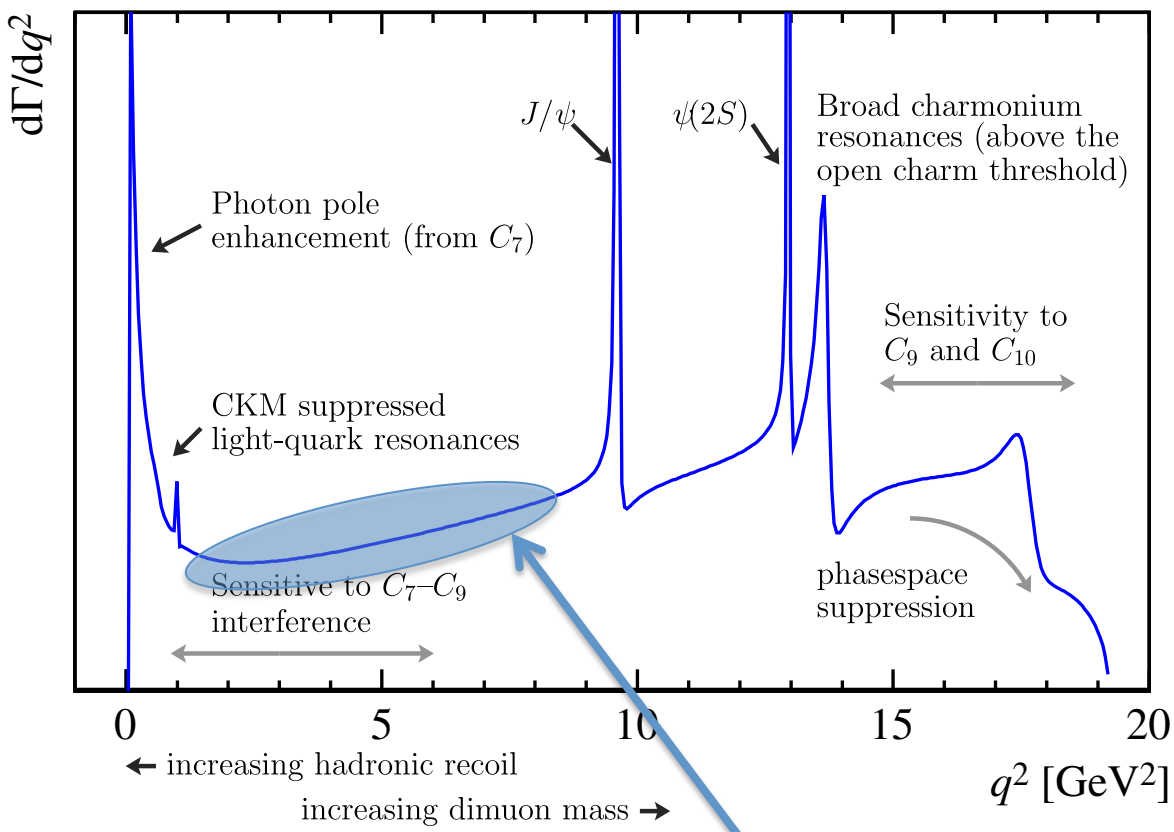
Long-distance effect?



$$\mathcal{O}_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)$$

Long-distance effect?

Lyon, Zwicky, arXiv:1406.0566

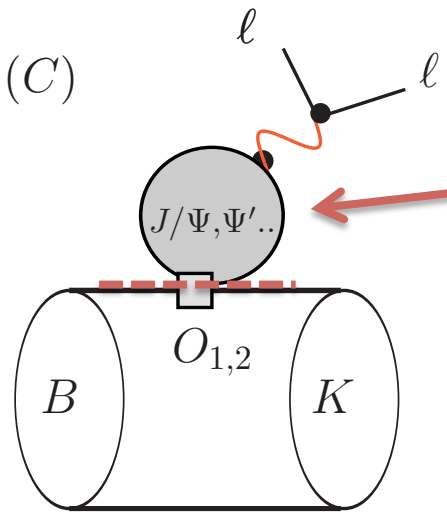


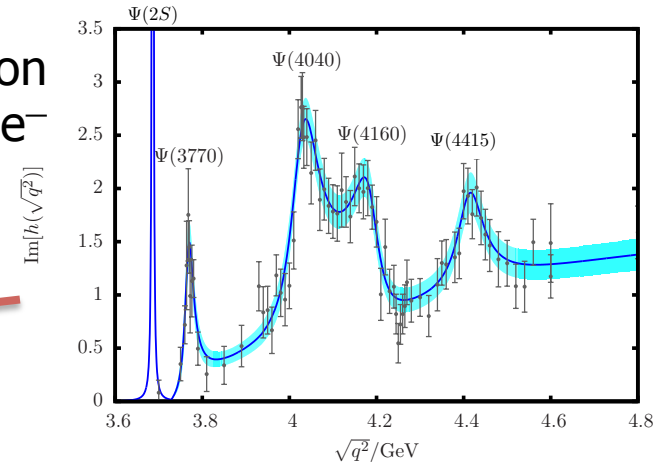
Insensitive to the resonance effects? If so, how much?

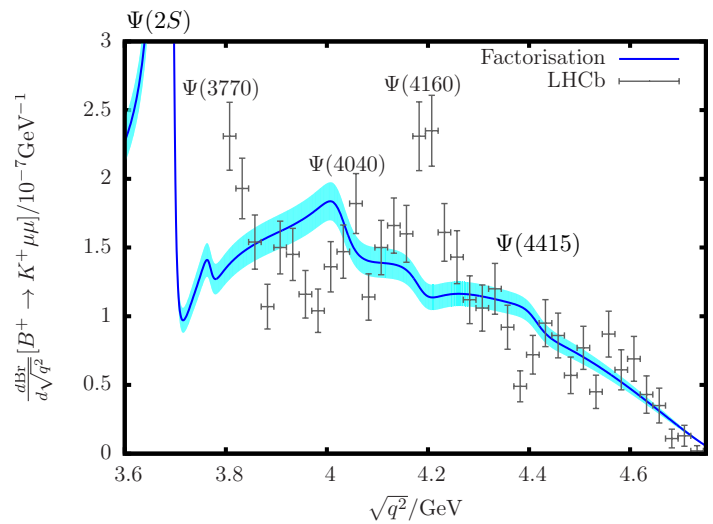
Factorization goes bad...

Lyon, Zwicky, arXiv:1406.0566

vacuum polarization
from e^+e^-



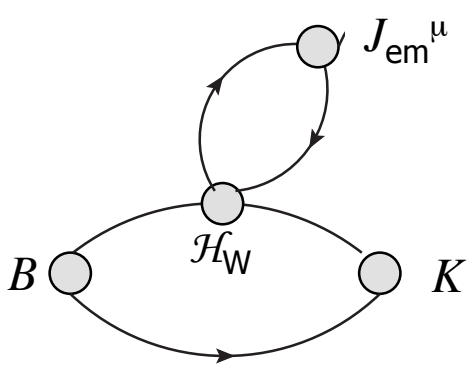




Lattice calculation?

First attempt:

Nakayama, Fri, 14:40



Formulation similar to "long-distance effects to $K \to \pi ll$"
Christ et al (RBC/UKQCD), PRD92, 094512 (2015); PRD94, 114516 (2016).

$$\int d^4x\, e^{iqx} \langle K(\mathbf{q}) | \mathrm{T}[J^{\mu}_{(\mathrm{em})}(x) \mathcal{H}_W(0)] | B(\mathbf{0}) \rangle$$

Euclidean

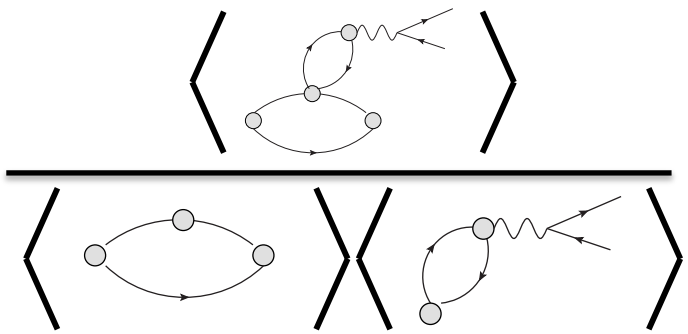
$$\int_0^{\infty} dt\, e^{\omega t} \int d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} \langle K(\mathbf{q}) | J^{\mu}_{(\mathrm{em})}(x) \mathcal{H}_W(0) | B(\mathbf{0}) \rangle$$

energy specified

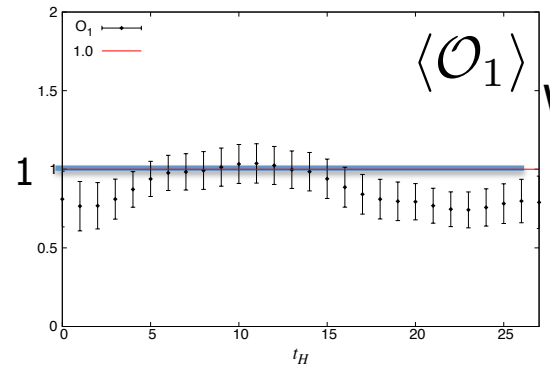
momentum inserted for charmonium

$$(\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma_\mu P_L c)$$

Factorization satisfied?

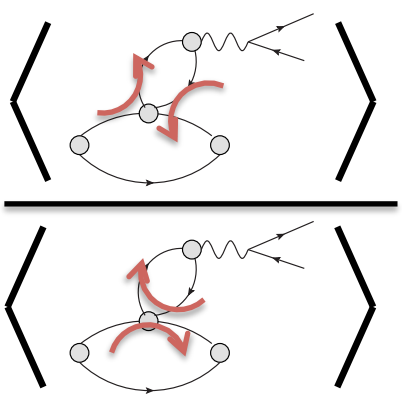


$$\frac{\langle P_K(t_K)\{(\bar{c}^i\gamma_\mu P_L b^j)(\bar{s}^j\gamma_\mu P_L c^i)\}(t_H)J_\alpha^{(\mathrm{em})}(t_J)P_B(t_B)\rangle}{\langle P_K(t_K)\{\bar{s}\gamma_\mu P_L b\}(t_H)P_B(t_B)\rangle\langle\{\bar{c}\gamma_\mu c\}(t_H)J_\alpha^{\mathrm{em}}(t_J)\rangle}$$



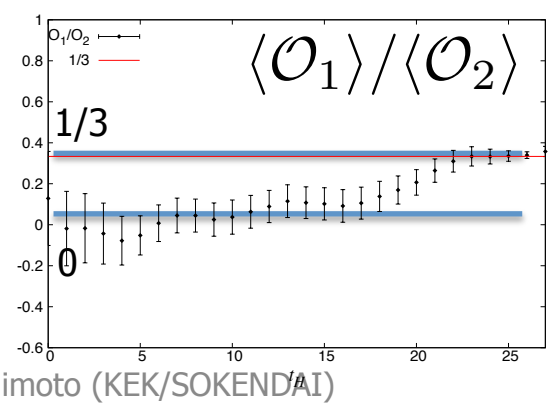
Well satisfied for

$$(\bar{c}^i\gamma_\mu P_L b^j)(\bar{s}^j\gamma_\mu P_L c^i) = (\bar{c}^i\gamma_\mu P_L c^i)(\bar{s}^j\gamma_\mu P_L b^j)$$



= 1/3 ?

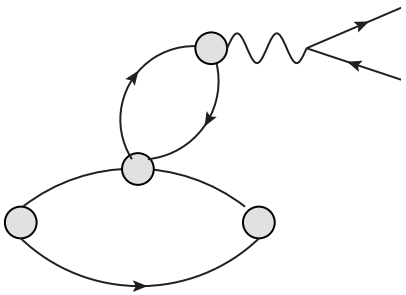
$$\frac{\langle P_K(t_K)\{(\bar{c}^i\gamma_\mu P_L b^i)(\bar{s}^j\gamma_\mu P_L c^j)\}(t_H)J_\alpha^{(\mathrm{em})}(t_J)P_B(t_B)\rangle}{\langle P_K(t_K)\{(\bar{c}^i\gamma_\mu P_L b^j)(\bar{s}^j\gamma_\mu P_L c^i)\}(t_H)J_\alpha^{(\mathrm{em})}(t_J)P_B(t_B)\rangle}$$

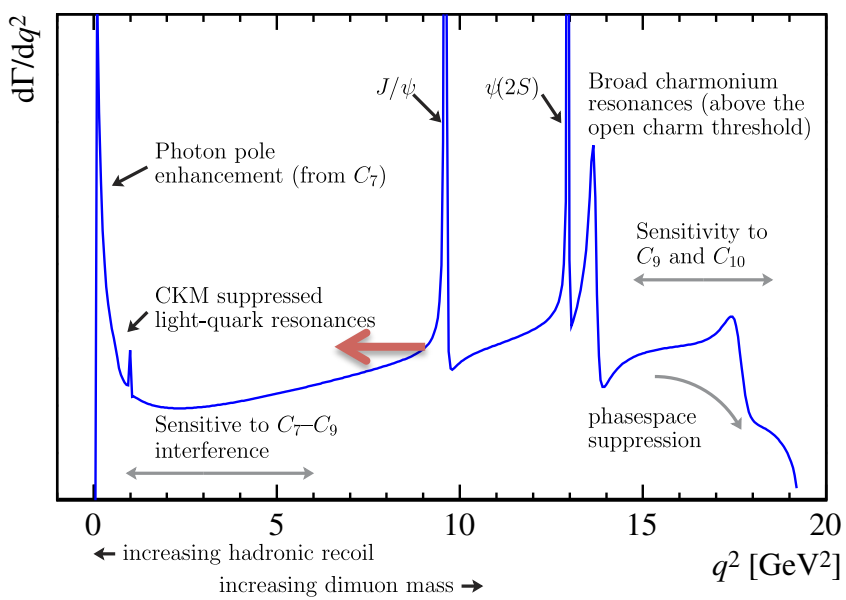


Significantly violated?

See also, K→ππ, RBC/UKQCD (2012)

Just a beginning…





- Only the region below J/ψ is accessible.
  - Otherwise one need to subtract the J/ψ contribution (as in K→πll)
- Recoil momentum too low ~ 0.5 GeV compared to > 2.5 GeV (physical)
  - How does the amplitude depend on the momentum?

A lot of challenges remain…

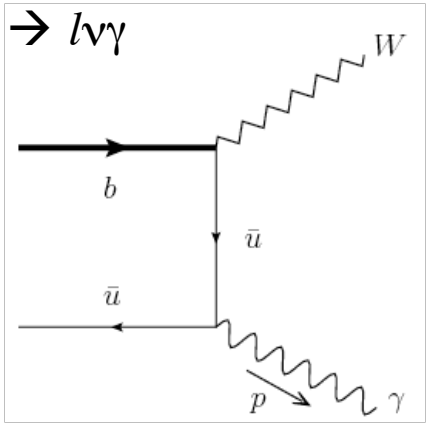
5. More challenges

5.3 Non-local matrix elements

Radiative leptonic

Soni, Fri, 18:10

$B \rightarrow l\nu\gamma$



- One can avoid the helicity suppression for $B \rightarrow \mu\nu$ by adding γ. Another way to test the lepton flavor universality.
- Lattice calculation involves two current insertions.

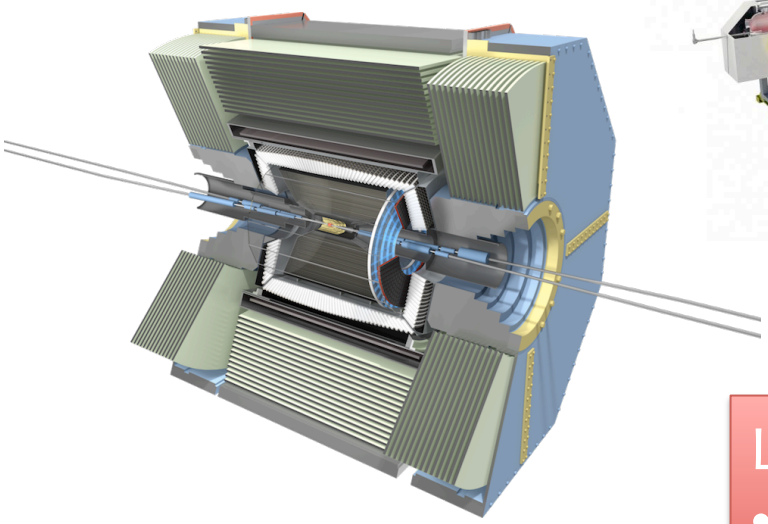
Sachrajda, Wed, 14:40

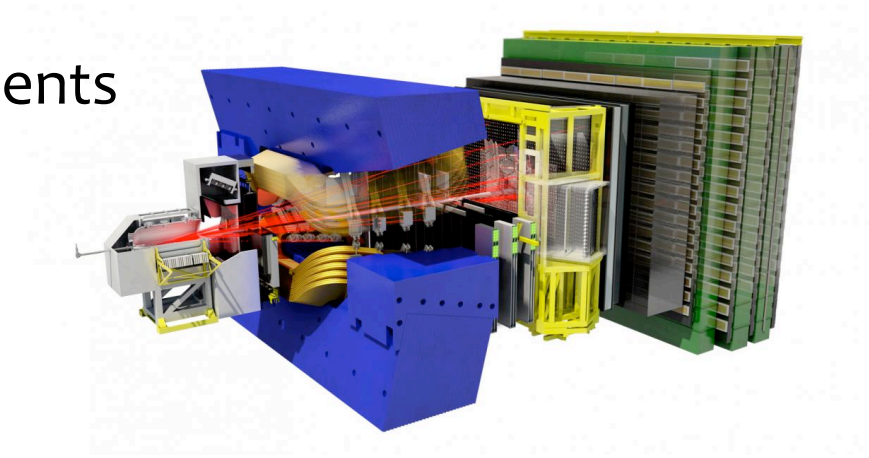
- Formulation for the QED corrections

6. Conclusion

Timely!

Precision experiments now online.





Lattice techniques have matured
- for precision calculations
- for unveiling unknowns

More adventures ahead of us!